The implication of BICEP2 result on Peccei-Quinn Symmetry from Anomalous U(1) gauge symmetry

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Introduction

- Axion as a dynamical solution to the strong CP problem

The Standard Model of the particle physics, even though confirmed by lots of experiments, has many unanswered questions.

One of them, the strong CP problem, asks why the CP violation in the strong interaction is so small (Strong CP Problem):

CP violating term $\theta \frac{g_s^2}{32\pi^2} G_a^{\mu\nu} \tilde{G}_{a\mu\nu}$

Electric dipole moment of nucleon $|d_n| < 3 \times 10^{-26}\, e\,\mathrm{cm}$ implies $\theta \lesssim 10^{-9}$

As a dynamical solution, an anomalous U(1) global symmetry is suggested. (Peccei-Quinn(PQ) symmetry)

R. D. Peccei, H. R. Quinn, Phys. Rev. Lett. 38(1977) 1440; Phys. Rev. D16 (1977) 1791

- PQ symmetry is spontaneously broken, then the goldstone boson, axion, which transform under the PQ symmetry as $a(x) \to a(x) + \alpha f_a$ is broken by the QCD instanton effect.

Axion to QCD instasnton coupling is given by $\frac{\xi}{32\pi^2}\frac{a}{f_a}G^a_{\mu\nu}\tilde{G}^{a\,\mu\nu}$

As a result, axion is not exactly massless,

$$m_a = m_\pi \frac{f_\pi}{f_a}\frac{\sqrt{m_u m_d}}{m_u + m_d} \simeq 6\mathrm{eV}\frac{10^6\mathrm{GeV}}{f_a}$$

but develops mass and stabilized at its vanishing value.

(Simply, in the Euclidean action, $G\tilde{G}$ term is pure imaginary, so vanishing value of this term minimize the effective potential for axion)

So almost vanishing θ is natural.

C. Vafa, E. Witten, Nucl. Phys. B234 (1984) 173; Phys. Rev. Lett. 53 (1984) 535

- In order to illuminate the physical properties of axion, it is essential to nail down the PQ breaking scale f_a and astrophysical bound says that it is likely to be in the intermediate scale.

Lower bound is coming from star cooling, $f_a > 10^9 \mathrm{GeV}$

Upper bound is coming from overcolsure of the Universe, but slightly subtle.

If PQ symmetry is broken after inflation, we need to take topological defects(cosmic string, domain wall…) contribution into account as well.

$$\begin{aligned}\Omega_a h^2 &= (\Omega_{\rm mis} + \Omega_{\rm string} + \Omega_{\rm wall})\, h^2 \\ &\simeq \Big(0.58 + (2.0 \pm 1.0) + (5.8 \pm 2.8)\Big) \times \left(\frac{\Lambda_{\rm QCD}}{400\,\mathrm{MeV}}\right)\left(\frac{f_a(t_0)}{10^{12}\,\mathrm{GeV}}\right)^{1.19}\end{aligned}$$

*initial misalighment angle is averaged over the causally disconnected patches: root-mean square value $\theta_0^2 \simeq \frac{1.85\pi^2}{3}$ is taken.

From $\Omega_a \leq \Omega_{\rm DM}$, the upper bound to be $(2-4)\times 10^{10}\ \mathrm{GeV}$

T. Hiramatsu, M. Kawasaki, K. ‘I Saikawa, T. Sekiguchi, Phys. Rev. D85 (2012) 105020; JCAP 1301(2013) 001.

If PQ symmetry is broken during or before inflation, such topological defects are diluted away. Instead, fluctuation along the massless axion direction (isocurvature perturbation) becomes the issue.

