Recent developments in supersymmetric unification

Borut Bajc

J. Stefan Institute, Ljubljana, Slovenia

BB, Vasja Susič, 13

Babu, BB, Vasja Susič, work in progress

Introduction

	non renormalizable	*renormalizable (low energy susy)*	*renormalizable (arbitrary susy)*
$SU(5)$	YES	NO	YES (?)
$SU(6)$	YES	NO	NO
...	...	...	...
$SO(10)$	YES	NO	YES (?)
...	...	...	...
E_6	YES (?)	*this talk*	

Motivations for E_6:

- Matter-Higgs unification

$$27 = 16 + 10 + 1$$

Not really; hard to make it work

But for sure theory of heavy vectorlike matter:

$$3 \times (\bar{5}(16) + \bar{5}(10))$$

- Heavy Higgs - light Higgs unification

  in SO(10):

$$\begin{aligned} 210 + \overline{126} + 126 &= \text{heavy Higgses} \\ 10 &= \text{light Higgs} \end{aligned}$$

  In principle possible without 10, but then just one Yukawa matrix

  In E_6 multiplets involved are big and contain both SM singlets (heavy Higgses) and weak doublets (light Higgses)

- In SO(10) term

$$\frac{c}{M_{Planck}} 16^4$$

needs $c \lesssim 10^{-7}$

In E_6 no 27^4 invariant $\rightarrow$ dangerous $d = 4$ operator suppressed by

$$\frac{M_{intermediate}}{M_{Planck}}$$

But this important only in non-renormalizable versions of the E_6 model.

The lowest dimensional representations of E_6:

$$
\begin{array}{rcl}
27^\mu & \ldots & \text{fundamental} \\
\overline{27}_\mu & \ldots & \text{anti-fundamental} \\
78^\mu{}_\nu & \ldots & \text{adjoint } (= (t^A)^\mu{}_\nu 78^A) \\
351^{\mu\nu} = -351^{\nu\mu} & \ldots & \text{two indices antisymmetric} \\
\overline{351}_{\mu\nu} = -\overline{351}_{\nu\mu} & \ldots & \text{two indices antisymmetric} \\
351'^{\mu\nu} = +351'^{\nu\mu} & \ldots & \text{two indices symmetric } (d_{\lambda\mu\nu} 351'^{\mu\nu} = 0) \\
\overline{351'}_{\mu\nu} = +\overline{351'}_{\nu\mu} & \ldots & \text{two indices symmetric } (d^{\lambda\mu\nu} \overline{351'}_{\mu\nu} = 0) \\
650^\mu{}_\nu & \ldots & (650^\mu{}_\mu = (t^A)^\nu{}_\mu 650^\mu{}_\nu = 0)
\end{array}
$$

Symmetry breaking

Look for the pattern $E_6 \to \text{SM}$.

The simplest renormalizable superpotential made of
$351' + \overline{351}' + 27 + \overline{27}$

$$
\begin{aligned}
W &= m_{351'}\, \overline{351}'\, 351' + \lambda_1\, 351'^3 + \lambda_2\, \overline{351'}^3 \\
&+ m_{27}\, \overline{27}\, 27 + \lambda_3\, 27\, 27\, \overline{351}' + \lambda_4\, \overline{27}\, \overline{27}\, 351' \\
&+ \lambda_5\, 27^3 + \lambda_6\, \overline{27}^3
\end{aligned}
$$

The SM singlets:

$$
\begin{aligned}
27 &\quad : \quad c_1, c_2 \\
\overline{27} &\quad : \quad d_1, d_2 \\
351' &\quad : \quad e_1, e_2, e_3, e_4, e_5 \\
\overline{351'} &\quad : \quad f_1, f_2, f_3, f_4, f_5
\end{aligned}
$$

