# Cabibbo's dream

Belén Gavela

(Alonso, Gavela, D.Hernandez, Merlo, Rigolin)

(Alonso, Gavela, Isidori, Maiani)

# Cabibbo's dream:

# dynamical origin of mass

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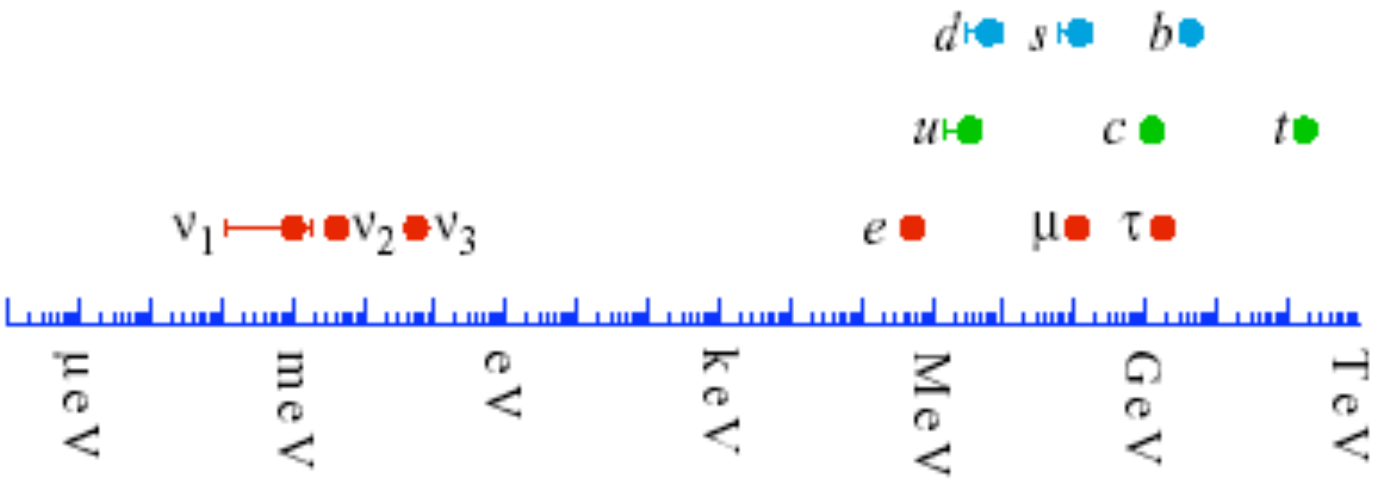
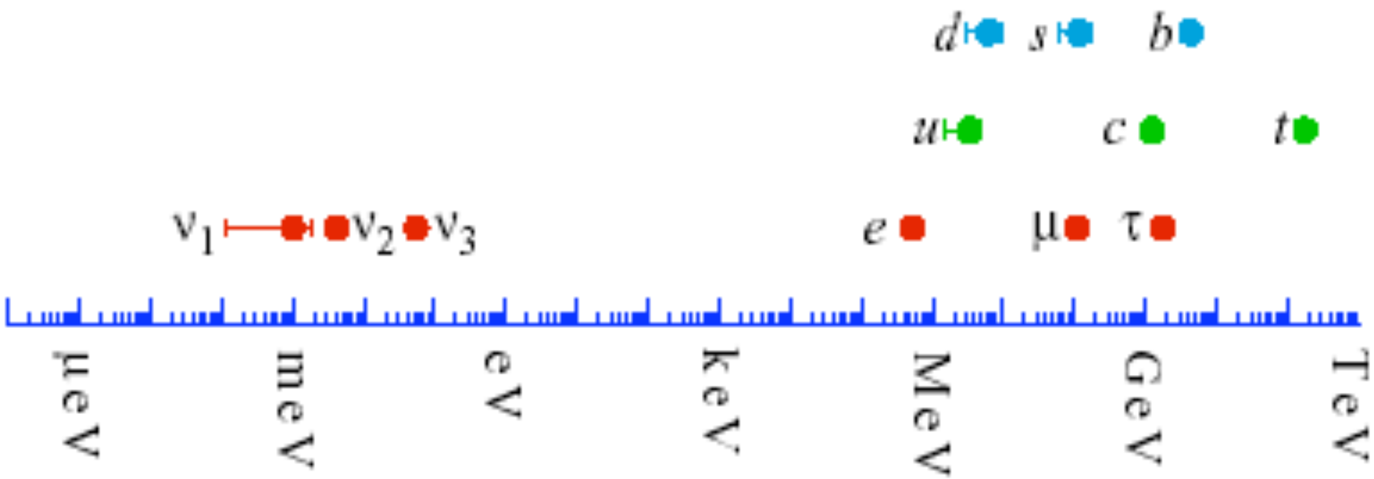
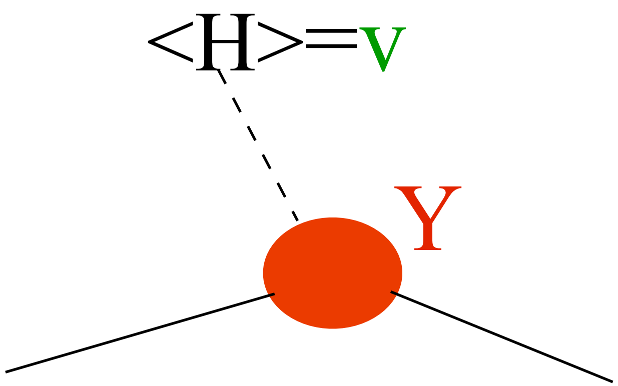
## Masses of the matter particles of the visible universe



## Masses of the matter particles of the visible universe



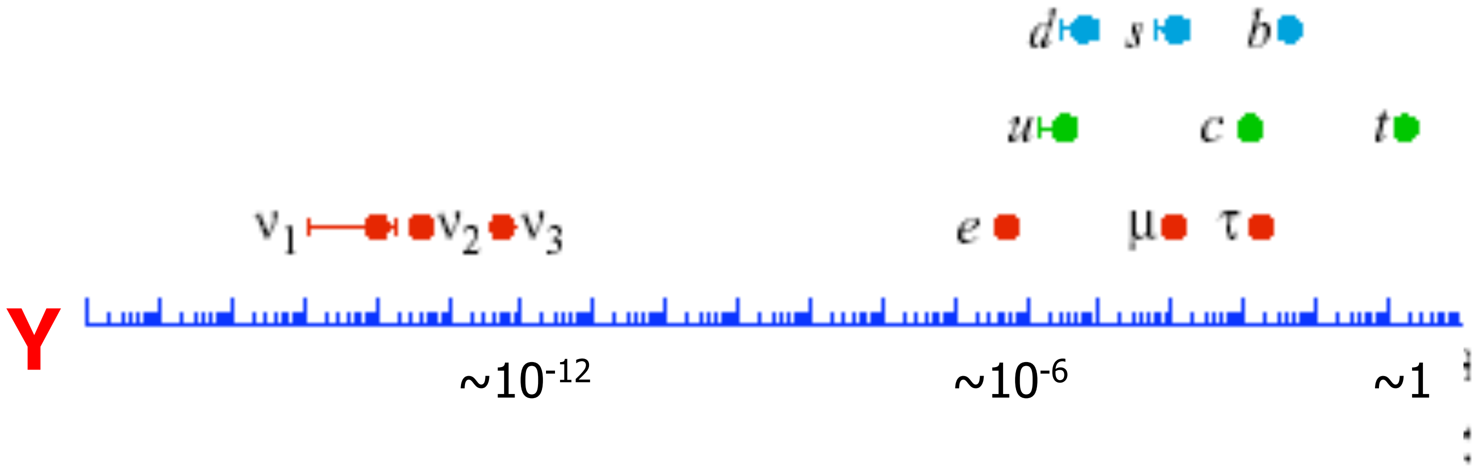

## Fermion masses in SM:



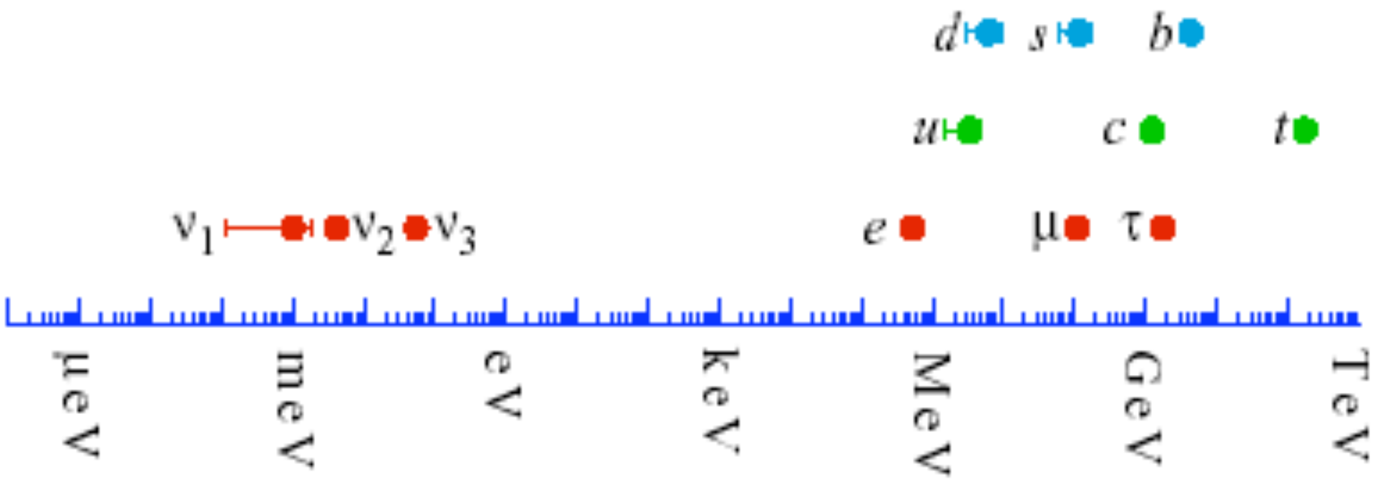

$$m \sim Y\, v$$

**YUKAWAs** are numbers

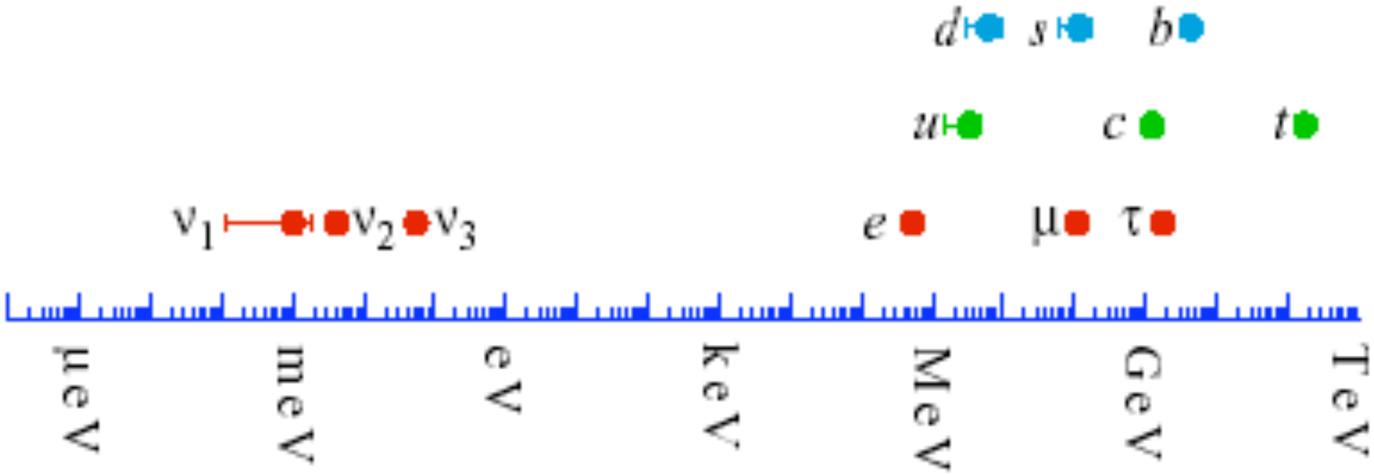
## The mass spectrum in terms of YUKAWA couplings



# Neutrino light on flavour ?




**Neutrinos lighter because Majorana?**

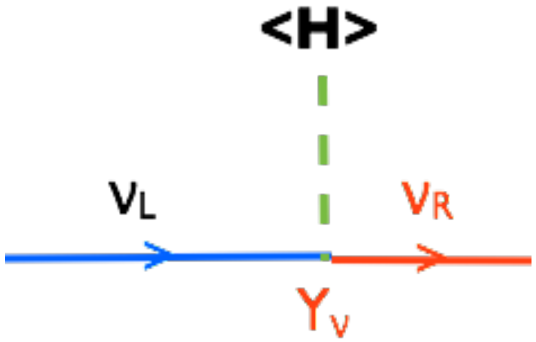



**Neutrinos lighter because Majorana?**

$$\nu = \bar{\nu}$$

# Simple case: add right-handed neutrino to SM

$$\delta\mathcal{L}_m = Y_\nu \, \overline{L} \, H \, \nu_R + h.c.$$



S
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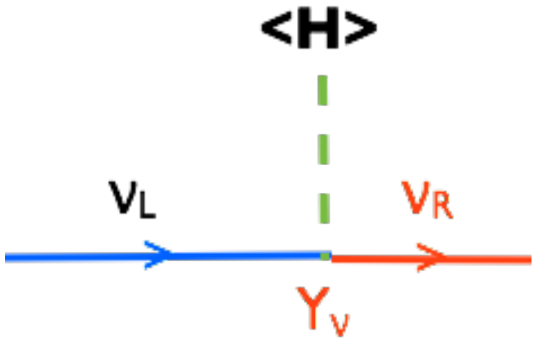
# Simple case: add right-handed neutrino to SM

$$\delta\mathcal{L}_m = Y_\nu \, \overline{L} \, H \, \nu_R + h.c. + M \, \overline{{\nu_R}^C} \, \nu_R$$

allowed by SM gauge invariance



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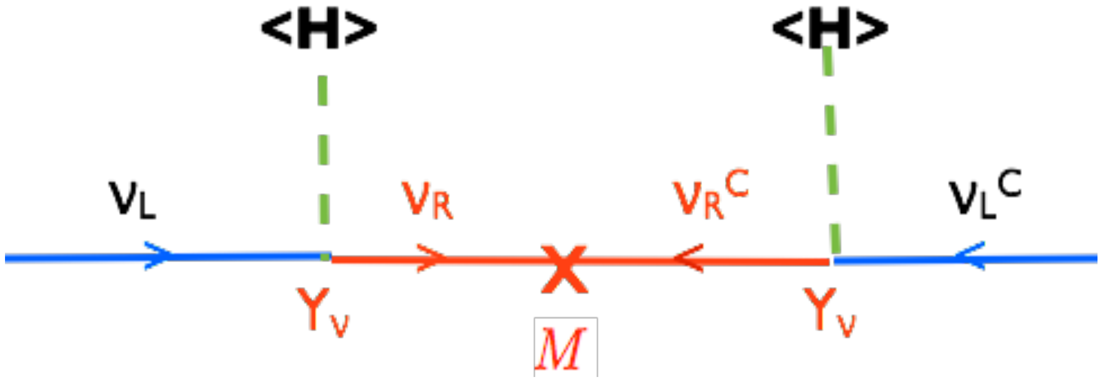

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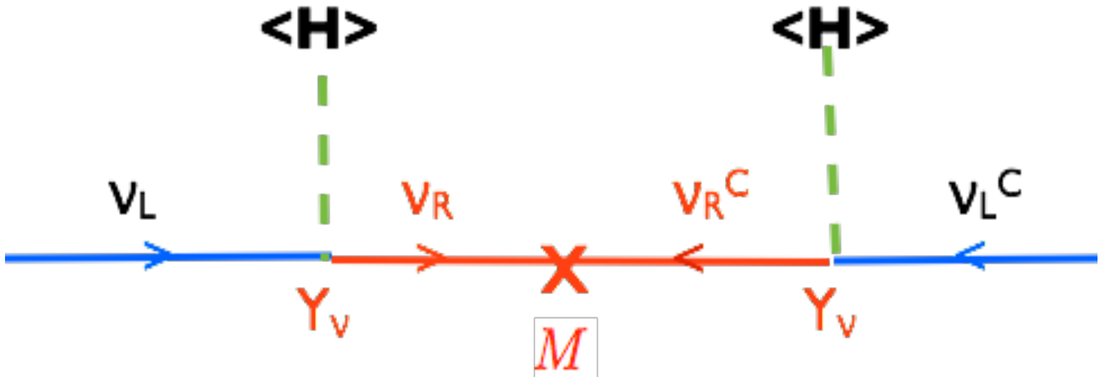


