Theory and Cosmology of Massive Gravity and Beyond

Gregory Gabadadze

New York University

Overview, and recent work with R. Kimura and D. Pirtskhalava

May 26, 2014

Motivation:

General Relativity (GR) is a very successful theory:

$$G_{\mu\nu} = \left(10^{-33}\,\mathrm{eV}\right)^2 g_{\mu\nu} + 8\pi G_N T_{\mu\nu}^{\mathrm{dm,m,rad,..}}$$

is a good parametrization of data.

Motivation:

General Relativity (GR) is a very successful theory:

$$G_{\mu\nu} = \left(10^{-33}\,\mathrm{eV}\right)^2 g_{\mu\nu} + 8\pi G_N T_{\mu\nu}^{\mathrm{dm,m,rad,..}}$$

is a good parametrization of data. An alternative:

$$G_{\mu\nu} = \left(10^{-33}\,\mathrm{eV}\right)^2 X_{\mu\nu} + 8\pi G_N T_{\mu\nu}^{\mathrm{dm,m,rad,..}}$$

is by construction more complex. Why is this worth considering?

Motivation:

General Relativity (GR) is a very successful theory:

$$G_{\mu\nu} = \left(10^{-33}\,\mathrm{eV}\right)^2 g_{\mu\nu} + 8\pi G_N T_{\mu\nu}^{\mathrm{dm,m,rad,..}}$$

is a good parametrization of data. An alternative:

$$G_{\mu\nu} = \left(10^{-33}\,\mathrm{eV}\right)^2 X_{\mu\nu} + 8\pi G_N T_{\mu\nu}^{\mathrm{dm,m,rad,..}}$$

is by construction more complex. Why is this worth considering?

For further tests of GR, good to have an alternative theory to compare with, and test both against the data. The Brans-Dicke theory was introduced for that purpose in 1960s. Good to have an alternative that is observationally (slightly) different.

The CC needs an incredible fine tuning, or the landscape. In the alternative, while the big CC could be put to zero (by, e.g., some nonlocal mechanism affecting CC but nothing else), the physical scale of dark energy, $10^{-33}\ eV$, might be a stable scale where GR is modified – technical naturalness.

The mass and potential terms:

The idea of an extension of GR by a mass term is arguably the easiest to articulate and explain.

Yet, such an extension had been a problem for long time. This problem a good enough motivation for a theorist to ask the questions: *what is the potential for gravity?*

The mass and potential terms for a scalar:

Kinetic term $= -(\partial_\mu \Phi)^2$

Adding mass + potential $= m^2\Phi^2 + \lambda\Phi^4$

in general enables solutions with $p \simeq -\rho$.

GR Extended by Mass and Potential Terms

Previous no-go statements invalid: *de Rham, GG, '10*

The Lagrangian of the theory: *de Rham, GG, Tolley, '10*

Using $g_{\mu\nu}(x)$ and 4 scalars $\phi^a(x)$, $a = 0, 1, 2, 3$, define

$$\mathcal{K}^\mu_\nu(g,\phi) = \delta^\mu_\nu - \sqrt{g^{\mu\alpha} f_{\alpha\nu}} \qquad f_{\alpha\nu} \equiv \partial_\alpha \phi^a \partial_\nu \phi^b \eta_{ab}$$

The Lagrangian is written using notation $tr(\mathcal{K}) \equiv [\mathcal{K}]$:

$$\mathcal{L} = M_{\rm pl}^2 \sqrt{g} \left(R + m^2 \left(\mathcal{U}_2 + \alpha_3\, \mathcal{U}_3 + \alpha_4\, \mathcal{U}_4 \right) \right)$$

$$\mathcal{U}_2 = [\mathcal{K}]^2 - [\mathcal{K}^2]$$

$$\mathcal{U}_3 = [\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3]$$

$$\mathcal{U}_4 = [\mathcal{K}]^4 - 6[\mathcal{K}^2][\mathcal{K}]^2 + 8[\mathcal{K}^3][\mathcal{K}] + 3[\mathcal{K}^2]^2 - 6[\mathcal{K}^4]$$

