Global F-theory models with $U(1)$ Symmetries

Sakura Schäfer-Nameki

King's College, London

Planck 2014, May 26, 2014, Paris

Synopsis of Talk

- F-theory allows for a systematic approach to string model building
  $\Rightarrow$ Bottom up approach (all the way to global models)
- Phenomenological requirements: $U(1)$ Symmetriess
  $\Rightarrow$ Local constraints need to be supplemented with global ones.
- Systematic construction of global models with $U(1)$
  $\Rightarrow$ Complete survey of models within F-theory

Main theme for geometric engineering in F-theory:

Geometric constraints $\Rightarrow$ Phenomenological constraints

Plan

1. Geometric engineering in F-theory
   - Pheno Input
   - Case for F-theory
   - Geometric engineering
2. F-theory models with $U(1)$ symmetries
   - Relevance of $U(1)$s
   - Anomalies
   - Systematics of global models with $U(1)$s

1. F-theory Geometric Engineering

Phenomenological Input

N=1 SUSY GUTs. Unification groups

$$E_6 \supset SO(10) \supset SU(5) \supset SU(3) \times SU(2) \times U(1)$$

Minimal setup with $SU(5)$.

Matter content for an N=1 SUSY GUT with gauge group $SU(5)$:

$$\mathbf{10}_M = \begin{pmatrix} Q \sim (\mathbf{3},\mathbf{2})_{+1/6} \\ U^c \sim (\bar{\mathbf{3}},\mathbf{1})_{-2/3} \\ E^c \sim (\mathbf{1},\mathbf{1})_{+1} \end{pmatrix}, \qquad \bar{\mathbf{5}}_M = \begin{pmatrix} D^c \sim (\bar{\mathbf{3}},\mathbf{1})_{+1/3} \\ L \sim (\mathbf{1},\mathbf{2})_{-1/2} \end{pmatrix}$$

$$\mathbf{5}_H = \begin{pmatrix} H_u \sim (\mathbf{1},\mathbf{2})_{+1/2} \\ H_u^{(3)} \sim (\mathbf{3},\mathbf{1})_{-1/3} \end{pmatrix}, \qquad \bar{\mathbf{5}}_H = \begin{pmatrix} H_d \sim (\mathbf{1},\mathbf{2})_{-1/2} \\ H_d^{(3)} \sim (\bar{\mathbf{3}},\mathbf{1})_{+1/3} \end{pmatrix}$$

Superpotential generating Yukawa couplings:

$$W \sim (\lambda_t)_{ij}\, \mathbf{5}_H \times \mathbf{10}_M^i \times \mathbf{10}_M^j + (\lambda_b)_{ij}\, \bar{\mathbf{5}}_H \times \bar{\mathbf{5}}_M^i \times \mathbf{10}_M^j$$

Pitfalls of GUT models: Exotica

- Additional GUT gauge bosons need to be lifted:

$$SU(5) \to SU(3) \times SU(2) \times U(1)_Y$$

$$\mathbf{24} \to \underbrace{(\mathbf{8},\mathbf{1})_0 \oplus (\mathbf{1},\mathbf{3})_0 \oplus (\mathbf{1},\mathbf{1})_0}_{\text{Gauge Fields}} \oplus \underbrace{(\mathbf{3},\overline{\mathbf{2}})_{-5} \oplus (\overline{\mathbf{3}},\mathbf{2})_{+5}}_{\text{Exotics}}$$

- Higgs triplets

$$SU(5) \to SU(3) \times SU(2) \times U(1)_Y$$

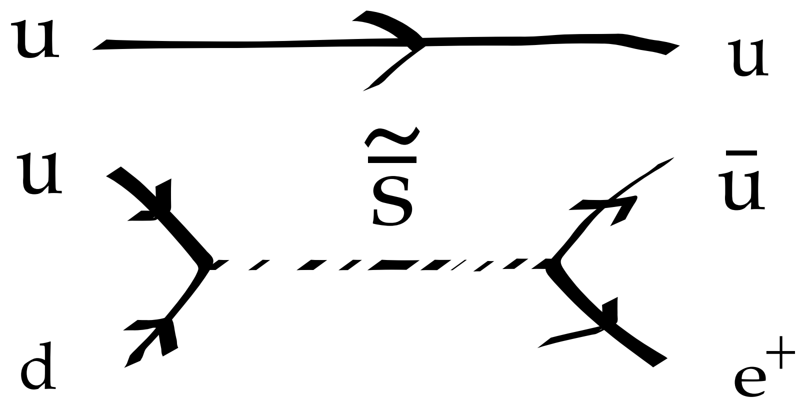
$$\mathbf{5}_H \to \underbrace{(\mathbf{1},\mathbf{2})_{+1/2}}_{H_u} \oplus \underbrace{(\mathbf{3},\mathbf{1})_{-1/3}}_{\text{Exotics}}$$

Pitfalls of GUT models: Rapid Proton Decay

Protect model from Proton Decay: half-life $> 10^{36}$ years:
Models generically contain B/L-violating operators (R-parity violating)

$$W \supset U^c D^c D^c + Q D^c L + L E^c L$$

Induce unhealthy $p^+ \to \pi^0 + e^+$



Furthermore need to forbid dim 5 coupling

$$W_{\mathrm{dim5}} \sim \frac{1}{\Lambda} Q^3 L \,, \qquad \Lambda > 10^{27}\,\mathrm{GeV}$$

$\Rightarrow$ Fobid by extra symmetries: $U(1)$ Symmetries to control spectrum

Why Embed into String Theory?

Why bother?

- The classic stuff (UV completion, inclusion of (super) gravity)
- GUT gauge groups are natural in string theory: $E_8 \times E_8$ heterotic, ADE singularities in Type II and M-theory
- Symmetries (continuous and discrete) are abundant
- Novel ways of breaking GUT group:
  - Wilson Lines: flat line bundle
  - Background flux in $U(1)_Y$ direction
- Potentially interesting flavor structure

Systematic Approaches

Goal: Systematic analysis of possible UV completions

- Top-Down:
  Mini-Landscape: comprehensive list in a small corner of the landscape

- Bottom-Up: [Aldazabal, Ibanez, Quevedo, Uranga]
  Take energy scales as guide-line
  1. Local engineering of realistic models
  2. Systematic inclusion of global constraints

  "Find necessary conditions for embedding"
  $\Rightarrow$ Stepwise systematic implementation of global constraints

Bottom-up best applicable when gauge dof's are localized, i.e. Type II.

