# Self-complete chaotic inflation

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HML, 1403.5602 [hep-ph];
G. Giudice & HML, Phys. Lett. B733 (2014) 58.

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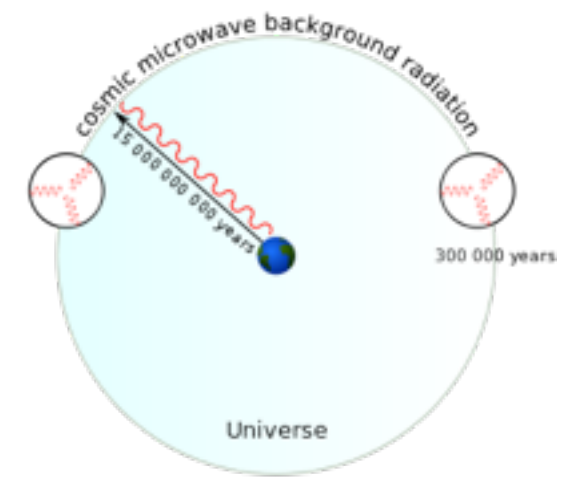
# Outline

- Introduction
- Starobinsky-like inflation and unitarity
- Quadratic inflation for BICEP2
- Conclusions

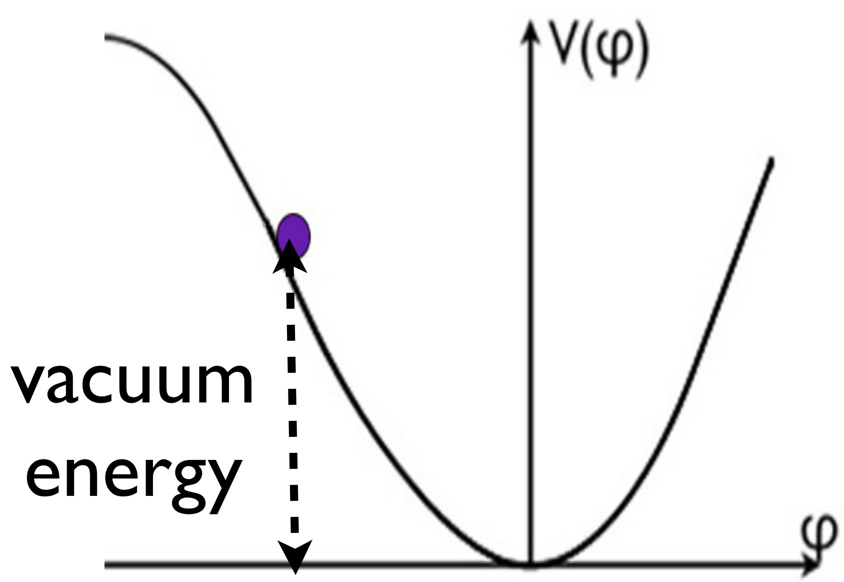
# Cosmic inflation

- Problems of horizon, homogeneity, flatness, and structure formation in $\Lambda$CDM.

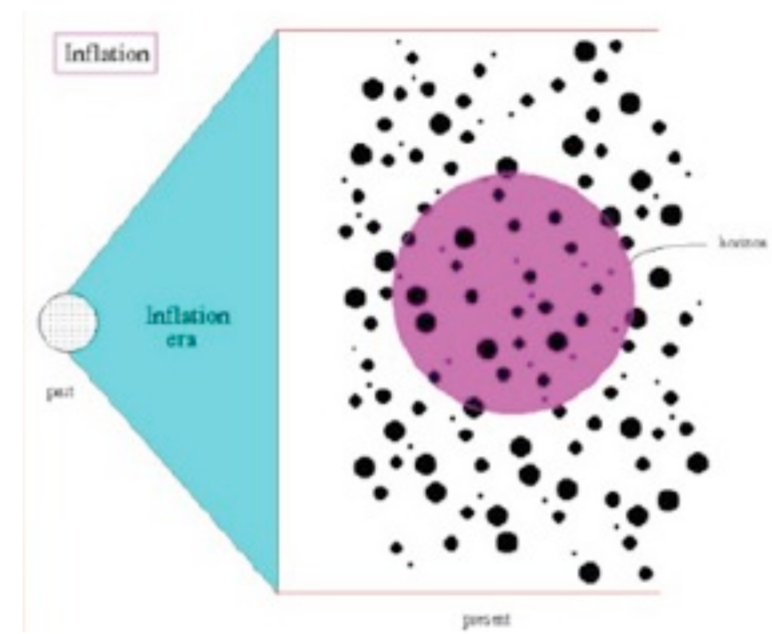
  Too small universe problem.




- Cosmic Inflation solves the problems by an exponential expansion with a scalar field just after Big Bang.

