What does gravity do with *(QCD)* *axions?*

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why that question?

⇔ can axion-CDM be distinguished from WIMPs using Large Scale Structure data?

(? maybe?: the stress energy tensors are different... Sikivie
...but need numerics for galaxy formation with classical field DM)

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(? maybe?: the stress energy tensors are different...
...but need numerics for galaxy formation with classical field DM) Sikivie

Tempest in the Axionpot: Sikivie proposes that to differ from WIMPs in LSS,
a) axions needs to be in a Bose Einstein Condensate not in my opinion
b) gravity drives them there not in my opinion

Sikivie+Yang
Erken,Sikivie,Tam,Yang
Bannik+Sikivie
Saikawa+ Yamaguchi
Saikawa etal
...

summary in my opinion: axions = (free) scalar cpled to gravity = simple!

Outline: what does gravity do with axions?

1. review axion cosmology
   - two axion populations: classical field + cold particles
2. how to answer the question
   - variables
   - equations
3. what do you rediscover?
   - axion field+ particles are CDM: expand U, and grow linear $\delta\rho$ like WIMPs
   - misalignement axion field has extra pressure, compared to WIMPs
   - (how is that different from calns of BE condensation via grav thermalisation)
4. what do you learn?
   - not need Bose Einstein Condensate — extra pressure arises for classical field
   - gravity does not move many axions between field and particles

⇒ axions are simple (scalar cpled to gravity).

⇒ write a galaxy formation code with classical field dark matter, to know if LSS different for axions vs WIMPs.

Review: axion in astro-particle physics

you all know the axion: trade CPV θ for dynamical field a, for which $\langle a \rangle_t \to 0$
phase of a complex SM-singlet scalar Φ, who gets big vev

Peccei Quinn
DineFischlerSrednicki,Zhitnitsky
Kim,ShifmanVainshteinZakharov

$$\Phi \to f_{PQ} e^{ia/f_{PQ}} \qquad\qquad f_{PQ} \sim 10^{11}\ \mathrm{GeV}$$

$\Rightarrow$ only new particle at low-energy is the (pseudo-) goldstone a

$$\text{mixes to pion} \quad : \quad m_a \sim \frac{m_\pi f_\pi}{f_{PQ}} \simeq\ 6 \times 10^{-5}\ \frac{10^{11}\ \mathrm{GeV}}{f_{PQ}}\ \mathrm{eV}$$

...
Srednicki NPB85

$$\text{(feeble) couplings to SM} \quad \propto \quad \frac{1}{f_{PQ}} \propto m_a$$

$\Rightarrow$ produce in sun, He-burning stars(g_{ae}), supernovae(g_{aN})...
upper bound on coupling to avoid rapid stellar energy loss:

Raffelt...

$$m_a \lesssim 10^{-2}\ \mathrm{eV} \qquad\qquad (f_{PQ} \gtrsim 10^{9}\ \mathrm{GeV})$$

Review: non-thermal axion production gives $Cold$ Dark Matter!

1. Suppose inflation before Peccei-Quinn Phase Trans.

   avoid CMB bounds on isocurvature fluctuations $\delta a/a \sim H_I/(2\pi f_{PQ})$

   Planck $\Rightarrow H_I \lesssim 10^7 \sqrt{f/10^{12}}$ GeV

   $or\ non-canonical\ kin.terms\ for\ a...$

WantzShellard
HanannHRW
FolkertsCristianoRedondo

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2. then at PQPT, in each horizon, $\Phi \rightarrow f_{PQ}e^{ia/f_{PQ}}$
   $*$ a massless, random $-\pi f_{PQ} \leq a_0 \leq \pi f_{PQ}$ from one horizon to the next
   $*$ ...one string/horizon