F-theory geometric engineering

F-theory = non-perturbative Type IIB

- Coupling: $\tau = C_0 + ie^{-\phi}$
- S-duality of Type IIB = $SL_2\mathbb{Z}$

$$\tau \longrightarrow \frac{a\tau + b}{c\tau + d}$$

- [Vafa] Geometrize τ= complex structure of curve of torus, consistent with $SL_2\mathbb{Z}$

$d = 4$, $N = 1$ theories: Elliptically fibered Calabi-Yau 4-folds

$$\begin{array}{ccc} T^2_\tau & \longrightarrow & Y_4 \\ & & \downarrow \\ & & B_3 \end{array}$$

Various ways to reach F-theory

- Non-perturbative IIB theory
- F-theory on K3-fibered CY4 is dual to heterotic on elliptic CY3
- Duality to M-theory: useful approach to learn about effective theory

$$M/S^1_A \times S^1_B \xrightarrow{R_A \to 0} IIA/S^1_B \xrightarrow{R_B \to 0} IIB$$

$$R_A, R_B \to 0\,, \qquad g_s = R_A/R_B = \text{fixed}$$

More generally: F-theory from M-theory on $\mathbb{E}_\tau$

$$\text{Elliptic curve} \qquad \mathbb{E}_\tau \sim S^1_A \times S^1_B : \qquad \begin{cases} \mathrm{Im}(\tau) = g_s = \text{fixed} \\ \mathrm{Vol}(\mathbb{E}_\tau) \to 0 \end{cases}$$

7-branes in F-theory

7-branes in IIB sources F_9: $z =$ direction perpendicular to 7-brane

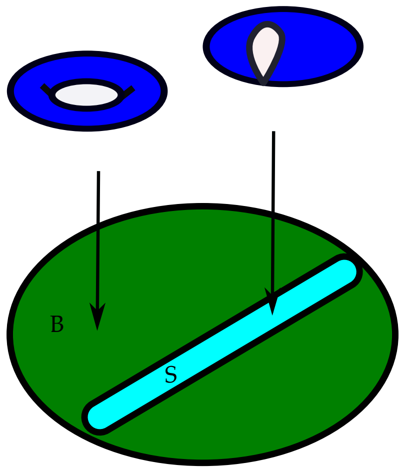
$$d \star F_9 = \delta(z - z_0) \quad \Rightarrow \quad \oint_{S^1} dC_0 = 1$$

which has solution locally

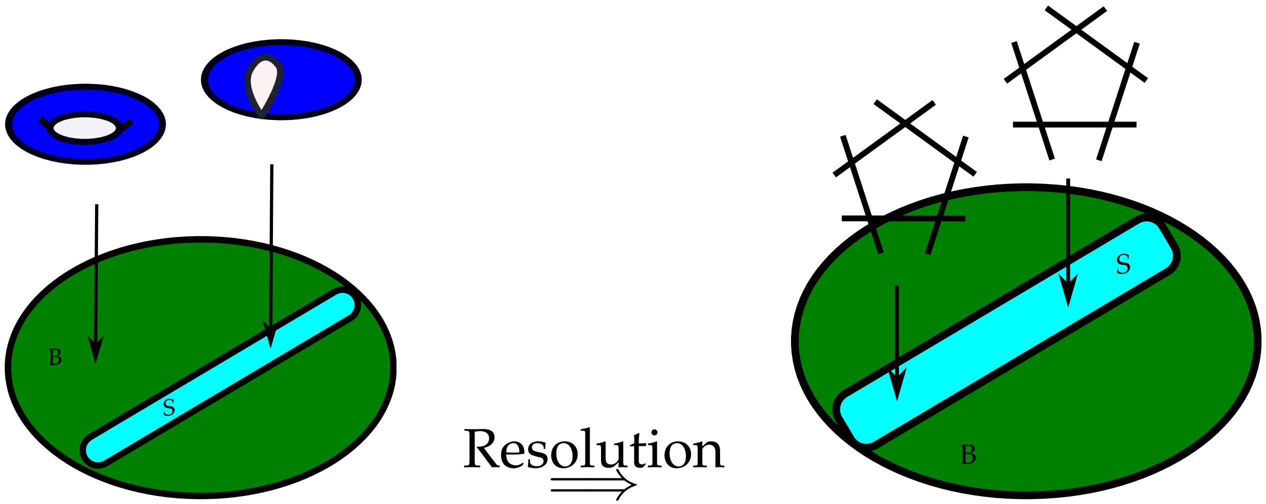
$$\tau(z) = \tau(z_0) + \frac{1}{2\pi i} \log(z - z_0) + \cdots$$

$\Rightarrow$ Monodromy: $\tau \to \tau + 1$

- (p, q) 7-branes with $SL_2\mathbb{Z}$ monodromy
- τ diverges at location of 7-brane
- Location of 7-branes are loci where fiber is singular



Gauge degrees of freedom from Singular Fibers



F/M-theory duality:

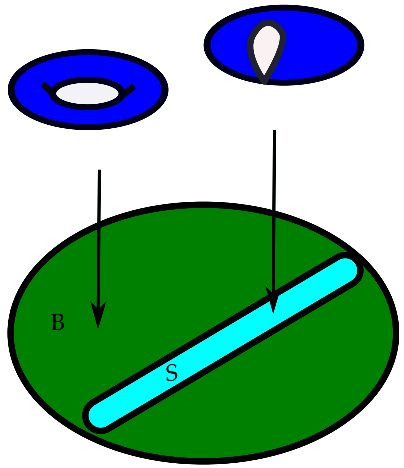
$C_3 = A_i \wedge \omega_i^{(1,1)}$ and M2 wrapping modes give rise to gauge degrees of freedom.

Singular elliptic fibrations

$\mathbb{E}_\tau \longrightarrow B_3$, given by a Weierstrass form

$$y^2 = x^3 + f x + g$$

f and g are functions on the base B.



Singular Fibers in codimension 1:

$\Rightarrow z = 0$, Surface $S \subset B$

$\Rightarrow f = \sum_i f_i z^i$, $g = \sum_i g_i z^i$

Singular along $z = 0$: $\quad \Delta = 4f^3 + 27g^2 = \delta_n z^n + O(z^{n+1})$

Classification of singular fiber types: Kodaira-Neron.

