Fine Tuning in the Holographic Minimal Composite Higgs Models.

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Introduction

- Higgs has been discovered, experimental evidence exists for its scalar nature and non zero VEV, but no discovery of deviation from standard model.

  Is this the birth of the hierarchy problem?

- Higgs is not the first light scalar to be discovered. Pion has mass ($\sim$ 140 MeV) slightly lower than QCD scale ($\sim$ 200 MeV) and masses of first resonances ($m_\rho \sim$ 700 MeV).

  Is the reason for the Higgs being lighter than the scale of new physics, simply that it is a pseudo Nambu-Goldstone boson analogous to the Pion?

- Two important differences between Higgs and pion:
  1. Relative difference between Higgs mass and scale of new physics appears to be greater than between the pion mass and QCD scale.
  2. Higgs needs to have effective potential such that it gains a non-zero VEV and breaks electroweak symmetry, i.e a negative Higgs squared term.

  How plausible / reasonable / natural / finely-tuned is this?

  To answer this we need an effective theory valid to scales at which we can reliably calculate the Higgs potential.

In the limit of massless up and down quarks

QCD Lagrangian invariant under $\mathrm{SU_L}(2) \times \mathrm{SU_R}(2)$.

Upon Chiral symmetry breaking, $\langle \bar{q}_R q_L \rangle \neq 0 \approx 200$ MeV,

$$\mathrm{SU_L}(2) \times \mathrm{SU_R}(2) \to \mathrm{SU_V}(2).$$

Gives rise to three massless Goldstone bosons, π^0 and $\pi^{\pm}$.

A brief QCD recap

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Upon Chiral symmetry breaking, $\langle \bar{q}_R q_L \rangle \neq 0 \approx 200$ MeV,

$$\mathrm{SU_L}(2) \times \mathrm{SU_R}(2) \to \mathrm{SU_V}(2).$$

Gives rise to three massless Goldstone bosons, π^0 and $\pi^{\pm}$.

Except chiral symmetry already explicitly broken by up and down masses.

$\mathrm{SU_L}(2) \times \mathrm{SU_R}(2)$ only approximate symmetry, broken by Yukawa couplings. Pions are massive pseudo-Goldstone bosons

$$m_\pi^2 = (m_u + m_d)\frac{M^2}{f_\pi} \approx 140 \text{ MeV}$$

where $M \approx 400$ MeV, is determined by Pion potential and $f_\pi = 93$ MeV.

Pion mass determined by Yukawa couplings that violate global symmetry.

Analogously in MCHM, the coefficients of the Higgs potential will be determined by the operators that violate the global symmetry.

We need an effective theory to work with

Any 5D theory of Gauge-Higgs unification can be expressed, in the holographic basis, as a gauge theory plus a non-linear sigma field. (Hosotani '83, Contino, Nomura & Pomarol '03, Panico, Serone & Wulzer '05 plus more)

$$S = \int d^5x \sqrt{G} \left(-\frac{1}{4} F^a_{MN} F^{MN\,a} + \mathcal{L}_{G.F} \right)$$

$$= \int d^4x \sum_n -\frac{1}{4} F^{(n)a}_{\mu\nu} F^{\mu\nu\,a}_{(n)} + \frac{1}{2} m_n^2 A^a_{\mu(n)} A^{\mu(n)}_a + \frac{1}{2} |D_\mu \Phi^{\hat{a}}|^2 \qquad \text{"KK basis"}$$

$$= \int \frac{d^4p}{(2\pi)^4} \frac{f_\pi^2}{2} |D_\mu \Sigma|^2 - \frac{P_t^{\mu\nu}}{2} \left(\tilde{A}^{a'}_\mu \Pi^{(+)}(p) \tilde{A}^{a'}_\nu + \tilde{A}^{\hat{a}'}_\mu \Pi^{(-)}(p) \tilde{A}^{\hat{a}'}_\nu \right) \qquad \text{"holographic basis"}$$

where $\tau^{\hat{a}}$ are broken generators, $\pm$ refer to one BC, $P_t^{\mu\nu} = \eta^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}$, $\Sigma = \exp\left[i \frac{\tau^{\hat{a}}\, \tilde{\Phi}_{\hat{a}}}{f_\pi} \right]$