More than one solution. For example:

$$c_2 = e_2 = e_4 = 0, \qquad d_2 = f_2 = f_4 = 0$$

$$d_1 = \frac{m_{351'} m_{27}}{2\lambda_3 \lambda_4 c_1}$$

$$e_1 = -\frac{m_{351'}}{6\lambda_1^{2/3}\lambda_2^{1/3}}, \qquad f_1 = -\frac{m_{351'}}{6\lambda_1^{1/3}\lambda_2^{2/3}}$$

$$e_3 = -\lambda_3 {c_1}^2 / m_{351'}, \qquad f_3 = -\frac{m_{351'} m_{27}^2}{4\lambda_3^2 \lambda_4 {c_1}^2}$$

$$e_5 = \frac{m_{351'}}{3\sqrt{2}\lambda_1^{2/3}\lambda_2^{1/3}}, \qquad f_5 = \frac{m_{351'}}{3\sqrt{2}\lambda_1^{1/3}\lambda_2^{2/3}}$$

with

$$0 = |m_{351'}|^4 |m_{27}|^4 + 2|m_{351'}|^4 |m_{27}|^2 |\lambda_3|^2 |c_1|^2 - 8|m_{351'}|^2 |\lambda_3|^4 |\lambda_4|^2 |c_1|^6 - 16|\lambda_3|^6 |\lambda_4|^2 |c_1|^8$$

Generic Yukawa sector in E_6

In all generality three types of Yukawas

$$W = 27_i \left(Y_{27}^{ij}\, 27 + Y_{\overline{351}'}^{ij}\, \overline{351}' + Y_{\overline{351}}^{ij}\, \overline{351} \right) 27_j$$

$$Y_{27,\overline{351}'} = Y_{27,\overline{351}'}^T \qquad \text{symmetric}$$

$$Y_{\overline{351}} = -Y_{\overline{351}}^T \qquad \text{antisymmetric}$$

Completely analogous to SO(10):

$$W = 16_i \left(Y_{10}^{ij}\, 10 + Y_{\overline{126}}^{ij}\, \overline{126} + Y_{120}^{ij}\, 120 \right) 16_j$$

$$Y_{10,\overline{126}} = Y_{10,\overline{126}}^T \qquad \text{symmetric}$$

$$Y_{120} = -Y_{120}^T \qquad \text{antisymmetric}$$

In fact

$$
\begin{aligned}
27 &= 1 + 10 + 16 \\
\overline{351}' &= 1 + 10 + \overline{16} + 54 + \overline{126} + 144 \\
\overline{351} &= 10 + \overline{16} + 16 + 45 + 120 + 144
\end{aligned}
$$

The antisymmetric $\overline{351}$ contribution (similar as 120 in SO(10)) seems less promising so we will concentrate on the symmetric 27 and $\overline{351}'$ from now on.

$$W = \begin{pmatrix} 16 & 10 & 1 \end{pmatrix} Y_{27} \begin{pmatrix} 10 & 16 & 0 \\ 16 & 1 & 10 \\ 0 & 10 & 0 \end{pmatrix} \begin{pmatrix} 16 \\ 10 \\ 1 \end{pmatrix}$$

$$+ \begin{pmatrix} 16 & 10 & 1 \end{pmatrix} Y_{\overline{351}'} \begin{pmatrix} \overline{126} + 10 & 144 & \overline{16} \\ 144 & 54 & 10 \\ \overline{16} & 10 & 1 \end{pmatrix} \begin{pmatrix} 16 \\ 10 \\ 1 \end{pmatrix}$$

- several new Higgs doublets (not only in 10 and $\overline{126}$)
- some fields have large $\mathcal{O}(M_{GUT})$ vevs $\to$
  - mixing between $\bar{5} \in 16$ and $\bar{5} \in 10$ (d^c, L)
  - mixing between $1 \in 1$ and $1 \in 16$ (ν^c)
- $M^U_{3\times 3}$, $M^D_{6\times 6}$, $M^E_{6\times 6}$, $M^N_{15\times 15} \to$ light $(M_{U,D,E,N})_{3\times 3}$

This case seems really minimal: 27 and $\overline{351}'$ that participate to symmetry breaking could contribute to Yukawa terms!

Can the weak doublets with $Y = \pm 1$ in 27 and $\overline{351'}$ be the Higgses $H, \bar{H}$ of the MSSM?

Since E_6 is a GUT, this means:

Can we make the doublet-triplet splitting with the massless eigenvector living in both 27 and $\overline{351'}$?

The doublet-triplet splitting

In our E_6 case doublets and triplets live in $351'$, $\overline{351'}$, 27, $\overline{27}$.