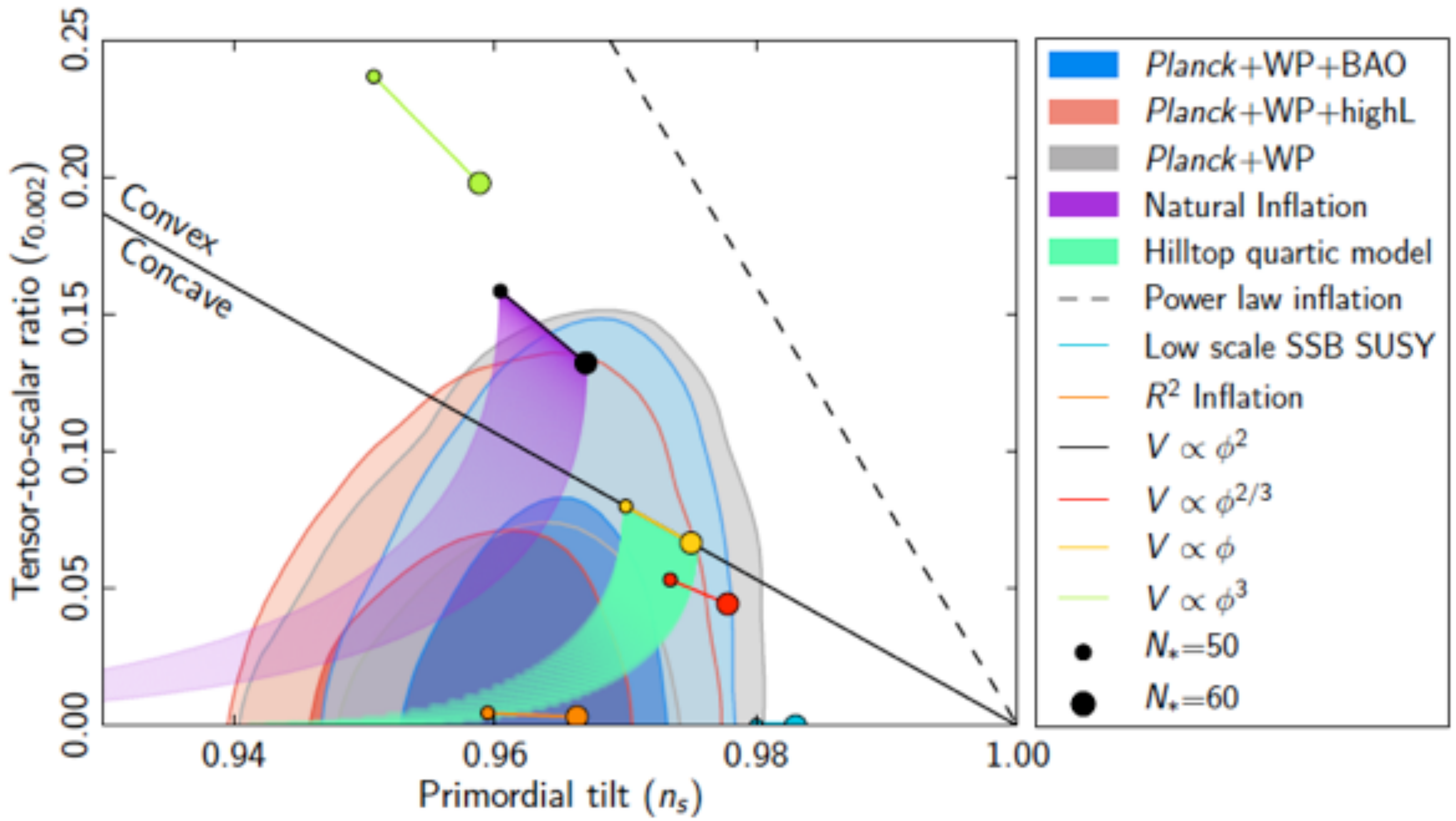



"exponentially" large universe




- Quantum fluctuation of inflaton generates CMB anisotropies, galaxies, and clusters.

# Inflation before BICEP2




$$n_s = 0.9603 \pm 0.0073$$

$$r_{0.002} < 0.12$$

(Planck+WP)

- Inflaton fluctuations lead to almost scale-invariant CMB.

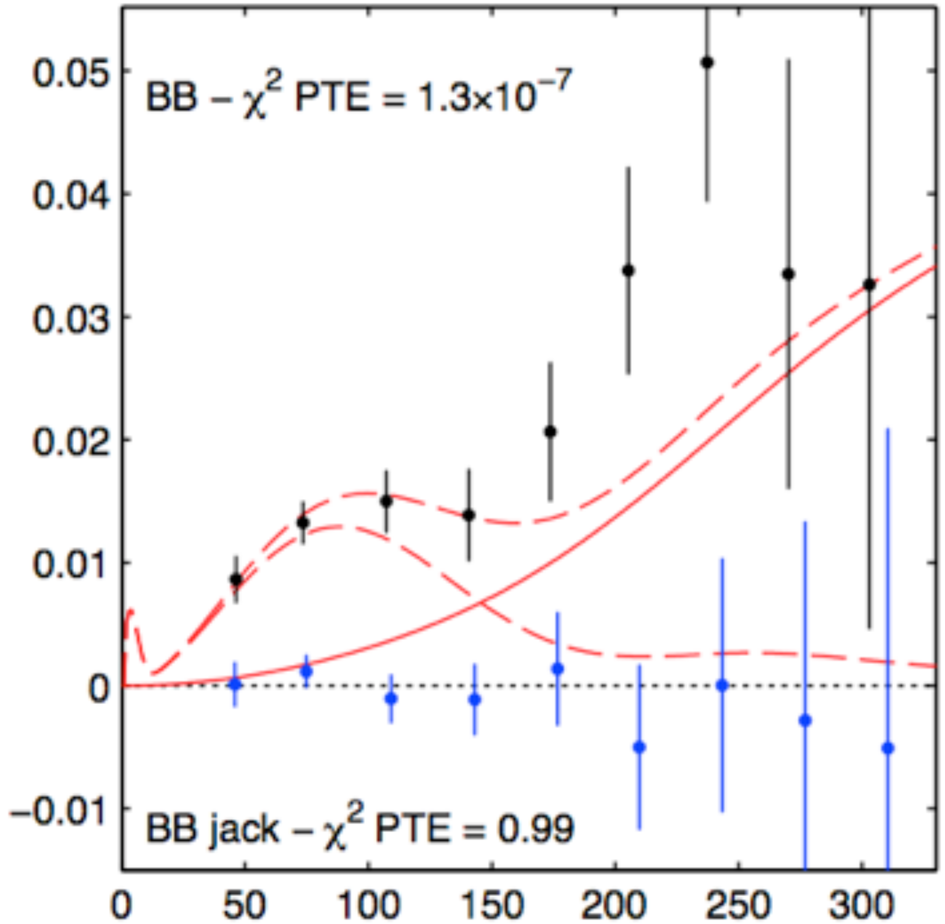
$$\langle \mathcal{R}(\vec{k})\mathcal{R}(\vec{k}')\rangle = \delta^3(\vec{k}-\vec{k}')\,\frac{2\pi^2}{k^3}\mathcal{P}_{\mathcal{R}}(k), \qquad \mathcal{P}_{\mathcal{R}}(k) \propto k^{n_s-1},$$