Lagrangian Rewritten via Levi-Civita Symbols:

de Rham, GG, Heisenberg, Pirtskhalava '11 (decoupling limit)
Koyama, Niz, Tasinato; Th. Nieuwenhuizen; '11 (full theory)

$$\mathcal{L} = M_{\rm pl}^2 \sqrt{g}\left(R + m^2\left(\mathcal{U}_2 + \alpha_3\,\mathcal{U}_3 + \alpha_4\,\mathcal{U}_4\right)\right)$$

$$\mathcal{U}_2 = \epsilon_{\mu\nu\alpha\beta}\epsilon^{\rho\sigma\alpha\beta}\mathcal{K}^\mu_\rho\mathcal{K}^\nu_\sigma$$

$$\mathcal{U}_3 = \epsilon_{\mu\nu\alpha\gamma}\epsilon^{\rho\sigma\beta\gamma}\mathcal{K}^\mu_\rho\mathcal{K}^\nu_\sigma\mathcal{K}^\alpha_\beta$$

$$\mathcal{U}_4 = \epsilon_{\mu\nu\rho\sigma}\epsilon^{\alpha\beta\gamma\delta}\mathcal{K}^\mu_\alpha\mathcal{K}^\nu_\beta\mathcal{K}^\rho_\gamma\mathcal{K}^\sigma_\delta$$

$$\mathcal{K}^\mu_\nu(g,\phi) = \delta^\mu_\nu - \sqrt{g^{\mu\alpha}f_{\alpha\nu}} \quad \text{unitary gauge} \quad f_{\alpha\nu} = \eta_{\alpha\mu}$$

Hamiltonian construction: *Hassan, Rachel A. Rosen, '11,'12; Deffayet, Mourad, Zahariade, '12*
Other proofs: *Mirbabayi, '12; Hinterbichler, R.A. Rosen, '12; Golovnev, 12; Kugo, Ohta, '13*

The vierbein formulation: Hinterbichler and Rachel A. Rosen '12
Fully Diffeomorphism and Local Lorentz Invariant vierbein formulation: GG, Hinterbichler, Pirtskhalava, Shang, '13

$$\mathcal{L}_\Lambda \sim M_{\rm pl}^2 \Lambda \, \epsilon^{\mu\nu\alpha\beta}\epsilon_{abcd} \, e_\mu{}^a e_\nu{}^b e_\alpha{}^c e_\beta{}^d$$

The mass and potentials

$$\mathcal{L}_2 \sim M_{\rm pl}^2 m^2 \, \epsilon^{\mu\nu\alpha\beta}\epsilon_{abcd} \, e_\mu{}^a e_\nu{}^b k_\alpha{}^c k_\beta{}^d$$
$$\mathcal{L}_3 \sim \alpha_3 M_{\rm pl}^2 m^2 \, \epsilon^{\mu\nu\alpha\beta}\epsilon_{abcd} \, e_\mu{}^a k_\nu{}^b k_\alpha{}^c k_\beta{}^d$$
$$\mathcal{L}_4 \sim \alpha_4 M_{\rm pl}^2 m^2 \, \epsilon^{\mu\nu\alpha\beta}\epsilon_{abcd} \, k_\mu{}^a k_\nu{}^b k_\alpha{}^c k_\beta{}^d$$

where $k_\mu{}^a \equiv e_\mu{}^a - \lambda^a{}_{\bar{a}}\partial_\mu\phi^{\bar{a}}$, and $\lambda^a_{\bar{a}}$ transforms w.r.t. $SO(3,1)$'s.
The mass terms can be promoted to the locally $SL(4)$ symmetric structures by promoting λ's to $SL(4)$! Hence the mass terms can have a larger local symmetry group than the EH term does.
Deffayet, Mourad, Zahariade '13: vierbein *vs.* metric formulation

Massive graviton on Minkowski background

GG, Hinterbichler, Pirtskhalava, Shang, '13

$$g_{\mu\nu} = \eta_{\mu\nu}, \qquad \partial_\mu \phi^a = \delta^a_\mu$$

Symmetry breaking pattern

$$ISO(3,1)_{\rm GCT} \times ISO(3,1)_{\rm INT} \to ISO(3,1)_{\rm DIAG}$$

Linearized theory: 3 NG Bosons eaten up by the tensor field that becomes massive. The theory guarantees unitary 5 degrees of freedom on (nearly) Minkowski backgrounds.

Nonlinear interactions are such that there are 5 degrees of freedom on any background. However, there is no guarantee that some of these 5 degrees of freedom aren't bad on certain backgrounds, thus destabilizing those backgrounds.