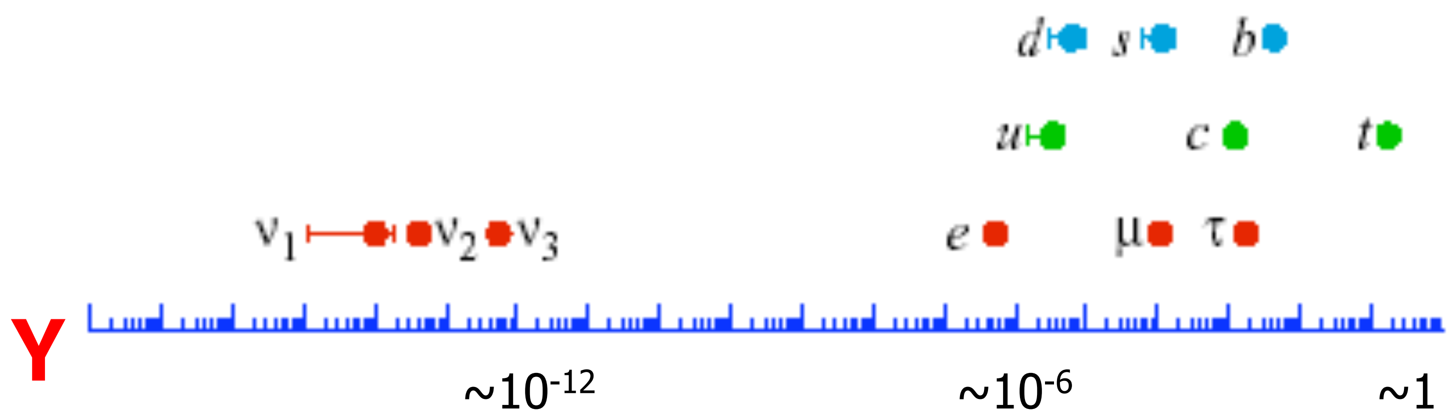

Seesaw type-I

$$\longrightarrow m_\nu \sim \frac{Y_\nu^2 \, v^2}{2M}$$

Which allows $Y_\nu \sim 1$ --> $M \sim M_{Gut}$

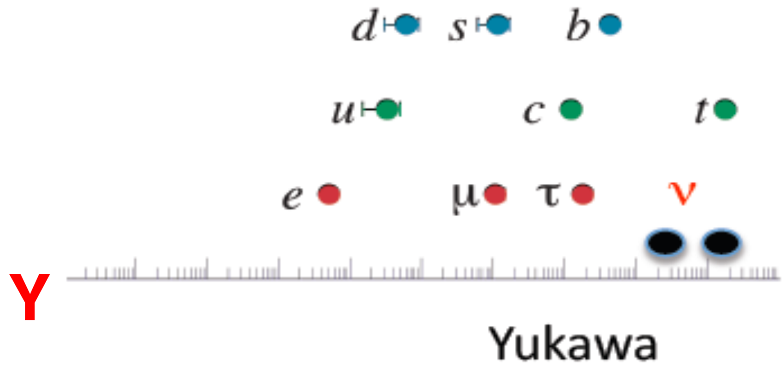
$Y_\nu \sim 10^{-6}$ --> $M \sim TeV$

## In pure SM, the mass spectrum in terms of YUKAWA couplings:

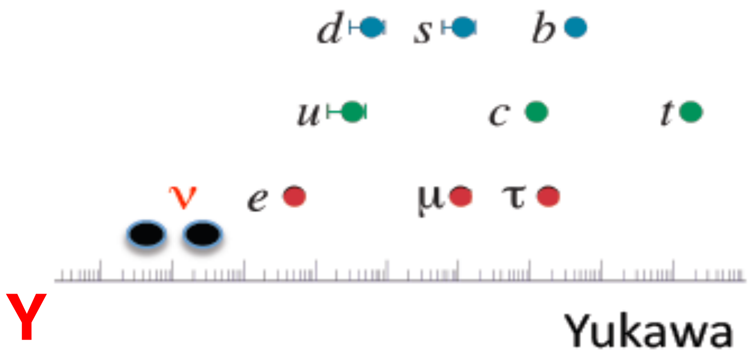


**Within seesaw**, the size of neutrino Yukawa couplings is similar to that for other fermions:

M $\leq$ GUT



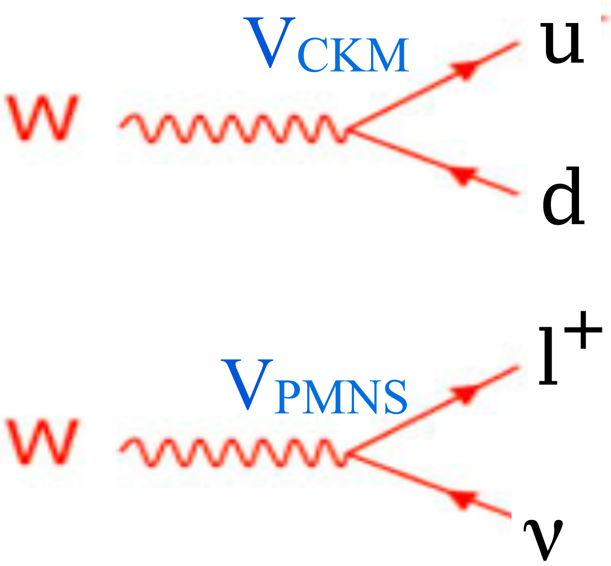

M = TeV




Minkowski; Gell-Mann, Ramond Slansky; Yanagida, Glashow...

# Neutrino oscillations demonstrated

# leptonic flavour violation in nature



$$U=\begin{pmatrix} c_{13}c_{12} & s_{12}c_{13} & s_{13}e^{i\delta} \\ -s_{12}c_{23}-s_{23}s_{13}c_{12}e^{-i\delta} & c_{12}c_{23}-s_{23}s_{13}s_{12}e^{-i\delta} & s_{23}c_{13} \\ s_{23}s_{12}-c_{23}s_{13}c_{12}e^{-i\delta} & -s_{23}c_{12}-c_{23}s_{13}s_{12}e^{-i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} e^{i\alpha} & & \\ & e^{i\beta} & \\ & & 1 \end{pmatrix}$$

DIRAC O MAJORANA

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**Leptons**

$$V_{PMNS} = \begin{pmatrix} 0.8 & 0.5 & \sim 9^{o} \\ -0.4 & 0.5 & -0.7 \\ -0.4 & 0.5 & +0.7 \end{pmatrix}$$

**Quarks**

$$V_{CKM} = \begin{pmatrix} \sim 1 & \lambda & \lambda^3 \\ \lambda & \sim 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & \sim 1 \end{pmatrix} \quad \lambda \sim 0.2$$

**Why so different?**

**Leptons**

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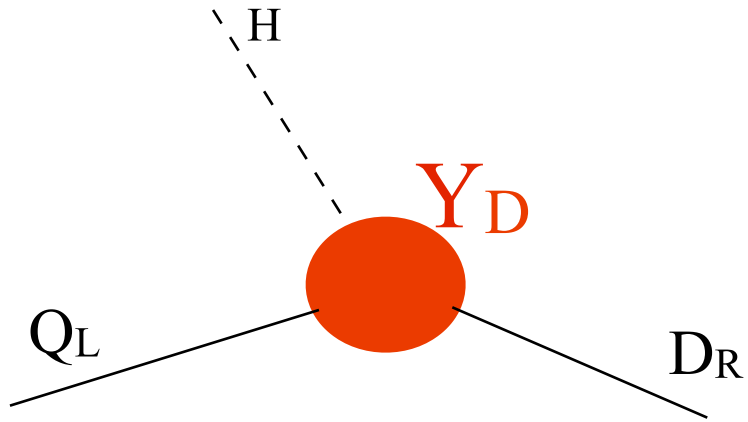
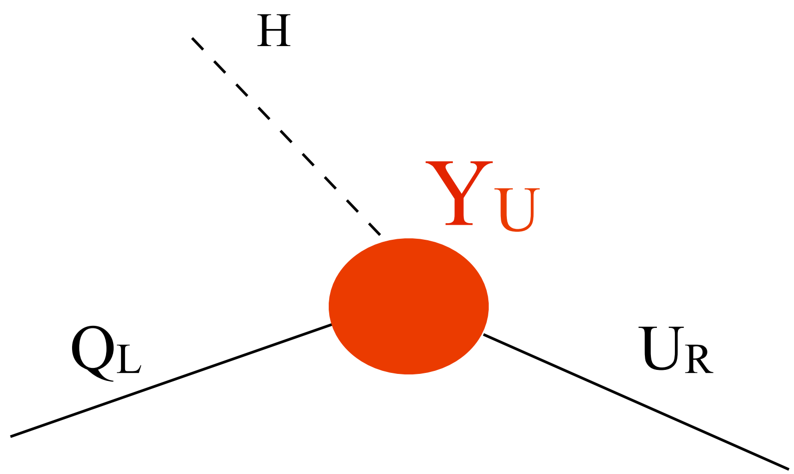
**Perhaps also because $\nu_s$ may be Majorana?**

- Dynamical Yukawas

# Yukawa couplings are the source of flavour in the SM

$$\delta\mathcal{L}_m = Q\, Y_d\, H\, d \; + Q\, Y_u \tilde{H}\, u + \text{h.c.}$$

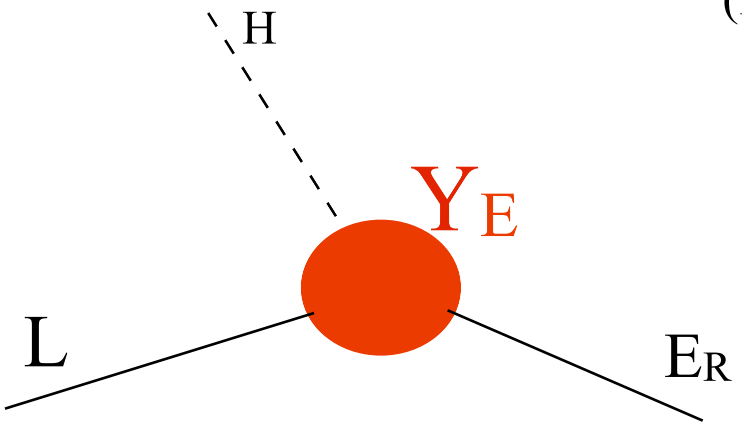


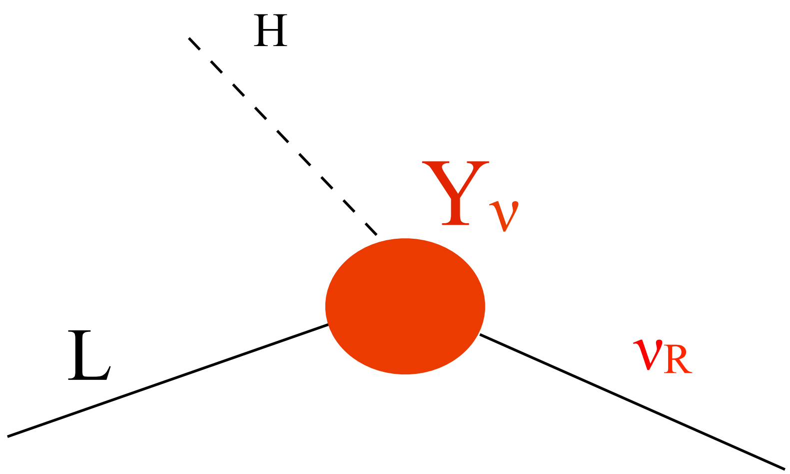



# Yukawa couplings are the source of flavour in the νSM

$$\delta\mathcal{L}_m = Y_\nu \, \overline{L}\, H\, \nu_R + h.c. + M\, \overline{\nu_R{}^C}\, \nu_R$$

(i.e. Seesaw type-I)



**May they correspond to dynamical fields (e.g. vev of fields that carry flavor) ?**

Many attempts: discrete symmetries,
continuous symmetries (Frogatt-Nielsen !!)...

**Instead of inventing an ad-hoc symmetry group,**

**why not use the continuous flavour group**

**suggested by the SM itself?**

**We have realized that the different pattern for**

**quarks versus leptons**

**may be a simple consequence of the**

**continuous flavour group of the SM (+ seesaw)**

(Alonso, Gavela, D.Hernandez, Merlo, Rigolin)

(Alonso, Gavela, Isidori, Maiani)

**We have realized that the different pattern for**

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**may be a simple consequence of the**

**continuous flavour group of the SM (+ seesaw)**

Our guideline is to use:

- maximal symmetry
- minimal field content

(Alonso, Gavela, Isidori, Maiani)

# Global flavour symmetry of the SM

* QCD has a global -chiral- symmetry in the limit of massless quarks.
  For n generations:

$$\mathcal{L}_{QCD}^{\text{fermions}} = \bar{\Psi}(i\not{D} - m)\Psi \rightarrow \bar{\Psi} i\not{D}\Psi = \overline{\Psi_L} i\not{D}\Psi_L + \overline{\Psi_R} i\not{D}\Psi_R$$

$$SU(n)_L \times SU(n)_R \times U(1)'s$$

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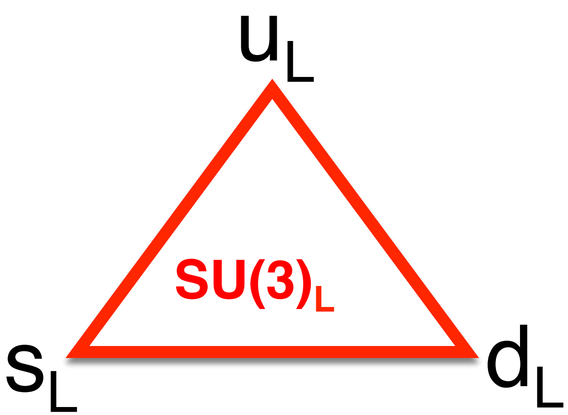
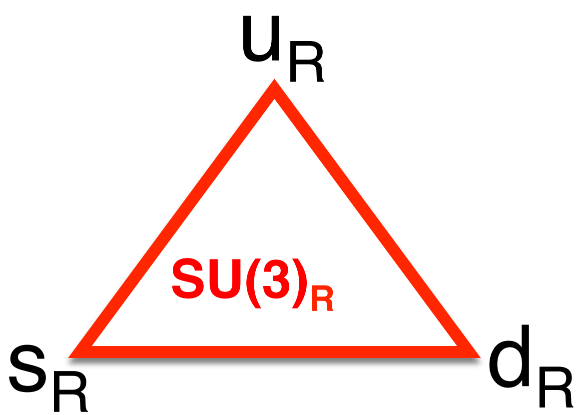
$$SU(n)_L \times SU(n)_R \times U(1)'s$$

e.g., for n=3 : u, d, s ,  massless QCD Lag. is invariant interchanging:




or



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$$SU(n)_L \times SU(n)_R \times U(1)'s$$

* In the SM, fermion masses and mixings result from Yukawa couplings. For massless quarks, the SM has a global flavour symmetry:

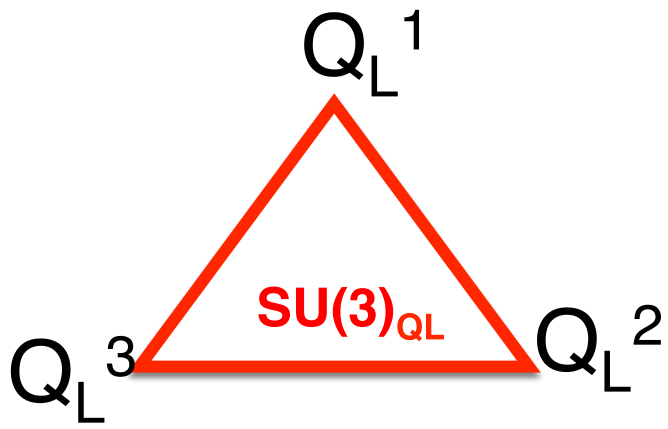
Quarks

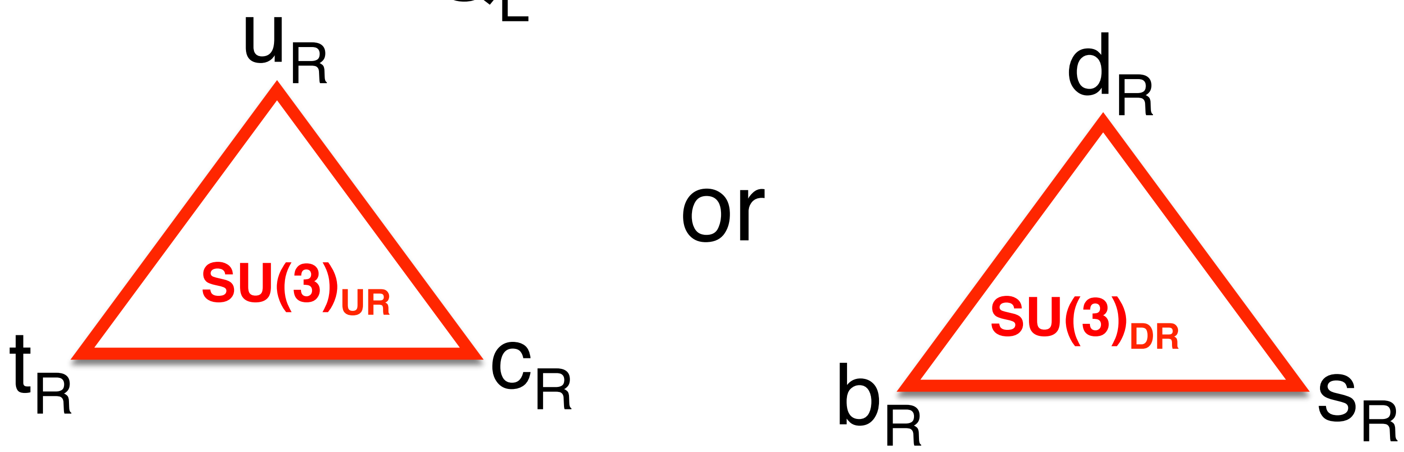
$$\mathcal{L}^{\text{fermions}}_{\text{SM}} = i \sum_{\psi=Q_L}^{D_R} \overline{\psi} \not{D} \psi \qquad G_{\text{flavour}} = U(n)_{Q_L} \times U(n)_{U_R} \times U(n)_{D_R}$$