Tate form

Dictionary between gauge symmetry and geometry:

Gauge degrees of freedom $\Leftrightarrow$ Singular elliptic fibrations

$\Updownarrow$

Tate form

Theorem:

$N = 1$ 4d gauge theory, with gauge group G

Then any singular elliptic fibration that engineers this can be globally written in Tate form [Bershadskya et al],[Katz, Morrison, SSN, Sully]

$$y^2 = x^3 + a_1 xy + a_2 x^2 + a_3 y + a_4 x + a_6$$

Fiber type encoded in

$$a_n = z^{i_n} b_n \,, \qquad b_n = O(1)$$

Type	Group	a_1	a_2	a_3	a_4	a_6	Δ
I_1	—	0	0	1	1	1	1
I_2	$SU(2)$	0	0	1	1	2	2
I_3^{ns}	$Sp(1)$	0	0	2	2	3	3
I_3^{s}	$SU(3)$	0	1	1	2	3	3
I_{2n}^{ns}	$Sp(n)$	0	0	n	n	$2n$	$2n$
I_{2n}^{s}	$SU(2n)$	0	1	n	n	$2n$	$2n$
I_{2n+1}^{ns}	$Sp(n)$	0	0	$n+1$	$n+1$	$2n+1$	$2n+1$
I_{2n+1}^{s}	$SU(2n+1)$	0	1	n	$n+1$	$2n+1$	$2n+1$
III	$SU(2)$	1	1	1	1	2	3
IV^{ns}	$Sp(1)$	1	1	1	2	2	4
IV^{s}	$SU(3)$	1	1	1	2	3	4
$I_0^{*\,ns}$	G_2	1	1	2	2	3	6
$I_0^{*\,ss}$	$SO(7)$	1	1	2	2	4	6
$I_0^{*\,s}$	$SO(8)^*$	1	1	2	2	4	6
$I_1^{*\,ns}$	$SO(9)$	1	1	2	3	4	7
$I_1^{*\,s}$	$SO(10)$	1	1	2	3	5	7
$I_2^{*\,ns}$	$SO(11)$	1	1	3	3	5	8
$I_2^{*\,s}$	$SO(12)^*$	1	1	3	3	5	8
$I_{2n-3}^{*\,ns}$	$SO(4n+1)$	1	1	n	$n+1$	$2n$	$2n+3$
$I_{2n-3}^{*\,s}$	$SO(4n+2)$	1	1	n	$n+1$	$2n+1$	$2n+3$
$I_{2n-2}^{*\,ns}$	$SO(4n+3)$	1	1	$n+1$	$n+1$	$2n+1$	$2n+4$
$I_{2n-2}^{*\,s}$	$SO(4n+4)^*$	1	1	$n+1$	$n+1$	$2n+1$	$2n+4$
$IV^{*\,ns}$	F_4	1	2	2	3	4	8
$IV^{*\,s}$	E_6	1	2	2	3	5	8
III^*	E_7	1	2	3	3	5	9
II^*	E_8	1	2	3	4	5	10
non-min	—	1	2	3	4	6	12

Tate Form for $SU(5)$

Tate form for an $SU(5)$ singularity

$$P_{Tate}: \qquad y^2 = x^3 + b_1 xy + b_2 z x^2 + b_3 z^2 y + b_4 z^3 x + b_6 z^5$$

$$\Delta = z^5 \delta_5 + z^6 \delta_6 + O(z^7)$$

Intuition:

$z^5 \cong$ five 7-branes giving rise to $SU(5)$ gauge dof's

$z^6 \cong$ extra flavor 7-brane intersecting $SU(5)$ stack

$\Rightarrow$ local enhancement to $SU(6)$, or $SO(10)$

$\Rightarrow$ matter

Higher codimension singularities for CY 4-folds

For $SU(5)$ discriminant has expansion

$$\Delta = z^5 \delta_5 + z^6 \delta_6 + O(z^7)$$

Higher order term:

$$\begin{array}{rll} \text{Gauge} & \text{codim } 1: & z = 0 \\ \text{Matter} & \text{codim } 2: & z = \delta_5 = 0 \\ \text{Yukawa} & \text{codim } 3: & z = \delta_5 = \delta_6 = 0 \end{array}$$

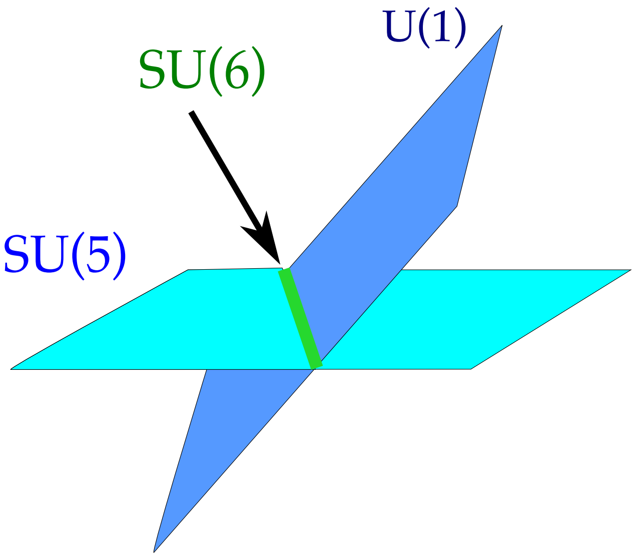
[Beasley, Heckman, Vafa], [Donagi, Wijnholt]

$\Rightarrow$ Singularity type changes along higher codim loci

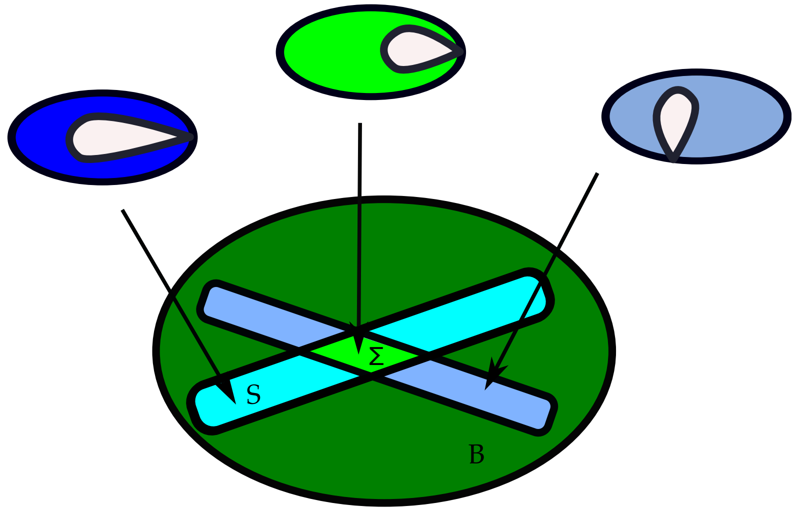
$\Rightarrow$ Resolution to determine fiber structure, and thereby physics.