Focus on slice of AdS$_5$ geometry and appeal to AdS/CFT conjecture

Local symmetries in bulk dual to global symmetries of strongly coupled CFT.

$$ds^2 = \frac{R^2}{r^2} \left(\eta^{\mu\nu} dx_\mu dx_\nu - dr^2 \right)$$

with $R \leqslant r \leqslant R'$, two free parameters

$$M_{\mathrm{KK}} = \frac{1}{R'} \sim \mathcal{O}(\mathrm{TeV}) \qquad \text{and} \qquad \Omega = \frac{R'}{R} \sim 10^{15}$$

See also following talk by Nicholas Setzer.

But Goldstone bosons (Higgs) still massless

Now work with unspecified, strongly coupled, approximately conformal, **effective** theory valid up to some 'warped down' cut-off.

As in QCD, Higgs mass and potential determined by operators that violate global symmetry or equivalently brane localised operators. Should consider all permitted operators - large parameter space.

For Gauge fields consider

$$\mathcal{L}_G = -\frac{1}{4}F^a_{MN}F^{MNa} + \sum_{i=\mathrm{IR,\ UV}} \frac{\delta(r-r_i)r}{R}\left(-\frac{\tilde{\theta}_i}{4}F^a_{\mu\nu}F^{\mu\nu a} - \frac{\tilde{\zeta}_i}{2}F^a_{5\mu}F^{5\mu a}\right)$$

For two fermions, ψ and χ, in same representation under unbroken symmetry, consider just IR localised operators consistent with low energy chiral theory.

$$\mathcal{L}_\Psi = \sum_{\Psi=\psi,\chi}\left(i\bar{\Psi}_i\Gamma^M \overleftrightarrow{\nabla}_M \delta^{ij}\Psi_j - M^i_\Psi \bar{\Psi}_i\delta^{ij}\Psi_j\right)$$
$$+\frac{\delta(r-R')r}{R}\left(i\theta^\Psi_{ij}R'\bar{\Psi}^i\not{\partial}\Psi^j + i\zeta_{ij}R'\bar{\psi}^i\not{\partial}\chi^j - m_{ij}\bar{\psi}^i\chi^j\right) + h.c.$$

Note: Boundary operators only source of flavour and CP violation.

Tachyons and Pion Decay Constants

After Wick rotating, $p \to ip_E$, pole or zero in form factors, $\Pi(p_E)$, would correspond to Tachyonic mode. For gauge fields occur when

$$\frac{\theta_{\rm IR}+\zeta_{\rm IR}}{1+\zeta_{\rm IR}}<0 \quad \text{and} \quad \frac{\theta_{\rm UV}+\zeta_{\rm UV}}{1-\zeta_{\rm UV}}<0.$$

while for fermions

$$4\zeta^2+4m^2\zeta^2+\theta^{\chi 2}+\theta^{\psi 2}-2\theta^{\psi}\theta^{\chi}-4m^2\theta^{\psi}\theta^{\chi}>0.$$

Two other important parameters are uniquely fixed by the W/Z mass and the geometry.

$$\boxed{f_\pi^2=\frac{4M_{\rm KK}^2}{g^2\left(\log(\Omega)+\frac{\theta_{\rm IR}+\zeta_{\rm IR}}{1+\zeta_{\rm IR}}+\frac{\theta_{\rm UV}+\zeta_{\rm UV}}{1-\zeta_{\rm UV}}\right)},}$$

and (also often denoted by ξ)

$$\boxed{s_h^2=\frac{v^2}{f_\pi^2}=\frac{m_W^2}{M_{\rm KK}^2}\left(\log(\Omega)+\frac{\theta_{\rm IR}+\zeta_{\rm IR}}{1+\zeta_{\rm IR}}+\frac{\theta_{\rm UV}+\zeta_{\rm UV}}{1-\zeta_{\rm UV}}\right)}$$

where $v\approx 246$ GeV and $m_W^2=\frac{g^2v^2}{4}$.

