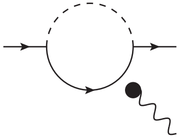
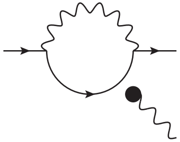
Dipole operator constraints on composite Higgs models

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Matthias König

May 27th, 2014





Dipole operators:

- emerge from one-loop correction to fermion-photon coupling $f_{iR} \to f_{jL}\gamma/g$.
- Various observables are governed by dipole operatores such as electric dipole moments (EDMs), flavour- and CP-violating quark transitions
  $\rightarrow$ well-measured observables provide important constraints on parameter space of NP models!

[MK, M. Neubert, D. M. Straub (*2014*), 1403.2756]

- New physics effects of dipoles in CHM studied throughout literature

[Agashe et al (*2005*), Phys.Rev. D71 016002, Agashe et al (*2004*), Phys.Rev.Lett. 93 201804, Gedalia et al (*2009*), Phys.Lett. B682 200–206, Delaunay et al (*2013*), JHEP 1301 027, N. Vignaroli (*2012*), Phys.Rev. D86 115011, Csaki et al (*2011*), Phys.Rev. D83 073002, Blanke et al (*2012*), JHEP 1208 038, Beneke et al (*2013*), JHEP 1308 010, ...]

- We investigate impact of choice of flavour structure and heavy quark representations on the NP effects
- Minor mistakes found in literature (see paper for details)

Outline

1. Setup
   - Two-site models with partial compositeness
   - Fermion representations and custodial protections
   - Flavour structure

2. Dipole operators in composite Higgs models
   - Relevant phenomenology
   - Results

Setup

Two-site models with partial compositeness

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[Contino et al (*2006*), hep-ph/0612180]

Lagrangian is split into three sectors:

$$\mathcal{L}_{CHM} = \mathcal{L}_{elementary} + \mathcal{L}_{composite} + \mathcal{L}_{mixing}$$

- Simplified two-site model: SM-like elementary sector
- Composite sector with $[SU(3) \times SU(2)_L \times SU(2)_R \times U(1)_X]$ symmetry
- Linear mass mixing between sectors realize partial compositeness

The doublet model

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- Fermion representations as in MCHM4

[Agashe et al (*2004*), hep-ph/0412089]

- Two doublets of heavy fermions:

$$Q = \begin{pmatrix} T \\ B \end{pmatrix} \sim (2,1)_{1/6} \quad R = (U, D) \sim (1,2)_{1/6}$$

- Quark mass Lagrangian:

$$\mathcal{L}_{m,\text{quarks}} = -\bar{Q} m_Q Q - \bar{R} m_Q R - \left(Y \bar{Q}_L \mathcal{H} R_R + \tilde{Y} \bar{R}_L \mathcal{H}^* Q_R + \text{h.c.} \right)$$

$$\mathcal{L}_{\text{mix}} = \lambda_L \bar{q}_L Q_R + \lambda_{Ru} \bar{U}_L t_R + \lambda_{Rd} \bar{D}_L b_R$$

- Quark mass matrix:

$$\begin{array}{c} \\ b_L \\ B_L \\ D_L \end{array} \begin{array}{c} \begin{array}{ccc} b_R & B_R & D_R \end{array} \\ \begin{pmatrix} 0 & -\lambda_L & 0 \\ 0 & m_Q & -\frac{Yv}{\sqrt{2}} \\ -\lambda_{Rd} & -\frac{\tilde{Y}v}{\sqrt{2}} & m_R \end{pmatrix} \end{array}, \quad m_b = Yv \frac{\lambda_L}{m_Q} \frac{\lambda_R}{m_R}$$

The triplet model (TS10)

- Fermion representations as in MCHM10
- Heavy fermions come in one bidoublet and two triplets:

$$Q = \begin{pmatrix} T & T_{\frac{5}{3}} \\ B & T_{\frac{2}{3}} \end{pmatrix}_{2/3}, \quad R = \begin{pmatrix} U_{\frac{5}{3}} \\ U \\ D \end{pmatrix}_{2/3}, \quad R' = \begin{pmatrix} U'_{\frac{5}{3}} & U' & D' \end{pmatrix}_{2/3}$$