[Georgi, Chivukula, 1987]

There are n=3 quark families. **With null Yukawa couplings,** the SM Lag. is invariant under $SU(3)_{QL} \times SU(3)_{UR} \times SU(3)_{DR} \times U(1)$'s

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \begin{pmatrix} c_L \\ s_L \end{pmatrix} \begin{pmatrix} t_L \\ b_L \end{pmatrix}$$



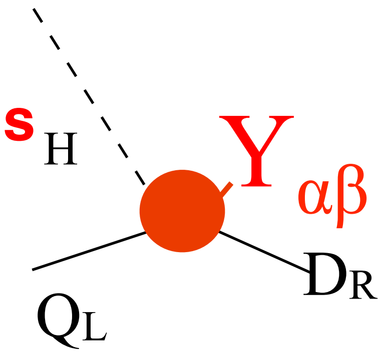
**This continuous global symmetry of the SM**

$$G_{\text{flavour}} = U(n)_{Q_L} \times U(n)_{U_R} \times U(n)_{D_R}$$

**is phenomenologically very successful and**

**at the basis of** *Minimal Flavour Violation*

in which the Yukawa couplings are only **spurions**




(Chivukula+Georgi 87; Hall+Randall; D'Ambrosio+Giudice+Isidori+Strumia; Cirigliano+Isidori+Grisntein+Wise; Davidson+Pallorini; Kagan+G. Perez + Volanski+Zupan,... )

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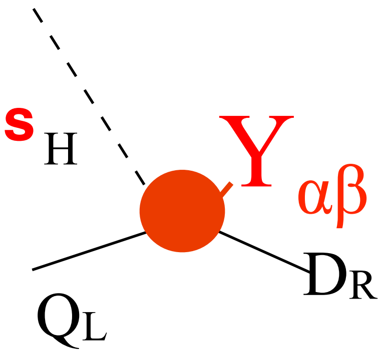
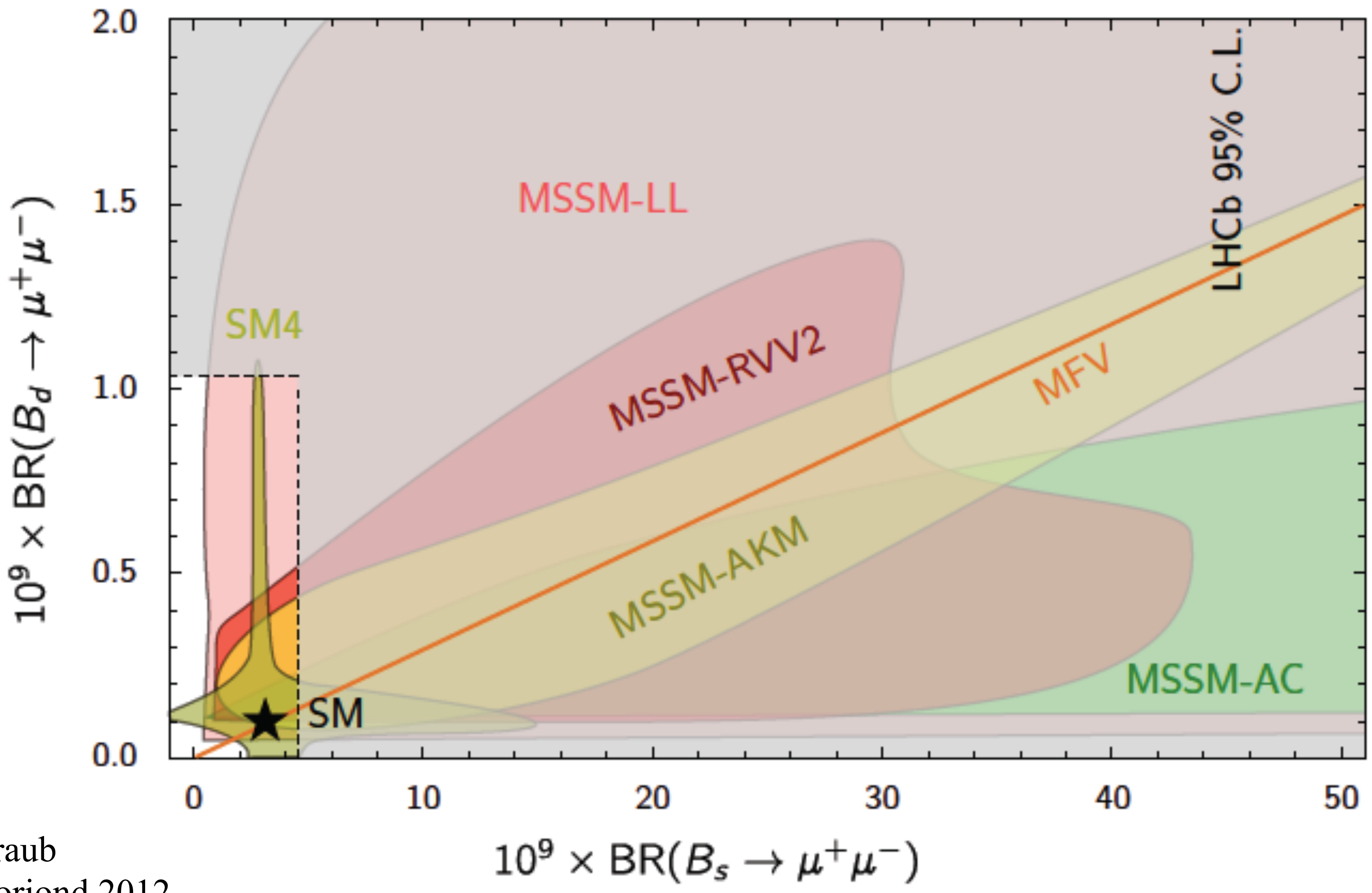



$$\frac{Y_{\alpha\beta}^{+}\, Y_{\delta\gamma}}{\Lambda_f^2}\, \overline{Q}_\alpha\, \gamma_\mu Q_\beta\, \overline{Q}_\gamma\, \gamma^\mu\, Q_\delta$$

(Chivukula+Georgi 87; Hall+Randall; D'Ambrosio+Giudice+Isidori+Strumia; Cirigliano+Isidori+Grisntein+Wise; Davidson+Pallorini; Kagan+G. Perez + Volanski+Zupan,... )

**AFTER LHCb 2012**




Straub
Moriond 2012

## One step further :

dynamical $Y$s

(Alonso, Gavela, D.Hernandez, Merlo, Rigolin)

(Alonso, Gavela, Isidori, Maiani)

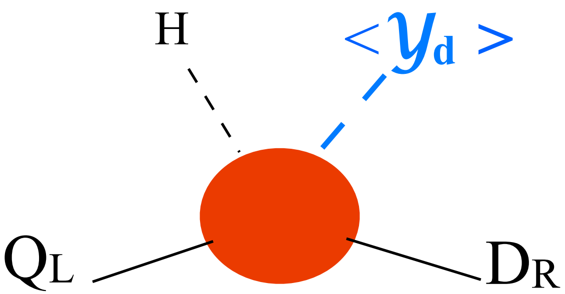
# Quarks

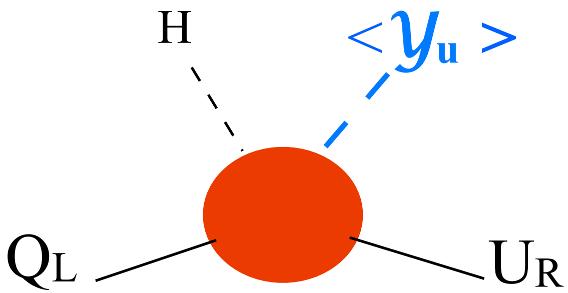
For this talk:

## each $Y_{SM}$ -- >one single field $\mathcal{Y}$

$$Y_{SM} \sim \frac{<\mathcal{Y}>}{\Lambda_f}$$

quarks:




Anselm+Berezhiani ; Berezhiani+Rossi ... Alonso+Gavela+Merlo+Rigolin ...

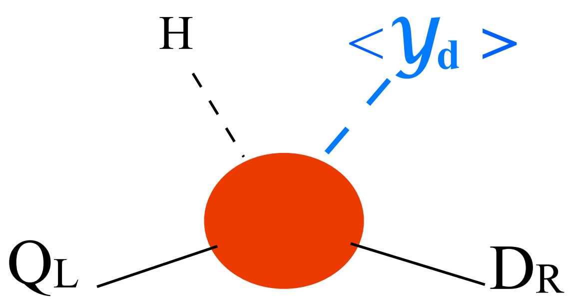
$$G_{flavour} = SU(3)_{Q_L} \times SU(3)_{U_R} \times SU(3)_{D_R} \quad ....$$

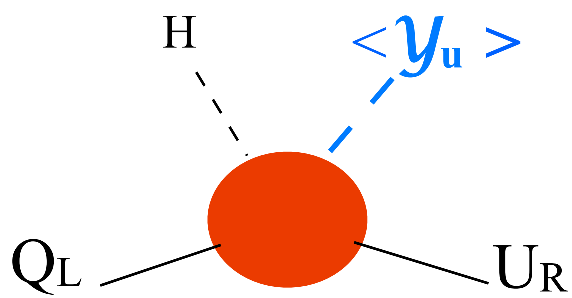
For this talk:

each $Y_{SM}$ -- >one single field $\mathcal{Y}$

$$Y_{SM} \sim \frac{<\mathcal{Y}>}{\Lambda_f}$$

quarks:




$\mathcal{Y}_d \sim (3,1,\bar{3})$ "bifundamentals" $\mathcal{Y}_u \sim (3,\bar{3},1)$

$G_{flavour} = SU(3)_{Q_L} \times SU(3)_{U_R} \times SU(3)_{D_R}$ ....

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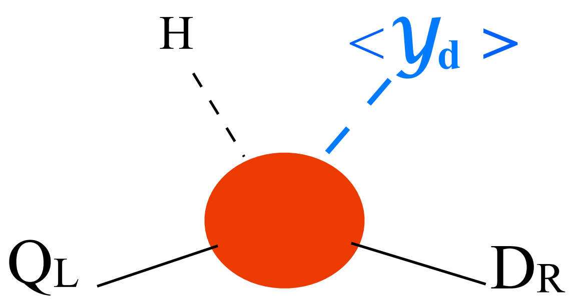
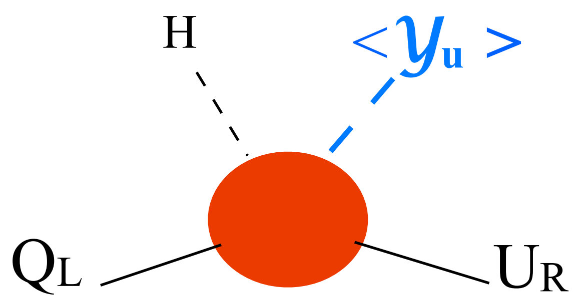
$\mathcal{Y}_d \sim (3,1,\bar{3})$ $\qquad\qquad$ $\mathcal{Y}_u \sim (3,\bar{3},1)$

**That is, two dynamical scalars**



$$G_{flavour} = SU(3)_{Q_L} \times SU(3)_{U_R} \times SU(3)_{D_R} \; ....$$

$\mathcal{Y}_d \sim (3,1,\bar{3})$     $\mathcal{Y}_u \sim (3,\bar{3},1)$

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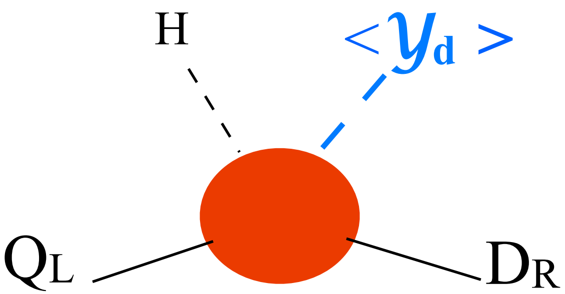
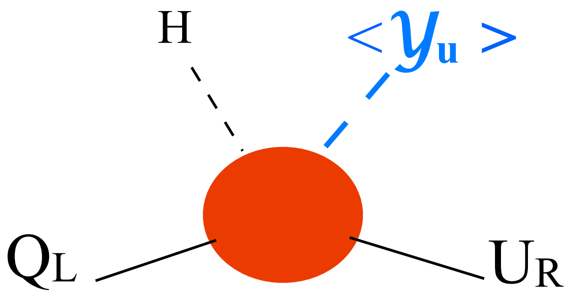


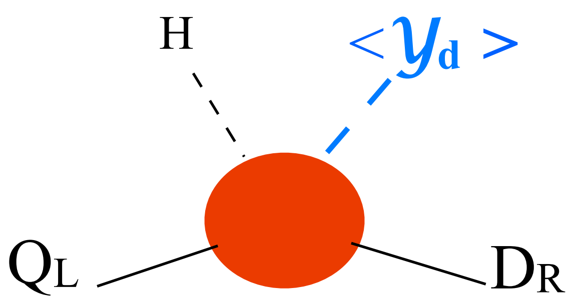



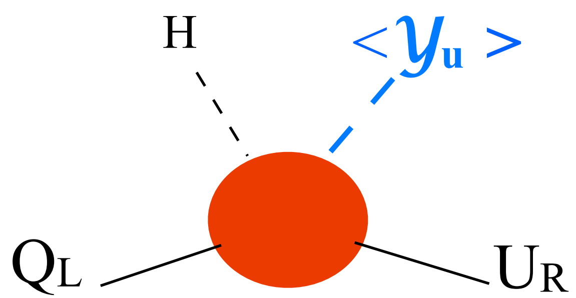

## ¿$V(\mathcal{Y}_d,\ \mathcal{Y}_u)$?

$$G_{flavour} = SU(3)_{Q_L} \times SU(3)_{U_R} \times SU(3)_{D_R} \quad \ldots.$$

$\mathcal{Y}_d \sim (3,1,\bar{3})$ $\qquad\qquad$ $\mathcal{Y}_u \sim (3,\bar{3},1)$

**That is, two dynamical scalars**




*** Does the minimum of the scalar potential justify the observed masses and mixings?**

$$\mathcal{Y}_d \sim (3, \bar{3}, 1) \qquad\qquad \mathcal{Y}_u \sim (3, 1, \bar{3})$$

$$\boxed{\frac{<\mathcal{Y}_d>}{\Lambda_f} = Y_D = V_{CKM} \begin{pmatrix} y_d & 0 & 0 \\ 0 & y_s & 0 \\ 0 & 0 & y_b \end{pmatrix}}, \quad \boxed{\frac{<\mathcal{Y}_u>}{\Lambda_f} = Y_U = \begin{pmatrix} y_u & 0 & 0 \\ 0 & y_c & 0 \\ 0 & 0 & y_t \end{pmatrix}}$$

$$V(\mathcal{Y}_d, \mathcal{Y}_u)$$

* Invariant under the SM gauge symmetry

* Invariant under its global flavour symmetry $G_{flavour}$

$$G_{flavour} = U(3)_{Q_L} \times U(3)_{U_R} \times U(3)_{D_R}$$

**The basis of the game is to find the minima of the invariants that you can construct out of Yukawa couplings**

L. Michel+Radicati 70, Cabibbo+Maiani71 for the spectrum of masses

List of possible invariants for quarks: Hanani, Jenkins, Manohar 2010

**The basis of the game is to find the minima of the invariants that you can construct out of Yukawa couplings**

L. Michel+Radicati 70, Cabibbo+Maiani71 for the spectrum of masses

# Cabibbo's dream

$$\mathbf{V}(\mathcal{Y}_d,\ \mathcal{Y}_u)$$

* Invariant under the SM gauge symmetry

* Invariant under its global flavour symmetry $G_{flavour}$

$$G_{flavour} = U(3)_{Q_L} \times U(3)_{U_R} \times U(3)_{D_R}$$

**There are as many independent invariants I as physical variables**

$$\mathbf{V}(\mathcal{Y}_d,\ \mathcal{Y}_u) = \mathbf{V}(\ \mathbf{I}(\mathcal{Y}_d,\ \mathcal{Y}_u))$$

# Minimization

a variational principle fixes the vevs of Fields

$$\delta V = 0$$

$$\sum_j \frac{\partial I_j}{\partial y_i}\frac{\partial V}{\partial I_j} \equiv J_{ij}\frac{\partial V}{\partial I_j} = 0\,,$$

masses, mixing angles etc.