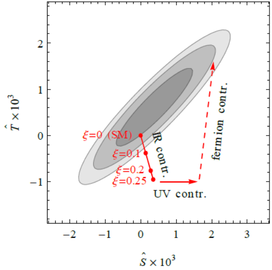
From here on in, focus on Minimal Composite Higgs Model (MCHM). Four broken generators, just complex Higgs doublet. (Agashe, Contino, Pomarol '04)

$$\mathrm{SO}(5) \times \mathrm{U}(1)_X \to \mathrm{SO}(4) \times \mathrm{U}(1)_X \cong \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_X$$

Custodial symmetry @ LO $\Rightarrow$.

$$S \approx \frac{2\pi v^2}{M_{\mathrm{KK}}^2}\left(\frac{3}{4} + \frac{\theta_{\mathrm{IR}} + \zeta_{\mathrm{IR}}}{1+\zeta_{\mathrm{IR}}}\right) + \mathcal{O}(\Omega^{-2})$$

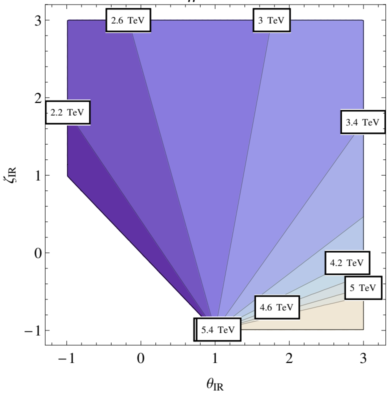
with $T = U = 0$



Taken from (Grojean, Matsedonskyi, Panico '13)

Lower bound on M_{KK} from S with $U \neq 0$,

$\Rightarrow s_h^2 \lesssim 0.07$.



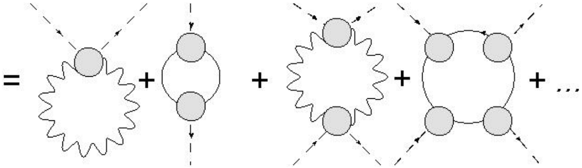
$T = U = 0 \Rightarrow s_h^2 \lesssim 0.015$

Analysis of Higgs Potential

LO contribution to Higgs potential generated at one loop by Coleman-Weinberg mechanism, see also (Barnard, Gherghetta, Medina & Ray '13) for NLO contribution.

For $\mathcal{L} = -\frac{P_t^{\mu\nu}}{2g^2} A_\mu^{\hat{a}} \Pi_G^{\hat{a}\hat{b}}(p) A_\nu^{\hat{b}} + \bar{\Psi}^I \Pi_\Psi^{IJ}(p) \Psi^J \ldots$

$$V(h) = \frac{3}{2} \int \frac{d^4 p_E}{(2\pi)^4} \log\left[\det\left(\Pi_G(p_E)\right)\right] - 2 \int \frac{d^4 p_E}{(2\pi)^4} \log\left[\det\left(\Pi_\Psi(p_E)\right)\right]$$



$$= -(\alpha_G + \alpha_\Psi) s_h^2 + (\beta_G + \beta_\Psi) s_h^4 + \mathcal{O}(s_h^6) = -\alpha s_h^2 + \beta s_h^4 + \mathcal{O}(s_h^6)$$

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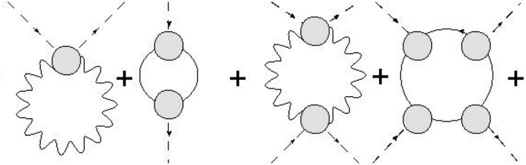
=  + ...

