MINIMAL MODELS FOR INFLATION FROM MINIMAL SUPERGRAVITY

Sergio Ferrara
C.E.R.N.

PLANCK 2014, PARIS, 29 MAY 2014

RECEN EXPERIMENTS (PLANCK, BICEP 2)

SEEM TO FAVOR SIMPLE ONE FIELD COSMOLOGICAL

MODELS FOR INFLATION EVEN IF THERE IS

TENSION BETWEEN THE TWO EXPERIMENTS.

WHILE FOR THE SLOW ROLL PARAMETER n_s

(SPECTRAL INDEX OF SCALAR PERTURBATIONS) THE

SAME FORMULA AGREES

$$n_s = 1 - \frac{2}{N} \simeq 0,96$$

FOR THE OTHER SLOW ROLL PARAMETER r (RATION

OF TENSOR TO SCALAR PERTURBATIONS) DIFFERENT

MODELS SEEM TO BE FAVORITE

$r = \frac{12}{N^2}$ (STAROBINSKY INFLATION, HIGGS INFLATION)

$r < 0.08$ (PLANCK)

OR

$r = \frac{8}{N}$ (CHAOTIC INFLATION)

$r \sim 0.2$ (BICEP2)

THE TWO CLASSES OF MODELS GIVE A RATHER DIFFERENT INFLATON POTENTIAL $V(\varphi)$ WHICH IS OF EXPONENTIAL TYPE FOR STAROBINSKY INFLATION AND OF POLYNOMIAL TYPE (QUADRATIC POWER) FOR CHAOTIC INFLATION (LINDE)

HERE WE PRESENT THE SUPERGRAVITY EMBEDDING OF THESE TWO MODELS WHICH IS MINIMAL IN TWO RESPECTS: IT USES THE MINIMAL SET OF MULTIPLETS NEEDED TO DESCRIBE THE MODELS. IT ALSO USES THE MINIMAL OFF-SHELL REPRESENTATION OF THE UNDERLYING LOCAL SUPERSYMMETRY ALGEBRA. THE LATTER INTRODUCES NEW FIELDS WHICH ARE "AUXILIARY" (NOT PROPAGATING) IN THE STANDARD EINSTEIN SUPERGRAVITY BUT BECOME PROPAGATING WHEN HIGHER CURVATURE TERMS ARE INTRODUCED.

THIS FORMULATION IS NEEDED TO DESCRIBE THE PURE SUPERGRAVITY SIDE OF STAROBINSKY INFLATION. IN THIS THEORY THE INFLATON IS "DUAL" TO A PURE SCALAR GRAVITATIONAL MODE, THE "SCALARON" WHICH EMERGES IN A $R + \alpha R^2$ THEORY OF GRAVITY. IT IS REMARKABLE THAT SUCH A THEORY IS "DUAL" TO EINSTEIN GRAVITY (WHITT) COUPLED TO A SCALAR FIELD WITH AN INFLATIONARY POTENTIAL

$$\mathcal{L} = -\frac{1}{2} R - \frac{1}{2}(\partial_\mu \varphi)^2 - \frac{1}{16\alpha}\left(1 - \exp\left(-\sqrt{\tfrac{2}{3}}\,\varphi\right)\right)^2$$

(STAROBINSKY MODEL)

THE MINIMAL SUPERGRAVITY EXTENSION OF SUCH A MODEL WAS DERIVED IN THE LATE EIGHTIES (CECOTTI, CECOTTI, SF, PORRATI, SABHARWAL) IN TWO DIFFERENT FORMS DEPENDING OF TWO DIFFERENT OFF-SHELL COMPLETION OF THE SUPERGRAVITY MULTIPLET

a) V_μ^a, ψ_μ, A_μ, S, P

b) V_μ^a, ψ_μ, A_μ, $b_{\mu\nu}$ $\qquad A_\mu \to A_\mu + \partial_\mu a$, $b_{\mu\nu} \to b_{\mu\nu} + \partial_\mu \xi_\nu - \partial_\nu \xi_\mu$

THE SIX BOSONIC DEGREES OF FREEDOM WHICH MAKE THE GRAVITY MULTIPLET TO HAVE THE SAME NUMBER OF BOSONS AND FERMIONS (12b + 12f) GIVE TWO DIFFERENT SUPERSYMMETRIC EXTENSION OF THE STAROBINSKY M.