$351'$ has 8 doublets (9 triplets)

$\overline{351}'$ has 8 doublets (9 triplets)

27 has 3 doublets (3 triplets)

$\overline{27}$ has 3 doublets (3 triplets)

All together 22 doublets (11 with $Y = +1$ and 11 with $Y = -1$):
doublet matrix M_D is 11×11

All together 24 triplets (12 with $Y = +2/3$ and 12 with $Y = -2/3$):
triplet matrix M_T is 12×12

analysis complicated by presence of would-be-Goldstones in
$16 + \overline{16} \in 78$

$\rightarrow M_{T,D}$ have automatically one zero eignevalue

We need the determinant without the zero-modes:

$$Det(M) \equiv \prod_{i=2}^{n} m_i$$

We would like to get

$$Det(M_D) = 0 \quad , \quad Det(M_T) \neq 0$$

But after long calculation the result is:

$$Det(M_T) = \# Det(M_D)$$

i.e doublet-triplet splitting impossible !

Bizarre situation: all was ok, we seems to fail on doublet-triplet splitting. And not because we don't like fine-tuning, we cannot even fine-tune!

Simplest solutions:

- add another $27 + \overline{27}$ pair with coupling

$$W_{DT} = m_{27}\, 27\, \overline{27} + \kappa_1\, 27\, 27\, \overline{351'} + \kappa_2\, \overline{27}\, \overline{27}\, 351' + \kappa_3\, 27\, 27\, 27 + \kappa_4\, \overline{27}\, \overline{27}\, \overline{27}$$

  with $\langle 27 \rangle\, ,\, \langle \overline{27} \rangle = \mathcal{O}(m_Z)$

  DT splitting now possible: MSSM Higgs live only in $27\, ,\, \overline{27}$

  In spite of this 3 Yukawa matrices involved.

- add another 78: although it does not contribute to Yukawas, it changes the symmetry breaking pattern (not being needed) thus relaxing constraints on DT.

  DT now possible in the old sector: MSSM Higgses live also in $\overline{351}'$ and 27 !

  This possibility more minimal, only 2 Yukawas.

The minimal model: $351' + \overline{351}' + 27 + \overline{27} + 78$

$$\begin{aligned} W_{Higgs} &= m_{351'}\,\overline{351}'\,351' + \lambda_1\,351'^3 + \lambda_2\,\overline{351'}^3 \\ &+ m_{27}\,\overline{27}\,27 + \lambda_3\,27^2\,\overline{351}' + \lambda_4\,\overline{27}^2\,351' \\ &+ \lambda_5\,27^3 + \lambda_6\,\overline{27}^3 \\ &+ m_{78}78^2 + \lambda_7\,27\,78\,\overline{27} + \lambda_8\,351'\,78\,\overline{351}' \end{aligned}$$

Other SM singlets:

$$78 \quad : \quad a_1, a_2, a_3, a_4, a_5$$

Solution with $a_i \neq 0$ found explicitly: disconnected with the previous one (no limit gives the previous solution with $a_i \to 0$)

$$W_{Yukawa} = 27_M \left(27\, Y_{27} + \overline{351'}\, Y_{\overline{351'}}\right) 27_M$$

All together the total number of real parameters

$$\underbrace{(11 \times 2 - 5)}_{\text{Higgs sector}} + \underbrace{(3 + 6 \times 2)}_{\text{Yukawa sector}} + \underbrace{1}_{\text{gauge coupling}} = 33$$

7 more parameters than in the minimal supersymmetric renormalizable SO(10).

Yukawa sector in the minimal E_6 model

As an example of what happens let's see the down sector:

$$\begin{pmatrix} d^{cT} & d'^{cT} \end{pmatrix} \begin{pmatrix} \bar{v}_2 Y_{27} + \left(\frac{1}{2\sqrt{10}}\bar{v}_4 + \frac{1}{2\sqrt{6}}\bar{v}_8\right) Y_{\overline{351}'} & c_2 Y_{27} \\ -\bar{v}_3 Y_{27} - \left(\frac{1}{2\sqrt{10}}\bar{v}_9 + \frac{1}{2\sqrt{6}}\bar{v}_{11}\right) Y_{\overline{351}'} & \frac{1}{\sqrt{15}} f_4 Y_{\overline{351}'} \end{pmatrix} \begin{pmatrix} d \\ d' \end{pmatrix}$$