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Higgs mass and VEV

Minimising potential leads to (with $c_h^2 = 1 - s_h^2$)

$$s_h^2 \approx \frac{\alpha}{2\beta} \qquad \text{and} \qquad m_H^2 \approx \frac{8\beta s_h^2 c_h^2}{f_\pi^2}$$

Hence small s_h^2 requires

$$\boxed{2\beta \gg \alpha > 0 \quad \text{but without fine tuning} \quad \beta \lesssim \alpha}$$

For MCHM $\mathrm{SO}(5)/\mathrm{SO}(4)$, with $s_x^2 = s_w^2/c_w^2$

$$\alpha_G = -\frac{3}{2}\int \frac{d^4 p_E}{(2\pi)^4}\left(\frac{3\Pi_1}{2\Pi_0} + \frac{s_x^2\Pi_1}{2\Pi_B}\right)$$

and

$$\beta_G = -\frac{3}{2}\int \frac{d^4 p_E}{(2\pi)^4}\left(\frac{3\Pi_1^2}{8\Pi_0^2} + \frac{s_x^2\Pi_1^2}{4\Pi_0\Pi_B} + \frac{s_x^2\Pi_1^2}{8\Pi_B^2}\right)$$

where $\Pi_1 = \Pi^{(-)} - \Pi^{(+)}$. Except LO contribution to S parameter

$$\boxed{S = -\frac{8\pi s_h^2}{g_5^2}\,\partial_{p^2}\Pi_1(p)\Big|_{p=0}}$$

A suppression in the gauge contribution to potential is typically correlated with a enhancement with the contribution to the S parameter.

Fermion Contribution to Potential

- **Very Model Dependant**. Must embed Fermions in representations of SO(5).
- Here consider minimal MCHM_5, embed Fermions in four **5**'s in fundamental representation, with one generation. (Contino, Da Rold, Pomarol '06)

$$\alpha_t = 2N_C \int \frac{d^4 p_E}{(2\pi)^4} \left(\frac{\Pi_1^q}{2\Pi_0^q} + \frac{\Pi_1^u}{2\Pi_0^u} + \frac{(M_1^u)^2}{2p_E^2 \Pi_0^q \Pi_0^u} \right)$$

$$\beta_t = N_C \int \frac{d^4 p_E}{(2\pi)^4} \left[\left(\frac{\Pi_1^q}{2\Pi_0^q} + \frac{\Pi_1^u}{2\Pi_0^u} + \frac{(M_1^u)^2}{2p_E^2 \Pi_0^q \Pi_0^u} \right)^2 + \frac{(M_1^u)^2}{p_E^2 \Pi_0^q \Pi_0^u} - \frac{\Pi_1^q \Pi_1^u}{2\Pi_0^q \Pi_0^u} \right]$$

$\Rightarrow$ without fine tuning, $\boxed{\beta_t < \alpha_t}$, while α_t more sensitive to Fermion resonance masses, m_Ψ, than β_t.

- Also related to top mass

$$m_t = \frac{s_h c_h}{\sqrt{2}} \frac{M_1^u(0)}{\sqrt{\Pi_0^q(0) + \frac{s_h^2}{2}\Pi_1^{q1}(0)}\sqrt{\Pi_0^u(0) + \frac{s_h^2}{2}\Pi_1^u(0)}},$$