- Lagrangian:

$$\begin{aligned} \mathcal{L}_{m,\text{quarks}} &= -\bar{Q} m_Q Q - \bar{R} m_R R - \bar{R}' m_R R' \\ &\quad - \big[Y \bar{Q}_L \mathcal{H} R_R + Y \bar{Q}_L \mathcal{H} R'_R \\ &\quad + \tilde{Y} \bar{R}_L \mathcal{H}^* Q_R + \tilde{Y} \bar{R}'_L \mathcal{H}^* Q_R + \text{h.c.} \big] \\ \mathcal{L}_{\text{mix}} &= \lambda_L \bar{q}_L L_R + \lambda_{Ru} \bar{U}_L t_R + \lambda_{Ru} \bar{D}_L b_R \end{aligned}$$

- b_L mixes with $P_{L/R}$ eigenstate $B \to Z$ coupling is protected

[Agashe et al (*2006*), Phys. Lett. B 641 p62]

- Fermion representations as in MCHM5
- Two bidoublets and two singlets:

$$Q_u = \begin{pmatrix} T & T_{\frac{5}{3}} \\ B & T_{\frac{2}{3}} \end{pmatrix}_{2/3}, \; Q_d = \begin{pmatrix} B_{-\frac{1}{3}} & T' \\ B_{-\frac{4}{3}} & B' \end{pmatrix}_{-1/3}, \; U, \; D$$

- Lagrangian:

$$\mathcal{L}_{m,\text{quarks}} = -\bar{Q}_U m_{Qu} Q_U - \bar{U} m_R U - \left[Y \bar{Q}_{U,L} \mathcal{H} U_R + \tilde{Y} \bar{U}_L \mathcal{H}^* Q_{U,R} + \text{h.c.} \right] + (U \to D)$$

$$\mathcal{L}_{\text{mix}} = \lambda_{Lu} \bar{q}_L L_{U,R} + \lambda_{Ru} \bar{U}_L u_R + (U \to D)$$

- B' no $P_{L/R}$ eigenstate $\Rightarrow \lambda_{Ld} \overset{!}{\ll} \lambda_{Lu} \Rightarrow m_b \ll m_t$ naturally

Towards three generations

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- Up till now, only considered only one generation of quarks.
- Find ways to generalize expressions to three generations.
- Every "parameter" is a matrix in flavour space, diagonalization no more feasible.
- Reconstruct expressions via mass insertion.
  $\Rightarrow$ e.g. $y_d \approx \lambda_{Ld} m_Q^{-1} Y m_R^{-1} \lambda_{Rd}$
- More or less exact depending on structure of matrices!
- We consider: flavour anarchy and $U(N)$ symmetric models.

[Barbieri et al (*2012*), 1211.5085]

Flavour anarchic models

- Yukawa matrices are assumed to be structureless.
- Composite-elementary mixings λ are hierarchical:

$$\lambda = \mathrm{diag}\left(\lambda_1, \lambda_2, \lambda_3\right)$$

- Think of all Y as an *average* Yukawa coupling.
  $\rightarrow$ Diagrammatic approach plagued by $O(1)$ uncertainties.
- Constraints from tree level meson-antimeson mixing.

Flavour symmetric models

Impose global symmetry on strong sector

- Consider the coice of $U(3)^3$ symmetry:

$$Y_{ij} = Y\delta_{ij},\ m_{ij} = m\delta_{ij}$$

- Mixings λ (either left-handed or right-handed) break the flavour symmetry.
  λ_L break $\rightarrow$ "right-handed compositeness"
  λ_R break $\rightarrow$ "left-handed compositeness"
- Breaking mixings need to reproduce the SM Yukawas.
  $\rightarrow$ need different λ for up- and down-type quarks
  $\rightarrow$ right-handed compositeness only possible in bidoublet model!

[Cacciapaglia et al (*2007*), 0709.1714]

[M. Redi, A. Weiler (*2014*), 1106.6357v4]

Flavour symmetric models

$U(3)^3$ flavour model successfully suppresses FCNC processes.
But predicts large degrees of compositeness for (one chirality of) light quarks $\rightarrow$ strongly constrained!

Solution: only assume first two generations to transform under flavour symmetry $U(2)^3$.

- Larger parameter space:

$$Y = \mathrm{diag}(Y, Y, Y_3),\ m_Q = \mathrm{diag}(m_Q, m_Q, m_{Q3})$$

- Allows for small degree of compositeness for the first two generations.