Exact Lagrangian in the Decoupling Limit (high energy limit)

For helicity 2 and helicity 0: *de Rham, GG, '10*

Helicity 1: GG,Hinterbichler, Pirtskhalava, Shang; Ondo, Tolley, 13

$$\mathcal{L} = -\frac{1}{2} h^{\mu\nu} \hat{\mathcal{E}}^{\alpha\beta}_{\mu\nu} h_{\alpha\beta} + h^{\mu\nu} \left(X^{(1)}_{\mu\nu} + \frac{\alpha}{\Lambda_3^3} X^{(2)}_{\mu\nu} + \frac{\beta}{\Lambda_3^6} X^{(3)}_{\mu\nu} \right)$$

$$\Pi_{\mu\nu} \equiv \partial_\mu \partial_\nu \pi, \qquad X^{(1)}_{\mu\nu} = \epsilon_{\mu\alpha} \epsilon_{\nu\beta} \Pi^{\alpha\beta}$$

$$X^{(2)}_{\mu\nu} = \epsilon_{\mu\alpha\rho} \epsilon_{\nu\beta\sigma} \Pi^{\alpha\beta} \Pi^{\rho\sigma}$$

$$X^{(3)}_{\mu\nu} = \epsilon_{\mu\alpha\rho\gamma} \epsilon_{\nu\beta\sigma\delta} \Pi^{\alpha\beta} \Pi^{\rho\sigma} \Pi^{\gamma\delta}$$

*Invariant, under linear diffs (up to a total derivative), under galilean transformations of π *The scalar part is similar to Galileons but also significant differences from them

Quantum corrections: The nonlinear terms do not get renormalized by quantum loops *de Rham, GG, Heisenberg, Pirtskhalava, 13 (see also de Rham, Heisenberg, Ribeiro 13)*

$$\mathcal{L} \;=\; -\frac{1}{2}h^{\mu\nu}\hat{\mathcal{E}}^{\alpha\beta}_{\mu\nu}h_{\alpha\beta} + h^{\mu\nu}\left(aX^{(1)}_{\mu\nu} + \frac{\alpha}{\Lambda_3^3}X^{(2)}_{\mu\nu} + \frac{\beta}{\Lambda_3^6}X^{(3)}_{\mu\nu}\right)$$

Quantum loop calculations: due to specific structure of the vertices loops do no renormalize a, α, β.

For the full theory, this implies that a choice of the value of m, and the two parameters α and β, is technically natural.

However, the loops induce other terms. Effective field theory below Λ_3, needs completion above that scale (or a nonperturbative method to make it calculable).

Superluminality vs. Acausality: In the high energy limit, $E, p >> m$, the theory reduces to certain Galileons. Galileons in general are known to lead to superluminal phase and group velocities. For some parameter space there is no superuminality for massive gravity Galileons, at least for the spherically symmetric solutions due to specific nature of these theories:

$$-(\partial\pi)^2 + \frac{\pi\epsilon\epsilon\partial\partial\pi\partial\partial\pi}{m^2 M_{\rm pl}} + \frac{\pi\epsilon\epsilon\partial\partial\pi\partial\partial\pi\partial\partial\pi}{m^4 M_{\rm pl}^2}$$

(no cubic Galileon without the quartic one; special couplings to matter, superluminal solutions unstable, L. Berezhiani, G. Chkareuli, GG). However, in theories containing general Galileons and their relatives, and for a generic parameter choice one finds superluminal phase and group velocities. Does this mean that these theories are acausal? Chronology protection due to strong coupling Burrage, de Rham, L. Heisenberg, Tolley, '11. (A)causality is determined by the front velocity, which is affected by the strong coupling regime' more careful studies needed: Works to apear

A well-known example of GR + QED: Drummond Hathrell, '80

$$\mathcal{L}_{\rm GR+QED} = M_{\rm pl}^2 eR + e\left(-\frac{1}{4}FF + \bar{\psi}(i\hat{D} - m_e)\psi\right)$$

A good effective theory below $M_{\rm pl}$ (other charged particles included in the standard way).

At energies below the electron mass $E, p << m_e$, via one loop vacuum polarization diagram one gets an effective theory

$$\mathcal{L}_{\rm eff} = M_{\rm pl}^2 e(R + c\frac{\alpha_{\rm em}}{m_e^2}RFF) - e\frac{1}{4}FF\cdots$$

Among the *RFF* terms is *RiemannFF* term that renormalizes the photon kinetic term in an external gravitational field (e.g., of the Earth), and gives superluminal phase and group velocities.
However, this does not mean that $\mathcal{L}_{\rm GR+QED}$ gives a acausal theory, in fact it gives a good causal effective theory below $M_{\rm pl}$.
Reconciliation – extensive discussions by Hollowood and Shore

