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Model Independent Bounds in Direct DM Searches



Paolo Panci



based on:

P.Panci,

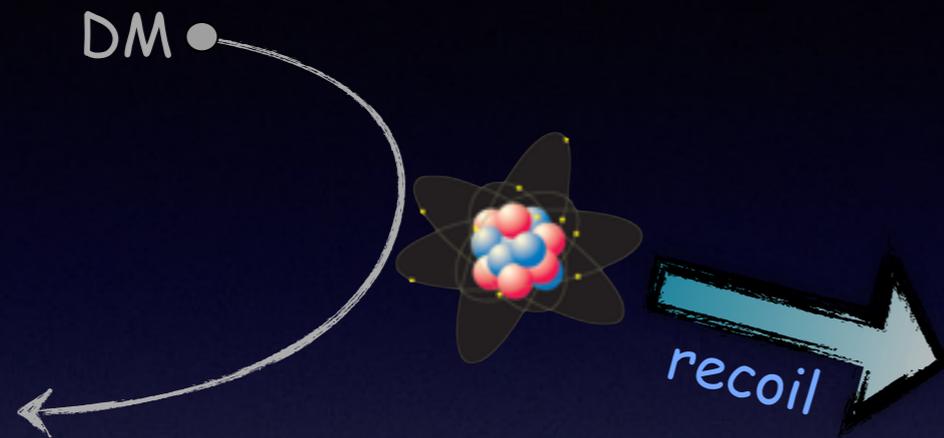
Review in Adv.High Energy.Phys. [arXiv: 1402.1507]

M.Cirelli, E.Del Nobile, P.Panci

JCAP 1310 (2013), 019, [arXiv: 1307.5955]

Direct Detection: Overview

Direct searches aim at detecting the **nuclear recoil** possibly induced by:



- elastic scattering:

$$\chi + \mathcal{N}(A, Z)_{\text{rest}} \rightarrow \chi + \mathcal{N}(A, Z)_{\text{recoil}}$$

- inelastic scattering:

$$\chi + \mathcal{N}(A, Z)_{\text{rest}} \rightarrow \chi' + \mathcal{N}(A, Z)_{\text{recoil}}$$

DM signals are **very rare events** (less than 1 cpd/kg/keV)

Experimental priorities for DM Direct Detection:

- the detectors must work deeply underground in order to reduce the background of cosmic rays
- they use active shields and very clean materials against the residual radioactivity in the tunnel (γ , α and neutrons)
- they must discriminate multiple scattering (DM does not scatter twice in the detector)

Direct Detection: Overview

DM local velocity $v_0 \sim 10^{-3}c$ \Rightarrow the collision between χ & \mathcal{N}
occurs in deeply non relativistic regime

$$E_R = \underbrace{\frac{1}{2}m_\chi v^2}_{\text{DM kinetic energy}} \underbrace{\frac{4m_\chi m_{\mathcal{N}}}{(m_\chi + m_{\mathcal{N}})^2}}_{\text{Kinematics factor}} \left(\frac{1 - \frac{v_t^2}{2v^2} - \sqrt{1 - \frac{v_t^2}{v^2}} \cos \theta}{2} \right),$$

θ scatter angle

$$\begin{cases} v_t = 0 & \text{elastic} \\ v_t = \sqrt{\frac{2\delta}{\mu_{\chi\mathcal{N}}}} \neq 0 & \text{inelastic} \end{cases}$$

v_t threshold velocity

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scatter angle threshold velocity

Theoretical differential rate of nuclear recoil in a given detector

$$\frac{dR_\mathcal{N}}{dE_R} = N_\mathcal{N} \frac{\rho_\odot}{m_\chi} \int_{v_{\min}(E_R)}^{v_{\text{esc}}} d^3v |\vec{v}| f(\vec{v}) \frac{d\sigma}{dE_R}$$

- $N_\mathcal{N} = N_a/A_\mathcal{N}$: Number of target
 $v_{\min}(E_R) = \sqrt{\frac{m_\mathcal{N} E_R}{2\mu_{\chi\mathcal{N}}^2} \left(1 + \frac{\mu_{\chi\mathcal{N}} \delta}{m_\mathcal{N} E_R} \right)}$: Minimal velocity
- ρ_\odot/m_χ : DM number density
 v_{esc} : DM escape velocity (450 - 650 km/s)

Differential Cross Section

$$\frac{d\sigma}{dE_R}(v, E_R) = \frac{1}{32\pi} \frac{1}{m_\chi^2 m_N} \frac{1}{v^2} |\mathcal{M}_N|^2 \longrightarrow \text{Matrix Element (ME) for the DM-nucleus scattering}$$

$v \ll c \Rightarrow$ the framework of relativistic quantum field theory is not appropriate

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Non relativistic (NR) operators framework