P. Fox, A. Pierce, S. D. Thomas, arxiv:hep-th/0409059

$$\mathcal{P}_S = 2\left(\frac{\Omega_a}{\Omega_{\rm DM}}\right)^2 \frac{2\theta_0^2 + (H_I/2\pi f_a(t_I))^2}{(\theta_0^2 + (H_I/2\pi f_a(t_I))^2)^2}\left(\frac{H_I}{2\pi f_a(t_I)}\right)^2$$
$$\simeq \frac{0.44}{x}\left(\frac{\Omega_a}{\Omega_{\rm DM}}\right)\left(\frac{H_I}{2\pi f_a(t_I)}\right)^2\left(\frac{f_a(t_0)}{10^{11}\mathrm{GeV}}\right)^{1.19},$$
$$x \equiv 2\frac{\theta_0^2 + (H_I/2\pi f_a(t_I))^2}{2\theta_0^2 + (H_I/2\pi f_a(t_I))^2} = 1-2$$

Then overclosure constrains becomes

$$\frac{\Omega_a}{\Omega_{\rm DM}} \simeq 0.11\left(\theta_0^2 + \left(\frac{H_I}{2\pi f_a(t_I)}\right)^2\right)\left(\frac{\Lambda_{\rm QCD}}{400\,\mathrm{MeV}}\right)\left(\frac{f_a(t_0)}{10^{11}\mathrm{GeV}}\right)^{1.19} \leq 1$$

while isocurvature constraint is $\frac{\mathcal{P}_S}{\mathcal{P}_\zeta} < 0.041\ (95\%\ \mathrm{C.L.})$

Planck Collaboration, arXiv:1303.5082

$\mathcal{P}_\zeta \simeq 2.19 \times 10^{-9}$ is the power spectrum of the curvature perturbations.

$$\left(\frac{\Omega_a}{\Omega_{\rm DM}}\right)\left(\frac{f_a(t_0)}{10^{11}\mathrm{GeV}}\right)^{1.19} < 2\times 10^{-10}\,x\left(\frac{H_I}{2\pi f_a(t_I)}\right)^{-2}$$

These constraints have restricted the PQ scale to be less than $10^{12}\mathrm{GeV}$ for order one initial misalignment angle, and can be larger if we allow the randomness of the initial misalignment angle. (Situation before BICEP2)

Recently, BICEP result shows that Hubble constant during inflation is

BICEP2 Collaboration, arXiv:1403.3985

$$H_I \simeq 10^{14}\ \mathrm{GeV}$$

and it gives another constraint, as will be shown later.

Question 1.: Is there a mechanism to generate such intermediate PQ breaking scale?
Especially, intermediate scale has a numerical relation,

$$v \sim (m_{\mathrm{SUSY}} M_{Pl}^n)^{1/(n+1)} \quad (n=1,2,\ldots)$$

Question 2.: What is the origin of the Peccei-Quinn symmetry?

- In this talk, we investigate the PQ symmetry during inflation if we accept the result of $H_I \simeq 10^{14}\ \mathrm{GeV}$ when we adopt the scenario that the PQ symmetry is originated from the anomalous U(1) gauge symmetry and its spontaneous breaking is induced by the SUSY breaking in order to answer to two questions.
- This is based on recent work, arXiv:1404.3880

String theoretic QCD axions

in the light of PLANCK and BICEP2

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PQ symmetry from anomalous U(1)

- In general, axion is a linear combination of fields which transform as shift under PQ symmetry.
- In general, they include not just pseudoscalar(imaginary part) of matter fields, but also of moduli.
- When we consider theory with extra dimension, gauge symmetry in the extra dimension can appear as a global shift symmetry of a modulus. This is the way that modulus obey the PQ symmetry
- For instance, string theory contains p-form fields C_p which has a gauge symmetry
$$C_p \to C_p + d\Lambda_{p-1}$$
- For compactification involving a p-cycle α_p in the internal space, axion-like field $\theta_{\rm st}$ is defined as

$$C_p(x, y) = \theta_{\rm st}(x)\omega_p(y)$$

$$\int_{\alpha_p} \omega_p = 1$$

x: 4-dim Minkowski space coordinates

y: internal space coordinate

$$C_p(x,y) = \theta_{\rm st}(x)\omega_p(y)$$

$\omega_p(y) = \ d\Omega_{p-1}(y)$ locally, the shift symmetry

$$U(1)_{\rm shift}\ :\ \theta_{\rm st}(x) \to \theta_{\rm st}(x) + \text{constant}$$

locally, but not globally: broken by non-perturbative effect.