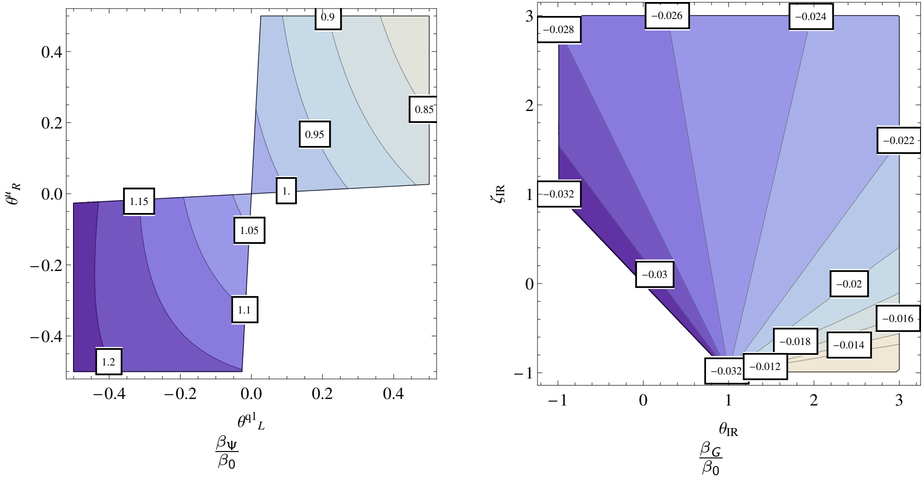
- Similar for **10**'s, but for **14**'s this leaves $\beta_t \sim \int \frac{d^4 p_E}{(2\pi)^4} \frac{\Pi_1}{\Pi_0}$

$\Rightarrow$ $\boxed{\beta_t \sim \alpha_t}$ and both α_t and β_t equally sensitive to m_ψ.

(see Panico, Redi, Tesi & Wulzer '12, Pappadopulo, Thamm & Torre '13, Carena, Da Rold, Ponton '14)

Consider a Benchmark MCHM$_5$ Point.

To see how this works for the MCHM$_5$, consider viable benchmark point with zero kinetic terms i.e. At $\alpha_0 = \alpha(\theta = \zeta = 0)$ and $\beta_0 = \beta(\theta = \zeta = 0)$ get correct VEV, m_H, m_t, m_b. Here $M_{\rm KK} \approx 2.4$ TeV, $s_h^2 = 0.037$ and $\Delta_{\rm B.G} \approx 43$.



β takes a value that is not too finely tuned. Dominated by fermion contribution.

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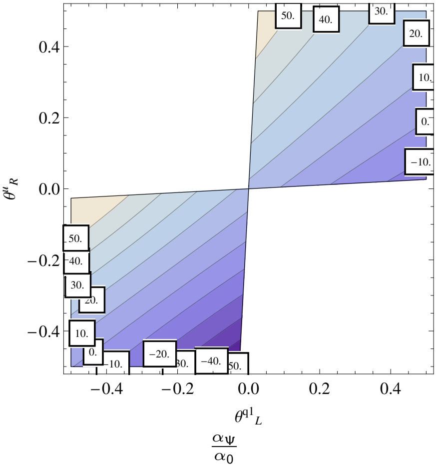
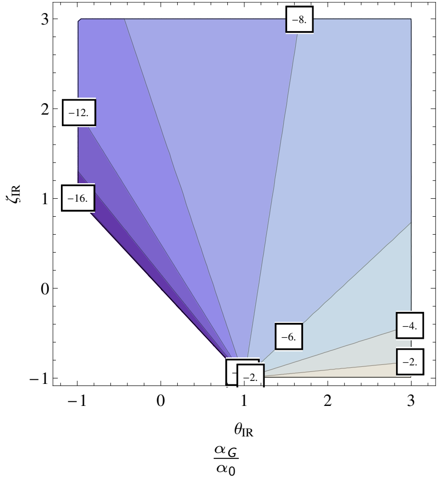




α requires tuning such that $\alpha < \alpha_t$.

How much fine tuning is required?