[Barbieri et al (*2012*), 1211.5085]

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Dipole operators in composite Higgs models

Dipole operators

Work in EFT approach, effective Hamiltonian:

$$\mathcal{H}_{eff} = -\sum_{i,j,q,V} \mathcal{C}_{q_i q_j V} \mathcal{Q}_{q_i q_j V} + \mathcal{C}'_{q_i q_j V} \mathcal{Q}'_{q_i q_j V}$$

with $q = u, d$ and $V = \gamma, g$

Effective operators:

$$\mathcal{Q}_{q_i q_j \gamma} = \frac{e m_{q_i}}{16\pi^2} \left(\bar{q}_j \sigma^{\mu\nu} P_R q_i\right) F_{\mu\nu}, \quad \mathcal{Q}_{q_i q_j g} = \frac{g_s m_{q_i}}{16\pi^2} \left(\bar{q}_j T^a \sigma^{\mu\nu} P_R q_i\right) G_{a\mu\nu}$$

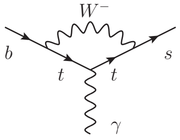
$$\mathcal{Q}'_{q_i q_j \gamma} = \frac{e m_{q_i}}{16\pi^2} \left(\bar{q}_j \sigma^{\mu\nu} P_L q_i\right) F_{\mu\nu}, \quad \mathcal{Q}'_{q_i q_j g} = \frac{g_s m_{q_i}}{16\pi^2} \left(\bar{q}_j T^a \sigma^{\mu\nu} P_L q_i\right) G_{a\mu\nu}$$

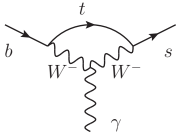
Observables involving dipole operators we considered:

Observable	Effective operators involved
Neutron EDM	$\mathcal{Q}_{qq\gamma}$, $\mathcal{Q}_{qqg}$
$B \to X_s\gamma$	$\mathcal{Q}^{(\prime)}_{bs\gamma}$, $\mathcal{Q}^{(\prime)}_{bsg}$
$B \to X_d\gamma$	$\mathcal{Q}^{(\prime)}_{bd\gamma}$, $\mathcal{Q}^{(\prime)}_{bdg}$
ϵ'/ϵ	$\mathcal{Q}^{(\prime)}_{sdg}$
ΔA_{CP}	$\mathcal{Q}'_{cug}$

Example: $b \to s\gamma$

Standard model:





Contributions from new physics:

- Heavy particles in the loop (composite fermions, composite gauge bosons)
- Contributions from $O(v^2)$ corrections to gauge couplings
- Diagrams with Higgs (flavour violating Higgs couplings allowed!)

Calculation of NP effects:

- Matching of relevant diagrams to the operators
  $\rightarrow$ obtain general formula for Wilson coefficients
- Mass-diagonalize Lagrangian
- Plug masses and couplings in mass-eigenbasis into Wilson coefficients

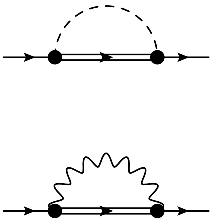
$\rightarrow$ Analytical expression for $\mathcal{C}_{q_i q_j V}$

- Perform RG running
- Derive bounds

Wilson coefficients: Leading contribution

Leading correction:

$$\Delta\mathcal{C}_{qqV} = a_{qV}\frac{Y\,\tilde{Y}}{m_Q m_R}$$



with a_{qV}: numerical factor
Generated by diagrams with

- a Higgs boson and a heavy fermion
- a W or Z boson and a heavy fermion

in the loop.
Three generations:

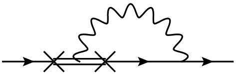
$$\mathcal{C}_{qqV} = \frac{a_{qV}}{m_{q_i}}\Delta^q_{ij}$$

$$\Delta^q_{ij} = \frac{v}{\sqrt{2}} U^\dagger_{Ld}\lambda_L m_Q^{-1} Y m_R^{-1} \tilde{Y} m_Q^{-1} Y m_R^{-1} \lambda_{Rd} U_{Rd}$$

Wilson coefficients: Subleading corrections

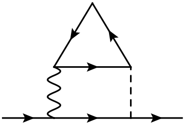
Especially interesting for models with $\tilde{Y} = 0$, contributions arising from:

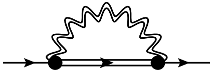
- Same diagrams as leading order: $\propto \frac{Y^2 m_h^2}{m_\psi^4}$ (from expansion of loop function)
- Same diagrams as leading order: $\propto \frac{Y^2 \lambda^2}{m_\psi^4}$ (from expansion in λ/m_ψ)
- SM quarks and W or Z in the loop, $\mathcal{O}(v^2)$ correction to gauge coupling: $\propto \frac{Y^2 \lambda^2}{m_\psi^4}$



Wilson coefficient: More subleading terms

- Barr-Zee type diagrams: $\propto \frac{g^2}{16\pi^2}\frac{Y^2}{m_\psi^2}$
- Heavy vector resonances and heavy fermions in loop: $\propto \frac{g_\rho^2}{M_\rho^2}$
  $\sim$ flavour-diagonal and real $\rightarrow$ ignore





- Possible contributions from higher-dimensional operators not included in this simplified setup

Bounds on parameter space

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Bounds for the flavour anarchic case:

	bound on:	$\left(\frac{m_Q m_R}{Y\tilde{Y}}\right)^{1/2}$			$\left(\frac{m_Q^2}{s_{Rt}^2 Y^2}\right)^{1/2}$	$\left(\frac{m_Q m_R}{Y}\right)^{1/2}$
	operator	doublet	triplet	bidoublet	(estimate)	(estimate)
d_n	$\mathcal{Q}_{ddV}$	**3.6 TeV**	**5.1 TeV**	**4.1 TeV**		0.8 TeV
	$\mathcal{Q}_{uuV}$	1.3 TeV	0.6 TeV	1.4 TeV		0.3 TeV
	$\mathcal{Q}_{ccg}$	1.1 TeV	1.7 TeV	1.5 TeV		0.5 TeV
	$\mathcal{Q}_{bbg}$	0.6 TeV	0.9 TeV	0.8 TeV		0.3 TeV
	$\mathcal{Q}_{ttg}$	0.3 TeV	0.4 TeV	0.4 TeV		0.2 TeV
$B \to X_s\gamma$	$\mathcal{Q}_{bsV}$	0.4 TeV	0.5 TeV	0.2 TeV	0.6 TeV	0.3 TeV
	$\mathcal{Q}'_{bsV}$	0.7 TeV	1.0 TeV	0.4 TeV	1.1 TeV	0.3 TeV
$B \to X_d\gamma$	$\mathcal{Q}_{bdV}$	0.2 TeV	0.3 TeV	0.1 TeV	0.3 TeV	0.2 TeV
	$\mathcal{Q}'_{bdV}$	0.6 TeV	0.8 TeV	0.3 TeV	0.9 TeV	0.3 TeV
ϵ'/ϵ	$\mathcal{Q}_{sdg}$	1.1 TeV	1.6 TeV	1.6 TeV		0.5 TeV
	$\mathcal{Q}'_{sdg}$	1.1 TeV	1.6 TeV	1.6 TeV		0.5 TeV
ΔA_{CP}	$\mathcal{Q}_{cug}$	0.9 TeV	1.4 TeV	1.3 TeV		0.4 TeV
	$\mathcal{Q}'_{cug}$	0.2 TeV	0.3 TeV	0.2 TeV		0.2 TeV

Bounds for flavour symmetric models

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Neutron electric dipole moment:

- Can show that only physical phase beside CKM reside in wrong-chirality $\tilde{Y}$
  $\rightarrow$ relevant bounds only for models with $\tilde{Y} \neq 0$!
- Bound on: $\frac{m_\psi^2}{Y \, \text{Im} \, \tilde{Y}}$ (and $\frac{m_{\psi 3}^2}{Y_3 \, \text{Im} \, \tilde{Y}_3}$ in $U(2)^3$ for third generation)

operator	doublet	triplet	bidoublet
$\mathcal{Q}_{ddV}$	**3.6 TeV**	**5.1 TeV**	**4.1 TeV**
$\mathcal{Q}_{uuV}$	1.3 TeV	0.6 TeV	1.4 TeV
$\mathcal{Q}_{ccg}$	1.1 TeV	1.7 TeV	1.5 TeV
$\mathcal{Q}_{bbg}$	0.6 TeV	0.8 TeV	0.8 TeV
$\mathcal{Q}_{ttg}$	0.3 TeV	0.4 TeV	0.4 TeV

Flavour violating observables:

- In $U(3)^3$: Leading contribution $\propto Y\tilde{Y}$ vanishes due to $Y, \tilde{Y} \propto \mathrm{Id}$
- In $U(2)^3$: Leading contribution is $\propto \left(\frac{Y\tilde{Y}}{m_\psi^2} - \frac{Y_3\tilde{Y}_3}{m_{\psi 3}^2}\right)$ for left-handed compositeness!

operator	doublet	triplet	bidoublet
$\mathcal{Q}_{bsV}$	0.37 TeV	0.52 TeV	0.22 TeV

- Next-to-leading correction: $\frac{Y^2}{m_\psi^2}\frac{s_{Rt}}{s_{Lt}Y} \lesssim \left(\frac{1}{0.6\mathrm{TeV}}\right)^2$

Conclusions

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- Leading contribution in models with $\tilde{Y} \neq 0$ comes from diagrams with a heavy fermion and a W, Z or Higgs
- Next-to-leading contributions suppressed by degree of compositeness $\rightarrow$ important for 3rd generation quarks

Anarchic models:

- Stringent bounds from neutron EDM ($m_\psi \gtrsim 4$ TeV for $Y \sim \tilde{Y} \sim 1$), even stronger for greater Y, $\tilde{Y}$!
- Several bounds from flavour-violating observables in the 1-2 TeV range

Flavour-symmetric models:

- Also strong bounds from neutron EDM, but on $Y \,\mathrm{Im}\, \tilde{Y}$ instead of $Y\tilde{Y}$ $\rightarrow$ can be avoided
- Bounds from flavour-violating observables are mild

Outlook

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- Several experiments in construction to improve accuracy for neutron EDM $\rightarrow$ stronger bound!

[Hewett et al (*2012*), 1205.2671]

- Flavour violating top-quark transitions $t \rightarrow q\gamma, g$ not yet strongly constrained $\rightarrow$ future LHC runs!
- Analysis to be done in more fundamental theory (pNGH), leading order effects should be similar
- Lepton sector? (eg. $\mu \rightarrow e\gamma$, d_{e^-}, $\ldots$)
- Full numerical analysis including all $\Delta F = 1$ and $\Delta F = 2$ processes and electroweak constraints...

[Barbieri et al (*2012*), 1211.5058]

Thank you for your attention!

Backup slides

Mass matrices

TS4:

$$M_\psi^u = \begin{array}{c} \\ t_L \\ T_L \\ U_L \end{array} \begin{array}{c} \begin{array}{ccc} t_R & T_R & U_R \end{array} \\ \begin{pmatrix} 0 & -\lambda_L & 0 \\ 0 & m_Q & -\frac{Y v}{\sqrt{2}} \\ -\lambda_{Ru} & -\frac{\tilde{Y} v}{\sqrt{2}} & m_R \end{pmatrix} \end{array}$$

TS5:

$$M_\psi^u = \begin{array}{c} \\ t_L \\ T_L \\ T'_L \\ T_{2/3L} \\ U_L \end{array} \begin{array}{c} \begin{array}{ccccc} t_R & T_R & T'_R & T_{2/3R} & U_R \end{array} \\ \begin{pmatrix} 0 & -\lambda_{Lu} & -\lambda_{Ld} & 0 & 0 \\ 0 & m_{Qu} & 0 & 0 & -\frac{Y v}{\sqrt{2}} \\ 0 & 0 & m_{Qd} & 0 & 0 \\ 0 & 0 & 0 & m_{Qu} & -\frac{Y v}{\sqrt{2}} \\ -\lambda_{Ru} & -\frac{\tilde{Y} v}{\sqrt{2}} & 0 & -\frac{\tilde{Y} v}{\sqrt{2}} & m_U \end{pmatrix} \end{array}$$