NR d.o.f. for elastic scattering

\vec{v} : DM-nucleon relative velocity

\vec{q} : exchanged momentum

\vec{s}_N : nucleon spin ($N = (p, n)$)

\vec{s}_χ : DM spin

The DM-nucleon ME can be constructed from Galileian invariant combination of d.o.f.

$$|\mathcal{M}_N| = \sum_{i=1}^{12} c_i^N(\lambda, m_\chi) \mathcal{O}_i^{\text{NR}}$$

functions of the parameters of your favorite theory (e.g. couplings, mixing angles, mediator masses), expressed in terms of NR operators

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Contact interaction ($q \ll \Lambda$)

$$\begin{aligned} \mathcal{O}_1^{\text{NR}} &= \mathbb{1} , \\ \mathcal{O}_3^{\text{NR}} &= i \vec{s}_N \cdot (\vec{q} \times \vec{v}^\perp) , & \mathcal{O}_4^{\text{NR}} &= \vec{s}_\chi \cdot \vec{s}_N , \\ \mathcal{O}_5^{\text{NR}} &= i \vec{s}_\chi \cdot (\vec{q} \times \vec{v}^\perp) , & \mathcal{O}_6^{\text{NR}} &= (\vec{s}_\chi \cdot \vec{q})(\vec{s}_N \cdot \vec{q}) , \\ \mathcal{O}_7^{\text{NR}} &= \vec{s}_N \cdot \vec{v}^\perp , & \mathcal{O}_8^{\text{NR}} &= \vec{s}_\chi \cdot \vec{v}^\perp , \\ \mathcal{O}_9^{\text{NR}} &= i \vec{s}_\chi \cdot (\vec{s}_N \times \vec{q}) , & \mathcal{O}_{10}^{\text{NR}} &= i \vec{s}_N \cdot \vec{q} , \\ \mathcal{O}_{11}^{\text{NR}} &= i \vec{s}_\chi \cdot \vec{q} , & \mathcal{O}_{12}^{\text{NR}} &= \vec{v}^\perp \cdot (\vec{s}_\chi \times \vec{s}_N) . \end{aligned}$$

Long-range interaction ($q \gg \Lambda$)