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3. QCD Phase Transition ($T \sim 200$ MeV): "tilt mexican hat"

$$V(a) \to f_\pi^2 m_\pi^2 [1 - \cos(a/f_{PQ})] \simeq \frac{m^2}{2} a^2 - \frac{m^2}{4! f_{PQ}^2} a^4 + \ldots$$

   $*$ ... at $H < m_a$, "misaligned" axion field starts oscillating around the minimum

   $*$ strings go away (radiate cold axion particles, $\vec{p} \sim H \lesssim 10^{-6} m_a$) Hiramatsu etal 1012.5502

PQPT after inflation $\Rightarrow$ **oscillating axion field + cold particles** redshift like CDM

Equations and Variables for studying axion-CDM +gravity

- $\mathcal{O}(G_N)$: Einsteins Eqns. Inside horizon(Newtonian gauge), grav. interactns described by

$$T^{\mu\nu}{}_{;\nu} = 0 \qquad , \qquad \nabla^2\Psi = 4\pi G_N \rho \qquad (\rho \to \delta\rho \text{ in linear regime})$$

NB: I want eqns for $T^{\mu\nu}$, not axion field eqns coupled to gravity (solvable in linear regime, and better handle on IR divs). Dynamics is equivalent (both obtained from $T^{\mu\nu}_{;\nu} = 0$ and Poisson Eqn).

$$\begin{aligned}
T^{\mu\nu}_{;\nu} &= \nabla_\nu[\nabla^\mu\phi\nabla^\nu\phi] - \nabla_\nu[g^{\mu\nu}\left(\frac{1}{2}\nabla^\alpha\phi\nabla_\alpha\phi - V(\phi)\right)] \\
&= (\nabla_\nu\nabla^\mu\phi)\nabla^\nu\phi + \nabla^\mu\phi(\nabla_\nu\nabla^\nu\phi) - g^{\mu\nu}\nabla_\nu\nabla^\alpha\phi\nabla_\alpha\phi + g^{\mu\nu}V'(\phi)\nabla_\nu\phi \\
0 &= \nabla^\mu\phi[(\nabla_\nu\nabla^\nu\phi) + V'(\phi)]
\end{aligned}$$

- $\mathcal{O}(G_N^2)$: covariantly quantised GR (F rules for graviton exchange) Dewitt

Two axion populations are classical field and distribution of cold particles. So variables = field and particle distribution.
$\Rightarrow$ What is their contribution to $T_{\mu\nu}$?

Calculating $T_{\mu\nu}$ for the axion field and particles...

Start from covariant 2nd quantised FT (in flat space!) — GR is covariant, cannot simultaneously describe classical field and particles without $\hbar$, and will want to calculate $\mathcal{O}(G_N^2)$.

Calculating $T_{\mu\nu}$ for the axion field and particles...

Start from covariant 2nd quantised FT — GR is covariant, cannot simultaneously describe classical field and particles without $\hbar$, and will want to calculate $\mathcal{O}(G_N^2)$.

1. in 2nd quantised FT, if write axion as complex (!) scalar field

$$\hat{\phi}(t,\vec{x}) = \int \frac{d^3k}{(2\pi)^3}\frac{1}{\sqrt{2k_0}}\left\{\hat{a}_{\vec{k}}e^{-ik\cdot x} + \hat{b}^{\dagger}_{\vec{k}}e^{ik\cdot x}\right\}$$

then the classical field can be represented as coherent state

Cohen-Tannoudji etal—the book

$$|\phi\rangle \quad \propto \quad \exp\left\{\int \frac{d^3p}{(2\pi)^3}\tilde{\phi}(\vec{p},t)\hat{a}^{\dagger}_{\vec{p}}\right\}|0\rangle \quad \text{such that} \quad \langle\phi|\hat{\phi}^n(t,\vec{x})|\phi\rangle = \phi^n(t,\vec{x})$$

and get

$$\langle\phi|\hat{T}_{\mu\nu}|\phi\rangle \quad = \quad T_{\mu\nu}(\phi)$$

2. in 2nd quantised field theory, can obtain a particle distribution $f(x,p)$ via Wigner transform: write stress-energy tensor as a 2-pt function (complex ϕ is axion!)