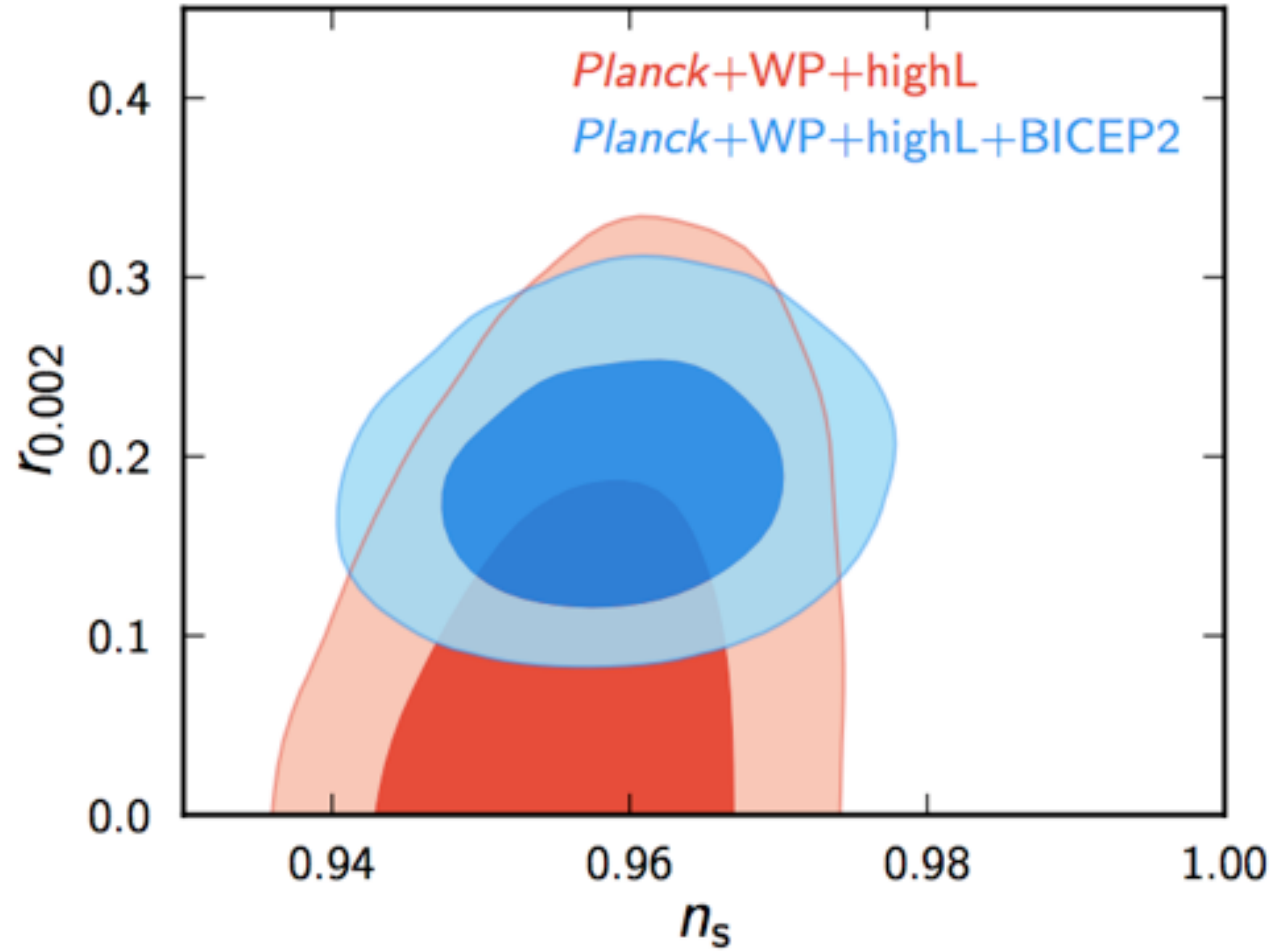
$$n_s = 1 - 6\epsilon + 2\eta, \qquad \epsilon = \frac{M_P^2}{2}\left(\frac{V'}{V}\right)^2 \ll 1,\ \eta = M_P^2\frac{V''}{V} \ll 1.$$

- Tensor perturbation (gravity-wave) in CMB less than 10%.
- Planck favored Starobinsky and Higgs inflations with small r.