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Differential Cross Section

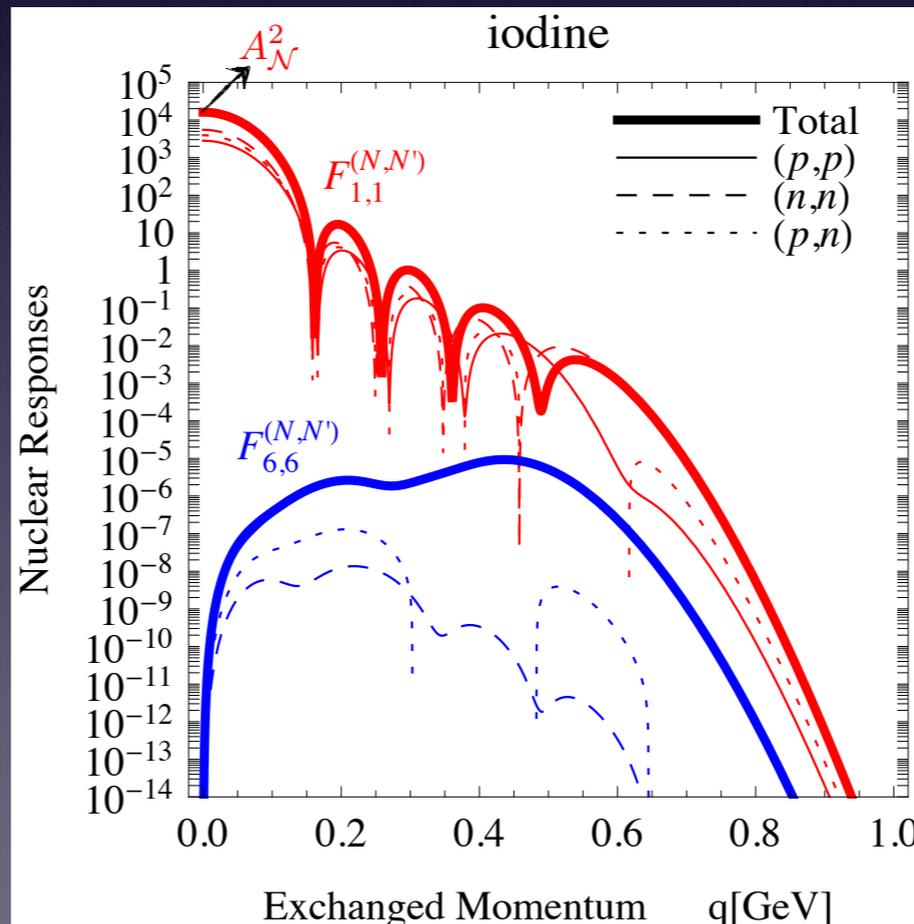
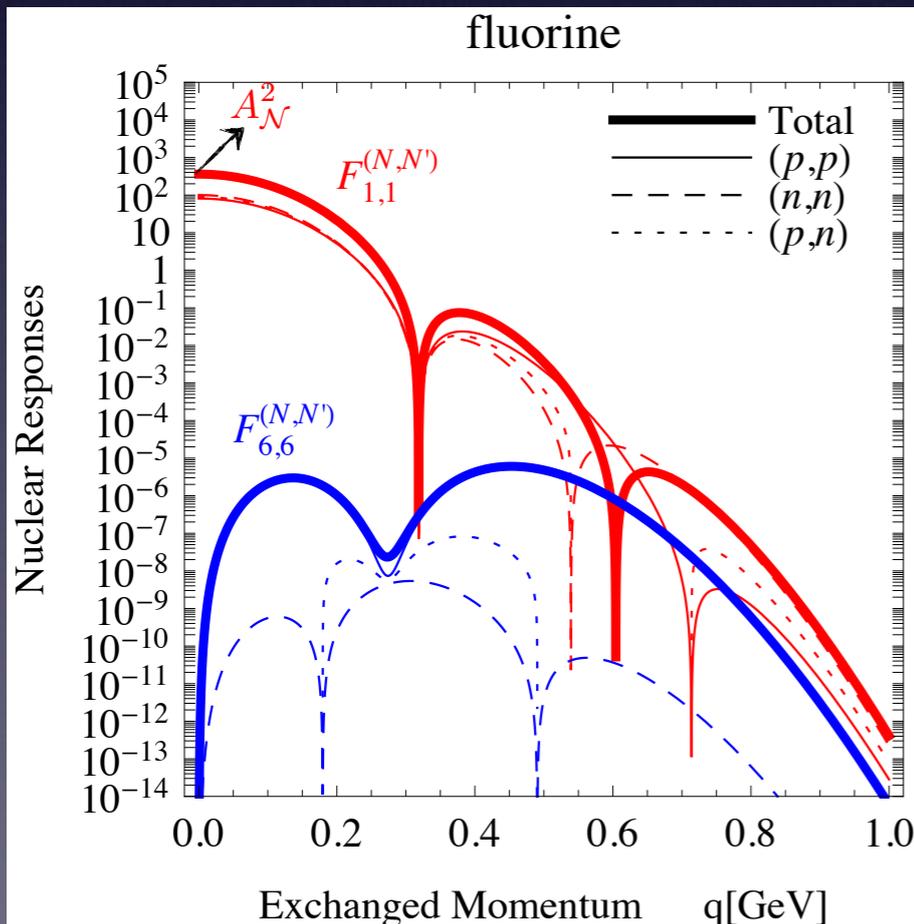
Nucleus is not point-like

There are different Nuclear Responses for any pairs of nucleons & any pairs of NR Operators

$$|\mathcal{M}_{\mathcal{N}}|^2 = \frac{m_{\mathcal{N}}^2}{m_N^2} \sum_{i,j=1}^{12} \sum_{N,N'=p,n} c_i^N c_j^{N'} F_{i,j}^{(N,N')}(v, q^2)$$

pairs of NR operators pairs of nucleons Nuclear response of the target nuclei

Nuclear responses for some common target nuclei in Direct Searches



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Rate of Nuclear Recoil

$$\frac{dR_{\mathcal{N}}}{dE_{\text{R}}} = N_{\mathcal{N}} \frac{\rho_{\odot}}{m_{\chi}} \frac{1}{32\pi} \frac{m_{\mathcal{N}}}{m_{\chi}^2 m_{\mathcal{N}}^2} \sum_{i,j=1}^{12} \sum_{N,N'=p,n} \mathbf{c}_i^N \mathbf{c}_j^{N'} \int_{v_{\min}(E_{\text{R}})}^{v_{\text{esc}}} d^3v \frac{1}{v} f_{\oplus}(v) F_{i,j}^{(N,N')}(v, q^2)$$

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exposure Comparison with the Experimental data

$$N_k^{\text{th}} = w_k \int_{\Delta E_k} dE_{\text{det}} \epsilon(E_{\text{det}}) \int_0^{\infty} dE_{\mathcal{R}} \sum_{\mathcal{N}=\text{Nucleus}} \mathcal{K}_{\mathcal{N}}(q_{\mathcal{N}} E_{\mathcal{R}}, E_{\text{det}}) \frac{dR_{\mathcal{N}}}{dE_{\mathcal{R}}}(E_{\mathcal{R}})$$

takes into account the response and energy resolution of the detector

runs over the different species in the detector (e.g. DAMA and CRESST are multiple-target)

quenching factor: accounts for the partial recollection of the released energy

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Uncertainties in Direct DM Searches