- Here define fine tuning for a set of observables $\mathcal{O}_i$, with input parameters X_j, by (Barbieri & Guidice '88).

$$\Delta_{\mathrm{B.G}} = \max_{i,j} \left| \frac{X_j}{\mathcal{O}_i} \frac{\partial \mathcal{O}_i}{\partial X_j} \right| \approx \max_{i,j} \left| \frac{X_j}{\mathcal{O}_i} \frac{\Delta \mathcal{O}_i}{\Delta X_j} \right|$$

- Naively anticipate VEV at scale f_π hence to get VEV at v estimate minimal tuning at

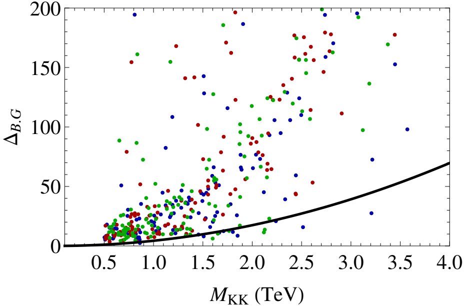
$$\Delta_{B.G}^{\min} \approx \frac{f_\pi^2}{v^2} = \frac{1}{s_h^2}.$$

- But mass scale of 5D theory is M_{KK} so perhaps

$$\Delta_{B.G} \approx \frac{M_{\mathrm{KK}}^2}{v^2} \approx \frac{4}{s_h^2} \quad (\text{with } \Omega = 10^{15})?$$

- Or possibly the masses of the fermion and vector resonances

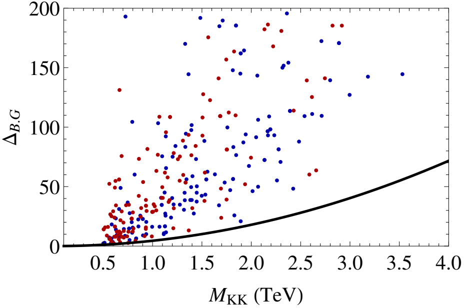
$$\Delta_{B.G} \approx \frac{m_{\Psi/\rho}^2}{v^2} \approx \frac{4}{s_h^2} \times \mathcal{O}(1)?$$



With no kinetic terms (green) or just gauge kinetic terms (blue $\theta_{\mathrm{IR}} = \zeta_{\mathrm{IR}} = 1$, red $\theta_{\mathrm{IR}} = 3$, $\zeta_{\mathrm{IR}} = 0$)
Least squares fit yields

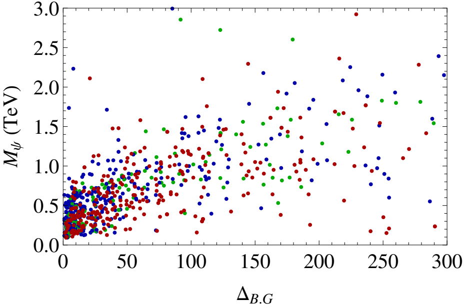
$$\Delta_{B.G} \approx \frac{5}{s_h^2}$$

with Coeff. of Determ. $R^2 \approx 0.8$.

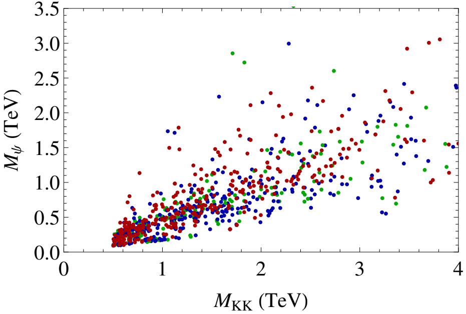


With anarchic Fermion kinetic terms $\theta \in [-0.5, 0.5]$ (blue) and $\theta \in [-1.5, 1.5]$ (red)
Least squares fit no longer meaningful ($R^2 = 0.04$), but fine tuning similar but slightly worse.

Typically anticipate a light fermion resonance

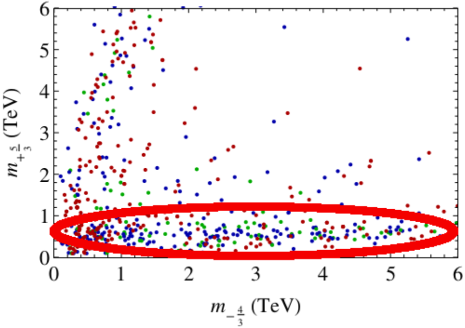


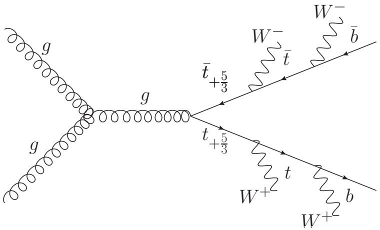
Also considerable variation when fine tuning compared with lightest fermion resonance.