$\bar{v}_{2,3,4,8,9,11} = \mathcal{O}(m_Z)$; $c_2, f_4 = \mathcal{O}(M_{GUT})$

$$\left.\begin{array}{l} d^c \in \bar{5}_{SU(5)} \in 16_{SO(10)} \\ d'^c \in \bar{5}_{SU(5)} \in 10_{SO(10)} \end{array}\right\} \text{mix}$$

$$d \in 10_{SU(5)} \in 16_{SO(10)}$$

$$d' \in 5_{SU(5)} \in 10_{SO(10)} \ldots \text{ heavy}$$

The matrix above has the form

$$\mathcal{M} = \begin{pmatrix} m_1 & M_1 \\ m_2 & M_2 \end{pmatrix}$$

with $m_{1,2} = \mathcal{O}(m_Z)$ and $M_{1,2} = \mathcal{O}(M_{GUT})$

All are 3×3 matrices.

the idea is to find a 6×6 unitary matrix $\mathcal{U}$ that

$$\mathcal{U} \begin{pmatrix} M_1 \\ M_2 \end{pmatrix} = \begin{pmatrix} 0 \\ \text{something} \end{pmatrix}$$

The solution is

$$\mathcal{U}=\begin{pmatrix}\left(1+XX^\dagger\right)^{-1/2} & -\left(1+XX^\dagger\right)^{-1/2}X\\ X^\dagger\left(1+XX^\dagger\right)^{-1/2} & \left(1+X^\dagger X\right)^{-1/2}\end{pmatrix}$$

with

$$X=M_1\,M_2^{-1}$$

so that

$$\mathcal{U}\mathcal{M}=\begin{pmatrix}\underbrace{\mathcal{O}(m_Z)}_{\text{light sector}} & 0\\ \mathcal{O}(m_Z) & \mathcal{O}(M_{GUT})\end{pmatrix}$$

For charged fermions they turn out to be

$$M_U = -v_1 Y_{27} + \left(\frac{1}{2\sqrt{10}} v_5 - \frac{1}{2\sqrt{6}} v_7\right) Y_{\overline{351}'},$$

$$M_D^T = \left(1 + XX^\dagger\right)^{-1/2} \left(\left(\bar{v}_2 - \bar{v}_3 X\right) Y_{27} + \left(\frac{1}{2\sqrt{10}}\left(\bar{v}_4 - \bar{v}_9 X\right) + \frac{1}{2\sqrt{6}}\left(\bar{v}_8 - \bar{v}_{11} X\right)\right) Y_{\overline{351}'}\right)$$

$$M_E = \left(1 + \frac{4}{9} XX^\dagger\right)^{-1/2} \left(\left(-\bar{v}_2 - \frac{2}{3}\bar{v}_3 X\right) Y_{27} + \left(-\frac{1}{2\sqrt{10}}\left(\bar{v}_4 + \frac{2}{3}\bar{v}_9 X\right) + \sqrt{\frac{3}{8}}\left(\bar{v}_8 + \frac{2}{3}\bar{v}_{11} X\right)\right) Y_{\overline{351}'}\right)$$

with

$$X = -3\sqrt{\frac{5}{3}}\,\frac{c_2}{f_4}\, Y_{27}\, Y_{\overline{351}'}^{-1},$$

$X \to 0$ gives minimal SO(10), but here not available ($c_2 \neq 0$) !

- Y_{27} and $Y_{\overline{351}'}$ symmetric $\rightarrow$ M_U symmetric

  Not true for X and so not for $M_{D,E}$, but we can always parametrize

$$X = M_U Y$$

with

$$Y = Y^T \qquad \text{symmetric}$$

- any function of a $2(3) \times 2(3)$ matrix M can be always written as

$$f(M) = \alpha + \beta M + \gamma M^2$$

with α, β, γ depend on f and the invariants of M.

$$
\begin{aligned}
M_D^T &= \left(1+(9/4)XX^\dagger\right)^{-1/2}\left(\alpha+\beta\,X+\gamma\,X^2\right)M_U \\
M_E &= \left(1+XX^\dagger\right)^{-1/2}\left(\alpha'+\beta'\,X+\gamma'\,X^2\right)M_U \\
M_N &= \left(1+XX^\dagger\right)^{-1/2}\left(\alpha''+\beta''\,X+\gamma''\,X^2\right)M_U\left(1+X^*X^T\right)^{-1/2}
\end{aligned}
$$

with $X=M_UY$ and $Y^T=Y$.