[Esole,Yau], [Marsano, SSN], [Lawrie, SSN]

Codim 2: Matter



$\Rightarrow$



$\Rightarrow$ Bifundamental matter is localized along codimension 2 loci: curves Σ given by $z = \delta_5 = 0$

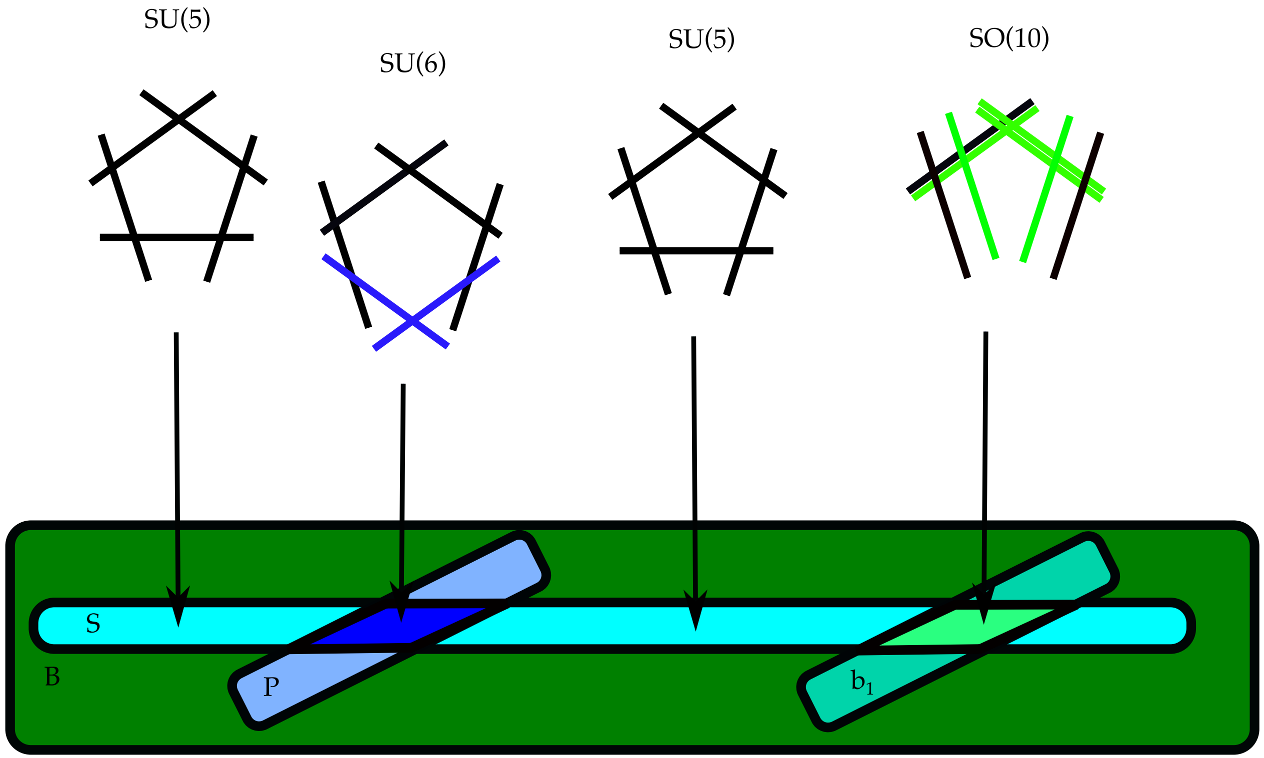
$\Rightarrow$ Matter type determined by singular fiber above Σ

$$G_\Sigma = SO(10) \text{ or } SU(6) \quad \rightarrow \quad SU(5) \times U(1)$$

Codim 2: Matter

What is fiber structure in higher codim? Recall: $\Delta \sim z^n \longrightarrow z^{n+i}$

Number of fiber components increases: matter dof's from wrapped M2



Along codim 2 curves:

$SU(5)$ fibers become reducible and enhance to $SO(10)$ or $SU(6)$ fibers.

Codim 3: Yukawas

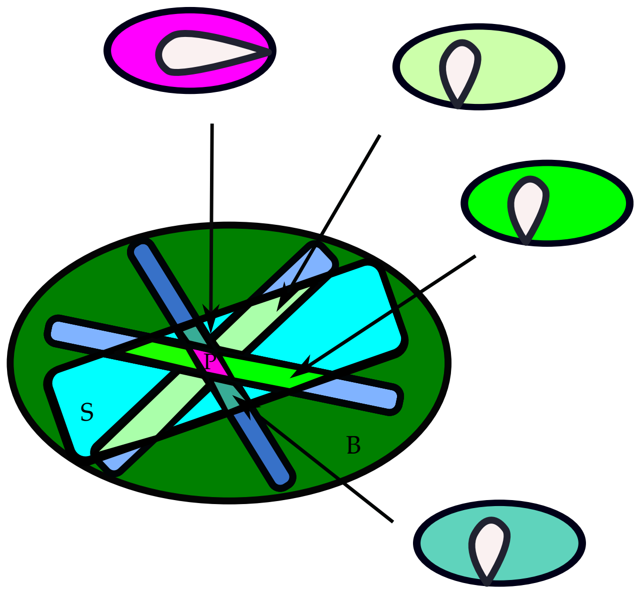
⇒ Yukawa couplings from codimension 3 points p in B_3

$$z = \delta_5 = \delta_6 = 0$$

⇒ Wrapped M2s above matter loci become homologically equivalent.