Mass matrices

TS10:

$$M_\psi^u = \begin{array}{c} \\ t_L \\ U_L \\ U'_L \\ T_L \\ T_{2/3L} \end{array} \begin{array}{c} \begin{array}{ccccc} t_R & U_R & U'_R & T_R & T_{2/3R} \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & -\lambda_L & 0 \\ -\lambda_{Ru} & m_R & 0 & -\frac{\tilde{Y}_V}{2} & \frac{\tilde{Y}_V}{2} \\ 0 & 0 & m_R & -\frac{\tilde{Y}_V}{2} & \frac{\tilde{Y}_V}{2} \\ 0 & -\frac{Y_V}{2} & -\frac{Y_V}{2} & m_Q & 0 \\ 0 & \frac{Y_V}{2} & \frac{Y_V}{2} & 0 & m_Q \end{pmatrix} \end{array} \qquad M_\psi^d = \begin{array}{c} \\ b_L \\ D_L \\ D'_L \\ B_L \end{array} \begin{array}{c} \begin{array}{cccc} b_R & D_R & D'_R & B_R \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & -\lambda_L \\ -\lambda_{Rd} & m_R & 0 & -\frac{\tilde{Y}_V}{\sqrt{2}} \\ 0 & 0 & m_R & -\frac{\tilde{Y}_V}{\sqrt{2}} \\ 0 & -\frac{Y_V}{\sqrt{2}} & -\frac{Y_V}{\sqrt{2}} & m_Q \end{pmatrix} \end{array}$$

$$M_\psi^{5/3} = \begin{array}{c} \\ T_{5/3L} \\ U_{5/3L} \\ U'_{5/3L} \end{array} \begin{array}{c} \begin{array}{ccc} T_{5/3R} & U_{5/3R} & U'_{5/3R} \end{array} \\ \begin{pmatrix} m_Q & -\frac{Y_V}{\sqrt{2}} & -\frac{Y_V}{\sqrt{2}} \\ -\frac{\tilde{Y}_V}{\sqrt{2}} & m_R & 0 \\ -\frac{\tilde{Y}_V}{\sqrt{2}} & 0 & m_R \end{pmatrix} \end{array}$$

Formulae

$$\mathcal{C}_{q_i q_j \gamma, g} = \sum_{\psi, X} \frac{1}{m_{q_i} m_X^2} \left(m_{q_i} V^{L*}_{i\psi X} V^{L}_{j\psi X} + m_{q_j} V^{R*}_{i\psi X} V^{R}_{j\psi X} \right) F^1_X(Q_\psi, Q_X, x)$$

$$+ \frac{1}{m_{q_i} m_X^2} \left(m_\psi V^{L*}_{i\psi X} V^{R}_{j\psi X} \right) F^2_X(Q_\psi, Q_X, x)\,,$$

$\mathcal{C}$	X	Q_ψ	Q_X	$\mathcal{C}$	X	Q_ψ	Q_X
$\mathcal{C}_{dd\gamma}$	h, Z, ρ^0	$-1/3$	0	$\mathcal{C}_{uu\gamma}$	h, Z, ρ^0	$2/3$	0
	W^+, ρ^+	$-4/3$	1		W^+, ρ^+	$-1/3$	1
	W^-, ρ^-	$2/3$	-1		W^-, ρ^-	$5/3$	-1
	G^*	$-4/9$	0		G^*	$8/9$	0
$\mathcal{C}_{ddg}$	h, W, ρ	1	0	$\mathcal{C}_{uug}$	h, W, ρ	1	0
	G^*	$-1/6$	$3/2$		G^*	$-1/6$	$3/2$

Loop functions

$$F_V^1(Q_\psi, Q_V, x) = Q_\psi \frac{\left(5x^4 - 14x^3 + 39x^2 - 18x^2 \log x - 38x + 8\right)}{24(x-1)^4} + Q_V \frac{\left(4x^4 - 49x^3 + 18x^3 \log x + 78x^2 - 43x + 10\right)}{24(x-1)^4},$$

$$F_V^2(Q_\psi, Q_V, x) = Q_\psi \frac{\left(-x^3 - 3x + 6x \log x + 4\right)}{4(x-1)^3} + Q_V \frac{\left(-x^3 + 12x^2 - 6x^2 \log x - 15x + 4\right)}{4(x-1)^3},$$

$$F_S^1(Q_\psi, Q_S, x) = Q_\psi \frac{\left(-x^3 + 6x^2 - 3x - 6x \log x - 2\right)}{24(x-1)^4} + Q_S \frac{\left(2x^3 + 3x^2 - 6x^2 \log x - 6x + 1\right)}{24(x-1)^4},$$

$$F_S^2(Q_\psi, Q_S, x) = Q_\psi \frac{\left(-x^2 + 4x - 2 \log x - 3\right)}{4(x-1)^3} + Q_S \frac{\left(x^2 - 2x \log x - 1\right)}{4(x-1)^3}.$$