Such non-perturbative effects should be small enough, not to move the axion minimum from almost vanishing value.

e.g. $\widetilde{V}(a) = -2M^4 \exp(-S_{inst}) \cos(a+\psi)$ $\qquad S_{inst} > 200$

And we have a axion to gauge field coupling

$$\int C_p \wedge F \wedge F \quad \to \quad \int_{M_4} \theta_{\rm st} F \wedge F \int_{\alpha_p} \omega_p.$$

Moreover, we can define modulus such that stringy axion is an imaginary part of it.

$$T = \frac{1}{2}\tau + i\theta_{\rm st}$$

- But such stringy axion typically requires a too large PQ scale:

(P. Svrcek, E. Witten, JHEP 06 (2006) 051)

Shift symmetry implies that Kaehler potential and holomorphic gauge coupling is of the form $K = K_0(T + T^*), \quad f_\alpha = T + \cdots$

and 4-dim effective potential

$$\begin{aligned} \mathcal{L}_{\rm eff} &= M_{Pl}^2 \frac{\partial^2 K_0}{\partial \tau^2} \partial_\mu \theta_{\rm st} \partial^\mu \theta_{\rm st} + \frac{1}{4} \theta_{\rm st} G^{\alpha\mu\nu} \tilde{G}^{\alpha}_{\mu\nu} + \cdots \\ &= \frac{1}{2} \partial_\mu a_{\rm st} \partial^\mu a_{\rm st} + \frac{1}{32\pi^2} \frac{a_{\rm st}}{f_a} G^{a\mu\nu} \tilde{G}^{a}_{\mu\nu} + \cdots , \end{aligned}$$

therefore, $f_a = \frac{1}{8\pi^2} \left(2 \frac{\partial^2 K_0}{\partial \tau^2} \right)^{1/2} M_{Pl}$ typically $f_a = \mathcal{O}\left(g_{\rm GUT}^2 M_{Pl}/8\pi^2\right)$

So not sufficient to obtain the intermediate scale. That means, axion may well be coming from matter fields, not modulus.

Then we have to ask the origin of the global PQ symmetry.

- One good suggestion is to regards the global PQ symmetry is a remnant of the anomalous U(1) gauge symmetry.

Consider the anomalous U(1)A symmetry,

$$\delta_{\rm GS} = \frac{1}{8\pi^2}\sum_i q_i {\rm Tr}(T_a^2(\phi_i))$$

$$U(1)_A : \quad V_A \to V_A + \Lambda + \Lambda^*, \quad T \to T + \delta_{\rm GS}\Lambda, \quad \phi_i \to e^{q_i\Lambda}\phi_i$$

Here, modulus plays a role of cancellation of the anomaly

(Green-Schwarz mechanism)

Moreover, as Kaehler potential depends on the gauge invariant combination

$$T + T^* - \delta_{\rm GS} V_A$$

Modulus can be a part of gauge transformation: eaten by gauge multiplet to make it massive.

So, gauge boson has two ways of being massive: eating modulus or matter.

- This is shown from the effective Lagrangian

$$\mathcal{L}_{\rm eff} = -\frac{1}{4g_A^2}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}\left((8\pi^2\delta_{\rm GS}f_{\rm st})^2 + v^2\right)\left(\frac{\partial_\mu\chi}{\sqrt{(8\pi^2\delta_{\rm GS}f_{\rm st})^2+v^2}} - A_\mu\right)^2$$
$$+\frac{1}{2}(\partial_\mu a)^2 + \frac{1}{32\pi^2}\frac{a}{f_a}G\tilde{G} + \frac{1}{2}g_A^2(\xi_{\rm FI} - v^2/2)^2 + \cdots,$$