- Local DM energy Density & Geometry of the Halo (e.g: spherically symmetric halos with isotropic or not velocity dispersion, triaxial models, co-rotating dark disk and so on.....)
- Nature of the interaction & Nuclear Responses (e.g: SI & SD scattering, long-range or point like character of the interaction and so on.....)
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Expected Number of Events

$$N_k^{\text{th}}(\lambda, m_\chi) = X \sum_{i,j=1}^{12} \sum_{N,N'=p,n} c_i^N(\lambda, m_\chi) c_j^{N'}(\lambda, m_\chi) \tilde{\mathcal{F}}_{i,j}^{(N,N')}(m_\chi, k)$$

Particle physics part

Model independent part

linear combination of integrated form factors

$$\frac{\rho_\odot}{m_\chi} \frac{1}{32\pi} \frac{1}{m_\chi^2 m_N^2}$$

$$w_k \sum_{\mathcal{N}=\text{Nucleus}} N_{\mathcal{N}} m_{\mathcal{N}} \int_{\Delta E_k} dE_{\text{det}} \epsilon(E_{\text{det}}) \int dE' \mathcal{K}(E_{\text{det}}, E') \int_{v_{\min}(\frac{E'}{q_{\mathcal{N}}})}^{v_{\text{esc}}} d^3v \frac{1}{v} f_{\oplus}(v) F_{i,j}^{(N,N')}(v, q^2)$$

once computed the integrated form factors, one can easily derive the expected number of events for any kinds of interactions, whose particle physics is completely encapsulated in the coefficient c_i^N

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Model independent Bounds in direct DM searches

- I'm going to present a new framework for "scaling" a bound given on a certain benchmark interaction to any other kinds of interactions
- For example the model dependent bounds presented by the experimental collaborations can also be applied to other class of models

Benchmark interaction

Contact interaction

$$\begin{aligned} \mathcal{O}_1^{\text{NR}} &= \mathbb{1}, \\ \mathcal{O}_3^{\text{NR}} &= i \vec{s}_N \cdot (\vec{q} \times \vec{v}^\perp), & \mathcal{O}_4^{\text{NR}} &= \vec{s}_\chi \cdot \vec{s}_N, \\ \mathcal{O}_5^{\text{NR}} &= i \vec{s}_\chi \cdot (\vec{q} \times \vec{v}^\perp), & \mathcal{O}_6^{\text{NR}} &= (\vec{s}_\chi \cdot \vec{q})(\vec{s}_N \cdot \vec{q}), \\ \mathcal{O}_7^{\text{NR}} &= \vec{s}_N \cdot \vec{v}^\perp, & \mathcal{O}_8^{\text{NR}} &= \vec{s}_\chi \cdot \vec{v}^\perp, \\ \mathcal{O}_9^{\text{NR}} &= i \vec{s}_\chi \cdot (\vec{s}_N \times \vec{q}), & \mathcal{O}_{10}^{\text{NR}} &= i \vec{s}_N \cdot \vec{q}, \\ \mathcal{O}_{11}^{\text{NR}} &= i \vec{s}_\chi \cdot \vec{q}, & \mathcal{O}_{12}^{\text{NR}} &= \vec{v}^\perp \cdot (\vec{s}_\chi \times \vec{s}_N). \end{aligned}$$

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Among all the NR interactions we choose the simplest:
(a model where DM interact with only protons with a constant cross section)

$$c_1^p = \lambda_B, \text{ while } c_1^N = 0$$

Benchmark DM-nucleon ME

$$|\mathcal{M}_{p,B}| = \lambda_B \mathcal{O}_1^{\text{NR}}$$



Events for the benchmark model

$$N_{k,B}^{\text{th}} = X \lambda_B^2 \tilde{\mathcal{F}}_{1,1}^{(p,p)}(m_\chi, k)$$

↓
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benchmark DM constant

Determination of the maximal value of λ_B allowed by the experimental data-set

Likelihood Ratio Test Statistic (TS)

$$\text{TS}(\lambda_B, m_\chi) = -2 \ln \left(\mathcal{L}(\vec{N}^{\text{obs}} | \lambda_B) / \mathcal{L}_{\text{bkg}} \right)$$

likelihood of obtaining the
set of observed data

↓ bkg.
likelihood

for any given value of m_χ , a 90% CL lower bound on
the free parameter can be obtained by solving