Cosmology of pure massive gravity. No flat FRW solution: *D'Amico, de Rham, Dubovsky, GG, Pirtskhalava, Tolley, '11* Exception: Open FRW selfaccelerated universe, *Gumrukcuoglu, Lin, Mykohyama 11*, regretfully, this is unstable Pseudo-homogeneous selfaccelerated solutions: In the dec limit: *de Rham, GG, Heisenberg, Pirtskhalava*. Exact solution: *Koyama, Niz, Tasinato (1,2,3), M. Volkov; L. Berezhiani, et al; ...*

For instance, *Koyama-Niz-Tasinato* solution:

$$ds^2 = -d\tau^2 + e^{m\tau}(d\rho^2 + \rho^2 d\Omega^2)$$

while, ϕ^0 and ϕ^ρ, are inhomogeneous functions. Selfacceleration is a generic feature of this theory, however, vanishing of the kinetic terms for some of the 5 modes is also a common feature of these solutions – too bad! Anisotropic solutions and fluctuations: Gumrukcuoglu, Lin, Mukohyama, '12.

Extensions beyond pure massive gravity are needed for cosmology, they are needed anyway to deal with the strong coupling.

Theory of Quasi-Dilaton: *D'Amico, GG, Hui, Pirtskhalava, '12*

Invariance of the action to rescaling of the reference frame coordinates ϕ^a w.r.t. the physical space coordinates, x^a, requires a field σ. In the Einstein frame:

$$\phi^a \to e^{\alpha}\ \phi^a, \qquad \sigma \to \sigma - \alpha M_{\rm Pl}$$

Hence we can construct the invariant action by replacing $\mathcal{K}$ by $\bar{K}$

$$\bar{K}^{\mu}_{\nu} = \delta^{\mu}_{\nu} - \sqrt{g^{\mu\alpha}\bar{f}_{\alpha\nu}} \qquad \bar{f}_{\alpha_\nu} = e^{2\sigma/M_{\rm Pl}}\partial_\alpha\phi^a\partial_\nu\phi^b\eta_{ab}$$

and adding the sigma kinetic term

$$\mathcal{L} = \mathcal{L}_{dRGT}\left(\mathcal{K} \to \bar{K}\right) - \omega\sqrt{g}(\partial\sigma)^2$$

and the term $\int d^4x\sqrt{-det\bar{f}}$ can also be added. In the Einstein frame σ does not couple to matter, but it does in the Jordan frame

Quasi-Dilaton: decoupling limit *GG, Kimura, Pirtskhalava '14*

$$\mathcal{L} = -\frac{1}{2} h^{\mu\nu} \left(\hat{\mathcal{E}} h\right)_{\mu\nu} + h^{\mu\nu} \left[-\frac{1}{2} \varepsilon_\mu \varepsilon_\nu \Pi + a_2 \varepsilon_\mu \varepsilon_\nu \Pi\Pi + a_3 \varepsilon_\mu \varepsilon_\nu \Pi\Pi\Pi \right]$$

Quasi-Dilaton: decoupling limit *GG, Kimura, Pirtskhalava '14*

$$\mathcal{L} = -\frac{1}{2} h^{\mu\nu} \left(\hat{\mathcal{E}} h\right)_{\mu\nu} + h^{\mu\nu} \left[-\frac{1}{2} \varepsilon_\mu \varepsilon_\nu \Pi + a_2 \varepsilon_\mu \varepsilon_\nu \Pi\Pi + a_3 \varepsilon_\mu \varepsilon_\nu \Pi\Pi\Pi \right]$$
$$- \omega \partial^\mu \sigma \partial_\mu \sigma + \sigma \left[\varepsilon\varepsilon\Pi + \tilde{a}_2 \varepsilon\varepsilon\Pi\Pi + \tilde{a}_3 \varepsilon\varepsilon\Pi\Pi\Pi + \tilde{a}_4 \varepsilon\varepsilon\Pi\Pi\Pi\Pi \right]$$