# Large r from Bicep2

- Large tensor-to-scalar ratio observed by Bicep2 disfavors Higgs and Starobinsky-like models.




$$r = 0.20^{+0.07}_{-0.05}$$

$$n_s = 0.9600 \pm 0.0071$$

$$r < 0.26$$ (Planck, with running)

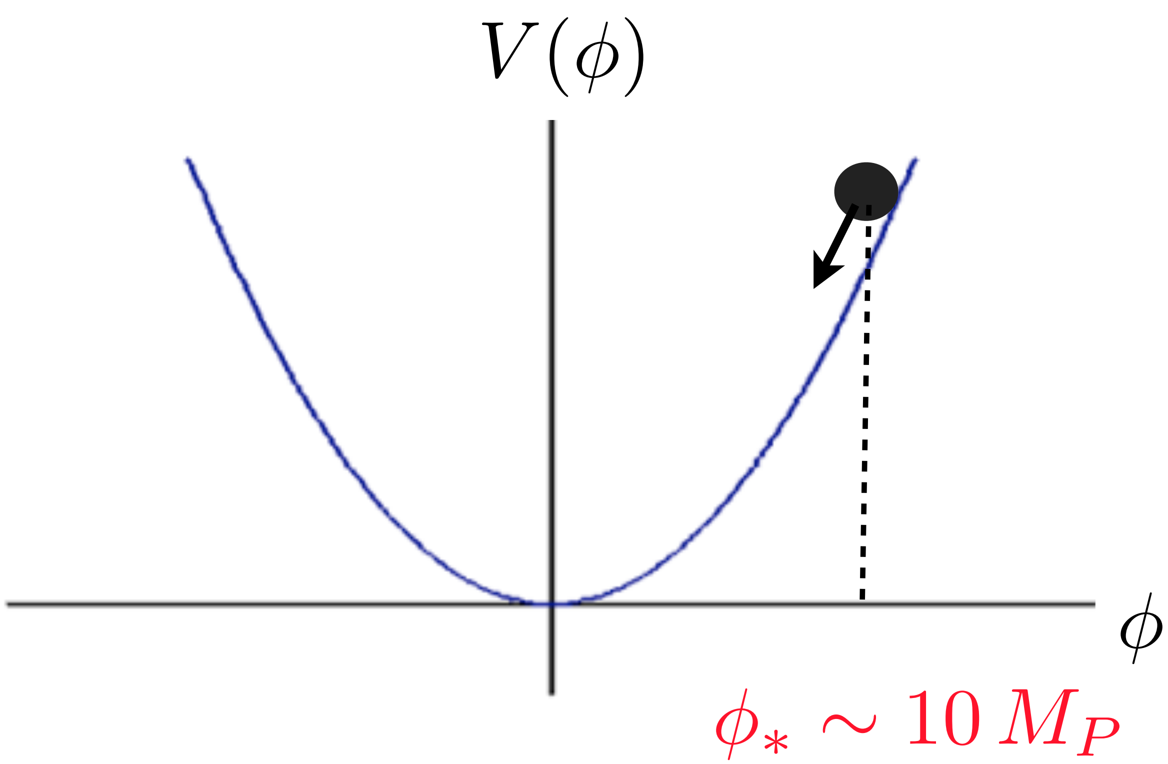
$r = 0.16^{+0.06}_{-0.05}$ with dust model.

Dust models must be confirmed by Planck.

# Quadratic inflation

[Linde (1983)]

- Monomial chaotic inflation models predict a large tensor-to-scalar ratio at the expense of super-Planckian inflation values.
- In particular, quadratic inflation is one of the best:




$$\epsilon_* = \eta_* = \frac{2}{\phi_*^2} = \frac{1}{2N+1}$$

$$n_s = 1 - 6\epsilon_* + 2\eta_* = 1 - \frac{4}{2N+1},$$

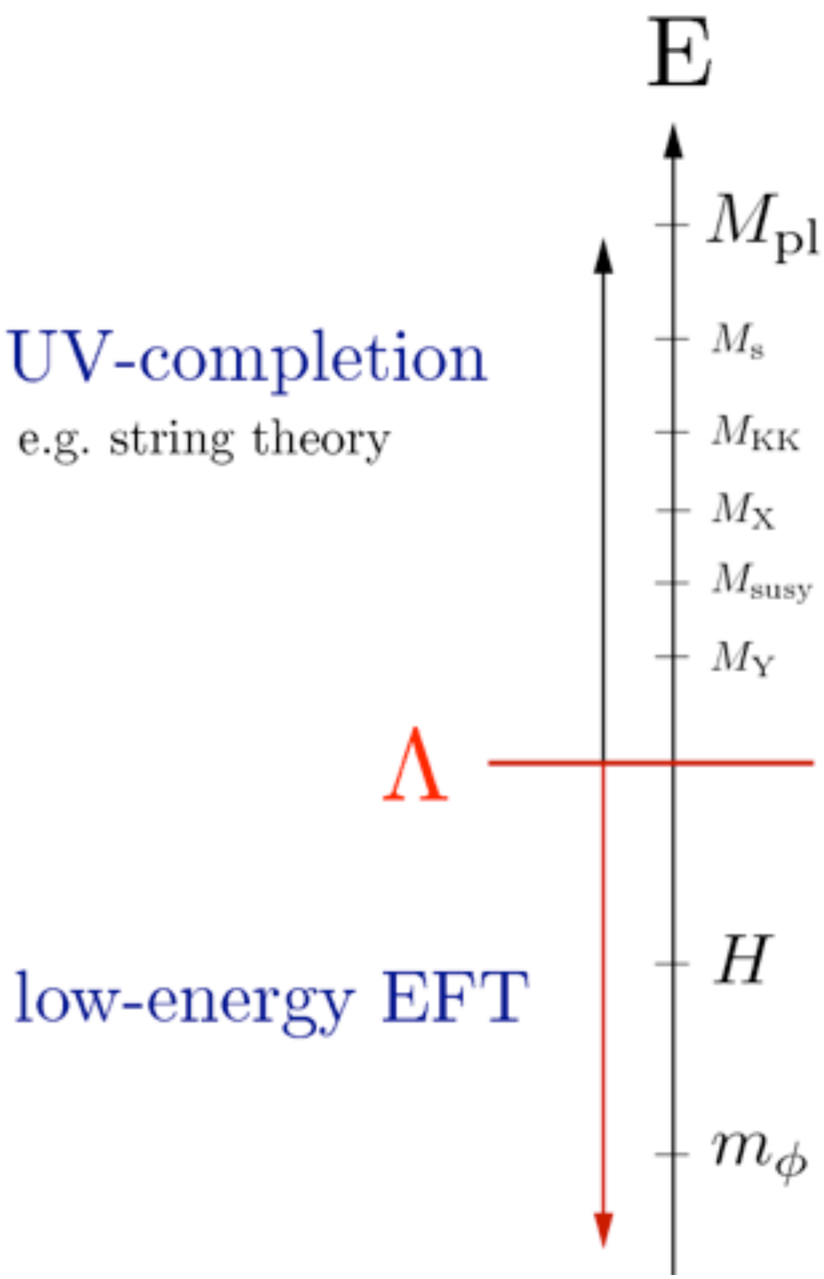
$$r = 16\epsilon_* = \frac{16}{2N+1}.$$

COBE:

$$m_\phi = 1.74(1.45) \times 10^{13}\,\mathrm{GeV}$$

$$\boxed{N = 50(60): \quad n_s = 0.960(0.967), \quad r = 0.158(0.132).}$$

# Effective field theory




- Effective field theory is valid at low energy much below the UV cutoff whose effect is suppressed by inverse powers.

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m_\phi^2\phi^2 - \frac{1}{3}B\phi^3 - \frac{1}{4}\lambda\phi^4 - \sum_{n=1}^{\infty}\frac{c_n\phi^{n+4}}{\Lambda^n}.$$

- Quantum effects of UV physics dominates @ E = Λ .

→ Higher order terms unconrollable for super-cutoff field values

# Large r & Lyth bound

[Lyth (1996)]

- Inflation field excursion during inflation can be related to the number of efolding:

$$\Delta\phi = M_P \int_{N_{\rm end}}^{N_{\rm cmb}} dN\sqrt{2\varepsilon} \quad > M_P N \sqrt{2\varepsilon_{\rm min}}$$

$$\gtrsim 5 M_P \sqrt{\frac{r}{0.1}}\,, \qquad \varepsilon_{\rm min} \sim \varepsilon_* .$$

- For r>0.1, an excursion of the inflaton field beyond the Planck scale is needed.

$\boxed{\text{Bicep2: r=0.20}}$ → "Effective field theory" could not be trusted.

$$\frac{\phi^{n+4}}{M_P^n}, \quad \frac{\phi^m}{M_P^m}(\partial\phi)^2, \cdots$$

# Loop holes ?

- Inflaton rolls slowly for most of time but at the CMB point for a large r ?

[Ben-Dayan, Brustein (2009); Shafi et al (2010); Majumdar(2011,2014)]

$\Longrightarrow$ non-monotonic $\varepsilon$ : $\varepsilon_{\rm min} \ll \varepsilon_* \ ?$

- Improved Lyth bound: [Antusch, Nolde (2014)]

$$\varepsilon_* - \varepsilon_{\rm min} = (\phi_* - \phi_{\rm min})\langle \eta - 2\varepsilon \rangle < \Delta\phi \, \langle \eta - 2\varepsilon \rangle$$

$$\varepsilon_* \simeq 0.11 \sqrt{\frac{r}{0.1}}, \qquad \varepsilon_{\rm min} < \frac{\phi_* - \phi_e}{N} \lesssim 0.02 \left( \frac{\Delta\phi}{M_P} \right)$$

$\Longrightarrow$
$$\frac{\Delta\phi}{M_P} \gtrsim \frac{0.11}{\langle \eta - 2\varepsilon \rangle} \sqrt{\frac{r}{0.1}} \ : \ \boxed{\Delta\phi \gtrsim M_P, \quad r \gtrsim 0.1}$$

$\varepsilon_{\rm min} \ll \varepsilon_*$

small number

# Starobinsky-like inflation and unitarity

# Starobinsky model

- Starobinsky model based on "pure gravity" is dual to a scalar-tensor theory. [Starobinsky (1984)]

$$\mathcal{L} = \sqrt{-g}\left(\frac{R}{2} + \xi^2 R^2\right) \Longleftrightarrow \mathcal{L} = \sqrt{-g}\left(\frac{R}{2} + 2\xi\phi R - \phi^2\right)$$

$$\Longrightarrow \quad \mathcal{L}_E = \sqrt{-g_E}\left(\frac{R}{2} - \frac{1}{2}(\partial_\mu\chi)^2 - \frac{1}{16\xi^2}\left(1 - e^{-\sqrt{\frac{2}{3}}|\chi|}\right)^2\right)$$