This is an homogenous linear equation;

if the rank of the Jacobian $J_{ij} = \partial I_j / \partial y_i,$ is:

**Maximum:**

then the only solution is: $\frac{\partial V}{\partial I_j} = 0\,,$

**Less than Maximum:**

then the number of equations reduces to a number equal to the rank

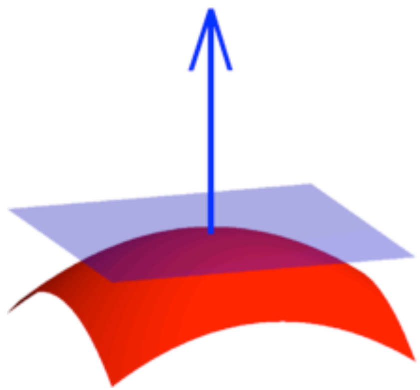
# Boundaries

for a reduced rank of the Jacobian,

$$\det(J) = 0$$

there exists (at least) a direction $\delta y_i$ for which a variation of the field variables does not vary the invariants

$$\delta I_j = \sum_i \frac{\partial I_j}{\partial y_i} \delta y_i = 0$$



that is a Boundary of the *I-manifold*

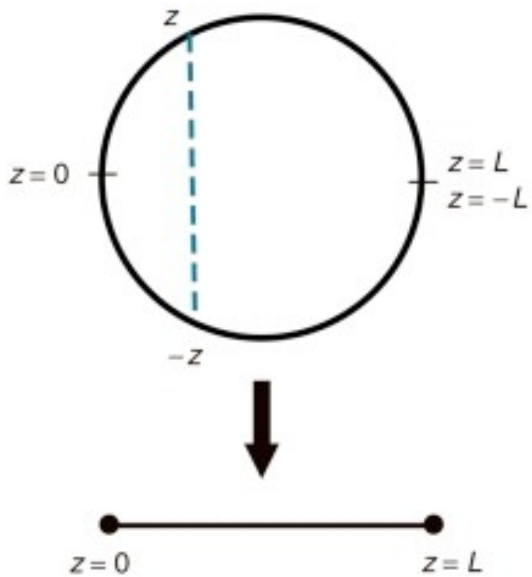
[Cabibbo, Maiani,1969]

$\boxed{\text{Boundaries Exhibit Unbroken Symmetry}}$ [Michel, Radicati, 1969]

(maximal subgroups)

# Boundaries Exhibit Unbroken Symmetry

## Extra-dimensions Example




*The smallest boundaries are*
*extremal points of any function*

[Michel, Radicati, 1969]

# Well before the electroweak SM: masses

## Jacobian Analysis: [40 years ago…]

Breaking of $SU(3) \times SU(3)$ [Cabibbo, Maiani]

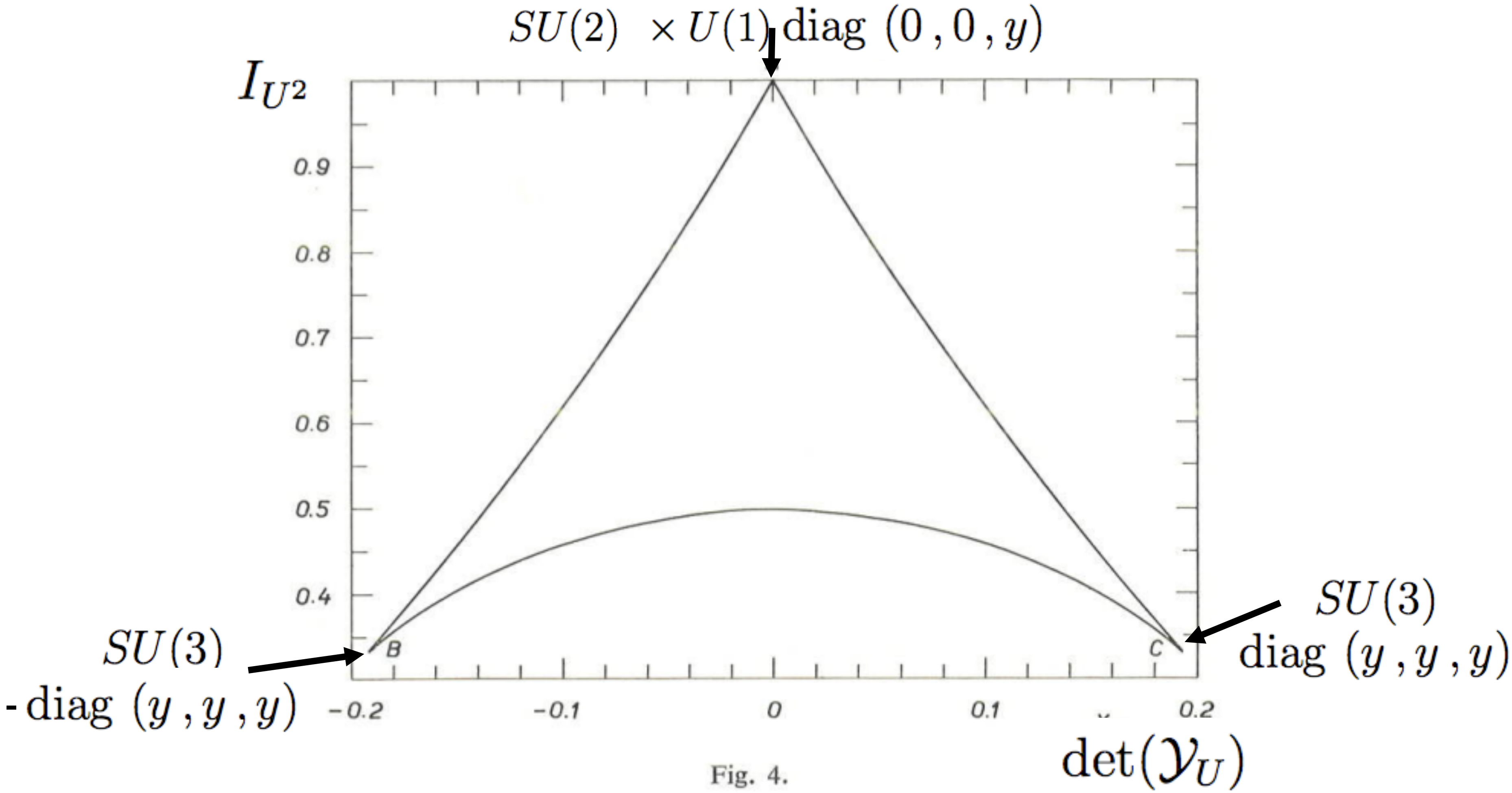



Fig. 4.

quark case

# Bi-fundamental Flavour Fields

For quarks: 10 independent invariants (because 6 masses+ 3 angles + 1 phase) that we may choose as

$$I_U = \mathrm{Tr}\left[\mathcal{Y}_U\mathcal{Y}_U^\dagger\right], \qquad I_D = \mathrm{Tr}\left[\mathcal{Y}_D\mathcal{Y}_D^\dagger\right],$$

$$I_{U^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_U\mathcal{Y}_U^\dagger\right)^2\right], \qquad I_{D^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_D\mathcal{Y}_D^\dagger\right)^2\right],$$

$$I_{U^3} = \mathrm{Tr}\left[\left(\mathcal{Y}_U\mathcal{Y}_U^\dagger\right)^3\right], \qquad I_{D^3} = \mathrm{Tr}\left[\left(\mathcal{Y}_D\mathcal{Y}_D^\dagger\right)^3\right],$$

$$I_{U,D} = \mathrm{Tr}\left[\mathcal{Y}_U\mathcal{Y}_U^\dagger\mathcal{Y}_D\mathcal{Y}_D^\dagger\right], \qquad I_{U,D^2} = \mathrm{Tr}\left[\mathcal{Y}_U\mathcal{Y}_U^\dagger\left(\mathcal{Y}_D\mathcal{Y}_D^\dagger\right)^2\right],$$

$$I_{U^2,D} = \mathrm{Tr}\left[\mathcal{Y}_U\mathcal{Y}_U^\dagger\left(\mathcal{Y}_D\mathcal{Y}_D^\dagger\right)^2\right], \qquad I_{(U,D)^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_U\mathcal{Y}_U^\dagger\mathcal{Y}_D\mathcal{Y}_D^\dagger\right)^2\right].$$

[Feldmann, Jung, Mannel; Jenkins, Manohar] quark case

# Bi-fundamental Flavour Fields

$$\mathrm{Tr}[\mathbf{y}_U \mathbf{y}_U] = \sum y_\alpha^2$$

$$I_U = \mathrm{Tr}\left[\mathcal{Y}_U \mathcal{Y}_U^\dagger\right] , \qquad I_D = \mathrm{Tr}\left[\mathcal{Y}_D \mathcal{Y}_D^\dagger\right] ,$$

$$I_{U^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger\right)^2\right] , \qquad I_{D^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_D \mathcal{Y}_D^\dagger\right)^2\right] ,$$

$$I_{U^3} = \mathrm{Tr}\left[\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger\right)^3\right] , \qquad I_{D^3} = \mathrm{Tr}\left[\left(\mathcal{Y}_D \mathcal{Y}_D^\dagger\right)^3\right] ,$$

only masses

$$I_{U,D} = \mathrm{Tr}\left[\mathcal{Y}_U \mathcal{Y}_U^\dagger \mathcal{Y}_D \mathcal{Y}_D^\dagger\right] , \qquad I_{U,D^2} = \mathrm{Tr}\left[\mathcal{Y}_U \mathcal{Y}_U^\dagger \left(\mathcal{Y}_D \mathcal{Y}_D^\dagger\right)^2\right] ,$$

$$I_{U^2,D} = \mathrm{Tr}\left[\mathcal{Y}_U \mathcal{Y}_U^\dagger \left(\mathcal{Y}_D \mathcal{Y}_D^\dagger\right)^2\right] , \qquad I_{(U,D)^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger \mathcal{Y}_D \mathcal{Y}_D^\dagger\right)^2\right] .$$

masses and mixings

[Feldmann, Jung, Mannel; Jenkins, Manohar
Alonso, Gavela, Isidori, Maiani]

# Jacobian Analysis: Mixing

$$\det(J_{UD}) = \left(y_u^2 - y_t^2\right)\left(y_t^2 - y_c^2\right)\left(y_c^2 - y_u^2\right)$$
$$\left(y_d^2 - y_b^2\right)\left(y_b^2 - y_s^2\right)\left(y_s^2 - y_d^2\right)$$
$$\times |V_{ud}||V_{us}||V_{cd}||V_{cs}|$$

the rank is reduced the most for:

$V_{CKM}$= PERMUTATION

no mixing: reordering of states

(Alonso, Gavela, Isidori, Maiani)

# Quark Natural Flavour Pattern

*Summarizing*, a possible and natural breaking pattern arises:

$$G_{\text{flavour (quarks)}} \quad : \quad U(3)^3 \to U(2)^3 \times U(1)$$

giving a hierarchical mass spectrum **without mixing**

$$< y_D > = \Lambda_f \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_b \end{pmatrix}, \quad < y_U > = \Lambda_f \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_t \end{pmatrix},$$

a good approximation to the observed Yukawas to order $(\lambda_C)^2$

And what happens for leptons ?

Any difference with Majorana neutrinos?

# Leptons

# Global flavour symmetry of the SM + seesaw

* In the SM, for quarks the maximal global symmetry in the limit of massless quarks was:

$$\mathscr{L}^{\text{quarks}}_{\text{SM}} = i \sum_{\psi=Q_L}^{D_R} \overline{\psi} \not{D} \psi \qquad G_{\text{flavour}} = U(n)_{Q_L} \times U(n)_{U_R} \times U(n)_{D_R}$$

* In SM +type I seesaw, for leptons:

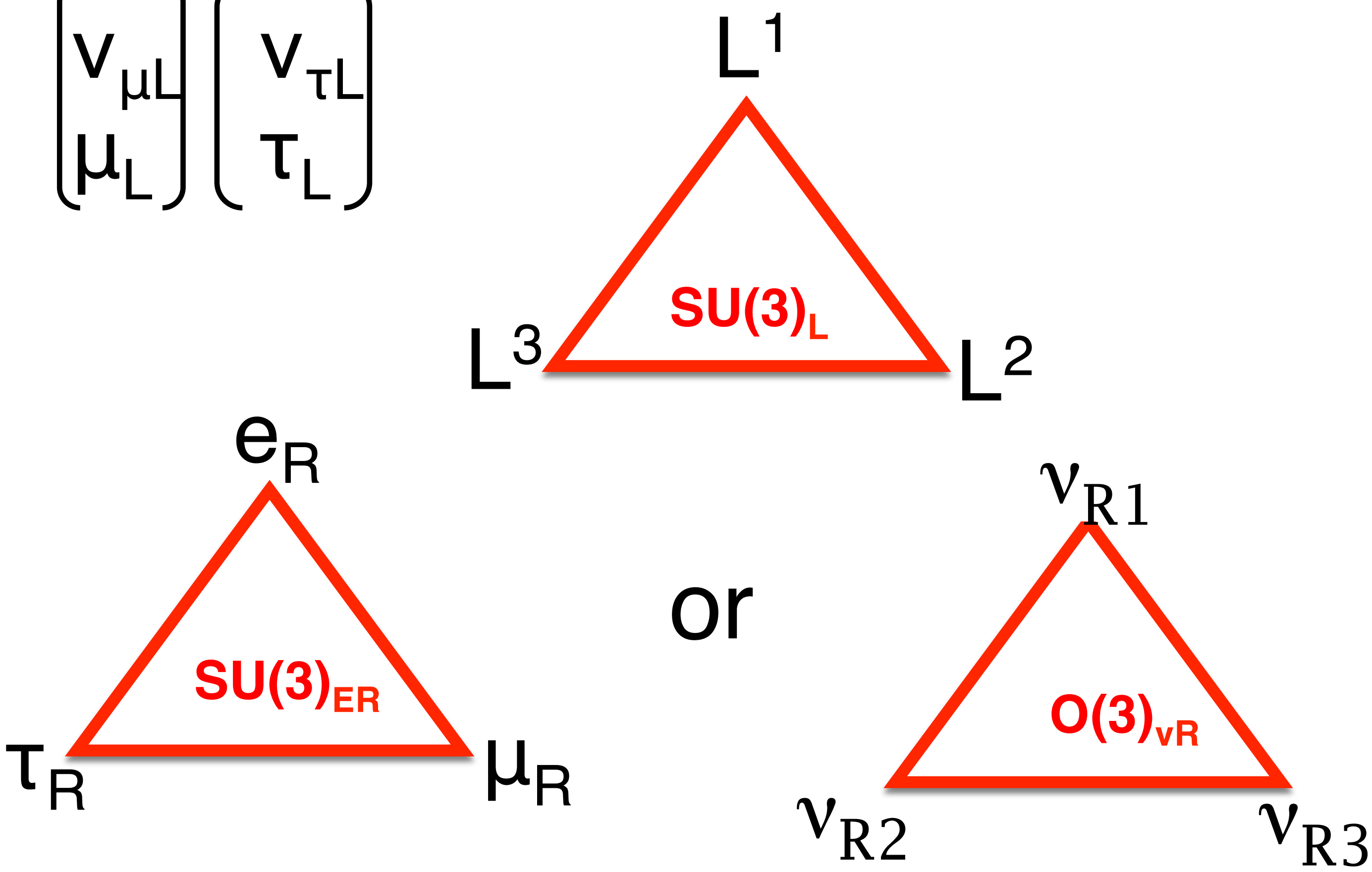
$$\mathcal{L} = \mathcal{L}_{\text{SM}} + Y_\nu \overline{L} H \nu_R + h.c. + M \overline{\nu_R{}^C} \nu_R$$

the maximal leptonic global symmetry in the limit of massless light leptons is $U(n)_L \times U(n)_{E_R} \times O(n)_{\nu_R}$

-> degenerate heavy neutrinos

There are n=3 lepton families. **With null Yukawa couplings,** the maximal symmetry of the Lag. is $SU(3)_L \times SU(3)_{ER} \times O(3)_{NR}$

$$L = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix} \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}$$



Ilustration: 2 families (Casas-Ibarra parametrization)

**for 2 generations**, the mixing terms in $\mathbf{V}(\mathcal{Y}_E, \mathcal{Y}_\nu)$ is :

## Leptons

$$Tr(\mathcal{Y}_E \mathcal{Y}_E^+ \mathcal{Y}_\nu \mathcal{Y}_\nu^+) \propto$$

$$(m_\mu^2 - m_e^2)\left[\cos 2\omega(m_{\nu_2} - m_{\nu_1})\cos 2\theta + 2\sin 2\omega\sqrt{m_{\nu_2} m_{\nu_1}}\, sin2\alpha \sin 2\theta\right]$$

Casas-Ibarra variable in R

where $U_{PMNS} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} e^{-i\alpha} & 0 \\ 0 & e^{i\alpha} \end{pmatrix}$

## Quarks

$$Tr(\mathcal{Y}_u \mathcal{Y}_u^+ \mathcal{Y}_d \mathcal{Y}_d^+) \propto (m_c^2 - m_u^2)(m_s^2 - m_d^2)\cos 2\theta$$

## * For instance for two generations: $O(2)_{NR}$

e.g. two families

$$m_\nu \sim Y_\nu \frac{v^2}{M} Y_\nu^T = y_1 y_2 \frac{v^2}{M} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\boxed{U_{PMNS} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} e^{i\pi/4} & 0 \\ 0 & e^{-i\pi/4} \end{pmatrix}}$$

Degenerate neutrino masses

**Generically, O(2) allows :**

- **one mixing angle maximal**
- **one relative Majorana phase of $\pi/2$**
- **two degenerate light neutrinos**

**Now for three generations and considering all possible independent invariants**

# Bi-fundamental Flavour Fields

Physical parameters
=Independent Invariants

Very direct results using the bi-unitary parametrization:

$$\mathbf{Y_\nu} = \frac{<\mathcal{Y}_\nu>}{\Lambda_f} = \mathcal{U}_L \mathbf{y}_\nu \mathcal{U}_R\,, \qquad \mathbf{Y_E} = \frac{<\mathcal{Y}_E>}{\Lambda_f} = \mathrm{y_E}$$

$$\mathcal{U}_L \mathcal{U}_L^\dagger = 1\,, \quad \mathcal{U}_R \mathcal{U}_R^\dagger = 1\,,$$