$$\begin{aligned}\hat{T}_{\mu\nu}(x_1,x_2) &= \partial_\mu\hat{\phi}^\dagger(x_1)\partial_\nu\hat{\phi}(x_2)+\partial_\nu\hat{\phi}^\dagger(x_1)\partial_\mu\hat{\phi}(x_2)\\ &\quad -g_{\mu\nu}\left(\partial^\alpha\hat{\phi}^\dagger(x_1)\partial_\alpha\hat{\phi}(x_2)-V(\hat{\phi}^\dagger(x_1)\hat{\phi}(x_2))\right)\end{aligned}$$

for $|x_1-x_2|\sim\delta\sim 1/|\vec{p}_a|\sim$ metre (in galaxy today). Then fourier transform wrt δ to get

$$\hat{T}_{\mu\nu}(X,Q)=\int\frac{d^4\delta}{(2\pi)^4}\hat{T}_{\mu\nu}(X-\delta/2,X+\delta/2)$$

so have X-dep $\hat{a}(X)$,

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$$\langle n|\hat{a}_k^\dagger(X)\hat{a}_p(X)|n\rangle = f(X,k)\delta^3(\vec{k}-\vec{p})(2\pi)^3$$

...ok provided there is separation of scales $X \gg \delta$. (δ will reappear at end of talk as an IR cut-off)

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So in the state describing cold particles and classical field, can calculate

$$\langle n,\phi|\hat{T}_{00}(X)|n,\phi\rangle=\rho_{\phi_{cl}}(X)+\rho_{part}(X)\overset{NR}{\simeq}m^2|\phi|^2+m\int\frac{d^3k}{(2\pi)^3}f(k,X)$$

NB: no transfer of axion particles between bath and coherent state at $\mathcal{O}(G_N)$

Rediscovering...stress-energy tensors

non-rel axion particles are dust, like WIMPs:

$$T_{\mu\nu} = \begin{bmatrix} \rho & \rho\vec{v} \\ \rho\vec{v} & \rho v_i v_j \end{bmatrix}$$

compare to perfect fluid: $T_{\mu\nu} = (\rho + P)U_\mu U_\nu - P g_{\mu\nu}$. $P_{int} \propto \lambda^2 \to 0$, nonrel $\Rightarrow P \ll \rho, U = (1, \vec{v}), |\vec{v}| \ll 1$

Complex (!) classical field ϕ : $T_{\mu\nu} = \partial_\mu \phi^\dagger \partial_\nu \phi + \partial_\nu \phi^\dagger \partial_\mu \phi - g_{\mu\nu} \mathcal{L}$. Take non-rel. limit $\phi \to \frac{1}{\sqrt{2m}} \sigma(x) e^{i\theta(x)} e^{-imt}$

$$T_{\mu\nu} = \begin{bmatrix} \rho & \rho\vec{v} \\ \rho\vec{v} & \rho v_i v_j + \Delta T_{ij} \end{bmatrix}$$

$$\rho = m|\sigma|^2 \qquad \vec{v} = \frac{\nabla\theta}{m}$$

$$\Delta T^i_j \sim \partial_i \eta \partial_j \eta \ , \ \lambda \eta^4$$

"extra" pressure with classical field— *not need Bose Einstein condensation!*

$\Rightarrow$ **is structure formation different?**

Rediscovering ... linearised structure formation with axions is like WIMPs

1. initial conditions: adiabatic density fluctuations inherited from surroundings at the QCDPT

2. Einsteins Eqns and $T^{\mu}_{\nu;\mu} = 0$: linear perturbations $\delta \equiv \delta\rho(\vec{k}, t)/\overline{\rho}(t)$ in dust or axion field have same behaviour on LSS scales:

   Ratra, Hwang+Noh

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G_N \overline{\rho}\delta + c_s^2 \frac{k^2}{R^2(t)}\delta = 0$$

   $(c_s \simeq \partial P/\partial \rho \to 0)$

   see that can $solve$ eqns for $T_{\mu\nu} \sim \phi^2$ in linear growth regime (whereas non-lin eqns for ϕ).