$$f_{\rm st} \equiv \frac{1}{8\pi^2}\sqrt{2\frac{\partial^2 K_0}{\partial\tau^2}}M_{Pl}$$

$$\xi_{\rm FI} = \delta_{\rm GS}\frac{\partial K_0}{\partial\tau}M_{Pl}^2$$

Then one combination of the stringy axion (imaginary part of the modulus) and matter axion (imaginary part of the matter) becomes the longitudinal part of the gauge boson

$$\chi = \frac{1}{\sqrt{(8\pi^2\delta_{\rm GS}f_{\rm st})^2+v^2}}\left((8\pi^2\delta_{\rm GS}f_{\rm st})^2\frac{\theta_{\rm st}}{\delta_{\rm GS}} + v^2\theta_\phi\right)$$

With the gauge boson mass

$$M_A = g_A\sqrt{(8\pi^2\delta_{\rm GS}f_{\rm st})^2+v^2},$$

And another combination becomes the physical axion

$$a = \frac{(8\pi^2\delta_{\rm GS}f_{\rm st})v}{\sqrt{(8\pi^2\delta_{\rm GS}f_{\rm st})^2+v^2}}\left(\frac{\theta_{\rm st}}{\delta_{\rm GS}} - \theta_\phi\right)$$

where PQ scale is given by $$f_a \equiv \frac{f_{\rm st} v}{\sqrt{(8\pi^2\delta_{\rm GS} f_{\rm st})^2 + v^2}}.$$

Which is given by the smaller value between v and fst and the axion is mainly composed of pseudoscalar associated with smaller scale.

Then we need small matter VEV, v.

Actually, when SUSY is broken, D term should have a small value

$$|D_A| = |\xi_{\rm FI} - v^2/2| \lesssim \mathcal{O}\left(\frac{m_{3/2}^2 M_{Pl}^2}{M_A^2}\right)$$

If it is obtained by cancellation between FI term $\xi_{\rm FI} = \delta_{\rm GS}\frac{\partial K_0}{\partial \tau} M_{Pl}^2$ and large matter VEV, we require

$$\xi_{\rm FI} \simeq v^2 = \mathcal{O}(\delta_{\rm GS} M_{Pl}^2) > f_{\rm st}^2 = \mathcal{O}(M_{Pl}^2/(8\pi^2)^2)$$

Then PQ scale is too large.

$$f_a = \frac{f_{\rm st} v}{\sqrt{(8\pi^2\delta_{\rm GS} f_{\rm st})^2 + v^2}} \simeq f_{\rm st}.$$

Therefore, we should investigate the vacuum with vanishing FI term when SUSY is conserved: small D-term is a result of tuning between small FI term and matter VEVs generated by SUSY breaking.

- Small matter VEV induced from the SUSY breaking

:Physically, it means that gauge boson get massive of order of $g_{\rm GUT}^2 M_{Pl}/8\pi^2$ and at low energy where gauge multiplets are integrated out, anomalous U(1) gauge symmetry appears as a global U(1) symmetry, which will be interpreted as a PQ symmetry.

- Suppose that matter field has an almost flat potential (set up by assigning PQ charges) such that potential is composed of non-renormalizable terms.
- When SUSY is broken, and if (some of) soft mass is tachyonic, we consider the potential in the form of, for example,

$$V(\phi) = -m_{\rm SUSY}^2 |\phi|^2 + \left| \frac{\partial W}{\partial \phi} \right|^2 = -m_{\rm SUSY}^2 |\phi|^2 + \frac{1}{M_{Pl}^{2n}} |\phi|^{4+2n}$$

which yields

$$v \sim (m_{\rm SUSY} M_{Pl}^n)^{1/(n+1)} \ll f_{\rm st}$$

Therefore,

$$f_a = \frac{f_{\rm st} v}{\sqrt{(8\pi^2 \delta_{\rm GS} f_{\rm st})^2 + v^2}} \simeq \frac{v}{8\pi^2 \delta_{\rm GS}}$$