Quasi-Dilaton: decoupling limit *GG, Kimura, Pirtskhalava '14*

$$
\begin{aligned}
\mathcal{L} = & -\frac{1}{2} h^{\mu\nu} \left(\hat{\mathcal{E}} h\right)_{\mu\nu} + h^{\mu\nu} \left[-\frac{1}{2}\varepsilon_\mu\varepsilon_\nu \Pi + a_2 \varepsilon_\mu\varepsilon_\nu \Pi\Pi + a_3 \varepsilon_\mu\varepsilon_\nu \Pi\Pi\Pi\right] \\
& - \omega \partial^\mu \sigma \partial_\mu \sigma + \sigma \left[\varepsilon\varepsilon\Pi + \tilde{a}_2 \varepsilon\varepsilon\Pi\Pi + \tilde{a}_3 \varepsilon\varepsilon\Pi\Pi\Pi + \tilde{a}_4 \varepsilon\varepsilon\Pi\Pi\Pi\Pi\right] \\
& - \frac{1}{4}\Big[2\varepsilon\varepsilon BB + b_2 \varepsilon\varepsilon BB\Pi - b_3 \varepsilon\varepsilon BB\Pi\Pi + 2\varepsilon\varepsilon B^2\Pi - 4a_2 \varepsilon\varepsilon B^2 \Pi\Pi \\
& - 4a_3 \varepsilon\varepsilon B^2 \Pi\Pi\Pi + 4\varepsilon\varepsilon B\partial A - 16 a_2 \varepsilon\varepsilon B\partial A \Pi - 24 a_3 \varepsilon\varepsilon B \partial A \Pi\Pi\Big]
\end{aligned}
$$

Both π and σ are Galileons: Shift symmetry of σ gets enhanced to Galilean symmetry in the decoupling limit!

Selfaccelerated solution with healthy perturbations

GG, Kimura, Pirtskhalava, '14: The background solution – de Sitter due to graviton mass; indistinguishable from cosm const

Perturbations are different though. Vector perturbations

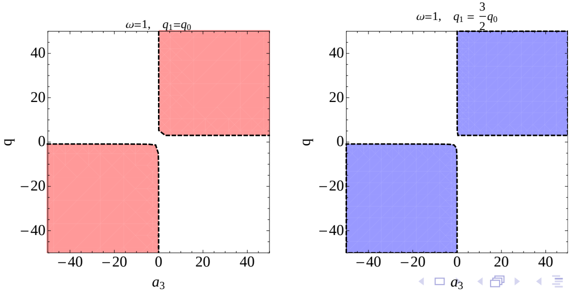
$$Q_1 B_{\mu\nu} B^{\mu\nu} + Q_2 B_{\mu\nu} F^{\mu\nu}$$

Scalar perturbations

$$A_1 \delta\dot{\pi}^2 - A_2 (\partial_i \delta\pi)^2 + B_1 \delta\dot{\pi}\delta\dot{\sigma} - B_2 \partial_i \delta\pi \partial_i \delta\sigma$$

No ghosts, tachyons, superluminalities, or gradient instabilities for

$$0 < \omega < 54, \qquad \mathrm{sgn}(a_3) = sgn(q)$$



Other developments and different extensions (subjective list):

Extended Quasidilaton: selfaccelerated solution with no ghosts (but is above the strong scale). De Felice, Mukohyama, '13; Mukohyama, '13; De Felice, Gumrukcuoglu, Mukohyama, '13.

Bigravity: Hassan, R.A. Rosen, '11, ... Cosmology e.g., De Felice, Gumrukcuoglu, Mukohyama, Tanahashi, Tanaka, 14 ....

Extended Massive Gravity: GG, Hinterbichler, Khoury, Pirtskhalava, Trodden, 13; Gumrukcuoglu, Hinterbichler, Lin, Mukohyama, Trodden 13

Black Hole solutions: Tasinato, Koyama, Niz '11,12; Berezhiani, Chkareuli, et al 12, M. S. Volkov 11,12,13, Babichev, Fabbri, 14, Kodama, Arraut '14

Interesting non-perturbative aspects: Sasaki, Yeom, Zhang, 12; Park, Sorbo, 12; Zhang, Saito, Yeom, Sasaki, 13

Conclusions:

- A classical theory that extends GR by the mass and potential term to a diff invariant non-linear theory of 5 degrees of freedom of a massive spin-2, does exist.
- This is a strongly coupled theory with a low scale (compare with the Electroweak theory without the Higgs mode), one neededs to make it tangible above the strong scale. The issue of (a)causality is entangled with the strong coupling issue.
- In the first order formulation the symmetry can be enhanced; a good staring point to think of the completion.
- Generic cosmological solutions have no FRW symmetries, but can approximate well FRW cosmologies.
- Selfaccelerated solutions emerge, but some fluctuations loose kinetic terms, and this is not acceptable, thus extensions of pure massive gravity are needed for this purpose too.
- A symmetry based extension – Quasi-dilaton. Has selfaccelerated solutions with nonvanishing kinetic terms for all the perturbations; no ghosts, no tachyons, no gradient instability, no superluminality on this selfaccelerated solution.