3. (very) small scale differences....
   - there is pressure and Jeans length $\sim 1/\sqrt{H(t)m_a}$ (and funny c_s on smaller scales?)
   - if PQPT after inflation, a random from one horizon to next, so $\delta\rho_a/\rho_a \sim \mathcal{O}(1)$ on QCDPT horizon scale (5km then, 0.1 pc today)... axion "miniclusters"

   Hogan,Rees

4. (the axion field does not turn into particles by parametric resonance)

   Kolb,Singh,Srednicki

Open question: distinguishing axions vs WIMPs in non-linear structure formation?

? non-linear dynamics:(black=eqns for dust)

$$T^{\mu}{}_{\nu;\mu} = 0 \quad \Leftrightarrow \quad \begin{cases} \partial_t \rho + \nabla \cdot (\rho \vec{v}) = 0 \\ \partial_t \vec{v} + (\vec{v} \cdot \nabla)\vec{v} = -\nabla \Psi \pm \text{ extra pressures from field} \end{cases}$$

⇒ write an axion field DM code and compare to dust code...

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$\Rightarrow$ write an axion field DM code and compare to dust code...

But need to know — does gravity move axions between the field and particle bath?

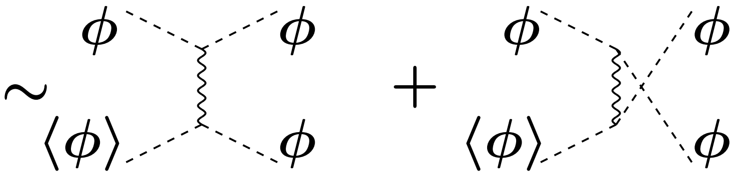
$\Leftrightarrow$ does gravity condense cold axion particles?

$\Leftrightarrow$ do cold axion particles condense?

Moving axions between field and bath with gravity? (in galaxy today)

not at $\mathcal{O}(G_N)$

$\mathcal{M} \sim$



at $\mathcal{O}(G_N^2)$, quantized GR ($v \sim 10^{-3}$ in cm frame)

$$\sigma = \frac{G_N^2 m^2 \pi}{8v^4} \int \sin\theta d\theta \left(\frac{1}{\sin^2(\theta/2)} + \frac{1}{\cos^2(\theta/2)} \right)^2$$

IR cutoff of graviton momenta $\sim H$?

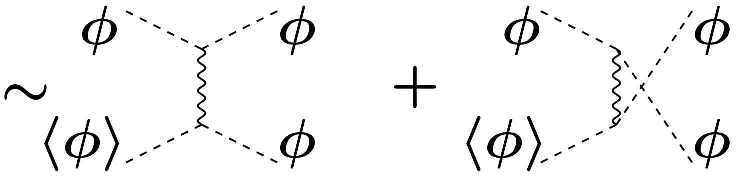
$$\sigma \sim \frac{G_N}{v^2}$$

...but this is for empty U containing two axions...

Moving axions between field and bath with gravity? (in galaxy today)

at $\mathcal{O}(G_N^2)$, quantized GR ($v \sim 10^{-3}$ in cm frame)

$\mathcal{M} \sim$



$$\sigma = \frac{G_N^2 m^2 \pi}{8v^4} \int \sin\theta d\theta \left(\frac{1}{\sin^2(\theta/2)} + \frac{1}{\cos^2(\theta/2)} \right)^2 \to 10^4 \frac{m^2}{m_{pl}^4} \qquad (m \sim 10^{-5} eV)$$

IR cutoff of graviton momenta $\sim mv$, because graviton only sees single axion inside box $\delta^3 \sim 1/(mv)^3$...

$$\text{probability} = \left| \sum \text{ indistinguisable amplitudes} \right|^2$$

so graviton of 10 metre wavelength interacts coherently with all axions in 10 metre cube $\leftrightarrow T_{\mu\nu}$. no axion particles in $T_{\mu\nu}$ on scales $> 1/(mv)$.