- Such a potential comes from the superpotential $\dfrac{\phi_1^{n+2}\phi_2}{M_{Pl}^n}$

H. Murayama, H. Suzuki, T. Yanagida, Phys. Lett. B291 (1992) 418

we expect the potential

$$V \simeq (n+2)^2\lambda^2\frac{|\phi_1|^{2(n+1)}|\phi_2|^2}{M_{Pl}^{2n}} + \lambda^2\frac{|\phi_1|^{2(n+2)}}{M_{Pl}^{2n}} + m_1^2|\phi_1|^2 + m_2^2|\phi_2|^2 - \left(\lambda A_\phi\frac{\phi_1^{n+2}\phi_2}{M_{Pl}^n} + \text{h.c.}\right)$$

Two fields have the opposite PQ charges. If soft mass is mainly coming from D-term mediation, $m_i^2 = -q_i D_A$ and if m1 is tachyonic, we have two fields are stabilized at intermediate scale,

$$v_1(t_0) \sim \left(\frac{m_{\text{SUSY}}M_{Pl}^n}{\lambda}\right)^{1/(n+1)},$$

$$v_2(t_0) \sim \frac{A_\phi v_1(t_0)}{m_{\text{SUSY}}} \ll v_1(t_0),$$

- This scenario is interesting because

1. It naturally explains the intermediate PQ scale
2. The potential shape is different from Mexican hat shaped potential

$$V(\phi) = \lambda(|\phi|^2 - f_a^2)^2$$

which is parametrized by only one scale.

This can cause different phenomenological result.

One important result is that PQ scale can be changed during the inflation.

Inflation and PQ breaking

- During the inflation, vacuum energy is dominated by the inflaton energy,

$$H_I^2 = \frac{8\pi G}{3}\rho \simeq \frac{V_I}{3M_{\rm Pl}^2}$$

and its nonzero, large value ($H_I \simeq 10^{14}~{\rm GeV}$ from BICEP2 implies that VI to be a GUT scale) means the large SUSY breaking scale.

$$F^I \simeq \sqrt{3} H_I M_{\rm pl}$$

So, we can replace $m_{\rm SUSY}$ by H_I.

As SUSY breaking scale is enhanced, D-term and matter VEVs would be enhanced.

In general, we can find a vacuum in which D-term is smaller than matter VEVs such that tuning between matter VEVs and FI term occurs.

- As an example, one may consider the following Kaehler- and superpotential realizing chaotic inflation from supergravity

(M. Kawasaki, M. Yamaguchi, T. Yanagida, Phys. Rev. Lett. 85 (2000) 3572)

$$K = \frac{M_{Pl}^2 \partial_\tau^2 K_0(\tau_0)}{2}(\tau - \tau_0 - \delta_{\rm GS} V_A)^2 + \phi_1^* e^{-V_A}\phi_1 + \phi_2^* e^{(n+2)V_A}\phi_2,$$

$$W = \lambda \frac{\phi_1^{n+2}\phi_2}{M_{Pl}^n},$$

$$K_{\rm SB} = |Z|^2 - \frac{|Z|^4}{\Lambda^2} + \frac{1}{2}(\Phi + \Phi^*)^2 + |X|^2,$$

$$W_{\rm SB} = \omega_0 + M^2 Z + \mu X \Phi.$$

$$\Delta K = (k|Z|^2 + \kappa|X|^2)(\tau - \tau_0 - \delta_{\rm GS} V_A) + \frac{k_i|Z|^2 + \kappa_i|X|^2}{M_{Pl}^2}\phi_i^* e^{-q_i V_A}\phi_i$$