THE "DUAL" STANDARD SUPERGRAVITY ACTION CONTAINS, IN THE a) FORMULATION TWO "MATTER" CHIRAL (MASSIVE) MULTIPLETS T, S (4b+4f) WHILE IN THE b) FORMULATION CONTAINS A "MASSIVE" VECTOR (OR TENSOR) MULTIPLET V (4b+4f). THE MAIN DIFFERENCE IS THAT IN THE a) THEORY WE ARE IN PRESENCE OF A "FOUR-FIELD" MODEL, IN THE b) THEORY WE HAVE A "SINGLE-FIELD" INFLATON MODEL SINCE THE OTHER THREE BOSONIC DEGREES OF FREEDOM COMBINE IN A MASSIVE VECTOR.

STANDARD SUPERGRAVITY FORMULAE ALLOW TO DESCRIBE THE a) THEORY IN TERMS OF A

KAHLER POTENTIAL K AND A SUPERPOTENTIAL W -

IT TURNS OUT THAT THEIR FORM IS

$$K = -3 \log \left(1 + T + \bar{T} - h(S, \bar{S}) \right) , \quad W = \lambda T S$$

(λ IS A CONSTANT RELATED TO THE α PARAMETER)

AND $h(S, \bar{S})$ IS AN ARBITRARY REAL FUNCTION WHICH STARTS WITH $h(S, \bar{S}) = S \bar{S} + O(S^3)$ TERMS

IT IS POSSIBLE TO CHOOSE THE FUNCTION $h(S, \bar{S})$ TO MAKE THE INFLATIONARY TRAJECTORY STABLE (WHERE THE "INFLATON" IS IDENIFIED WITH THE $\mathrm{Re}\, T$ SCALAR WHILE THE OTHER THREE SCALARS ARE "EXTREMIZED" - THE POTENTIAL FOR

$\mathrm{Re}\, T = \exp\left(-\sqrt{\frac{2}{3}}\varphi\right)$ IS THE STAROBINSKY POTENTIAL

IT CAN BE SHOWN THAT THIS THEORY, FOR ANY $h(S,\bar{S})$ IS "DUAL" TO A HIGHER CURVATURE SUPERGRAVITY THEORY.

THE SCALAR SUPERCURVATURE (S.F., ZUMINO) IS A CHIRAL SUPERFIELD $\bar{D}_{\dot{\alpha}} R = 0$ AND $h(S,\bar{S})$ CORRESPONDS TO TERMS OF THE FORM $h(R,\bar{R})$ IN THE SUPERGRAVITY SIDE. IT IS IMPORTANT TO NOTICE THAT THE INFLATON POTENTIAL IS AN "F TERM" POTENTIAL, WHICH MEANS IT COMES FROM THE STANDARD EXPRESSION

$$V(T,S) = e^{K}\left(D_i W D_{\bar{j}} \bar{W} K^{i\bar{j}} - 3|W|^2\right) \qquad (i,j = S,T)$$

AND THE INFLATON POTENTIAL IS

$$V(\varphi) = V(T,S)\Big|_{\partial V/\partial S = 0,\ \frac{\partial V}{\partial \operatorname{Im} T} = 0}$$

IT HAPPENS THAT ALL SUPERSYMMETRIC MODELS FOR THE INFLATON POTENTIAL CONSIDERED IN THE LITERATURE (KALLOSH, LINDE; ELLIS, NANOPOULOS, OLIVE; KALLOSH, LINDE, ROEST, ...) ARE MOSTLY DEFORMATION OF THE PREVIOS MODEL WITH MODIFICATION OF $K(T, \bar{T}, S, \bar{S})$ AND OF $W(T, S)$ BUT STILL KEEPING THE SAME (S, T) CHIRAL MULTIPLET CONTENT.

IT IS IN FACT POSSIBEE TO SHOW THAT AT LEAST TWO MULTIPLETS ARE NEEDED TO GET AN INFLATIONARY POTENTIIAL. IN FACT FORMER THEORIES WITH HIGHER SUPERCURVATURE TERMS OF F TERM TYPE ~~WITH~~ WITH CHIRAL FUNCTION $f(\mathcal{R})$ $(\bar{D} f = 0)$

WERE CONSIDERED IN THE PAST (KETOV) BUT WERE SHOWN (ELLIS, NANOPOULOS, OLIVE; S.F., KEHAGIAS, PORRATI) NOT TO PRODUCE AN INFLATIONARY POTENTIAL.