- Neutrino mass sum of type I and type II contributions
- α, β, γ, α', β', γ', α'', β'', γ'', are $f(c_a, f_b, v_i, \bar{v}_j, m_i, \lambda_j)$

$N_g = 2$ case

Unknowns (9):

α, β, α', β', α'', β'',

$\gamma = \gamma' = \gamma'' = 0$

$Y_1 \equiv Tr(Y)$, $Y_2 \equiv det(Y)$, $Z \equiv Tr(M_U Y)$

To fit (7):

m_s, m_b, m_μ, m_τ, V_{cb},

Δm^2_{23}, $\sin^2\theta_{23}$

Possible to fit, shown explicitly

$N_g = 3$ case

Unknowns (15):

α, β, γ, α', β', γ', α'', β'', γ'',

$Y_{1,2,3}$, $Z_{1,2,3}$

To fit (14):

m_d, m_s, m_b, m_e, m_μ, m_τ, $\theta^q_{1,2,3}$,

$\theta^l_{1,2,3}$, Δm^2_{23}, Δm^2_{12}

Looks possible to fit, but harder than before, not checked yet

Proton decay

Color triplet mass matrix is 12×12, but one eigenvalue is automatically zero (would-be Goldstone).

Just some of these triplets (in 27 and $\overline{351}'$) are coupled to MSSM matter fields.

Projection factors to light matter states must be included (different from usual cases without heavy vector like matter)

Full analysis complicated, but

- several triplets involved, just some elements of the inverse matrix relevant
- only some combination of parameters are fixed by the fitting of fermion masses and mixing angles, orthogonal combinations free
- we already know some examples of heavy vector like matter that helps in both fitting masses and getting long enough proton lifetime

Take for example the minimal renormalizable supersymmetric SU(5) with extra vector like matter-type $5_F + \bar{5}_F$

- of the four $\bar{5}_F$ only 3 combinations are light (chiral):

$$\bar{5}_F^a \left(\eta_a 24_H + \mu_a\right) 5_F$$

  the choice of these combinations breaks SU(5) by $\langle 24_H \rangle$: this corrects the bad relation $M_D = M_E$

- The combination of heavy triplets can account for the heavy color triplet that corrects RGE's. Since this is matter-type, it does not contribute to proton decay

- $M_D \neq M_E$ has the virtue to get an extra unitary matrix V in the $d = 5$ proton decay operators.

$$V = V(\theta_R) \quad \rightarrow \quad \text{arbitrary}$$

Babu, BB, Tavartkiladze, '12

Landau pole

Renormalizable GUT models face the following problem:

$$\frac{1}{M_{Planck}} \Sigma f^3 \sim \frac{M_{GUT}}{M_{Planck}} f^3$$

with

$$\frac{M_{GUT}}{M_{Planck}} \sim \frac{1}{10^{2-3}} \quad \gtrsim \quad \text{Yukawa } (2^{nd} \text{ generation})$$

$$\gg \quad \text{Yukawa } (3^{rd} \text{ generation})$$

But at least one could argue that gravity *for some reason* does not produce such terms.

Here the problem worse.

$$\beta_{E_6} = -159 \rightarrow \text{Landau pole}$$

$$M_{GUT} \lesssim \Lambda_{Landau\ pole} \lesssim 10\ M_{GUT} \ll M_{Planck}$$

Why terms $1/\Lambda_{\rm Landau\ pole}$ neglected?

- Large N expansion works pretty well even for $N = 3$.
- Similar assumptions quite often used, for example in R-parity in MSSM, constrained MSSM (minimal sugra), minimal flavor violation, ...

  Some terms assumed to be zero with good phenomenological but no good theoretical reasons.

  Nice, but just examples. More (experimental and/or theoretical) data needed.
- No idea what would be the UV completion and the superpotential does not get renormalized (zero remains zero)
- More speculative: some attempts to make sense of a Landau pole are on the market (Redmond,..., Bogolyubov, ..., Shirkov, ...)

BB, Ioannisian, work in progress

Conclusions

- E_6 a tractable (although cumbersome) theory
- examples of (so far) possibly realistic cases ($N_g = 2$)

Some open questions:

- Landau pole very close just above M_{GUT}. Any possibility to treat it ?
- Neutrino mass scale should be lower than M_{GUT}. To get it the full mass spectrum at that scale should be known and included in gauge couplings RGEs