$\Rightarrow$ * $m_{e,\mu,\tau} = v\, \mathrm{y_E}$

*But the relation of $\mathcal{Y}_\nu$ with light neutrino masses is through:

$$m_\nu = \mathbf{Y} \frac{v^2}{M} \mathbf{Y}^T$$

# Bi-fundamental Flavour Fields

Physical parameters
=Independent Invariants

Very direct results using the bi-unitary parametrization:

$$Y_\nu = \langle \frac{\mathcal{Y}_\nu}{\Lambda_f} \rangle = \mathcal{U}_L \mathbf{y}_\nu \mathcal{U}_R \,, \qquad Y_E = \langle \frac{\mathcal{Y}_E}{\Lambda_f} \rangle = \mathbf{y}_E$$

$$\mathcal{U}_L \mathcal{U}_L^\dagger = 1\,, \quad \mathcal{U}_R \mathcal{U}_R^\dagger = 1\,,$$

* $m_{e,\mu,\tau} = v\, y_E$

*But the relation of $\mathcal{Y}_\nu$ with light neutrino masses is through:

$$U_{PMNS}\, \mathbf{m}_\nu\, U_{PMNS}^T = \frac{v^2}{2M} \mathcal{U}_L\, \mathbf{y}_\nu\, \mathcal{U}_R \mathcal{U}_R^T\, \mathbf{y}_\nu\, \mathcal{U}_L^T\,,$$

# Bi-fundamental Flavour Fields

Physical parameters
=Independent Invariants

Very direct results using the bi-unitary parametrization:

$$Y_\nu = \frac{<\mathcal{Y}_\nu>}{\Lambda_f} = \mathcal{U}_L \mathbf{y}_\nu \mathcal{U}_R\,, \qquad Y_E = \frac{<\mathcal{Y}_E>}{\Lambda_f} = y_E$$

$$\mathcal{U}_L \mathcal{U}_L^\dagger = 1\,, \quad \mathcal{U}_R \mathcal{U}_R^\dagger = 1\,,$$

* $m_{e,\,\mu,\,\tau} = v\, y_E$

*But the relation of $\mathcal{Y}_\nu$ with light neutrino masses is through:

$\mathcal{U}_R$ is relevant for leptons

$$U_{PMNS}\,\mathbf{m}_\nu\,U^T_{PMNS} = \frac{v^2}{2M}\mathcal{U}_L\,\mathbf{y}_\nu\,\mathcal{U}_R\mathcal{U}_R^T\,\mathbf{y}_\nu\,\mathcal{U}_L^T\,,$$

Number of Physical parameters = number of Independent Invariants

15 invariants for $G_{\text{flavour (leptons)}} = U(3)_L \times U(3)_{E_R} \times O(3)_{N_R}$

Leptons

$$I_E = \mathrm{Tr}\left[\mathcal{Y}_E\mathcal{Y}_E^\dagger\right], \qquad I_\nu = \mathrm{Tr}\left[\mathcal{Y}_\nu\mathcal{Y}_\nu^\dagger\right],$$

$$I_{E^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_E\mathcal{Y}_E^\dagger\right)^2\right], \qquad I_{\nu^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_\nu\mathcal{Y}_\nu^\dagger\right)^2\right],$$

$$I_{E^3} = \mathrm{Tr}\left[\left(\mathcal{Y}_E\mathcal{Y}_E^\dagger\right)^3\right], \qquad I_{\nu^3} = \mathrm{Tr}\left[\left(\mathcal{Y}_\nu\mathcal{Y}_\nu^\dagger\right)^3\right],$$

$$I_L = \mathrm{Tr}\left[\mathcal{Y}_\nu\mathcal{Y}_\nu^\dagger\mathcal{Y}_E\mathcal{Y}_E^\dagger\right],$$

$$I_{L^2} = \mathrm{Tr}\left[\mathcal{Y}_\nu\mathcal{Y}_\nu^\dagger\left(\mathcal{Y}_E\mathcal{Y}_E^\dagger\right)^2\right],$$

$$I_{L^3} = \mathrm{Tr}\left[\mathcal{Y}_E\mathcal{Y}_E^\dagger\left(\mathcal{Y}_\nu\mathcal{Y}_\nu^\dagger\right)^2\right],$$

$$I_{L^4} = \mathrm{Tr}\left[\left(\mathcal{Y}_\nu\mathcal{Y}_\nu^\dagger\mathcal{Y}_E\mathcal{Y}_E^\dagger\right)^2\right],$$

$U_L$ and eigenvalues

$$I_R = \mathrm{Tr}\left[\mathcal{Y}_\nu^\dagger\mathcal{Y}_\nu\mathcal{Y}_\nu^T\mathcal{Y}_\nu^*\right],$$

$$I_{R^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_\nu^\dagger\mathcal{Y}_\nu\right)^2\mathcal{Y}_\nu^T\mathcal{Y}_\nu^*\right],$$

$$I_{R^3} = \mathrm{Tr}\left[\left(\mathcal{Y}_\nu^\dagger\mathcal{Y}_\nu\mathcal{Y}_\nu^T\mathcal{Y}_\nu^*\right)^2\right],$$

$U_R$ and eigenvalues

New Invariants wrt Quarks

$$I_{LR} = \mathrm{Tr}\left[\mathcal{Y}_\nu\mathcal{Y}_\nu^T\mathcal{Y}_\nu^*\mathcal{Y}_\nu^\dagger\mathcal{Y}_E\mathcal{Y}_E^\dagger\right], \qquad I_{RL} = \mathrm{Tr}\left[\mathcal{Y}_\nu\mathcal{Y}_\nu^T\mathcal{Y}_E^*\mathcal{Y}_E^T\mathcal{Y}_\nu^*\mathcal{Y}_\nu^\dagger\mathcal{Y}_E\mathcal{Y}_E^\dagger\right],$$

(Alonso, Gavela, Isidori, Maiani)

Number of Physical parameters = number of Independent Invariants

15 invariants for $G_{\text{flavour (leptons)}} = U(3)_L \times U(3)_{E_R} \times O(3)_{N_R}$

Leptons

$$I_E = \mathrm{Tr}\left[\mathcal{Y}_E \mathcal{Y}_E^\dagger\right], \qquad I_\nu = \mathrm{Tr}\left[\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger\right],$$
$$I_{E^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_E \mathcal{Y}_E^\dagger\right)^2\right], \qquad I_{\nu^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger\right)^2\right],$$
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$$I_{L^2} = \mathrm{Tr}\left[\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger \left(\mathcal{Y}_E \mathcal{Y}_E^\dagger\right)^2\right],$$
$$I_{L^3} = \mathrm{Tr}\left[\mathcal{Y}_E \mathcal{Y}_E^\dagger \left(\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger\right)^2\right],$$
$$I_{L^4} = \mathrm{Tr}\left[\left(\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger \mathcal{Y}_E \mathcal{Y}_E^\dagger\right)^2\right],$$

$\mathrm{U_L}$ and eigenvalues

$$I_R = \mathrm{Tr}\left[\mathcal{Y}_\nu^\dagger \mathcal{Y}_\nu (\mathcal{Y}_\nu^\dagger \mathcal{Y}_\nu)^{\mathrm{T}}\right]$$
$$I_{R^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_\nu^\dagger \mathcal{Y}_\nu\right)^2 \mathcal{Y}_\nu^T \mathcal{Y}_\nu^*\right],$$
$$I_{R^3} = \mathrm{Tr}\left[\left(\mathcal{Y}_\nu^\dagger \mathcal{Y}_\nu \mathcal{Y}_\nu^T \mathcal{Y}_\nu^*\right)^2\right],$$

$\mathrm{U_R}$ and eigenvalues

$$I_{LR} = \mathrm{Tr}\left[\mathcal{Y}_\nu \mathcal{Y}_\nu^T \mathcal{Y}_\nu^* \mathcal{Y}_\nu^\dagger \mathcal{Y}_E \mathcal{Y}_E^\dagger\right], \quad I_{RL} = \mathrm{Tr}\left[\mathcal{Y}_\nu \mathcal{Y}_\nu^T \mathcal{Y}_E^* \mathcal{Y}_E^T \mathcal{Y}_\nu^* \mathcal{Y}_\nu^\dagger \mathcal{Y}_E \mathcal{Y}_E^\dagger\right],$$

(Alonso, Gavela, Isidori, Maiani)

New Invariants wrt Quarks

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$$I_L = \mathrm{Tr}\left[\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger \mathcal{Y}_E \mathcal{Y}_E^\dagger\right],$$

$$I_{L^2} = \mathrm{Tr}\left[\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger \left(\mathcal{Y}_E \mathcal{Y}_E^\dagger\right)^2\right],$$

$$I_{L^3} = \mathrm{Tr}\left[\mathcal{Y}_E \mathcal{Y}_E^\dagger \left(\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger\right)^2\right],$$

$$I_{L^4} = \mathrm{Tr}\left[\left(\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger \mathcal{Y}_E \mathcal{Y}_E^\dagger\right)^2\right],$$

$U_L$ and eigenvalues

$$\mathrm{Tr}(\mathbf{y}_\nu^2 \mathcal{U}_R \mathcal{U}_R^T \mathbf{y}_\nu^2 \mathcal{U}_R^* \mathcal{U}_R^\dagger)$$

$$I_{R^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_\nu^\dagger \mathcal{Y}_\nu\right)^2 \mathcal{Y}_\nu^T \mathcal{Y}_\nu^*\right],$$

$$I_{R^3} = \mathrm{Tr}\left[\left(\mathcal{Y}_\nu^\dagger \mathcal{Y}_\nu \mathcal{Y}_\nu^T \mathcal{Y}_\nu^*\right)^2\right],$$

$U_R$ and eigenvalues

$$I_{LR} = \mathrm{Tr}\left[\mathcal{Y}_\nu \mathcal{Y}_\nu^T \mathcal{Y}_\nu^* \mathcal{Y}_\nu^\dagger \mathcal{Y}_E \mathcal{Y}_E^\dagger\right], \qquad I_{RL} = \mathrm{Tr}\left[\mathcal{Y}_\nu \mathcal{Y}_\nu^T \mathcal{Y}_E^* \mathcal{Y}_E^T \mathcal{Y}_\nu^* \mathcal{Y}_\nu^\dagger \mathcal{Y}_E \mathcal{Y}_E^\dagger\right],$$

New Invariants wrt Quarks

(Alonso, Gavela, Isidori, Maiani)

# Jacobian

$$J = \begin{pmatrix} \partial_{\mathbf{y}_E} I_{E^n} & 0 & 0 & \partial_{\mathbf{y}_E} I_{L^n} & \partial_{\mathbf{y}_E} I_{LR} \\ 0 & \partial_{\mathbf{y}_\nu} I_{\nu^n} & \partial_{\mathbf{y}_\nu} I_{R^n} & \partial_{\mathbf{y}_\nu} I_{L^n} & \partial_{\mathbf{y}_\nu} I_{LR} \\ 0 & 0 & \partial_{\mathcal{U}_R} I_{R^n} & 0 & \partial_{\mathcal{U}_R} I_{LR} \\ 0 & 0 & 0 & \partial_{\mathcal{U}_L} I_{L^n} & \partial_{\mathcal{U}_L} I_{LR} \\ 0 & 0 & 0 & 0 & \partial_{\mathcal{U}_L \mathcal{U}_R} I_{LR} \end{pmatrix},$$

$$\mathrm{Diag}(J) \equiv (J_E\,,\, J_\nu\,,\, J_{\mathcal{U}_R}\,,\, J_{\mathcal{U}_L}\,,\, J_{LR})$$

# Jacobian Analysis: Mixing

$$\det\left(J_{\mathcal{U}_L}\right) = \left(y_{\nu_1}^2 - y_{\nu_2}^2\right)\left(y_{\nu_2}^2 - y_{\nu_3}^2\right)\left(y_{\nu_3}^2 - y_{\nu_1}^2\right)$$
$$\left(y_e^2 - y_\mu^2\right)\left(y_\mu^2 - y_\tau^2\right)\left(y_\tau^2 - y_e^2\right)|\mathcal{U}_L^{e1}||\mathcal{U}_L^{e2}||\mathcal{U}_L^{\mu 1}||\mathcal{U}_L^{\mu 2}|\,.$$

same as for $V_{CKM}$

O(3) vs U(3)

$$\det J_{\mathcal{U}_R} = \left(y_{\nu_1}^2 - y_{\nu_2}^2\right)^3\left(y_{\nu_2}^2 - y_{\nu_3}^2\right)^3\left(y_{\nu_3}^2 - y_{\nu_1}^2\right)^3$$
$$\times \left|\left(\mathcal{U}_R\mathcal{U}_R^T\right)_{11}\right|\left|\left(\mathcal{U}_R\mathcal{U}_R^T\right)_{22}\right|\left|\left(\mathcal{U}_R\mathcal{U}_R^T\right)_{12}\right|$$

*the rank is reduced the most for* $\boxed{\mathcal{U}_R\mathcal{U}_R^T \textit{ being a permutation}}$

# Jacobian Analysis: Mixing

...which is now **not** trivial mixing...

$$\frac{v^2}{M}\begin{pmatrix} y_{\nu_1}^2 & 0 & 0 \\ 0 & 0 & y_{\nu_2}y_{\nu_3} \\ 0 & y_{\nu_2}y_{\nu_3} & 0 \end{pmatrix} = U_{PMNS}\begin{pmatrix} m_{\nu_1} & 0 & 0 \\ 0 & m_{\nu_2} & 0 \\ 0 & 0 & m_{\nu_2} \end{pmatrix} U_{PMNS}^T\,,$$

...in fact it allows maximal mixing:

# Jacobian Analysis: Mixing

...which is now **not** trivial mixing...

$$
- \frac{v^2}{M} \begin{pmatrix} y_{\nu_1}^2 & 0 & 0 \\ 0 & 0 & y_{\nu_2} y_{\nu_3} \\ 0 & y_{\nu_2} y_{\nu_3} & 0 \end{pmatrix} = U_{PMNS} \begin{pmatrix} m_{\nu_1} & 0 & 0 \\ 0 & m_{\nu_2} & 0 \\ 0 & 0 & m_{\nu_2} \end{pmatrix} U_{PMNS}^T ,
$$

...in fact it allows maximal mixing:

$$
U_{PMNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \end{pmatrix} , \quad m_{\nu 2} = m_{\nu 3} = \frac{v^2}{M} y_{\nu_2} y_{\nu_3} , \quad m_{\nu_1} = \frac{v^2}{M} y_{\nu_1}^2 .
$$

and maximal Majorana phase

# Jacobian Analysis: Mixing

...which is now **not** trivial mixing...

$$- \frac{v^2}{M} \begin{pmatrix} y_{\nu_1}^2 & 0 & 0 \\ 0 & 0 & y_{\nu_2} y_{\nu_3} \\ 0 & y_{\nu_2} y_{\nu_3} & 0 \end{pmatrix} = U_{PMNS} \begin{pmatrix} m_{\nu_1} & 0 & 0 \\ 0 & m_{\nu_2} & 0 \\ 0 & 0 & m_{\nu_2} \end{pmatrix} U_{PMNS}^T \, ,$$

...in fact it leads to one maximal mixing angle:

$\theta_{23} = 45°$;

Majorana Phase Pattern (-i,-i,1)

& at this level mass degeneracy: $m_{\nu 2} = m_{\nu 3}$

related to the O(2) substructure

[Alonso, Gavela, D. Hernández, L. Merlo;

[Alonso, Gavela, D. Hernández, L. Merlo, S. Rigolin]

## If the three neutrinos are quasidegenerate:

$$U_{PMNS} \begin{pmatrix} m_0 & 0 & 0 \\ 0 & m_0 & 0 \\ 0 & 0 & m_0 \end{pmatrix} U^T_{PMNS} = -\frac{y_\nu v^2}{M} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

This very simple structure is signaled by the extrema of the potential and

has eigenvalues (1,1,-1) → 3 degenerate light neutrinos + a maximal Majorana phase

and is diagonalized by a maximal θ = 45°

What is the symmetry in this boundary?

a very intriguing

$$\boxed{U(1)_{diag}}$$

With hierarchical charged leptons, in all cases

$SU(3)_L \times SU(3)_E \times O(3)_\nu \rightarrow SU(2)_E \times U(1)_{diag}$

# Generalization to any seesaw model

the effective Weinberg Operator

$$\bar{\ell}_L \tilde{H} \frac{C^{d=5}}{M} \tilde{H}^T \ell_L^c$$

shall have a flavour structure that breaks $U(3)_L$ to $O(3)$

$$\frac{v^2 \; C^{d=5}}{M} = - m_\nu \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

then the results apply to any seesaw model

First conclusion:

* at the same order in which the minimum of the potential

does NOT allow quark mixing,

it allows:

- hierarchical charged leptons
- quasi-degenerate neutrino masses
- one angle of ~45 degrees
- one maximal Majorana phase

**The conclusions hold irrespective of the**

**renormalizability of the potential,**

**and are thus stable under radiative corrections,**

**the symmetries protect the boundaries**

# Perturbations can produce a second large angle

if the three neutrinos are quasidegenerate, perturbations:

$$: U_{PMNS} \begin{pmatrix} m_0 & 0 & 0 \\ 0 & m_0 & 0 \\ 0 & 0 & m_0 \end{pmatrix} U_{PMNS}^T = - \frac{y_\nu v^2}{M} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

produce a second large angle and a perturbative one together with mass splittings