AN IMPORTANT DEFORMATION OF THE (S,T) MODEL FROM WHICH THE CONCEPT OF "ATTRACTORS" CAME FROM (KALLOSH, LINDE, ROEST) IS A SUPERPOTENTIAL OF THE TYPE $W(S,T) = S f(T)$ WHICH ALLOWS BOSONIC POTENTIALS CONTAINING ARBITRARY FUNCTIONS OF THE INFLATON $f(\tanh \frac{\varphi}{\sqrt{6}})$. THESE THEORIES ARE NO LONGER EQUIVALENT TO PURE HIGHER CURVATURE SUPERGRAVITY BUT IN CERTAIN CASES, TO HIGHER CURVATURE COUPLED TO A (SINGLE) CHIRAL MULTIPLET

FOR INSTANCE, TAKING K AS BEFORE

$K = -3\log(1+T+\bar{T}-h(S,\bar{S}))$ BUT NOW $W = S\,f(T)$

THE "DUAL" HIGHER DERIVATIVE SUPERGRAVITY IS

A "MATTER COUPLED THEORY WITH ((CECOTTI, KALLOSH)

$$\Phi = e^{-\frac{1}{3}K} = 1 + T - \frac{f(T)}{f'(T)} + \bar{T} - \frac{\bar{f}(\bar{T})}{\bar{f}'(\bar{T})}$$

AND A TERM $\frac{1}{|f'(T)|^2} R\bar{R}$

AND BOTH TERMS BECOME T INDEPENDENT IF $f(T) = aT$.

THE b) FORMULATION GIVES DIRECTLY A SINGLE-FIELD INFLATION MODEL WHERE A "D" TERM POTENTIAL FOR THE MASSIVE SUPERFIELD IS GENERATED.

THE MOST GENERAL SELF-INTERACTION OF SUCH MASSIVE VECTOR MULTIPLET WITH SPIN CONTENT $(1, 2(1/2), 0)$ RESIDES ON A "REAL FUNCTION" J OF A "REAL VARIABLE" C : $J(C)$ (van Proeyen)

THE BOSONIC PART OF THE SUPERGRAVITY ACTION IS

$$\mathcal{L} = -\frac{1}{2}R - \frac{1}{4}F_{\mu\nu}(B)F^{\mu\nu}(B) + \frac{g^2}{2}J''(C)B_\mu B^\mu + \frac{1}{2}J''(C)(\partial_\mu C)^2 - \frac{g^2}{2}J'(C)^2$$

SO THE POTENTIAL IS $V(C) = \frac{g^2}{2}J'^2(C)$

NOTE THAT THE LAGRANGIAN ONLY DEPENDS ON

J', J'' SO A LINEAR TERM IN J SHIFTS J' BY A CONSTANT BUT LEAVES J'' INVARIANT. THIS CONSTANT IS THE SO CALLED FAYET-ILIOPOULOS TERM.

BY USING THE STUECKELBERG TRICK ONE WRITES THIS LAGRANGIAN AS A GAUGE THEORY BY SHIFTING $A_\mu = B_\mu + \frac{1}{g}\partial_\mu a$ SO THAT

$$\frac{g^2}{2} J''(C)\, B_\mu B^\mu = \frac{g^2}{2} J''(C)\left(A_\mu + \frac{1}{g}\partial_\mu a\right)^2$$

IN THE LIMIT $g \to 0$ THE THEORY BECOMES

$$\mathcal{L} = -\frac{1}{2}R - \frac{1}{4}F_{\mu\nu}(A)^2 + \frac{1}{2}J''(C)\left((\partial_\mu a)^2 + (\partial_\mu C)^2\right)$$

THE (a, C) VARIABLES CAN BE COMPLEXIFIED $z = iC - a$ AND THE J FUNCTION CAN BE INTERPRETED AS KAHLER POTENTIAL $J = -\frac{1}{2}K(\mathrm{Im}\, z)$

THE HIGHER CURVATURE SUPERGRAVITY IN THE b)

FORMULATION IS "DUAL" TO A SELF-INTERACTING

MASSIVE VECTOR MULTIPLET WITH A VERY PRECISE

CHOICE OF $J(C) = \frac{3}{2}(\log -C + C)$ – (SF, C, P, S)

COMPUTATION OF THE POTENTIAL, FOR A CANONICALLY

NORMALIZED FIELD $C = -\exp\sqrt{\frac{2}{3}}\varphi$ ONE OBTAINS

(SF, Kellos, Linde, Porrati)