To estimate rate, account for high axion occupation # (in galaxy today)

to estimate evaporation/condensation rate, must take into account high occupation number of axions:

$$\frac{\partial}{\partial t}n \quad = \quad \int \Pi_i \widetilde{d^3 p_i} \tilde{\delta}^4 |\mathcal{M}|^2 \Big[f_1 f_2 (1 + f_3)(1 + f_4) - f_3 f_4 (1 + f_1)(1 + f_2) \Big]$$

$[...] \sim f^3$, so rate for individual axion to evaporate/condense

$$\Gamma \sim n_\phi \sigma_G f \sim 10^{13} \left(\frac{\rho_{DM}}{\rho_c} \right)^2 \left(\frac{m}{m_{pl}} \right)^3 H_0 \ll H_0$$

is negligeable...

Summary

the QCD axion solves the strong CP problem, and constitutes an interesting DM candidate. If the PQ phase transition after inflation, there are two populations: the classical "misalignment" field, and cold particles radiated by strings

to distinguish axion from WIMP CDM: direct detection, axion effects on γ propagation, maybe the extra pressures from the axion field give differences during non-linear structure formation? $\Rightarrow$*numerical galaxy formation*

1) there is a perception that axions need to be a Bose Einstein (BE) condensate, so as to differ from WIMPs

Since the BE condensate is described as a classical field, and misalignment axions are already a classical field, this seems unneccessary?

2) there is debate as to whether gravity can put axions in a BE condensate

most discussions for the misalignmentaxions...see 1)

for the cold axion particles from strings, no if you believe my IR cutoff...

Backup

What other people do

1. Use eqn for a scalar coupled to gravity, $\Rightarrow$ calculate the gravitational interaction rate of axions

$$i\frac{\partial}{\partial t}n_{\vec{q}}(t) \simeq 4\pi m G_N \sum_k \frac{R^2(t)}{|\vec{k}|^2}\delta\rho(\vec{k},t)\left\{\widetilde{a}^*(\vec{q}+\vec{k},t)\widetilde{a}(\vec{q},t) - \widetilde{a}^*(\vec{q},t)\widetilde{a}(\vec{q}-\vec{k},t)\right\}$$

greater than H in early U for $T_\gamma \lesssim$ keV. (beautiful calns of Saikawa etal)

2. Sikivie interprets as a thermalisation rate (à la Boltzman), and argues that "gravitational thermalisation" drives axions to a Bose E condensate.

1. I think not need a BE condensate —LO description of a BEC is a classical field, and have one already.
2. what are grav interactions *doing*?
   Solution of $T^{\mu\nu}_{;\nu} = 0$ parametrised with $\rho \sim \phi^2$ says gravity grows axion inhomogeneities...
   If gravity $\Rightarrow$ inverse cascade in axions, should see in eqns?
3. thermalisation $\Leftrightarrow$ entropy production.
   Rate for entropy-producing grav interactions of axions is $\ll H$

Davidson,Elmer

Why the axion:

gauge boson sector of QCD: input g_s,

$$-\frac{1}{4}G^A_{\mu\nu}G^{\mu\nu A} - \theta\frac{g_s^2}{32\pi^2}G^A_{\mu\nu}\widetilde{G}^{\mu\nu A} \qquad A:1..8, \quad \widetilde{G}^{\mu\nu} = \varepsilon^{\alpha\beta\mu\nu}G_{\alpha\beta}$$

neutron edm ⇒ $\theta \lesssim 10^{-10}$ ... *but instantons dynamically generate* $\theta \sim 1$?