# Perturbations can produce a second large angle

if the three neutrinos are quasidegenerate, perturbations:

$$U_{PMNS} \begin{pmatrix} m_0 & 0 & 0 \\ 0 & m_0 & 0 \\ 0 & 0 & m_0 \end{pmatrix} U^T_{PMNS} = - \frac{y_\nu v^2}{M} \begin{pmatrix} 1+\delta+\sigma & \epsilon+\eta & \epsilon-\eta \\ \epsilon+\eta & \delta+\kappa & 1 \\ \epsilon-\eta & 1 & \delta-\kappa \end{pmatrix}$$

produce a second large angle and a perturbative one together with mass splittings

$$\theta_{23} \simeq \pi/4 \ , \ \theta_{12} \ \text{large} \quad , \quad \theta_{13} \simeq \epsilon$$

Majorana phases: 1 maximal, 1 large

Dirac-like CP phase generically large

~ degenerate spectrum

# Perturbations can produce a second large angle

if the three neutrinos are quasidegenerate, perturbations:

$$U_{PMNS}\begin{pmatrix} m_0 & 0 & 0 \\ 0 & m_0 & 0 \\ 0 & 0 & m_0 \end{pmatrix} U^T_{PMNS} = - \frac{y_\nu v^2}{M}\begin{pmatrix} 1+\delta+\sigma & \epsilon+\eta & \epsilon-\eta \\ \epsilon+\eta & \delta+\kappa & 1 \\ \epsilon-\eta & 1 & \delta-\kappa \end{pmatrix}$$

produce a second large angle and a perturbative one together with mass splittings

$$\theta_{23} \simeq \pi/4 \ , \ \theta_{12} \text{ large} \ , \ \theta_{13} \simeq \epsilon$$

only this vanishes with the perturbations

Majorana phases: 1 maximal, 1 large

Dirac-like CP phase generically large

~ degenerate spectrum

This large value does not vanish with vanishing perturbations

# Perturbations can produce a second large angle

if the three neutrinos are quasidegenerate, perturbations:

$$U_{PMNS} \begin{pmatrix} m_0 & 0 & 0 \\ 0 & m_0 & 0 \\ 0 & 0 & m_0 \end{pmatrix} U_{PMNS}^T = - \frac{y_\nu v^2}{M} \begin{pmatrix} 1+\delta+\sigma & \epsilon+\eta & \epsilon-\eta \\ \epsilon+\eta & \delta+\kappa & 1 \\ \epsilon-\eta & 1 & \delta-\kappa \end{pmatrix}$$

produce a second large angle and a perturbative one together with mass splittings

$$\theta_{23} \simeq \pi/4 \;,\; \theta_{12} \text{ large} \;,\; \theta_{13} \simeq \epsilon$$

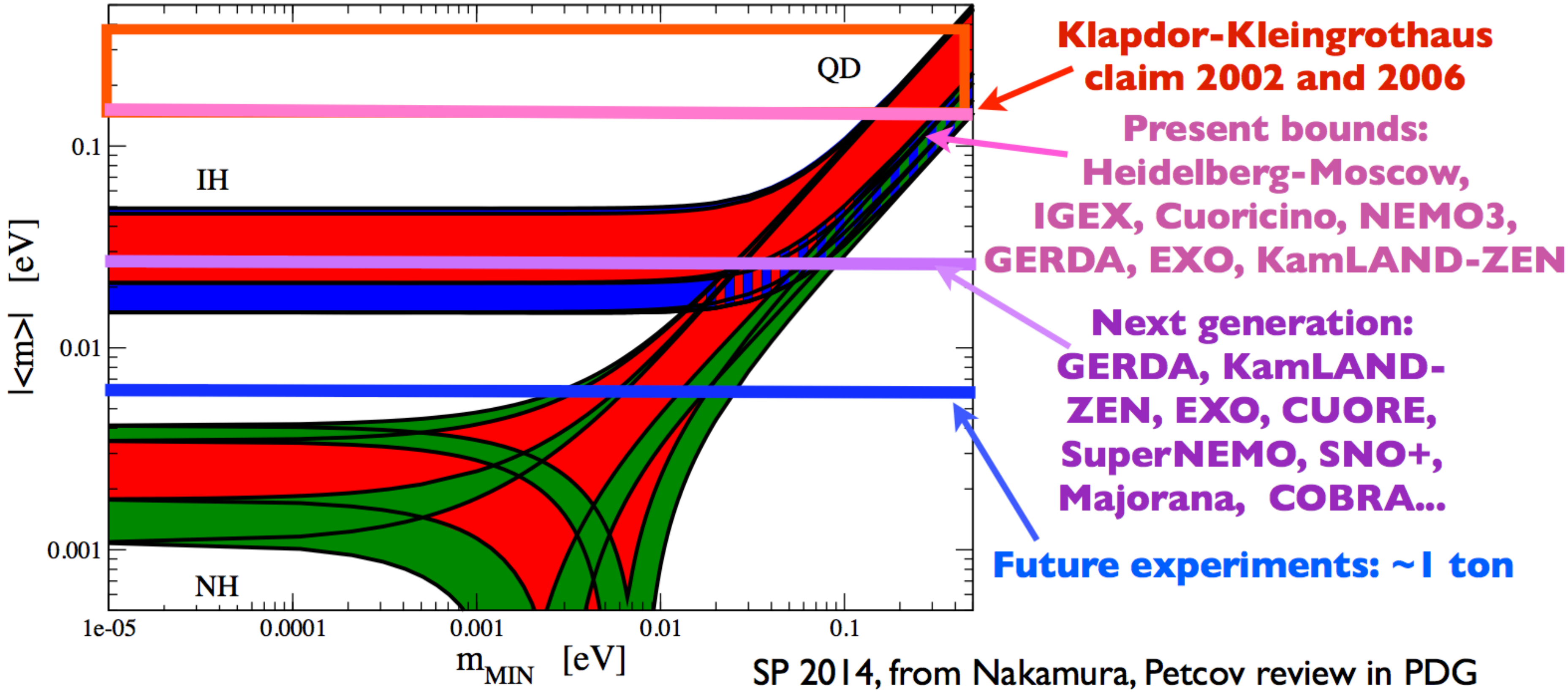
Majorana phases: 1 maximal, 1 large

Dirac-like CP phase  generically large

~ degenerate spectrum

*accommodation of angles requires degenerate spectrum*

at reach in future neutrinoless double β exps.!




SP 2014, from Nakamura, Petcov review in PDG

**Wide experimental program** for the future: **a positive signal would indicate that L is violated!**

courtesy S. Pascoli

*accommodation of angles requires degenerate spectrum*

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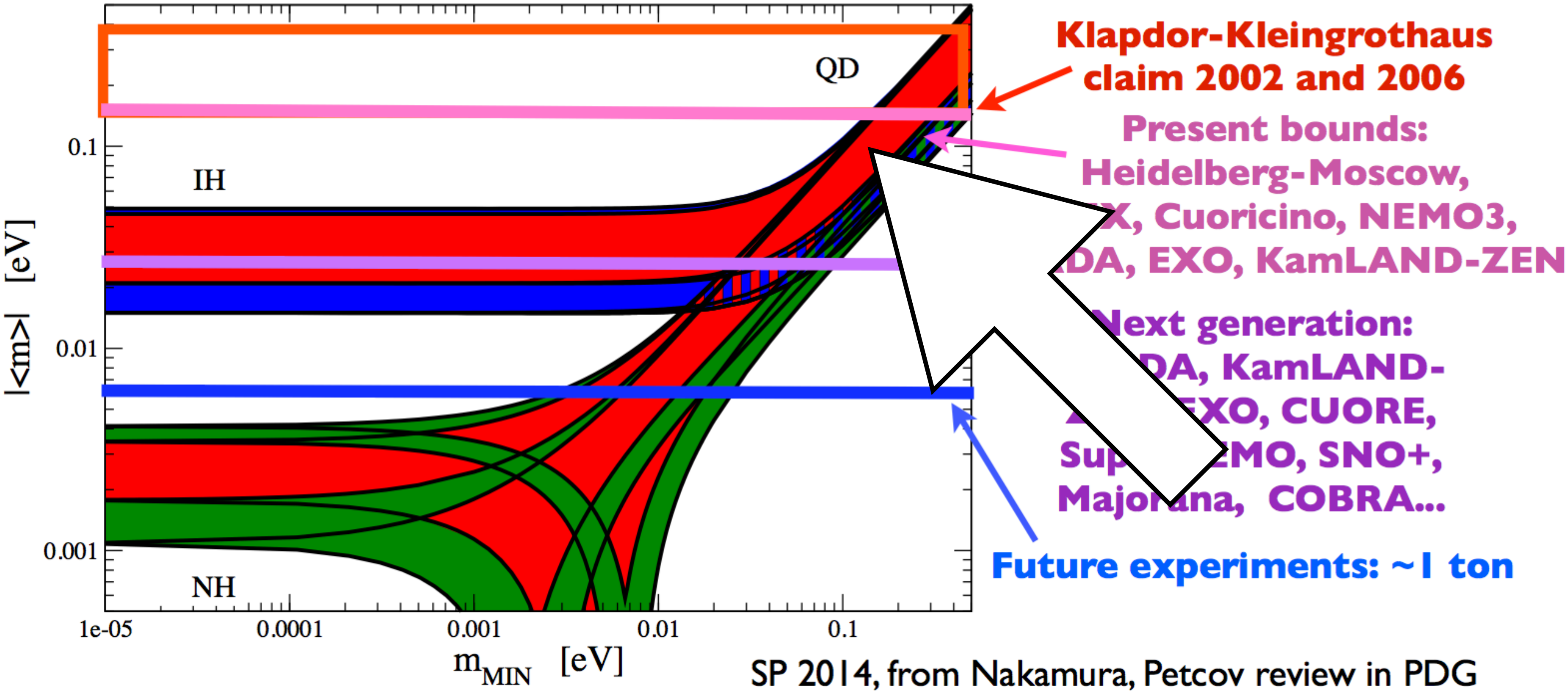
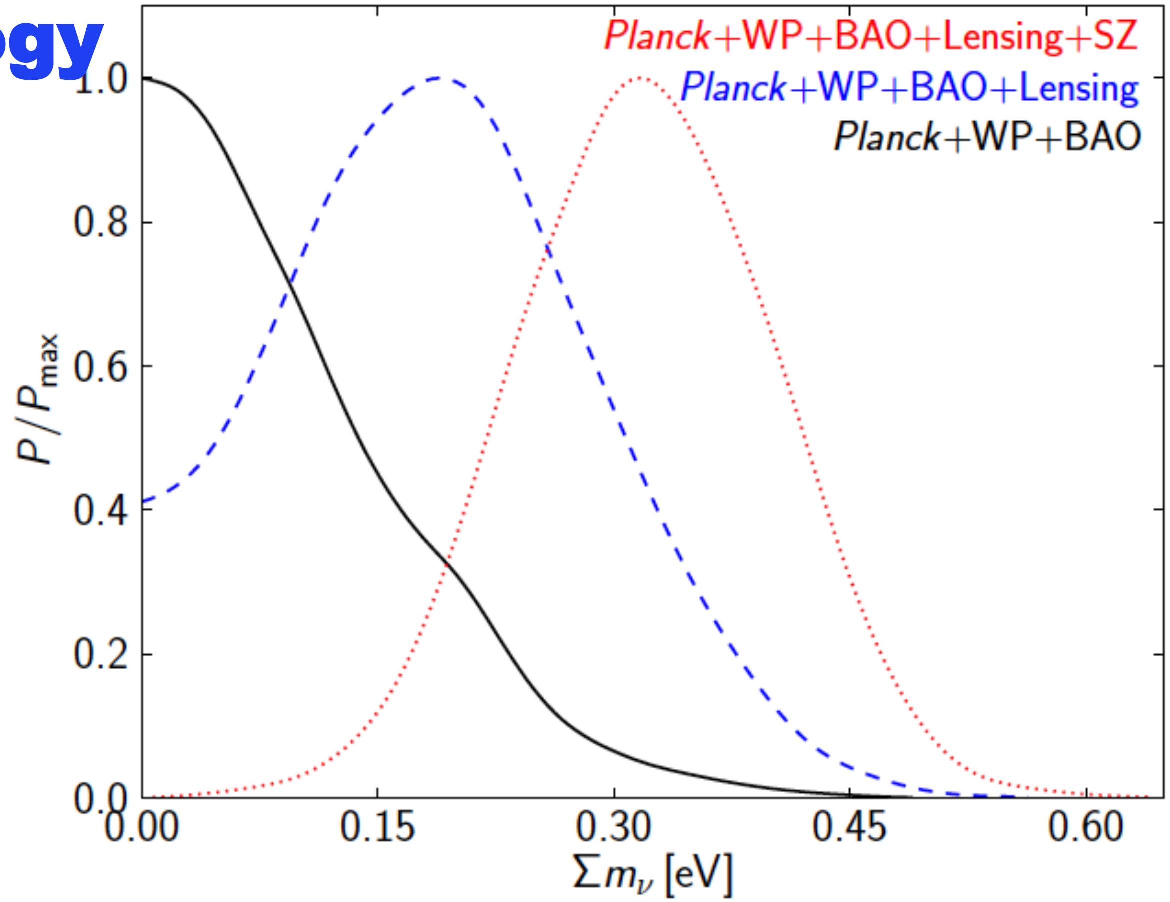



SP 2014, from Nakamura, Petcov review in PDG

**Wide experimental program** for the future: **a positive signal would indicate that L is violated!**

courtesy S. Pascoli

# Cosmology




Battye and Moss

$$\sum m_\nu = (0.320 \pm 0.081)\,\mathrm{eV}$$

FIG. 2: Marginalized likelihoods for $\sum m_\nu$. The datasets are colour coded in the legend, but the solid line is for (I), the dashed line is for (II) and the dotted line is for (III). It is clear that inclusion of lensing leads to a preference for $\sum m_\nu > 0$ which is compatible with that coming from the SZ cluster counts and that there is a strong preference ($\approx 4\sigma$) in the case of dataset (III).

# Where do the differences in Mixing originated?

in the symmetries of the

Quark and Lepton sectors

$\mathcal{G}_{\mathcal{F}}^{q} \sim U(3)^3$

$\mathcal{G}_{\mathcal{F}}^{l} \sim U(3)^2 \times O(3)$

for the type I seesaw employed here;

# Where do the differences in Mixing originated?

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for the type I seesaw employed here;

*in general* $U(n_g)$ vs $O(n_g)$

# Where do the differences in Mixing originated?

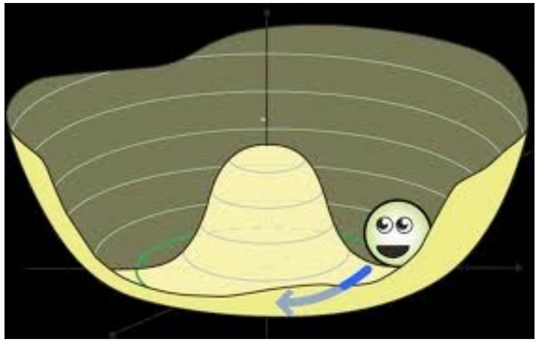
From the

MAJORANA vs DIRAC nature of fermions

# Flavour Symmetry Breaking

Spontaneous breaking of flavour symmetry dangerous

To prevent Goldstone Bosons the symmetry can be Gauged



$$Y_{SM} \sim \frac{1}{<y>}$$

[Grinstein, Redi, Villadoro
Guadagnoli, Mohapatra, Sung
Feldman]

# On going work:

- Gauging it, ~done (by same + Fernandez-Martinez…)
- Obtain the perturbations from dynamics: is the observed secondary patter of masses and mixings an option?
- GUT-compatibility, easy

# Conclusions

- Spontaneous Flavour Symmetry Breaking is a predictive dynamical scenario
- Simple solutions arise that resemble nature in first approximation
- The differences in the mixing pattern of Quarks and Leptons arise from their Dirac vs Majorana nature (U vs. O symmetries). O(2) singled out -> O(3).
- A correlation between large angles and degenerate spectrum emerges. Explicitly, for neutrinos we find: one maximal Majorana phases $(\alpha, 1, i)$ , $\theta_{23} = 45^\circ$, $\theta_{12}$ large, $\theta_{13}$ small and deg. $\nu$'s
- This scenario will be tested in the near future by $0\nu2\beta$ experiments (~.1eV).... or cosmology!!!

The prediction:

large mixing angles $\Leftrightarrow$ Majorana degenerate neutrinos

leads to neutrinoless double beta decay and CMB signals that could be observed in a not too distant future !!