THE STAROBINSKY POTENTIAL AND LAGRANGIAN (Farakos, Kehagias, Riotto)

$$\mathcal{L} = - - - - -\frac{1}{2}(\partial_\mu\varphi)^2 - \frac{9}{8}g^2\left(1-\exp -\sqrt{\tfrac{2}{3}}\varphi\right)^2$$

So THE SUPERSYMMETRIC GENERALIZATION JUST REPRODUCES

THE SINGLE-FIELD STAROBINSKY MODEL WITH

$\alpha \propto \frac{1}{g^2}$. IT IS INTERESTING TO OBSERVE THAT

THE PARTICULAR FORM OF $J(C)$ CORRESPONDS TO

AN $SU(1,1)/U(1)$ SYMMETRIC KAHLER MANIFOLD WITH A PARABOLIC ISOMETRY BEING GAUGED. FOR A KAHLER POTENTIAL $K = -3\alpha \log \mathrm{Im} z$ THE CURVATURE IS

$$R(C) = \frac{J'''(C)^2 - J'(C)\,J^{IV}(C)}{2\,J''(C)^2} = -\frac{2}{3\alpha}$$

AND FOR $\alpha \to \infty$ $R(C) \to 0$.

THE α DEPENDENT POTENTIAL BECOMES (SF, Kallosh, Linde, Porrati)

$$V(\varphi) = \frac{9}{8} g^2 \left(1 - \exp\sqrt{\frac{2}{3\alpha}}\,\varphi\right)^2 = \frac{9}{8} g^2 P(C)^2$$

NOTE THAT THE CANONICAL VARIABLE φ IS RELATED TO THE C VARIABLE BY THE EQUATION $(P(C) = J'(C))$

$$J''(C) = \left(\frac{d\varphi}{dC}\right)^2 = P'(C)$$

IT THEN FOLLOWS

$$\mathcal{P}'(C) = \mathcal{P}'(\varphi)\frac{d\varphi}{dC} = \left(\frac{d\varphi}{dC}\right)^2 \Rightarrow \mathcal{P}'(\varphi) = \frac{d\varphi}{dC} \quad \left(\mathcal{P}(\varphi) = \mathcal{P}(C(\varphi))\right)$$

$$C(\varphi) = \int d\varphi \, \frac{dC}{d\varphi} = \int d\varphi \, \frac{1}{\mathcal{P}'(\varphi)}$$

$$J(C) = \int dC \, J'(C) = \int \mathcal{P}(\varphi)\frac{dC}{d\varphi}\, d\varphi = \int \frac{\mathcal{P}(\varphi)}{\mathcal{P}'(\varphi)}\, d\varphi$$

THE KINETIC TERM OF THE KAHLER MANIFOLD IS
(SF, Fre, Sorin)

$$\frac{1}{2} J''(C)\left((\partial_\mu C)^2 + (\partial_\mu a)^2\right) = \frac{1}{2}\left[(\partial_\mu \varphi)^2 + (\mathcal{P}'(\varphi))^2(\partial_\mu a)^2\right]$$

THE PREVIOUS EQUATIONS ALLOW US TO COMPUTE

$C(\varphi)$ ONCE $\mathcal{P}'(\varphi) = \frac{d\varphi}{dC}$ IS SOLVED.

THE CURVATURE IN THE φ VARIABLE IS $R(\varphi) = -4\,\frac{\mathcal{P}'''(\varphi)}{\mathcal{P}'(\varphi)}$

THE ONE-FIELD SUPERGRAVITY MODEL FOR INFLATION CAN BE DEFORMED IN TWO WAYS:

1) SIMPLY CHANGE J(C) → CHANGE THE KAHLER MANIFOLD

2) DONT CHANGE THE MANIFOLD BUT CHANGE ITS GAUGED ISOMETRY.

FOR THE CASE OF SYMMETRIC SPACES THIS PROCEDURE GENERATES five MODELS. THREE WITH CONSTANT CURVATURE DEPENDING WHETHER A PARABOLIC, ELLIPTIC OR HYPERBOLIC ISOMETRY IS GAUGED - TWO WITH VANISHING CURVATURE WHERE THE PARABOLIC OR ELLIPTIC ISOMETRY IS GAUGED.