How to make θ unobservable? *Aha!* There are quarks and the axial anomaly: a chiral rotn through η contributes:

$$\delta\mathcal{L} \propto \eta\partial_\mu J_5^\mu = \eta\frac{g_s^2 N}{8\pi^2}G\widetilde{G} + \eta\sum_f m_f\overline{q}_f\gamma_5 q_f$$

(N ⇔ coloured fermion reps)

a chiral phase rotn moves θ onto (coloured) fermion mass matrix...still CPV

⇒ **solution**: add fields, such that “generalised” chiral rotns ($\equiv$ PQ sym) are a sym of classical theory.

Peccei Quinn

To build an (Invisible) axion model

ShifmanVainshteinZakharov
Srednicki NPB85

1. aim to obtain a “Peccei-Quinn” symmetry = a global symmetry of the classical Lagrangian, broken by colour anomalies ($\simeq$ some generalisation of chiral rotns)

2. for instance (SVZ), add a gauge-singlet scalar with $Q_{PQ} = 2$ and SU(2) singlet quarks $\Psi_{L,R}$ with $Q_{PQ} = \pm 1$, so

$$\mathcal{L} = \mathcal{L}_{SM} + \partial_\mu \Phi^\dagger \partial^\mu \Phi + i\overline{\Psi} \not{D} \Psi + \{\lambda \Phi \overline{\Psi} \Psi + h.c.\} + V(\Phi)$$

3. arrange to break the PQ sym spontaneously, at high scale, such that all new particles are heavy except the goldstone = axion

4. so can rotate θ to the phase of Φ...which is a dynamical field...who will get a mass and want to sit at zero.

   ...so if CDM is an oscillating axion field, the nedm oscillates at $m_a \sim 10^{10}$ s^{-1}

Analytic discussions of non-linear structure formation

Erken, Sikivie, Tam, Yang
Bannik+Sikivie

Sikivie:

1. at $T_\gamma \sim$ keV, "gravitational thermalisation" of axions drives them to a "Bose-Einstein Condensate"
2. axion field can support vortices, which allow caustics in the galactic DM distribution

Rindler-Daller + Shapiro:

1. find analytic solutions representing stable rotating galactic halos former of scalar field
2. vortices are energetically favoured, for self-interactions of opposite sign from axions (?or smaller masses?)

(recall $V(a) = f_{\rm PQ}^2 m_a^2 [1 - \cos(a/f_{\rm PQ})] \simeq \frac{1}{2} m_a^2 a^2 - \frac{1}{4!} \frac{m_a^2}{f_{\rm PQ}^2} a^4$)

What is a Bose Einstein condensate? (I don't know. Please tell me if you do!)

Important characteristics of a BE condensate seem to be

1. a classical field,
2. carrying a conserved charge,
3. ? whose fourier modes are concentrated at a particular value — most of the "particles" who condense, should coherently do the same thing (but not necc the zero-momentum mode)

consistent with

- BE condensation in equilibrium stat mech, finite T FT, alkali gases.
- LO theory of BE condensates (Boguliubov → Pitaevskii) as a classical field

Are the misalignment axions a BE condensate?

1. a classical field yes
2. carrying a conserved charge, in the NR limit, ≈ yes
3. ? whose fourier modes are concentrated at a particular value — most of the "particles" who condense, should coherently do the same thing (but not necc the zero-momentum mode) ....umm?

Two approaches:

A: Doesn't matter: misalignment axions are a classical field, gives extra pressure which allows axion CDM to differ from WIMPs.

B: Follow Sikivie = misalignment field is not a BE condensate, needs to be to differ from WIMPs, ⇒ does gravity put it there?

Saikawa+Yamaguchi+etal
Davidson+Elmer,...