## We set the perturbations by hand.
## Can we predict them also dynamically?

Fundamental Fields

May provide dynamically the perturbations

In the case of quarks they can give the right corrections:

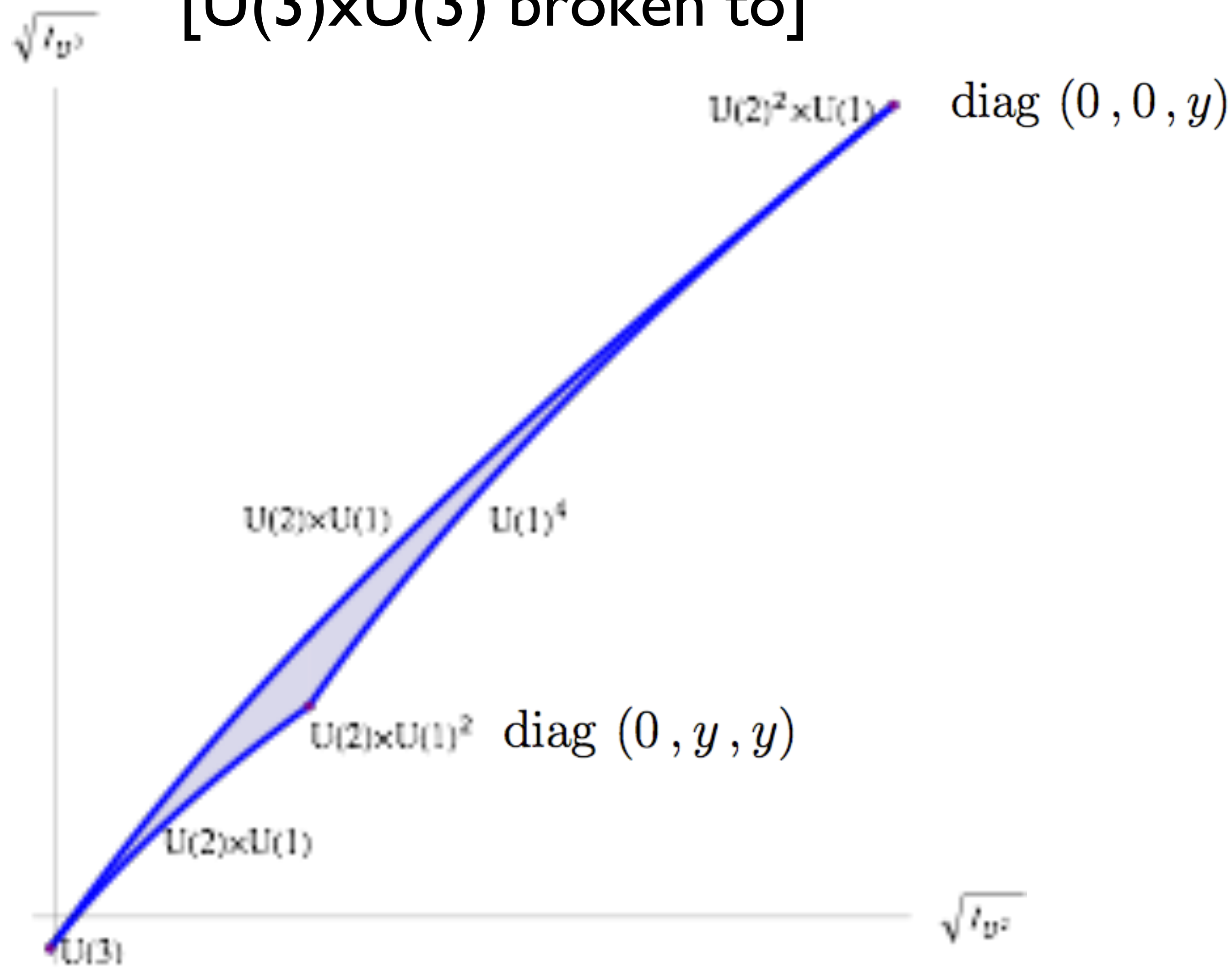
$$\frac{\mathcal{Y}_U}{\Lambda_f} + \frac{\chi_U^L \chi_U^{R\dagger}}{\Lambda_f^2} \sim \begin{pmatrix} 0 & \sin\theta_c\, y_c & 0 \\ 0 & \cos\theta_c\, y_c & 0 \\ 0 & 0 & y_t \end{pmatrix}$$

[Alonso, Gavela, Merlo, Rigolin]

under study in the lepton sector

# Jacobian Analysis: Masses

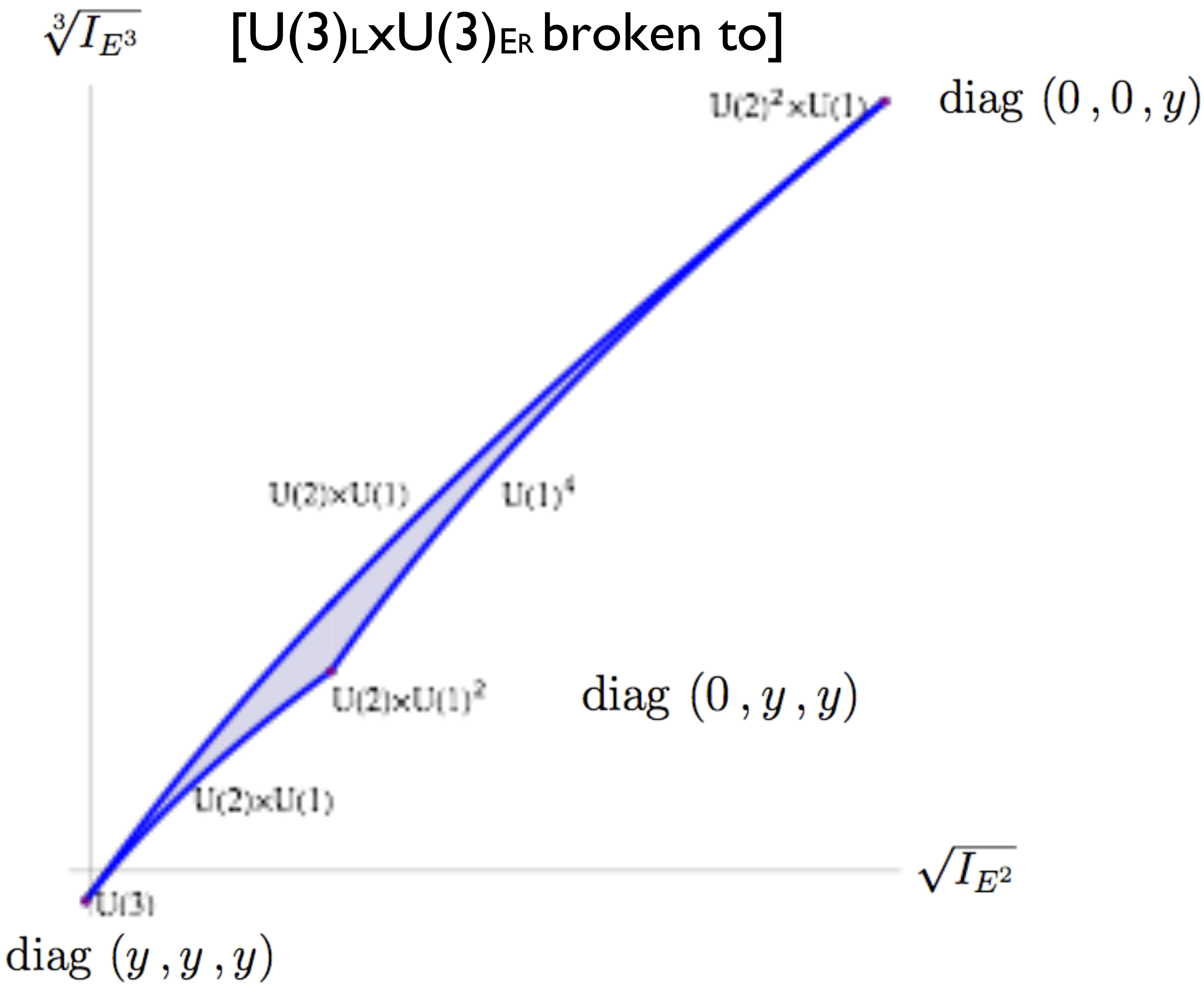
## [U(3)xU(3) broken to]




(Alonso, Gavela, Isidori, Maiani )

Jacobian Analysis: Eigenvalues

[U(3)LxU(3)ER broken to]



Renormalizable Potential

Invariants at the Renormalizable Level

$$I_U = \mathrm{Tr}\left[\mathcal{Y}_U \mathcal{Y}_U^\dagger\right] , \qquad I_D = \mathrm{Tr}\left[\mathcal{Y}_D \mathcal{Y}_D^\dagger\right] ,$$

$$I_{U^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger\right)^2\right] , \qquad I_{D^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_D \mathcal{Y}_D^\dagger\right)^2\right] ,$$

$$I_{U^3} = \mathrm{Tr}\left[\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger\right)^3\right] , \qquad I_{D^3} = \mathrm{Tr}\left[\left(\mathcal{Y}_D \mathcal{Y}_D^\dagger\right)^3\right] ,$$

$$\boxed{I_{U,D} = \mathrm{Tr}\left[\mathcal{Y}_U \mathcal{Y}_U^\dagger \mathcal{Y}_D \mathcal{Y}_D^\dagger\right] ,} \qquad I_{U,D^2} = \mathrm{Tr}\left[\mathcal{Y}_U \mathcal{Y}_U^\dagger \left(\mathcal{Y}_D \mathcal{Y}_D^\dagger\right)^2\right] ,$$

$$I_{U^2,D} = \mathrm{Tr}\left[\mathcal{Y}_U \mathcal{Y}_U^\dagger \left(\mathcal{Y}_D \mathcal{Y}_D^\dagger\right)^2\right] , \qquad I_{(U,D)^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger \mathcal{Y}_D \mathcal{Y}_D^\dagger\right)^2\right] .$$

Renormalizable Potential

with the definition

$$X \equiv (I_U, I_D)^T = \left(\mathrm{Tr}\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger\right), \mathrm{Tr}\left(\mathcal{Y}_D \mathcal{Y}_D^\dagger\right)\right)^T ,$$

the potential

$$V^{(4)} = -\mu^2 \cdot X + X^T \cdot \lambda \cdot X + g\, \mathrm{Tr}\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger \mathcal{Y}_D \mathcal{Y}_D^\dagger\right) + h_U \mathrm{Tr}\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger \mathcal{Y}_U \mathcal{Y}_U^\dagger\right) + h_D \mathrm{Tr}\left(\mathcal{Y}_D \mathcal{Y}_D^\dagger \mathcal{Y}_D \mathcal{Y}_D^\dagger\right) ,$$

mass spectrum

which contains 8 parameters

Renormalizable Potential

with the definition

$$X \equiv (I_U, I_D)^T = \left(\mathrm{Tr}\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger\right), \mathrm{Tr}\left(\mathcal{Y}_D \mathcal{Y}_D^\dagger\right)\right)^T ,$$

the potential

mixing

$$V^{(4)} = -\mu^2 \cdot X + X^T \cdot \lambda \cdot X + g\,\mathrm{Tr}\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger \mathcal{Y}_D \mathcal{Y}_D^\dagger\right) + h_U \mathrm{Tr}\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger \mathcal{Y}_U \mathcal{Y}_U^\dagger\right) + h_D \mathrm{Tr}\left(\mathcal{Y}_D \mathcal{Y}_D^\dagger \mathcal{Y}_D \mathcal{Y}_D^\dagger\right) ,$$

which contains 8 parameters

# Renormalizable Potential, mixing three families

Von Neumann Trace Inequality

$$y_u^2 y_b^2 + y_s^2 y_c^2 + y_d^2 y_t^2 \leq \mathrm{Tr}\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger \mathcal{Y}_D \mathcal{Y}_D^\dagger\right) \leq y_u^2 y_d^2 + y_s^2 y_c^2 + y_b^2 y_t^2 \,.$$

So the Potential selects:

coefficient in the potential

"normal" Hierarchy

$$g < 0\,, \quad V_{CKM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} ;$$

"inverted" Hierarchy

$$g > 0\,, \quad V_{CKM} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} .$$

No mixing, independently of the mass spectrum

Bi-fundamental Flavour Fields

Physical parameters
=Independent Invariants

$$\#\,\text{d.o.f. in } \mathcal{Y}_{U,D} \text{ - } \left(\dim(\mathcal{G}^q_{\mathcal{F}}) - 1_{U(1)_B}\right) = 10$$
$$2 \times 18 \qquad\qquad 3 \times 9 - 1$$

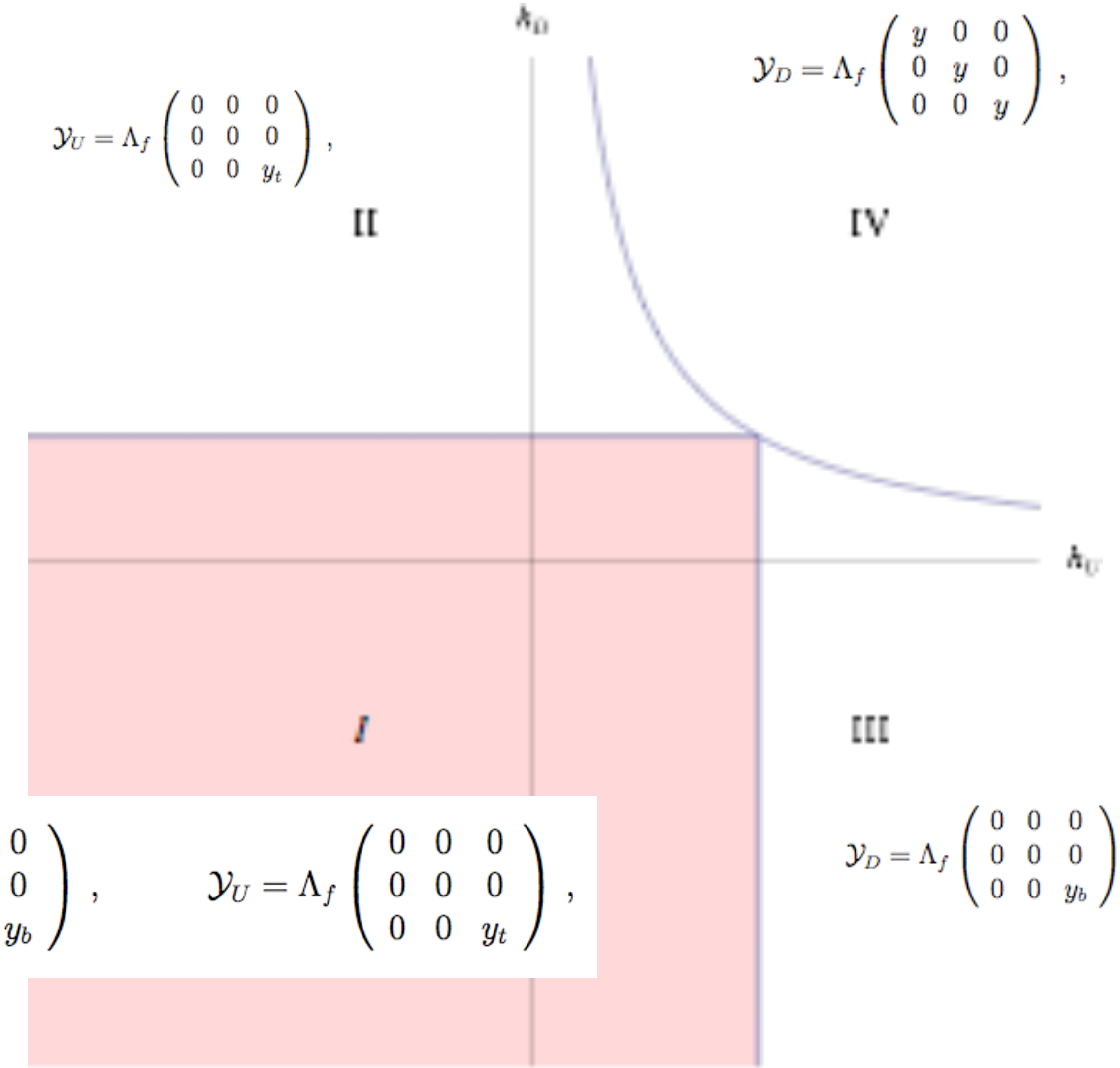
These are (proportional to):

3 masses in de up sector,
3 masses in de down sector,
4 mixing parameters in $V_{CKM}$

## Renormalizable Potential, masses

$$\mathcal{Y}_D = \Lambda_f \begin{pmatrix} y & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & y_b \end{pmatrix}, \qquad \mathcal{Y}_U = \Lambda_f \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_t \end{pmatrix},$$

$$\mathcal{Y}_D = \Lambda_f \begin{pmatrix} y & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & y \end{pmatrix}, \qquad \mathcal{Y}_U = \Lambda_f \begin{pmatrix} y' & 0 & 0 \\ 0 & y' & 0 \\ 0 & 0 & y' \end{pmatrix},$$




$$\mathcal{Y}_D = \Lambda_f \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_b \end{pmatrix}, \qquad \mathcal{Y}_U = \Lambda_f \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_t \end{pmatrix},$$