- From this, we have the potential

$$V \simeq \frac{g_A^2}{2} D_A^2 + V_0(\tau) + (n+2)^2 \lambda^2 \frac{|\phi_1|^{2(n+1)} |\phi_2|^2}{M_{Pl}^{2n}} + \lambda^2 \frac{|\phi_1|^{2(n+2)}}{M_{Pl}^{2n}}$$
$$+ m_1^2 |\phi_1|^2 + m_2^2 |\phi_2|^2 - \left(\lambda A_\phi \frac{\phi_1^{n+2} \phi_2}{M_{Pl}^n} + \text{h.c.} \right),$$

with

$$D_A = |\phi_1|^2 - (n+2)|\phi_2|^2 - \delta_{\rm GS} \frac{\partial K_0}{\partial \tau},$$

and V0 VEV is given by the inflaton vacuum energy, for example,

$$V_0 = e^{K_0} \left(\frac{M^4}{1 + k(\tau - \tau_0)} + \frac{\mu^2 \varphi^2}{1 + \kappa(\tau - \tau_0)} - 3|W_{\rm SB}|^2 \right) \qquad 3H_I^2 M_{Pl}^2 \simeq |F^X|^2 = \mu^2 \varphi^2 (t_I)$$

where $\frac{1}{1 + \kappa(\tau - \tau_0)}$ is the inverse Kaehler metric in the inflaton direction $K^{X\bar{X}}$

and κ is the coupling for Kaehler mixing between modulus and inflaton sector.

- Then the minimization condition for modulus

$$\partial_\tau V \;\propto\; g_A^2 D_A - \frac{1}{\delta_{\rm GS}\partial_\tau^2 K_0}\left(\frac{\partial V_0}{\partial\tau} + \cdots\right) = 0$$

is solved to be

$$g_A^2 D_A \simeq \frac{3}{\delta_{\rm GS}\partial_\tau^2 K_0}\left(\frac{|\phi_1|^2 - (n+2)|\phi_2|^2}{\delta_{\rm GS} M_{Pl}^2} - \kappa\right) H_I^2$$

Then, the soft mass is given by

$$\begin{aligned}\tilde{m}_i^2 \;&=\; m_i^2 + q_i g_A^2 D_A \\ &\simeq\; (1-\kappa_i)\frac{|F^X|^2}{M_{Pl}^2} + q_i g_A^2 D_A = \mathcal{O}\left((1-\kappa_i)H_I^2\right) + \mathcal{O}\left(8\pi^2\kappa H_I^2\right)\end{aligned}$$

And if some of it is tachyonic, PQ symmetry is broken by corresponding fields during inflation. If m1 is tachyonic, we have

$$\begin{aligned}v_1(t_I) \;&\sim\; \left(\frac{H_I M_{Pl}^n}{\lambda|\delta_{\rm GS}|^{1/2}}\right)^{1/(n+1)} \\ v_2(t_I) \;&\sim\; \frac{A_\phi v_1(t_I)}{|\delta_{\rm GS}|^{1/2} H_I},\end{aligned}$$

- Therefore, PQ scale during inflation can be quite enhanced, such that

$$f_a(t_I) \simeq \frac{f_{\rm st} v_1(t_I)}{\sqrt{(8\pi^2 \delta_{\rm GS} f_{\rm st})^2 + v_1^2(t_I)}} \sim {\rm Min}\big(f_{\rm st},\, v_i(t_I) \big)$$

$$v_1(t_I) \gtrsim f_{\rm st} = \frac{\sqrt{2\partial_\tau^2 K_0}}{8\pi^2} M_{Pl} = \mathcal{O}(10^{16} - 10^{17})\ {\rm GeV}$$