$$\text{TS}(\lambda_B, m_\chi) = \chi_{90\% \text{ CL}}^2 \simeq 2.71$$

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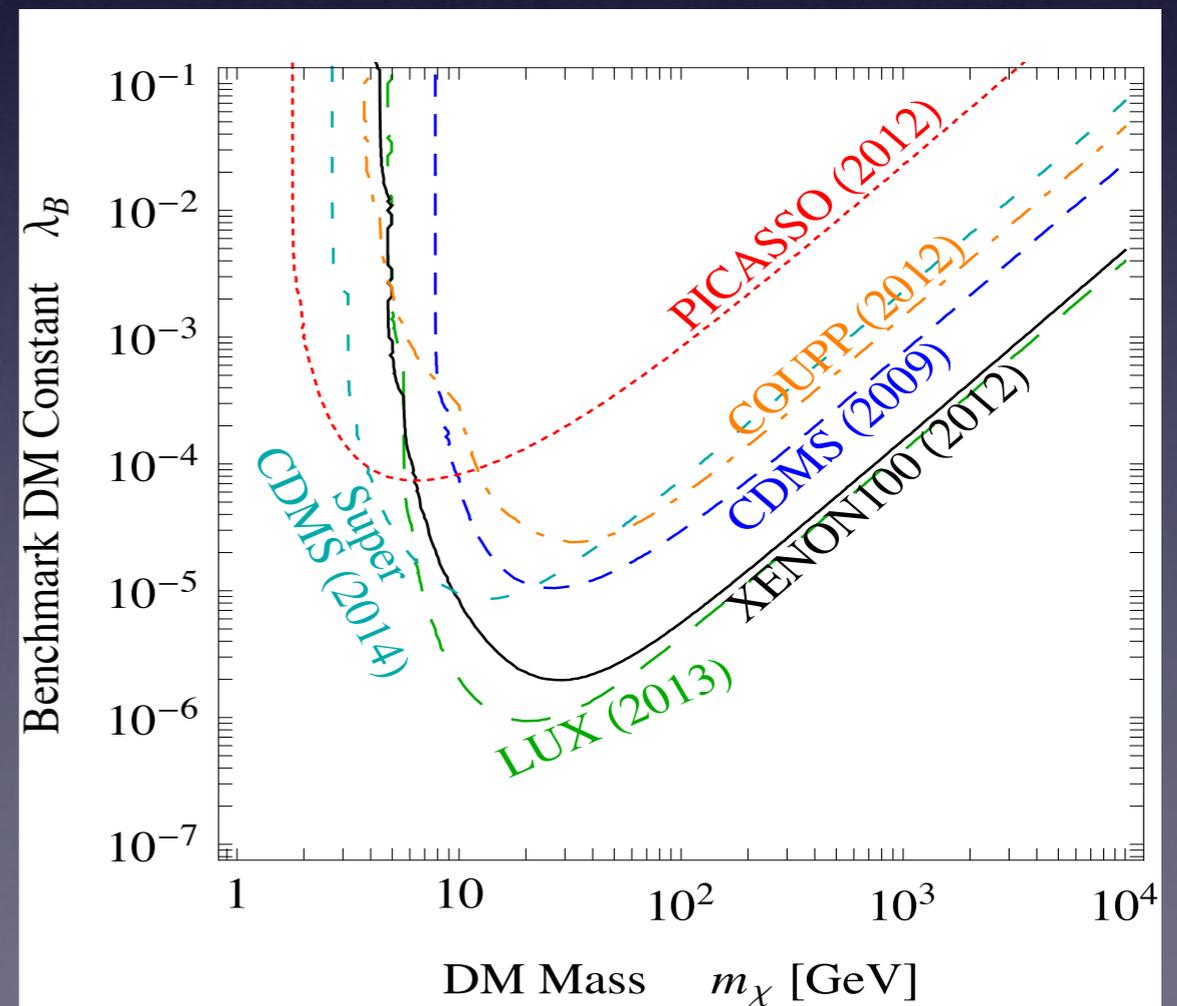
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The functions TS that allow the users to compute the bound λ_B^{CL} at the desired CL are provided here:

<http://www.marcocirelli.net/NROpsDD.html>



Rescaling Functions

For any model the bound
must be drawn at the same CL:

$$\text{TS}(\lambda, m_\chi) = \text{TS}(\lambda_B, m_\chi)$$

For null-results Exps. a solution is:

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$$\sum_{i,j=1}^{12} \sum_{N,N'=p,n} \underbrace{c_i^N(\lambda, m_\chi) c_j^{N'}(\lambda, m_\chi)}_{\text{Particle physics part}} \underbrace{\tilde{y}_{i,j}^{(N,N')}(m_\chi)}_{\text{Model independent}} = \lambda_B^2$$

$$\tilde{y}_{i,j}^{(N,N')}(m_\chi) = \frac{\sum_k \tilde{\mathcal{F}}_{i,j}^{(N,N')}(m_\chi, k)}{\sum_k \tilde{\mathcal{F}}_{1,1}^{(p,p)}(m_\chi, k)}$$