However generically expect to see fermionic resonances with masses below $M_{\mathrm{KK}} < m_\rho \approx 2.4\ M_{\mathrm{KK}}$.

Expect to see a fermion resonance at LHC.

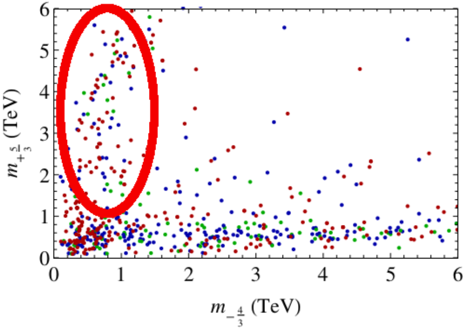




For 40-60% of considered parameter space, the lightest fermion resonance was a charged $+\frac{5}{3}$ fermion.

Could be seen in the relatively clean same-sign di-lepton channel.(Contino & Servant '08, Mrazek & Wulzer '09, Dissertori, Furlan Moortgat & Nef '10)

Currently excluded at $m_{+\frac{5}{3}} \lesssim 800$ GeV. It is anticipated with 300 (3000) fb^{-1}, LHC14 will exclude to $m_{+\frac{5}{3}} \lesssim 1.4(1.6)$ TeV, assuming SM like couplings.



More challenging to observe. Would decay with

$$\mathrm{Br}\left(B_{-\frac{4}{3}} \to W^- b\right) = 1$$

Nonetheless, still constrained by direct searches, for example (Atlas-Conf-2013-060) based on lepton plus $>$ 4 jets excludes

$$m_{-\frac{4}{3}} \lesssim 740 \text{ GeV}.$$

For 20-40% of considered parameter space, the lightest fermion resonance was a charged $-\frac{4}{3}$ fermion.

Conclusions

- Tightest constraints on tuning come from direct searches for fermion resonances and Electroweak precision tests.
- If you just consider oblique LO contribution to EW precision tests (i.e. $T = U = 0$), $\Rightarrow M_{\rm KK} \gtrsim 4$ TeV, $s_h^2 \lesssim 0.015$, $\Delta_{B.G}^{\rm min} \gtrsim 60$, typical $\Delta_{B.G} \gtrsim 300$, nothing at LHC ... probably too pessimistic!
- Any positive contribution to T or U would significantly alleviate EW constraints. E.g. with $U \neq 0, \Rightarrow M_{\rm KK} \gtrsim 2$ TeV, $s_h^2 \lesssim 0.07$, $\Delta_{B.G}^{\rm min} \gtrsim 15$ typical $\Delta_{B.G} \gtrsim 70$.
- Large and complicated parameter space, always possible to find points with much lower tuning.
- 4D models can break correlation between $\mathcal{O}(m_\psi) \sim \mathcal{O}(m_\rho)$, but have to be careful that one is still studying a meaningful strongly coupled theory and now also constrained by direct searches.
- In holographic MCHM$_5$, one would anticipate lightest fermion resonance to be charged $+\frac{5}{3}$ or $-\frac{4}{3}$.
- LHC14 will discover or exclude a fermion resonance with $m_\psi \lesssim 1.5$ TeV. In the case of exclusion, for the MCHM$_5$, this will exclude a significant proportion (but not all) of the parameter space with $\Delta_{B.G} \lesssim 200$.
- Here we do not attempt to address the question of whether $\Delta_{B.G}$ is a good judge of naturalness!