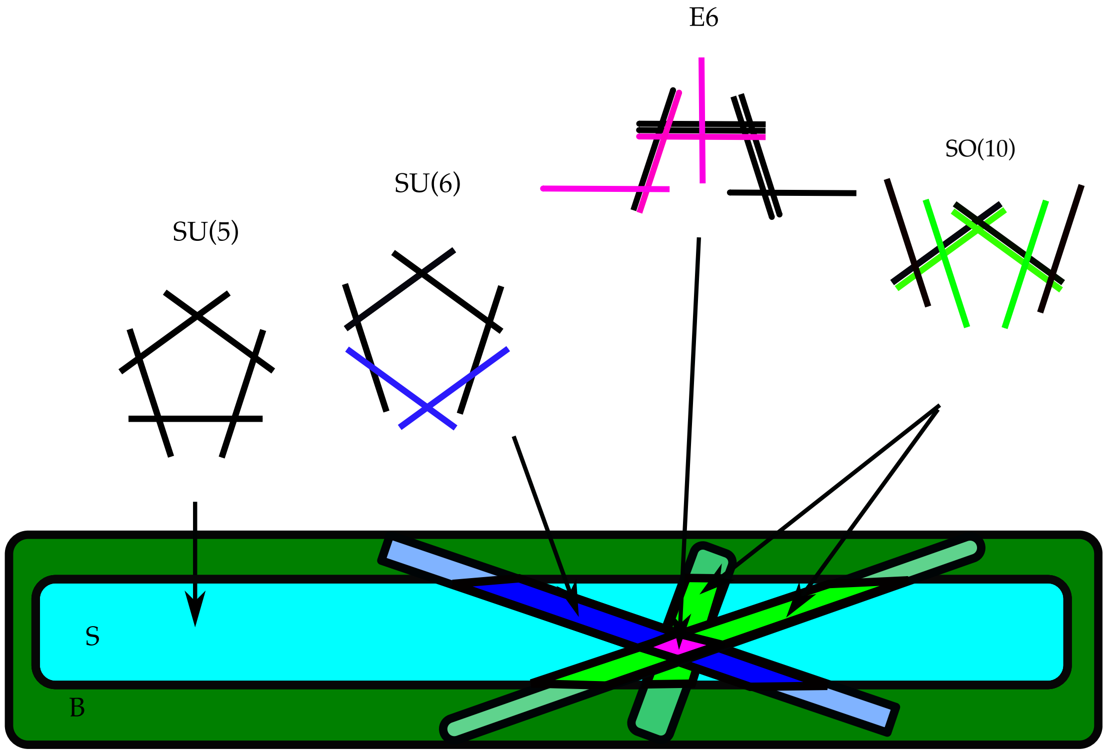
⇒ Type of Yukawa coupling determined from singular fiber above p

$$G_p \longrightarrow SU(5) \times U(1)_1 \times U(1)_2$$



$SO(12)$: $\bar{\mathbf{5}}_H \times \bar{\mathbf{5}}_M \times \mathbf{10}_M$ $\quad E_6$: $\mathbf{10}_M \times \mathbf{10}_M \times \mathbf{5}_H$ $\quad SU(7)$: $\mathbf{5} \times \bar{\mathbf{5}} \times \mathbf{1}$

Codimension 3 Fibers



NB: higher codimension fibers have beautiful geometric structure, generalizing classic Kodaira fiber story.

[Lawrie, SSN][Hayashi, Lawrie, SSN][Hayashi, Lawrie, Morrison, SSN][Esole, Shao, Yau]

Additional benefits from resolution: G-flux

G-flux encodes gauge field via ($\omega_i = (1,1)$ forms)

$$G_4 = dC_3 = F_i \wedge \omega_i$$

$\Rightarrow$ Key to get chirality

Four-form $G_4 \in H^{2,2}(Y_4)$, with one leg in fiber and satisfy

$$G \wedge J = 0\,, \qquad G + \frac{1}{2} c_2(Y_4) \in H^4(Y_4, \mathbb{Z})$$

Proper quantization requires c_2. $(2,2)$ forms are dual to surfaces: construct G-fluxes from exceptional divisors of resolution

Breaking the GUT group

$$SU(5) \to SU(3) \times SU(2) \times U(1)_Y$$

1. Wilson lines: flat connections
   $\Rightarrow$ not all 4-dimensional manifolds S allow for these:
   S=Enriques, 5 H_u and H_d [Marsano, Clemens, Pantev, Raby]
   $\Rightarrow$ approach is contrary to initial goal to extract generic features
   $\Rightarrow$ Also: these ALWAYS have exotics!

2. Hypercharge flux [Buican, Malyshev, Morrison, Verlinde, Wijnholt]
   $\Rightarrow$ background gauge field for $U(1)_Y$ (cunningly constructed not to mass up the hypercharge gauge boson) removes Higgs triplets and XY bosons $(\mathbf{3}, \overline{\mathbf{2}})_{-5} \oplus (\overline{\mathbf{3}}, \mathbf{2})_{+5}$
   $\Rightarrow$ Generic construction independent of S
   $\Rightarrow$ $\mathcal{L}_Y$ line bundle, in $U(1)_Y$ direction, such that no zero modes for triples, XY bosons
   $\Rightarrow$ Global constraint: masslessness of $U(1)_Y$

Hypercharge GUT breaking

Requirement: $U(1)_Y$ gauge boson remains massless!
Consider $\mathcal{L}_Y$ background flux. In F-theory CS coupling

$$\int_{Y\times\mathbb{R}^{1,3}} C_4 \wedge G_4 \wedge G_4$$

Expanding $G = (F_Y + c_1(\mathcal{L}_Y)) \wedge \omega_Y$ and $C_4 = C_2^i \wedge \omega^i$, $\omega^i \in H^2(B_3, \mathbb{Z})$

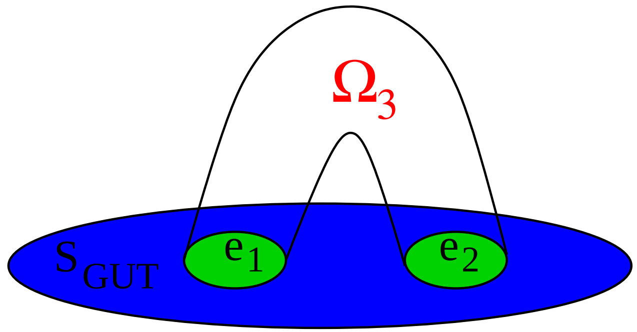
generates mass term for F_Y : $$\left[\mathrm{Tr}(T_Y^2) \int_S c_1(\mathcal{L}_Y) \wedge i^*\omega^i\right] \int_{\mathbb{R}^{1,3}} C_2^i \wedge F_Y$$

This can be avoided if

$$\int_S c_1(\mathcal{L}_Y) \wedge i^*\omega^i = 0 \qquad \forall \omega^i \in H^2(B_3, \mathbb{Z})$$

$U(1)_Y$ remains massless if there is a 3-chain Ω_3 in B_3 whose boundary is the dual inside S of $c_1(\mathcal{L}_Y)$

$$\partial\Omega_3 = e_1 \cup (-e_2)$$



2. F-theory models with $U(1)$ symmetries

$U(1)$ Symmetries

Use $U(1)$ symmetry to forbid tree-level

- dimension 5 proton decay operators (and dim 4 by B-L)

$$W_{dim5} = \frac{Q^3 L}{\Lambda}$$

$\Lambda > 10^{27}$ GeV.