"Scaling" Functions

- nuclear physics
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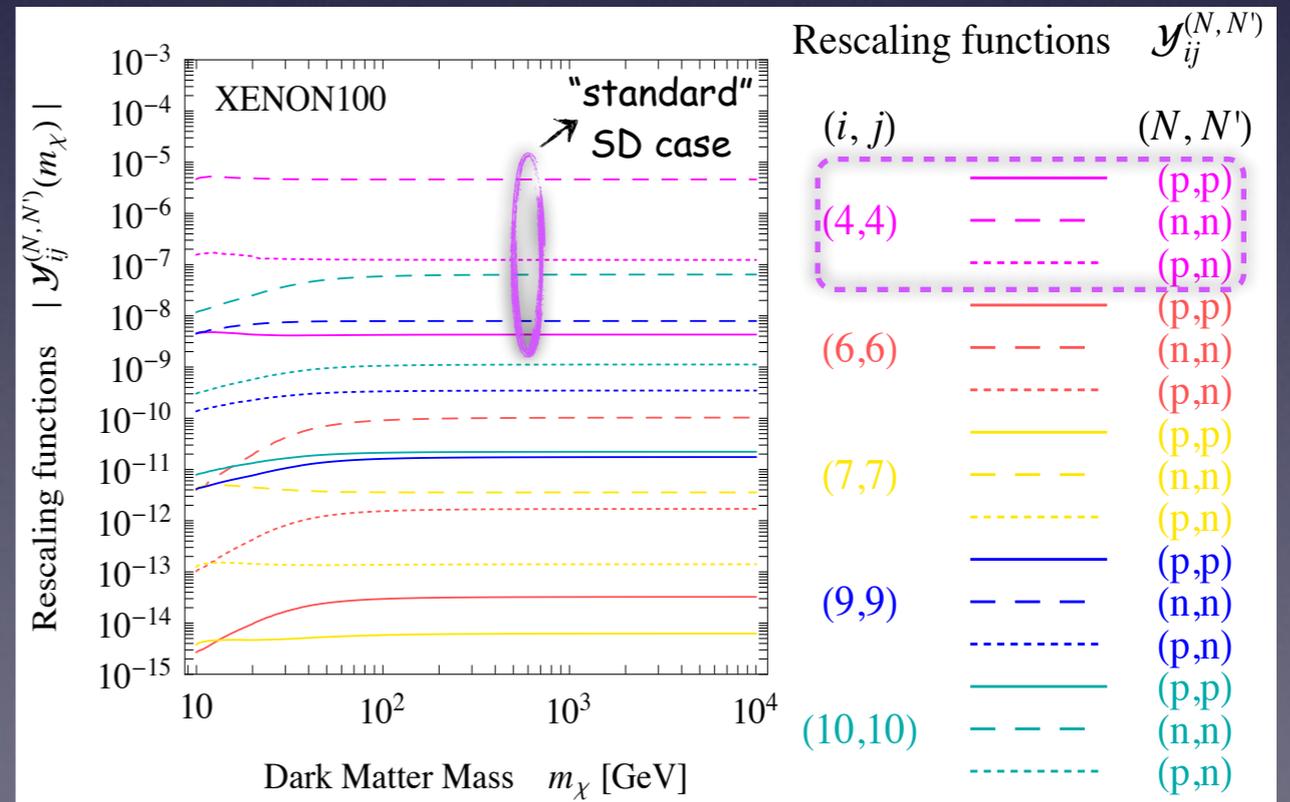
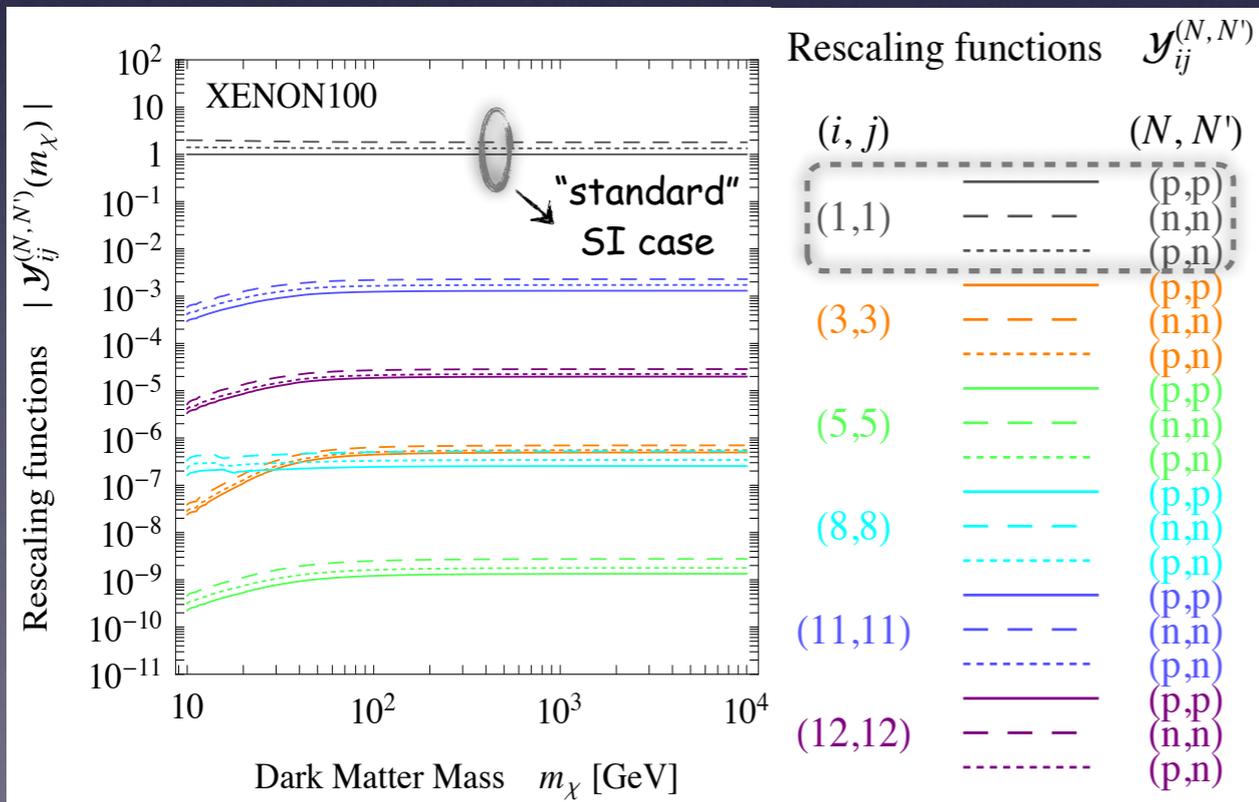
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Example: SI & SD Interactions