$g_{\mu\nu} \rightarrow g_{\mu\nu}/(1+4\xi\phi)$

$1 + 4\xi\phi = e^{\sqrt{\frac{2}{3}}|\chi|}$ in Einstein frame

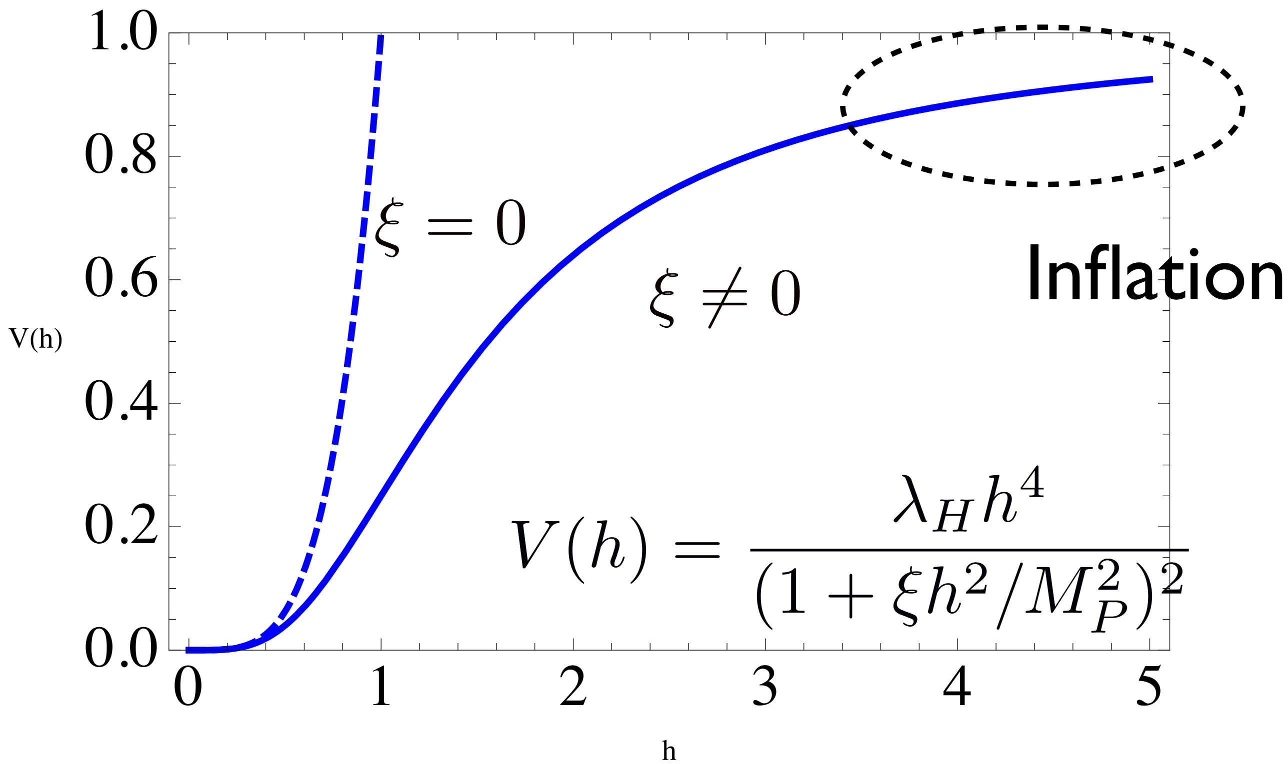
$\xi \sim 10^4$ needed for CMB.

- Starobinsky-like models are a best fit to Planck data.

$n_s = 1 - \dfrac{2}{N}, \; r = \dfrac{12}{N^2}$ : successful for N=40-80 at 95% C.L.

# Higgs inflation

- Non-minimal coupling for Higgs boson is introduced.

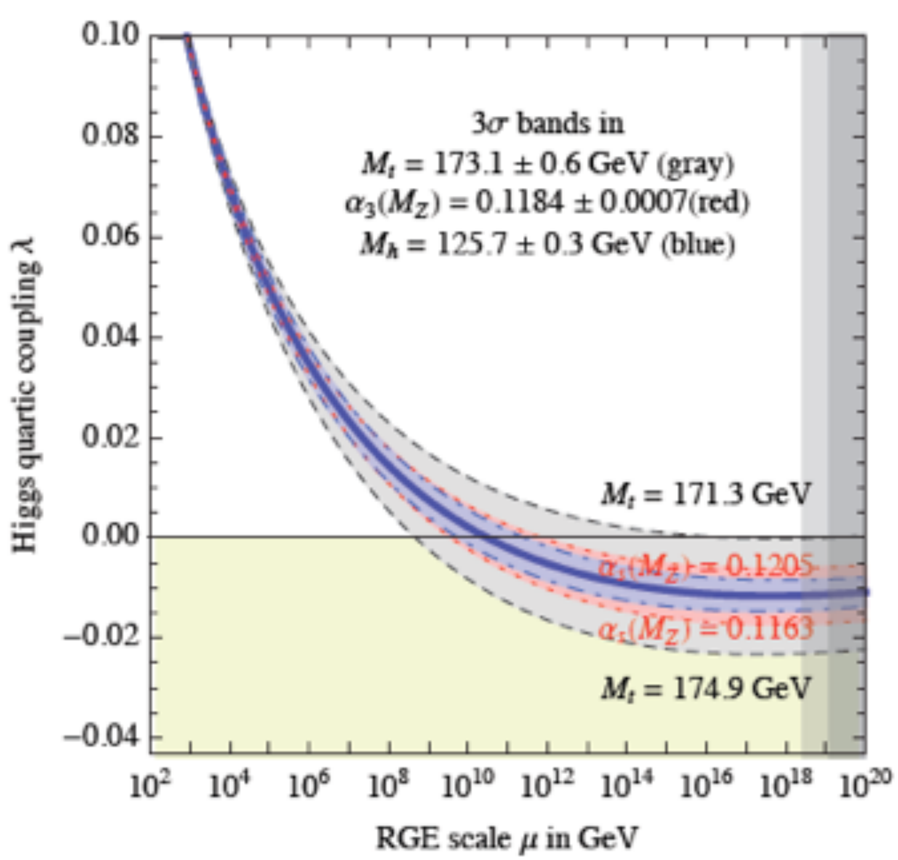



[Bezrukov, Shaposhnikov (2007)]

$$\mathcal{L} = \sqrt{-g}\left(\frac{R}{2} + \frac{1}{2}\xi h^2 R - \frac{1}{2}(\partial_\mu h)^2 - \frac{\lambda_H}{4}(h^2 - v^2)^2\right)$$

$$\boxed{\xi \sim 10^4 \ \text{needed.}}$$

Higgs inflation is classically equivalent to Starobinsky model.




But, loop corrections may change the potential form significantly.

[Buttazzo et al (2013)]

Universe at criticality?