(SF, FRE, SORIN)

CHAOTIC INFLATION: IN A "F" TERM MULTI-FIELD POTENTIAL TERM IT IS HARD TO OBTAIN (AT MOST IN SOME DIRECTIONS OF THE FIELD SPACE) A QUADRATIC POTENTIAL. ONE WAY IS TO IMPOSE A SHIFT SYMMETRY ON THE KAHLER POTENTIAL (KAWASAKI, YAMAGUCHI, YANAGIDA) (Kallosh, Linde, Westpha) (Ellis, Garcia, Nanopoulos, Olive)

IN TERMS OF THE (T, S) CHIRAL FIELDS THIS EXCHANGE THE ROLE OF (ImT, ReT) SINCE IT IS NOW ImT WHICH PLAYES THE ROLE OF INFLATON. IT IS THEN NATURAL, IN THE SUPERGRAVITY DUAL, TO CALL THIS SCENARIO IMAGINARY STAROBINSKY MODEL (SF, Kehagias, Riotto), EVEN IF A COUPLING TO MATTER IS NEEDED IN ORDER TO STABILIZE THE Re T COMPONENT.

CHAOTIC INFLATION IN THE b) SINGLE FIELD SUPERGRAVITY MINIMAL EMBEDDING

IN THIS CASE AN EXACT MODEL IS POSSIBLE SINCE WE CAN TAKE A FLAT-KAHLER SPACE WHERE WE GAUGE A PARABOLIC ISOMETRY (TRANSLATION).

THE ALTERNATIVE GAUGING OF AN ELLIPTIC ISOMETRY WOULD GIVE A QUARTIC POTENTIAL.

FOR THIS CASE $J''(C) = \text{const.}$ $\quad J(C) = \frac{1}{2}C^2 + \xi C$

BUT THE FI TERM IS IRRELEVANT IN THIS CASE

THEN $P(\varphi) = \varphi$ AND

$$V(\varphi) = \frac{1}{2} m^2 \varphi^2$$

THIS MODEL CAN ALSO BE OBTAINED FROM THE CONSTANT (α) CURVATURE CASE BY TAKING THE LIMIT $\alpha \to \infty$, $g^2 \to \infty$ WITH $m^2 \alpha \frac{g^2}{\alpha}$ FIXED

$$g^2 \left(1 - e^{-\sqrt{\frac{2}{3\alpha}}\varphi}\right)^2 \to \frac{m^2}{2}\varphi^2 .$$

CONCLUSIONS:

IF SUPERSYMMETRY IS A FUNDAMENTAL SPACE-TIME SYMMETRY IT SHOULD AT LEAST MANIFESTS ITSELF AT THE PLANCK SCALE SO IT SHOULD BE RELEVANT TO DISCUSS INFLATIONARY COSMOLOGY AND IN PARTICULAR RESTRICT THE PLETHORA OF INFLATON POTENTIAL MODELS.

HERE THE RESTRICTION HAS BEEN MADE TO CONFINE THE DEGREES OF FREEDOM OF INFLATION AS THOSE COMING FROM AN $R+\alpha R^2$ THEORY.

FROM THIS WE GET EITHER AN F TERM POTENTIAL (WITH TWO CHIRAL MULTIPLETS)

OR A D TERM POTENTIAL FROM A MASSIVE VECTOR MULTIPLET.

THE CLASS OF ALLOWED MODELS IS RESTRICTED BY THE FACT THAT THE INFLATON φ IS THE COORDINATE OF A COMPLEX KAHLER SPACE.

THE NATURE OF GAUGED ISOMETRIES CHANGES THE COMPLEX STRUCTURE OF THE MANIFOLD SINCE THE KILLING VECTOR MUST BE HOLOMORPHIC

$z = iC - a = w$ PARABOLIC $k^z\partial_z \Rightarrow k^z = iz$

$z = \exp -iw$ ELLIPTIC $k^z\partial_z \Rightarrow k^z = 1$

$z = i\tanh(-\frac{1}{2}w)$ HYPERBOLIC $k^z\partial_z \Rightarrow k^z = i(1+z^2)$

MOREOVER THE CURVATURE MUST BE NON SINGULAR

$P'''(\varphi)/P'(\varphi)$ NOT SINGULAR

FOR THE α-ATTRACTORS OF THE TYPE (KALLOSH, LINDE, ROEST)

$\mathcal{P}(\varphi) = \tanh^n(\varphi/\sqrt{6})$ THE EMBEDDING OF φ IN THE

KAHLER MANIFOLD SEVERELY RESTRICTS THE VALUES OF n

to $n = 1, 2$ ALL THE OTHER BEING INCOMPATIBLE WITH

THE MANIFOLD STRUCTURE.