SI DM-nucleon effective Lagrangian

$$\mathcal{L}_{\text{SI}}^N = \lambda_{\text{SI}} \cdot \bar{\chi} \chi \bar{N} N$$

$$\sigma_{\text{SI}}^p = \frac{\lambda_{\text{SI}}^2}{\pi} \mu_{\chi p}^2 \quad \text{Total SI DM-nucleon Cross section}$$

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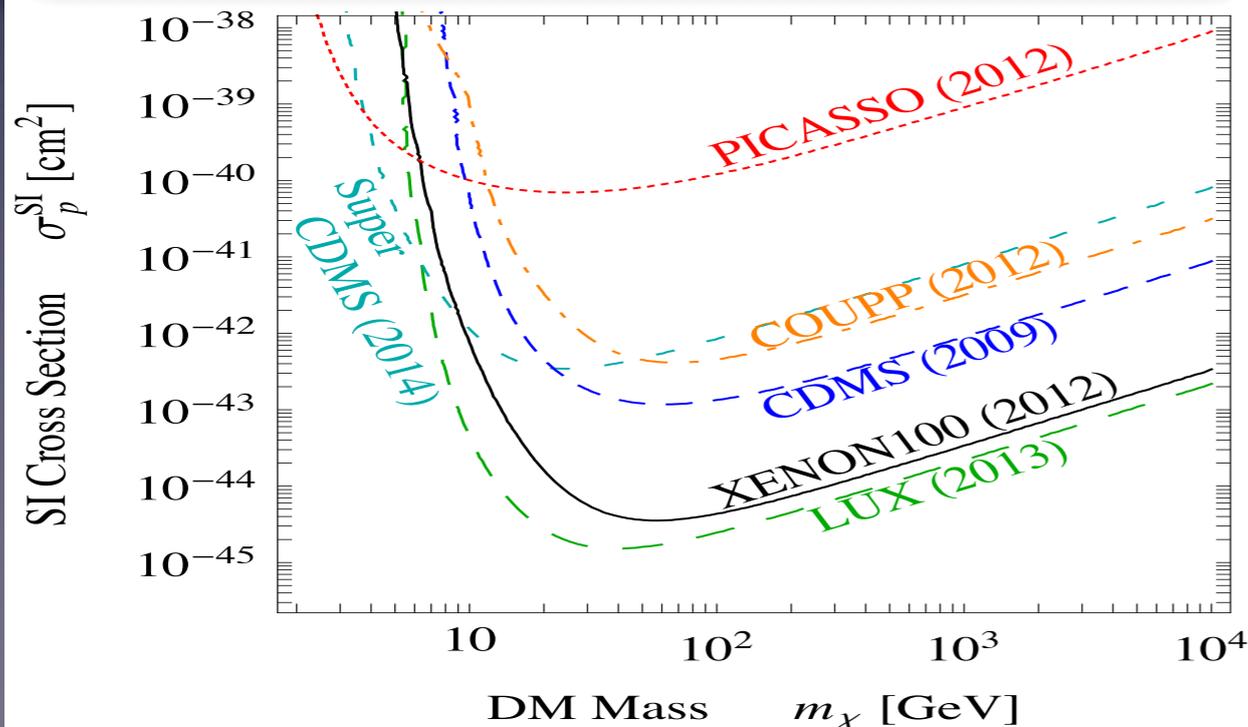
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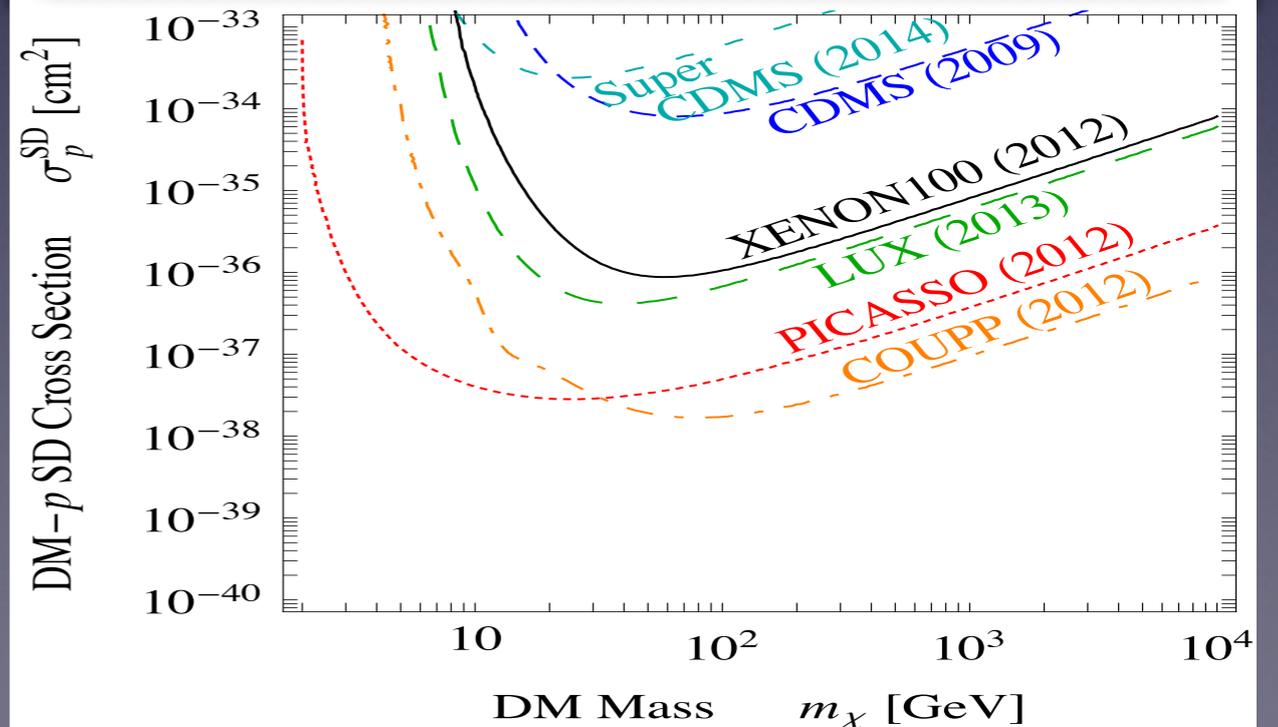
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$$\lambda_{\text{B}}^2 = \sigma_{\text{SI}}^p \sum_{N, N'=p, n} 16 \pi m_{\chi}^2 \frac{m_N^2}{\mu_{\chi p}^2} \tilde{y}_{1,1}^{(N, N')}(m_{\chi})$$



$$\lambda_{\text{B}}^2 = \sigma_{\text{SD}}^p \frac{256}{3} \pi m_{\chi}^2 \frac{m_N^2}{\mu_{\chi p}^2} \tilde{y}_{4,4}^{(p,p)}(m_{\chi})$$



Summary & Conclusions

I have described a **method** and a self-contained set of **numerical tools** to derive the bounds from some current experiments to virtually any arbitrary models of DM

- The **method** is based on the formalism of non-relativistic operators
- it incorporates into the nuclear responses all the necessary detector and astrophysical ingredients

Tools for model-independent bounds in direct dark matter searches

Data and Results from [1307.5955](#) [hep-ph], JCAP 10 (2013) 019.

If you use the data provided on this site, please cite:

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This is **Release 3.0** (April 2014). *Log of changes at the bottom of this page.*

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