[Bezrukov, Shaposhikov(2014);
Hama, Kawai, Oda, Park (2014) ]

# Starobinsky-like models

[Giudice, HML (2014)]

- General Lagrangian for a single real scalar field:

$$\mathcal{L} = \sqrt{-g}\left[\frac{\Omega(\phi)}{2}R - \frac{K(\phi)}{2}(\partial\phi)^2 - U(\phi)\right]$$

$$\Rightarrow \quad \mathcal{L} = \sqrt{-g}\left[\frac{R}{2} - \frac{1}{2}(\partial\chi)^2 - V(\chi)\right],$$

$$V(\chi) = \frac{U[\phi(\chi)]}{\Omega^2[\phi(\chi)]},$$

$$\frac{d\chi}{d\phi} = \sqrt{\frac{K}{\Omega} + \frac{3\Omega'^2}{2\Omega^2}}$$

- Slow-roll condition:

$$\epsilon \equiv \frac{1}{2}\left(\frac{1}{V}\frac{dV}{d\chi}\right)^2 = \frac{1}{3}\left(\frac{\Omega U'}{\Omega' U} - 2\right)^2 \ll 1: \quad \boxed{U(\phi) = V_I\, \Omega^2(\phi)\, [1 + O(\sqrt{\epsilon})]\,.}$$

- Starobinsky conditions:

$$\frac{\Omega'^2}{\Omega} \gg K,$$ scalar eom: $$\frac{\Omega'}{2}R = U' \quad \Rightarrow \quad \Omega = 1 + \frac{R}{4V_I}, \quad U = \frac{R^2}{16V_I}$$

$$\Rightarrow \quad \boxed{U = V_I\,(\Omega - 1)^2}$$

# Univeral attractors

[Kallosh, Linde (2013)]

- Frame function and potential are chosen:

$$K = 1\,, \qquad \Omega = 1 + \xi f(\phi)\,, \qquad U = \lambda f^2(\phi)$$

$\Rightarrow$ satisfies both slow-roll and Starobinsky conditions.

Inflaton phase: $V(\chi) = V_I \left(1 - e^{-\sqrt{\frac{2}{3}}|\chi|}\right)^2\,, \quad V_I = \frac{\lambda}{\xi^2}\,.$

- "Small" field values attained in vacuum.

$$f(\phi) = \phi^n : \qquad \frac{d\chi}{d\phi} = \sqrt{\boxed{\frac{1}{(1+\xi\phi^n)}} + \frac{3n^2\xi^2\phi^{2(n-1)}}{2(1+\xi\phi^n)^2}}$$

"K-term" dominant

$$\phi(\chi) = \chi\left[1 + F_1(\xi\chi^n, \xi^2\chi^{2(n-1)})\right] \qquad (\text{for } n > 1)$$

$\Rightarrow$ $V(\chi) = \lambda\chi^{2n}\left[1 + F_2(\xi\chi^n, \xi^2\chi^{2(n-1)})\right]$

Unitarity violates @ $\Lambda_{\rm UV} = \frac{M_P}{\xi^{\frac{1}{n-1}}} \ll M_P, \;\; n > 1.$

e.g. n=2: Higgs inflation. cf. n=1: $\Lambda_{\rm UV} = M_P.$

# Induced inflation

[Giudice, HML (2014)]

- Another frame function and potential are chosen:

$$K = 1\,, \quad \Omega = \xi f(\phi)\,, \quad U = \lambda\left[f(\phi) - \xi^{-1}\right]^2 .$$

$$f(\langle\phi\rangle) = \xi^{-1}, \text{ where } \langle\phi\rangle \text{ is the vacuum configuration.}$$

Inflaton phase: $V(\chi) = V_I\left(1 - e^{-\sqrt{\frac{2}{3}}|\chi|}\right)^2, \quad V_I = \frac{\lambda}{\xi^2}.$

- "Large" field values attained in vacuum.

$f(\phi) = \phi^n :$ $\quad \dfrac{d\chi}{d\phi} = \sqrt{\dfrac{1}{\xi\phi^n} + \boxed{\dfrac{3n^2}{2\phi^2}}}$ "induced K-term" dominant

$$\hookrightarrow \quad \frac{\phi}{v} = \exp\left(\sqrt{\frac{2}{3}}\,\frac{|\chi|}{n}\right) + O(v^2) \qquad v \equiv \langle\phi\rangle = \xi^{-1/n}.$$

$$\Rightarrow \quad V(\chi) = \frac{\lambda}{\xi^2}\left[1 - e^{-\sqrt{\frac{2}{3}}|\chi|} + O(\xi^{-2/n})\right]^2$$

Unitarity cutoff = Planck scale.

# Comparison between models

- Scalar kinetic term in Einstein frame is "background-dependent".

$$\frac{\mathcal{L}}{\sqrt{-g}} = -\frac{1}{2}\left(\frac{K}{\Omega}+\frac{3\Omega'^2}{2\Omega^2}\right)\Big|_{\phi=\langle\phi\rangle}(\partial\phi)^2 = -\frac{1}{2}(\partial\phi)^2 \quad \text{(universal attractors)}$$

$$\frac{\mathcal{L}}{\sqrt{-g}} = -\frac{1}{2}\left(\frac{K}{\Omega}+\frac{3\Omega'^2}{2\Omega^2}\right)\Big|_{\phi=\langle\phi\rangle}(\partial\phi)^2 = -\frac{3}{4}\,n^2\xi^{2/n}(\partial\phi)^2 \quad \text{(induced inflation)}\,.$$

- In induced inflation, the large wavefunction rescaling eliminates any positive powers of ξ in interaction terms.

→ "Induced kinetic term" dominates both in vacuum and in inflation background, leading to Planck-scale cutoff.