$$f_a(t_I) \simeq f_{\rm st}$$

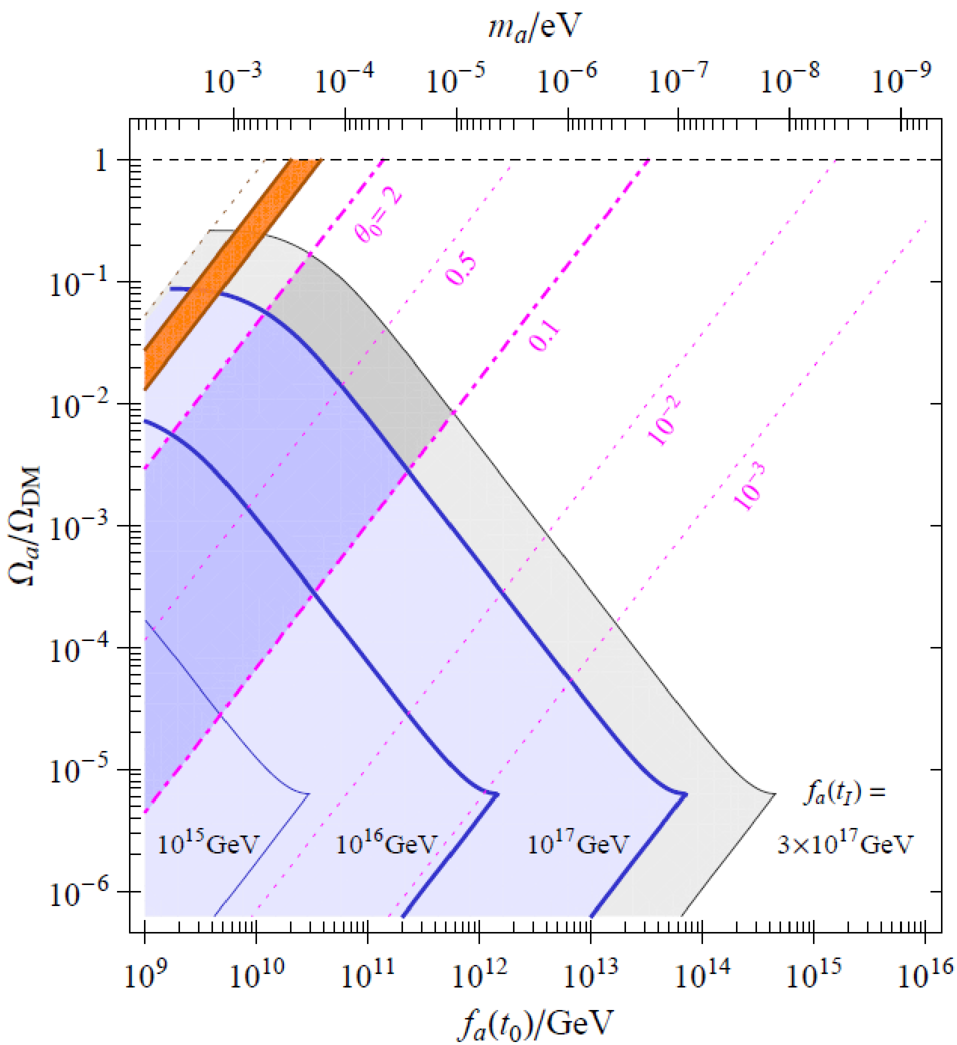
That means, during the inflation, axion is mainly composed of stringy axion, whereas matter pseudoscalar is mainly eaten by the gauge boson.

- Such enhancement in the PQ sclae is helpful in alleviating the isocurvature constraint (A. D. Linde, Phys. Lett. B 259 (1991) 38)

$$\left(\frac{\Omega_a}{\Omega_{\rm DM}}\right)\left(\frac{f_a(t_0)}{10^{11}{\rm GeV}}\right)^{1.19} < 2\times 10^{-10}\, x \left(\frac{H_I}{2\pi f_a(t_I)}\right)^{-2}$$

such that allowed (present Universe) PQ scale upper bound can be enhanced.

- But unfortunately, such alleviation is not enough such that axion explains the full dark matter in the Universe, as summarized in the figure, which shows 1/10 abundance can be allowed.



Summary

- If PQ symmetry is regarded as a vestige of the anomalous U(1) gauge symmetry, one can successfully construct the PQ symmetry fulfilling Intermediate PQ scale when PQ symmetry breaking is induced by SUSY breaking.
- During inflation, (if we accept the BICEP2 result) SUSY breaking is parametrized by Hubble scale during inflation. In this case, isocurvature perturbation constraint is alleviated, but not sufficient to allow for axion to explain the whole dark matter density abundance.