- μ-term (for natural EWSB $\mu \sim$ O(100)GeV, but why?!).

$$W_\mu = \mu H_u H_d$$

Both problem can be addressed with one $U(1)$ (assuming the $U(1)$ is compatible with Yukawa couplings)

$$\Rightarrow \qquad U(1)_{PQ}: \qquad q_{H_u} + q_{H_d} \neq 0$$

Refined Structures

So far we realized the gauge, matter and superpotential couplings for an $N = 1$ GUT model.

- $U(1)$ symmetries:
  Forbid Q^3L dim 5 proton decay operator. In addition: μ term
- GUT breaking:
  Mechanism to break $SU(5) \to SU(3) \times SU(2) \times U(1)$

Combined problem: Local analysis

Step 1 in bottom-up approach: engineer low energy physics of 7-branes

- $SU(5)$ GUT with chiral matter and Yukawas
  $\Rightarrow$ Local gauge theory model (Higgs bundle)

- Hypercharge GUT breaking

- $U(1)$ symmetry that forbids

$$W_\mu \sim \mu H_u H_d \quad \text{and} \quad W_{\text{dim5}} \sim \frac{1}{\Lambda} Q^3 L$$

$\Rightarrow$ Constraints arising from Anomalies and "Globalization".

Anomalies

[Dudas, Palti][Marsano][Dolan, Marsano,SSN][Palti]

F_Y restriction on $\Sigma_{\mathbf{10}}$ and $\Sigma_{\bar{\mathbf{5}}}$ generates chiral spectrum
$\Rightarrow$ Require $G^2_{MSSM} \times U(1)$ mixed anomaly cancellation for additional $U(1)$
$\Rightarrow$ Additional constraints from $U(1)_Y \times U(1)^2$ anomalies
Chiral index:

$$\delta_{\mathbf{R}} = n_{\mathbf{R}} - n_{\bar{\mathbf{R}}} = \int_\Sigma c_1(\mathcal{L}_\Sigma \otimes \mathcal{L}_Y^{Y_{\mathbf{R}}}) = \int_\Sigma \left(c_1(\mathcal{L}_\Sigma) + M_{\mathbf{R}} c_1(\mathcal{L}_Y^{Y_{\mathbf{R}}}) \right)$$

For the **10** and $\bar{\mathbf{5}}$ matter we can write the multiplicities in terms of the chiral index for multiplets in irreducible $SU(3) \times SU(2) \times U(1)_Y$ representations

$$\mathbf{10}: \qquad \delta_{(\mathbf{1},\mathbf{1})_{+1}} = M_{\mathbf{10}} + N_{\mathbf{10}}\,, \quad \delta_{(\mathbf{3},\mathbf{2})_{+1/6}} = M_{\mathbf{10}}\,, \quad \delta_{(\bar{\mathbf{3}},\mathbf{1})_{-2/3}} = M_{\mathbf{10}} - N_{\mathbf{10}}$$

$$\mathbf{5}: \qquad \delta_{(\bar{\mathbf{3}},\mathbf{1})_{+1/3}} = M_{\mathbf{5}}\,, \quad \delta_{(\mathbf{1},\mathbf{2})_{-1/2}} = M_{\mathbf{5}} - N_{\mathbf{5}}\,.$$

Anomalies for model with gauge group $SU(5)\times U(1)^n$ s

- Pure G_{MSSM}

$$\sum_i M_{\mathbf{10}^i} = \sum_j M_{\bar{\mathbf{5}}^j}\,, \qquad \sum_i N_{\mathbf{10}^i} = \sum_j N_{\bar{\mathbf{5}}^j} = 0\,.$$

- $G^2_{MSSM}\times U(1)$ [Marsano]

$$\sum_i q(\mathbf{10}^i)N_{\mathbf{10}^i} = \sum_j q(\bar{\mathbf{5}}^i)N_{\bar{\mathbf{5}}^j}\,.$$

  This constraint is automatically satisfied in spectral cover models.

  [Marsano, Saulina, SSN]

- $U(1)_Y\times U(1)\times U(1)'$: [Palti]

$$3\sum_{\mathbf{10}^i} q(\mathbf{10}^i)q'(\mathbf{10}^i)N_{\mathbf{10}^i} = \sum_j q(\bar{\mathbf{5}}^j)q'(\bar{\mathbf{5}}^j)N_{\bar{\mathbf{5}}^j}\,.$$

Constraints from Anomalies

[Dolan, Marsano, Saulina, SSN]

$G^2_{MSSM} \times U(1)$ mixed anomaly cancellation

$$\sum_i q(\mathbf{10}^i) N_{\mathbf{10}^i} = \sum_j q(\bar{\mathbf{5}}^i) N_{\bar{\mathbf{5}}^j}$$

- Minimal $SU(5)$ GUT
  $\Rightarrow$ the only $U(1)$ compatible is $U(1)_\chi$ ($B - L$ and Y)
  $\Rightarrow q_{H_u} + q_{H_d} = 0$
- If $U(1)$ forbids W_μ and $W_{\text{dim}\,5}$, i.e. $q_{H_u} + q_{H_d} \neq 0$
  $\Rightarrow$ there are always non-GUT exotics
  $\Rightarrow$ Can lift these by $U(1)$-charged GUT-singlet X

$$W_{\text{ex}} = \lambda X f_{\text{ex}} \bar{f}_{\text{ex}}$$

$\Rightarrow$ Highly constrained setup. $U(1)^3$ anomaly still to cancel.

$\Rightarrow$ Consistent global lift?

Elliptic Fibrations with $U(1)$s

Geometric engineering of $U(1)$ $\Leftrightarrow$ "Extra section" of elliptic fiberation

Section: Map $\sigma : B \longrightarrow \mathbb{E}_\tau$.

Why? $\sigma = 0$ divisor, dual to $(1,1)$ form in fiber:

$$C_3 = \sum_i \omega^{(1,1)} \wedge A \qquad \Rightarrow \quad U(1) \text{ gauge boson}$$

In practice: rational solutions to the elliptic curve equation. For Weierstrass in $\mathbb{P}^{123}[w, x, y]$ zero section

$$y^2 = x^3 + f x w^4 + g w^6 \qquad \sigma_0 : \quad w = 0,\ x = y = 1\,.$$

Construction of Extra Sections

Analog of Weierstrass models:

- $U(1)$: Embedding into $\mathbb{P}^{112}[w, x, y]$: [Morrison, Park]

$$y^2 + b_0 x^2 y = c_0 w^4 + c_1 w^3 x + c_2 w^2 x^2 + c_3 w x^3$$

Sections: $y = w = 0$ and $w = y + b_0 x^2 = 0$.