# Quadratic inflation for BICEP2

# Non-canonical kinetic term

- Consider a single-field inflation with non-canonical kinetic term:

$$\mathcal{L} = \sqrt{-g}\left[\frac{\Omega(\phi)}{2}R - \frac{K(\phi)}{2}(\partial\phi)^2 - U(\phi)\right],$$

$$K(\phi) = 1 + \xi f(\phi), \quad V(\phi) = \lambda(g(\phi) - g(\phi_0))^2, \quad \Omega = 1 + \zeta h(\phi), \quad \zeta = \mathcal{O}(1).$$

- There is a class of attractor solutions leading to quadratic inflation. [F. Takahaski et al (2010,2014); HML(2014)]

$$f(\phi) = \phi^n, \quad g(\phi) = \phi^m, \quad m = \frac{n+2}{2}.$$

m=2: quartic potential

$$\Rightarrow \quad \frac{\mathcal{L}}{\sqrt{-g}} \approx \frac{1}{2}R - \frac{1}{2}(\partial_\mu\chi)^2 - \boxed{\frac{\lambda}{\xi}\left(1+\frac{n}{2}\right)^2(\chi-\chi_0)^2}$$

$\chi \approx \sqrt{\xi}\phi^{n/2+1}/(n/2+1)$

COBE: $\frac{\lambda}{\xi} = \frac{16}{(n+2)^2} \times 6.38(4.43) \times 10^{-12}.$ $\boxed{\rightarrow \text{large } \xi \text{ for } \lambda \sim 1.}$

# Sub-Planckian inflaton

- During inflation, the inflaton is sub-Planckian for a large ξ>>1 in Jordan frame.

non-canonical: $\xi\phi^2 \gg 1$

slow-roll: $\chi \approx \sqrt{\xi}\phi^2/2 \gg 1 \quad (n = m = 2)$

$$\Rightarrow \boxed{\frac{\sqrt{2}}{\xi^{1/4}} \ll \phi < 1 \quad : \quad \text{a large room for sub-Planckian.}}$$

$$\left(m = (n+2)/2 : \quad \frac{1}{\xi^{1/(n+2)}} \ll \phi < 1\right)$$

- The same predictions for inflation as in quadratic inflation with canonical kinetic term.

# Unitarity violation

[HML (2014)]

- Expand the inflaton field around a large vacuum expectation value: $\phi = \phi_0 + \bar{\phi}$

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{1}{2}R - \frac{1}{2}\Big(1+\xi(\phi_0+\bar{\phi})^n\Big)(\partial_\mu\bar{\phi})^2 - \lambda\Big((\phi_0+\bar{\phi})^{1+n/2} - \phi_0^{1+n/2}\Big)^2$$

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{1}{2}R - \frac{1}{2}\left(1+\overbrace{\frac{\xi\phi_0^n}{1+\xi\phi_0^n}}^{\to 1}\left[\left(1+\boxed{\frac{\hat{\phi}}{\phi_0\sqrt{1+\xi\phi_0^n}}}\right)^n - 1\right]\right)(\partial_\mu\hat{\phi})^2$$

$$-\lambda\phi_0^{2+n}\left[\left(1+\boxed{\frac{\hat{\phi}}{\phi_0\sqrt{1+\xi\phi_0^n}}}\right)^{1+n/2} - 1\right]^2.$$

$$\hat{\phi} = \sqrt{1+\xi\phi_0^n}\,\bar{\phi}$$

canonical field

- A large inflaton VEV leads to Planck-scale cutoff.

$$\boxed{\Lambda_{UV} = \frac{\sqrt{\xi}\phi_0^{1+n/2}}{M_P^{n/2}} \sim M_P,}$$ for $\phi_0 \sim \dfrac{M_P}{\xi^{1/(n+2)}}$. $(\to \xi\phi^n \gtrsim 1)$

cf. Small VEV: $\xi\phi^n \ll 1:\ \Lambda_{UV} = \dfrac{M_P}{\xi^{1/n}}$ But, $\phi_I \gg \Lambda_{UV}$.

# Unitarity scales

**Starobinsky-like inflation:**

$$\mathcal{L} = \sqrt{-g}\left(\frac{R}{2} + \xi^2 R^2\right) \longleftrightarrow \mathcal{L} = \sqrt{-g}\left[\frac{\Omega(\phi)}{2}R - \frac{K(\phi)}{2}(\partial\phi)^2 - U(\phi)\right], \quad U = V_I\,(\Omega - 1)^2\,.$$

$$K = 1\,, \quad \Omega = 1 + \xi f(\phi)\,, \quad U = \lambda f^2(\phi) \Longrightarrow \boxed{\Lambda_{UV} = \frac{M_P}{\xi^{1/(n-1)}}.}$$

Higgs inflation: $f(\phi) = \phi^2 : \quad \Lambda_{UV} = \frac{M_P}{\xi}.$

[Burgess, HML, Trott (2009,2010); Barbon et al (2009)]

$$K = 1\,, \quad \Omega = \xi f(\phi)\,, \quad U = \lambda\left[f(\phi) - \xi^{-1}\right]^2$$

induced inflation $\Longrightarrow \boxed{\Lambda_{UV} = M_P}$

**Non-canonical quadratic inflation:**

$$K = 1 + \xi\phi^n, \quad V = \lambda(\phi^{1+n/2} - 1/\xi^{1/2})^2,$$

$$\Omega = 1 + \zeta f(\phi), \quad \zeta = \mathcal{O}(1). \Longrightarrow \boxed{\Lambda_{UV} = M_P.}$$

# Conclusions

- Large-field inflations are favored by gravity waves detected by BICEP2, but suffer a problem of higher order corrections at Planck scale.
- Attractor solutions for Starobinsky-like inflation or quadratic inflation are suggested.,
- A large coupling leads to sub-Planckian inflations in Jordan frame, while the rescaling of inflation field with a large VEV makes the model unitarity up to Planck-scale.

$\Rightarrow$ Higher order Planck-scale corrections in Jordan frame are under control.

"self-complete"