- $U(1)^2$: Embedding into $dP_2[w, x, y; l_1, l_2]$:

[Mayrhofer, Palti, Weigand][Cvetic, Klever, Piragua]

$$s_1 l_1^2 l_2^2 w^3 + s_2 l_1^2 l_2 w^2 x + s_3 l_1^2 w x^2 + s_5 l_1 l_2^2 w^2 y + s_6 l_1 l_2 w x y + s_7 l_1 x^2 y + s_8 l_2^2 w y^2 + s_9 l_2 x y^2$$

Sections: $l_1 = 0$; $l_2 = 0$; $x = s_9\,,\ y = -s_7$

GUT models with extra $U(1)$s: Toric Models

- Toric Tops: Using toric geometry methods, one can generate models e.g. in $\mathbb{P}^{112}$ that have realize along $z = 0$ an $SU(5)$. The extra section guarantees $U(1)$.

  [Braun, Grimm, Keitel][Mayrhofer, Palti, Weigand][Cvetic, Klever, Piragua]

- Only one **10** matter locus.
- Phenomenological implications studied in

  [Krippendorf, Pena, Oehlmann, Ruehle]

However, these are not a complete class of elliptic fibrations with extra section.

$\Rightarrow$ Systematic approach – Tate-like forms for models with extra sections

$$y^2 + b_0(z)x^2y = c_0(z)w^4 + c_1(z)w^3x + c_2(z)w^2x^2 + c_3(z)wx^3$$

[Kuentzler, SSN][Lawrie, SSN]

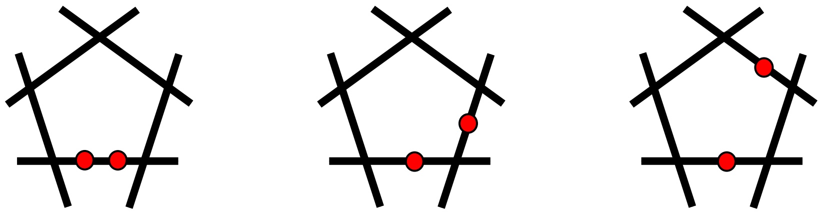
Tate-type models for $SU(5) \times U(1)$

[Kuentzler, SSN]

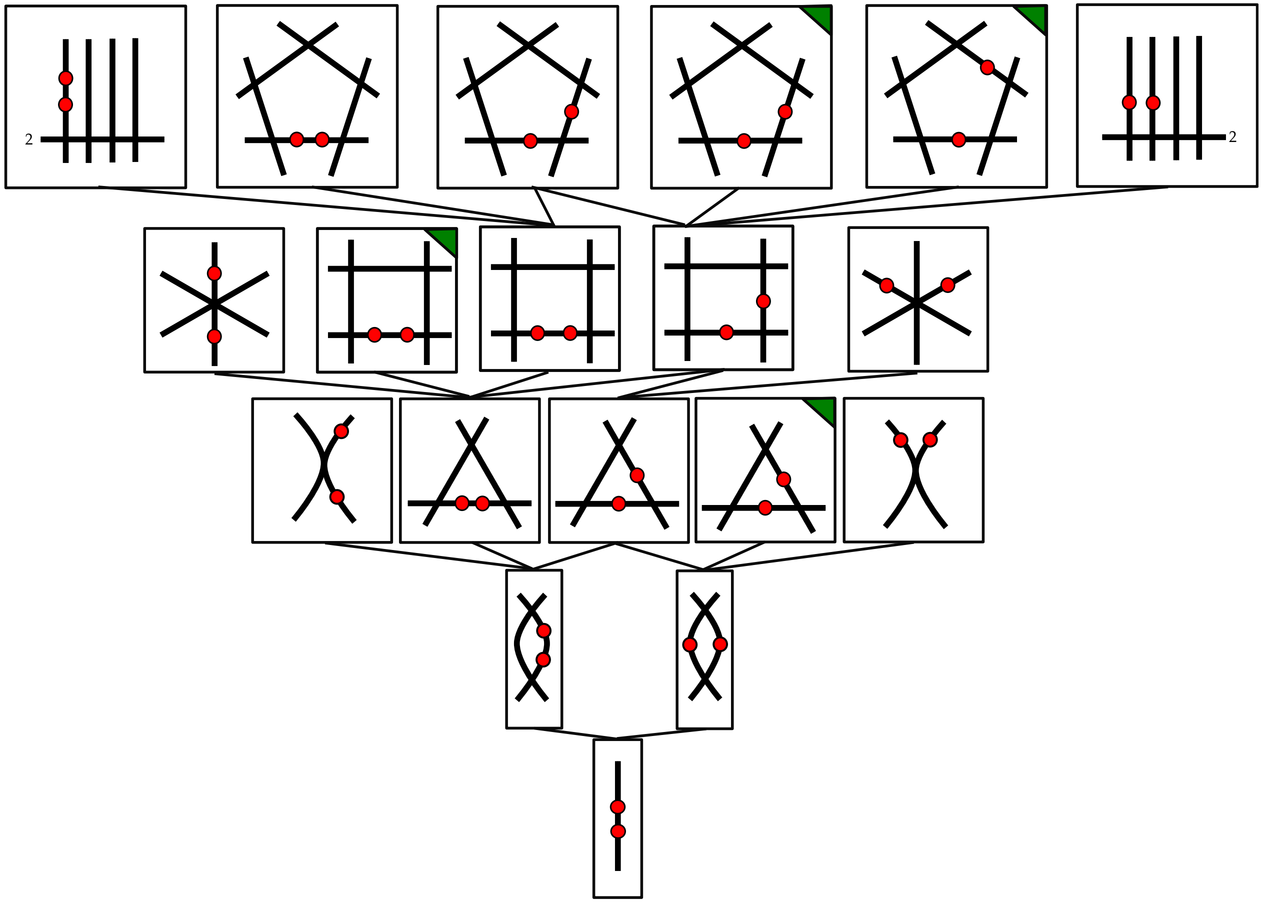
Any model with one $U(1)$ has embedding into $\mathbb{P}^{112}$

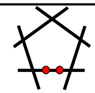
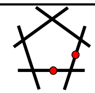
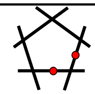
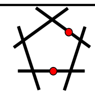
$$y^2 + b_0(z)x^2y = c_0(z)w^4 + c_1(z)w^3x + c_2(z)w^2x^2 + c_3(z)wx^3$$

- Proceed along enhancements of discriminant $\Delta(z)$:
  "solving polynomial eq over UFD"
- Canonical and non-canonical models:
  pure vanishing order (like for $\mathbb{P}^{123}$) or more general forms
- Fibers are characterized by Kodaira fibers and location of sections
  For instance for $SU(5)$:

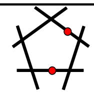
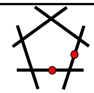


Tate Tree for $SU(5) \times U(1)$



Model	Representation	Fiber Type	Yukawa Couplings
$\mathcal{Q}\,(5,3,1,0,0,0,2)$	$\mathbf{10}_0+\overline{\mathbf{10}}_0$ $\mathbf{5}_{-5}+\overline{\mathbf{5}}_5$ $\mathbf{5}_5+\overline{\mathbf{5}}_{-5}$ $\mathbf{5}_0+\overline{\mathbf{5}}_0$		$\mathbf{10}_0\mathbf{10}_0\mathbf{5}_0$ $\mathbf{10}_0\overline{\mathbf{5}}_5\overline{\mathbf{5}}_{-5}$ $\mathbf{10}_0\overline{\mathbf{5}}_0\overline{\mathbf{5}}_0$
$\mathcal{Q}\,(4,2,1,1,0,0,2)$	$\mathbf{10}_2+\overline{\mathbf{10}}_{-2}$ $\mathbf{5}_6+\overline{\mathbf{5}}_{-6}$ $\mathbf{5}_{-4}+\overline{\mathbf{5}}_4$ $\mathbf{5}_1+\overline{\mathbf{5}}_{-1}$		$\mathbf{10}_2\mathbf{10}_2\mathbf{5}_{-4}$ $\mathbf{10}_2\overline{\mathbf{5}}_{-6}\overline{\mathbf{5}}_4$ $\mathbf{10}_2\overline{\mathbf{5}}_{-1}\overline{\mathbf{5}}_{-1}$
$\mathcal{Q}\,(4,3,2,1,0,0,1)$	$\mathbf{10}_{-3}+\overline{\mathbf{10}}_3$ $\mathbf{5}_6+\overline{\mathbf{5}}_{-6}$ $\mathbf{5}_{-4}+\overline{\mathbf{5}}_4$ $\mathbf{5}_1+\overline{\mathbf{5}}_{-1}$		$\mathbf{10}_{-3}\mathbf{10}_{-3}\mathbf{5}_6$ $\mathbf{10}_{-3}\overline{\mathbf{5}}_{-1}\overline{\mathbf{5}}_4$
$\mathcal{Q}\,(3,2,2,2,0,0,1)$	$\mathbf{10}_1+\overline{\mathbf{10}}_{-1}$ $\mathbf{5}_{-7}+\overline{\mathbf{5}}_7$ $\mathbf{5}_{-2}+\overline{\mathbf{5}}_2$ $\mathbf{5}_3+\overline{\mathbf{5}}_{-3}$		$\mathbf{10}_1\mathbf{10}_1\mathbf{5}_{-2}$ $\mathbf{10}_1\overline{\mathbf{5}}_2\overline{\mathbf{5}}_{-3}$

Models with two $\mathbf{10}$ curves

Model	Representation	Fiber Type	Yukawa Couplings
$\mathcal{Q}\,(3,2,1,1,0,0,1)\|_{P_1}$	$\mathbf{10}_1+\overline{\mathbf{10}}_{-1}$ $\mathbf{10}_{-4}+\overline{\mathbf{10}}_4$ $\mathbf{5}_{-7}+\overline{\mathbf{5}}_7$ $\mathbf{5}_{-2}+\overline{\mathbf{5}}_2$ $\mathbf{5}_3+\overline{\mathbf{5}}_{-3}$		$\mathbf{10}_1\mathbf{10}_1\mathbf{5}_{-2}$ $\mathbf{10}_1\overline{\mathbf{5}}_2\overline{\mathbf{5}}_{-3}$ $\mathbf{10}_{-4}\overline{\mathbf{5}}_7\overline{\mathbf{5}}_{-3}$ $\mathbf{10}_1\mathbf{10}_{-4}\mathbf{5}_3$ $\mathbf{10}_{-4}\overline{\mathbf{5}}_2\overline{\mathbf{5}}_2$
$\mathcal{Q}\,(3,2,1,1,0,0,1)\|_{P_2}$	$\mathbf{10}_2+\overline{\mathbf{10}}_{-2}$ $\mathbf{10}_{-3}+\overline{\mathbf{10}}_3$ $\mathbf{5}_6+\overline{\mathbf{5}}_{-6}$ $\mathbf{5}_{-4}+\overline{\mathbf{5}}_4$ $\mathbf{5}_1+\overline{\mathbf{5}}_{-1}$		$\mathbf{10}_2\mathbf{10}_2\mathbf{5}_{-4}$ $\mathbf{10}_2\overline{\mathbf{5}}_{-6}\overline{\mathbf{5}}_4$ $\mathbf{10}_2\overline{\mathbf{5}}_{-1}\overline{\mathbf{5}}_{-1}$ $\mathbf{10}_{-3}\mathbf{10}_{-3}\mathbf{5}_6$ $\mathbf{10}_{-3}\overline{\mathbf{5}}_4\overline{\mathbf{5}}_{-1}$ $\mathbf{10}_2\mathbf{10}_{-3}\mathbf{5}_1$

where $P_1=b_{1,0}^2c_{0,3}-b_{1,0}b_{2,1}c_{1,2}+b_{2,1}^2c_{2,1}$.

Pheno (in progress)

- Single $U(1)$ models with one **10**: equivalent to toric models
  $\Rightarrow$ can realize PQ but problems with hypercharge
- Single $U(1)$ models with multiple **10**: new models
  $\Rightarrow$ PQ symmetries and more freedom in assigning matter.
- Multiple $U(1)$ models: can reproduce toric models, but way more options with multiple **10**s.

Unlike local models: these are globally consistent, and the Tate-like models form a comprehensive list of all $G \times U(1)^n$ gauge groups with matter and charges in F-theory.

3. F-theory GUT model building

Global questions:

- Lifting local models:
  realization of GUT dof's, matter, Yukawas, chirality (G-flux)
  $\Rightarrow$ Resolution of singular elliptic CY4

- $U(1)$ symmetries:
  symmetries to protect from dim 5 proton decay, μ-term.
  $\Rightarrow$ Global $U(1)$ requires elliptic fibrations with extra sections
  ("ensures additional $\omega^{1,1}$ form")
  $\Rightarrow$ New Tate forms for elliptic fibrations with extra sections

- Global hypercharge flux:
  Open issue: resolve tension between trivial F_Y class in CY4, and
  required non-trivial restriction on Higgses.

Current F-theory slogan:

"to address pheno questions comprehensively, requires answering their reformulation in algebraic geometry"