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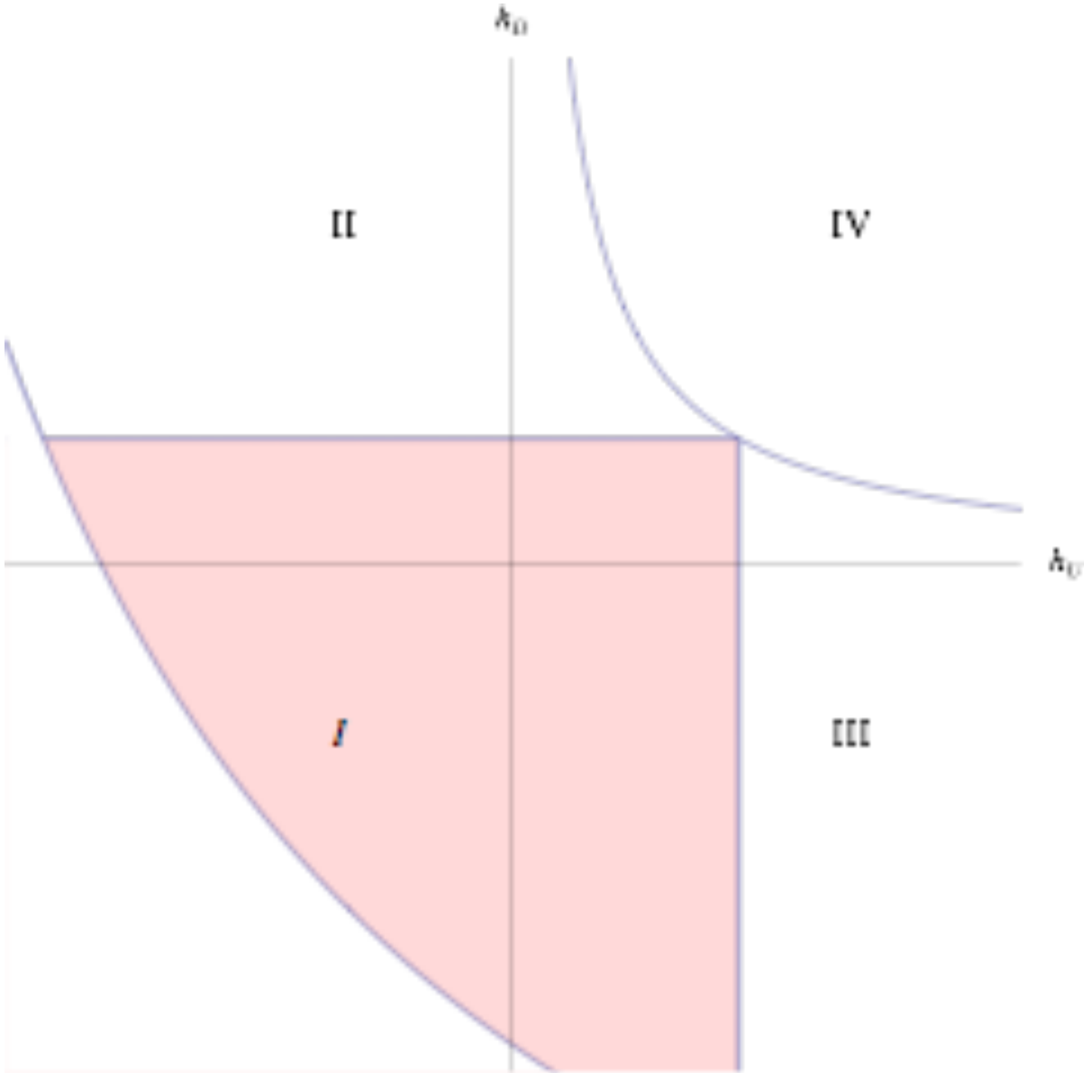
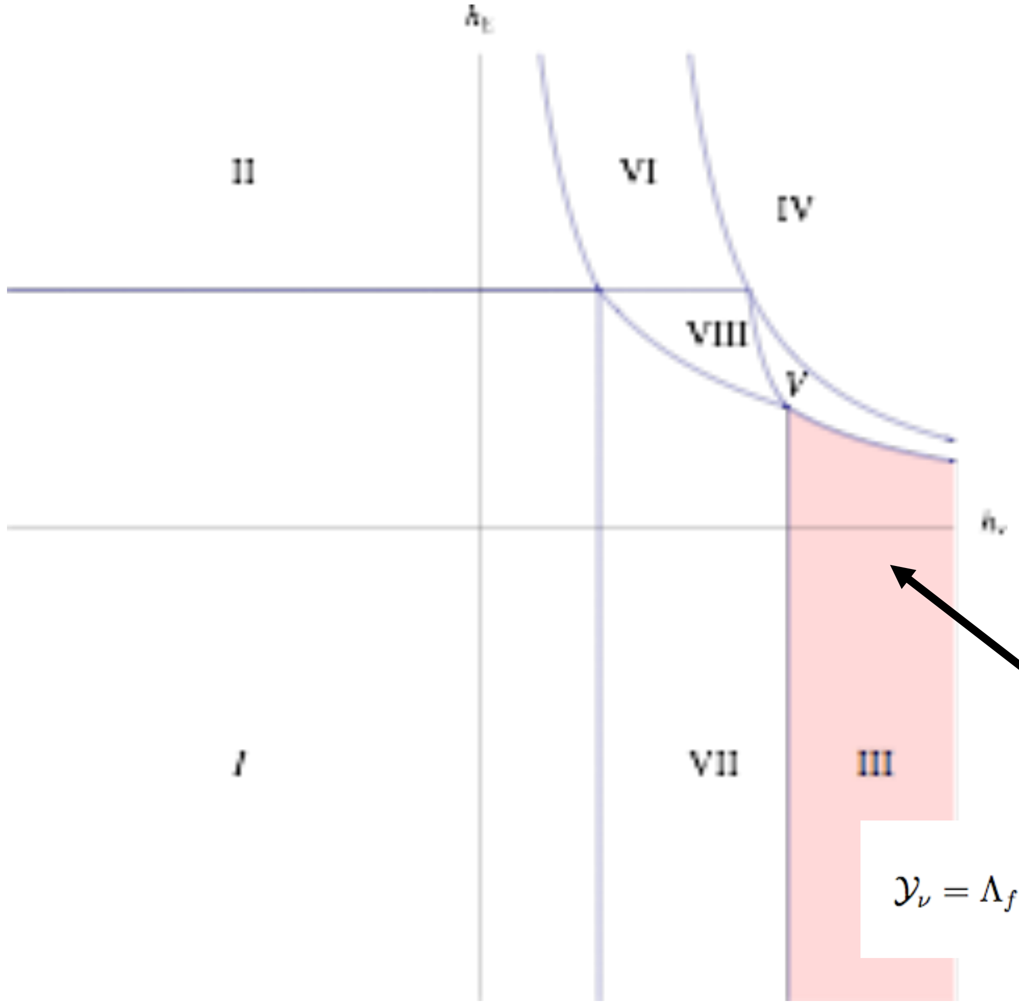
Renormalizable Potential, Stability




This region's size nonetheless depends on the rest of paramters (λ,g)

# Renormalizable Potential: Masses




$$\mathcal{Y}_E = \Lambda_f \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_\tau \end{pmatrix},$$

$$\mathcal{Y}_\nu = \Lambda_f \begin{pmatrix} y_{\nu_1}/\sqrt{2} & 0 & iy_{\nu_1}/\sqrt{2} \\ 0 & y_{\nu_2} & 0 \\ -y_{\nu_3}/\sqrt{2} & 0 & iy_{\nu_3}/\sqrt{2} \end{pmatrix}$$

* What is the role of the neutrino flavour group?

To analyze this in general, use common parametrization for quarks and leptons:

$$Y = U_L \; y^{diag.} U_R$$

* Quarks, for instance: $U_R$ unphysical, $U_L \dashrightarrow U_{CKM}$

$$Y_D = U_{CKM}\, \mathrm{diag}(y_d, y_s, y_b) \quad ; \quad Y_U = \mathrm{diag}(y_u, y_c, y_t)$$

* Leptons:

$$Y_E = \mathrm{diag}(y_e, y_\mu, y_\tau) \quad ; \quad Y_\nu = U_L \; y^{diag.} U_R$$

$U_{PMNS}$ diagonalize

$$\boxed{m_\nu \sim Y_\nu \frac{v^2}{M} Y_\nu^{\,T} = U_L\, y_\nu^{diag.}\, U_R \frac{v^2}{M} U_R^T\, y_\nu^{diag.}\, U_L^T}$$

* What is the role of the neutrino flavour group?

U(n)

i.e.: $U(3)_L \times U(3)_{E_R} \times U(2)_{N_R}$

or: $U(3)_L \times U(3)_{E_R} \times U(3)_{N_R}$

* What is the role of the neutrino flavour group?

e.g. $SU(n)_{N_R}$ ... leptons

e.g. generic seesaw

$$\mathcal{L} = \mathcal{L}_{SM} + i\overline{N_R}\partial\!\!\!/ N_R - \left[\overline{N_R} Y_N \tilde{\phi}^\dagger \ell_L + \frac{1}{2}\overline{N_R} M N_R{}^c + h.c.\right]$$

with M carrying flavour $\longrightarrow$ M spurion

More invariants in this case:

$Tr(\mathcal{Y}_E \mathcal{Y}_E^+)$ $Tr(\mathcal{Y}_E \mathcal{Y}_E^+)^2$ $Tr(\mathcal{Y}_E \mathcal{Y}_E^+ \mathcal{Y}_\nu \mathcal{Y}_\nu^+)$

$Tr(\mathcal{Y}_\nu \mathcal{Y}_\nu^+)$ $Tr(\mathcal{Y}_\nu \mathcal{Y}_\nu^+)^2$

$Tr(\mathcal{M}_N \mathcal{M}_N^+)$ $Tr(\mathcal{M}_N \mathcal{M}_N^+)^2$ $Tr(\mathcal{M}_N \mathcal{M}_N^+ \mathcal{Y}_\nu^+ \mathcal{Y}_\nu)$

At the minimum:

* $Tr(\mathcal{Y}_\nu \mathcal{Y}_\nu^+ \mathcal{Y}_E \mathcal{Y}_E^+) = Tr(U_L\, y_\nu^{diag.\,2}\, U_L^+\, y_l^{diag.\,2}) \longrightarrow U_L=1$
* $Tr(\mathcal{M}_N \mathcal{M}_N^+ \mathcal{Y}_\nu \mathcal{Y}_\nu^+) = Tr(U_R\, y_\nu^{diag.\,2}\, U_R^+\, M_i^{diag.\,2}) \longrightarrow U_R=1$

same conclusion for 3 families of quarks:

$$Y = U_L \ y^{diag.} \ U_R$$

* Quarks, for instance: $U_R$ unphysical, $U_L \dashrightarrow U_{CKM}$

$$Y_D = U_{CKM} \, \mathrm{diag}(y_d, y_s, y_b) \quad ; \quad Y_U = \mathrm{diag}(y_u, y_c, y_t)$$

$$\mathrm{Tr}\left( \mathcal{Y}_u \mathcal{Y}_u^+ \mathcal{Y}_d \mathcal{Y}_d^+ \right) = \mathrm{Tr}\left( U_L \, y_u^{diag.\,2} \, U_L^{\,+} \, y_d^{diag.\,2} \right)$$

$\longrightarrow$ $U_L = U_{CKM} \sim 1$ at the minimum

NO MIXING

# Minimal Flavour violation (MFV)

- Flavour data (i.e. B physics) consistent with all flavour physics coming from Yukawa

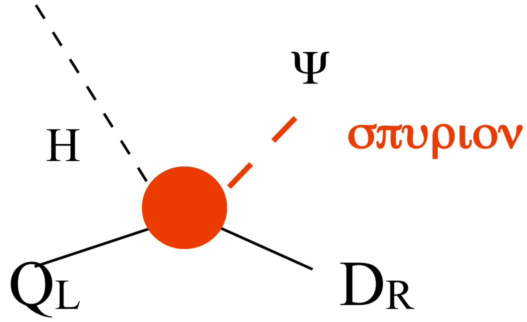
MFV Hypothesis ≡ The Yukawas are the only sources (*irreducible*) of flavour violation. in BSM  R. S. Chivukula and H. Georgi, Phys. Lett. B 188, 99 (1987).

It is very predictive for quarks:  $O^{d=6} \sim \overline{Q}_\alpha Q_\beta \overline{Q}_\gamma Q_\delta$

$$\mathcal{L} = \mathcal{L}_{SM} + c^{d=6} \frac{O^{d=6}}{\Lambda^2_{\text{flavour}}} + \ldots\ldots$$

known function of Yukawas

(D' Ambrosio, Cirigliano, Isidori, Grinstein, Wise….Buras….)




**Some good ideas:**

Minimal Flavour Violation:

- Use the flavour symmetry of the SM in the limit of massless fermions (Chivukula+ Georgi)

quarks: $G_{flavour} = U(3)_{Q_L} \times U(3)_{U_R} \times U(3)_{D_R}$

**Hybrid dynamical-non-dynamical Yukawas:**

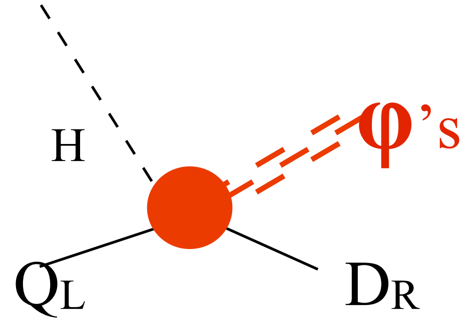
U(2) (Pomarol, Tomasini; Barbieri, Dvali, Hall, Romanino...)....

$U(2)^3$ (Craig, Green, Katz; Barbieri, Isidori, Jones-Peres, Lodone, Straub.. ..Sala)

$$\begin{pmatrix} U(2) & & \\ 0 & 0 & 1 \end{pmatrix}$$

Sequential ideas (Feldman, Jung, Mannel; Berezhiani+Nesti; Ferretti et al., Calibbi et al. ...)

## Some good ideas, based on continuous symmetries:




Frogatt-Nielsen '79: $U(1)_{flavour}$ symmetry

- Yukawa couplings are effective couplings,
- Fermions have $U(1)_{flavour}$ charges

$$\left(\frac{<\varphi>}{\Lambda}\right)^{n} Q\, H\, q_R \quad , \quad Y \sim \left(\frac{<\varphi>}{\Lambda}\right)^{n}$$

e.g. n=0 for the top, n large for light quarks, etc.

--> FCNC ?

# Current neutrino parameters

3 sizable mixing angles

NuFIT 1.2 (2013)

| | Free Fluxes + RSBL | | Huber Fluxes, no RSBL | |
|---|---|---|---|---|
| | bfp $\pm 1\sigma$ | $3\sigma$ range | bfp $\pm 1\sigma$ | $3\sigma$ range |
| $\sin^2\theta_{12}$ | $0.306^{+0.012}_{-0.012}$ | $0.271 \to 0.346$ | $0.313^{+0.013}_{-0.012}$ | $0.277 \to 0.355$ |
| $\theta_{12}/^\circ$ | $33.57^{+0.77}_{-0.75}$ | $31.37 \to 36.01$ | $34.02^{+0.79}_{-0.76}$ | $31.78 \to 36.55$ |
| $\sin^2\theta_{23}$ | $0.446^{+0.008}_{-0.008} \oplus 0.593^{+0.027}_{-0.043}$ | $0.366 \to 0.663$ | $0.444^{+0.037}_{-0.031} \oplus 0.592^{+0.028}_{-0.042}$ | $0.361 \to 0.665$ |
| $\theta_{23}/^\circ$ | $41.9^{+0.5}_{-0.4} \oplus 50.3^{+1.6}_{-2.5}$ | $37.2 \to 54.5$ | $41.8^{+2.1}_{-1.8} \oplus 50.3^{+1.6}_{-2.5}$ | $36.9 \to 54.6$ |
| $\sin^2\theta_{13}$ | $0.0231^{+0.0019}_{-0.0019}$ | $0.0173 \to 0.0288$ | $0.0244^{+0.0019}_{-0.0019}$ | $0.0187 \to 0.0303$ |
| $\theta_{13}/^\circ$ | $8.73^{+0.35}_{-0.36}$ | $7.56 \to 9.77$ | $9.00^{+0.35}_{-0.36}$ | $7.85 \to 10.02$ |
| $\delta_{\mathrm{CP}}/^\circ$ | $266^{+55}_{-63}$ | $0 \to 360$ | $270^{+77}_{-67}$ | $0 \to 360$ |
| $\dfrac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$ | $7.45^{+0.19}_{-0.16}$ | $6.98 \to 8.05$ | $7.50^{+0.18}_{-0.17}$ | $7.03 \to 8.08$ |
| $\dfrac{\Delta m^2_{31}}{10^{-3}\ \mathrm{eV}^2}$ (N) | $+2.417^{+0.014}_{-0.014}$ | $+2.247 \to +2.623$ | $+2.429^{+0.055}_{-0.054}$ | $+2.249 \to +2.639$ |
| $\dfrac{\Delta m^2_{32}}{10^{-3}\ \mathrm{eV}^2}$ (I) | $-2.411^{+0.062}_{-0.062}$ | $-2.602 \to -2.226$ | $-2.422^{+0.063}_{-0.061}$ | $-2.614 \to -2.235$ |

Gonzalez-Garcia, Maltoni, Salvado, Schwetz, 1209.3023

2 mass squared differences

# BSM because

**1) Experimental evidence** for new particle physics:

*** **Neutrino masses**
*** **Dark matter**
*** **CMB polarization ?**

## BSM because

1) **Experimental evidence** for new particle physics:

*** **Neutrino masses**
*** **Dark matter**
*** **(CMB polarization ????)**

2) **SM fine-tunings/uneasiness**, i.e. in electroweak:

*** **Hierarchy problem**
*** **Flavour puzzle**

$\boxed{\text{SU(3) x SU(2) x U(1) x classical gravity}}$

We ~understand ordinary particles= excitations over the vacuum

We DO NOT understand the vacuum = state of lowest energy:

- The **gravity** vaccuum: cosmological cte. $\Lambda$, $\Lambda \sim 10^{-123}\, M_{\mathrm{Planck}}^4$

* The **QCD** vaccuum : Strong CP problem, $\theta_{QCD} \leq 10^{-10}$

* The **electroweak** vaccuum: Higgs-mass, v.e.v.~O (100) GeV

The Higgs excitation has the quantum numbers of the EW vacuum

## BSM electroweak

* **HIERARCHY PROBLEM**

fine-tuning issue: if there is BSM physics,
why is the Higgs so light?

→SUSY ?, strong-int. Higgs ?, extra-dim ?….

In practice, none without further fine-tunings

• **FLAVOUR PUZZLE**: no progress

Understanding stalled since 30 years.
Only new B physics data AND neutrino masses and mixings

BSMs tend to make it worse

## The FLAVOUR WALL for BSM

BSM theories usually die or are very unsatisfactory when confronted with:

i) **Electric dipole moments** **(quarks and leptons)**

ii) **FCNC processes** **(quarks and leptons)**

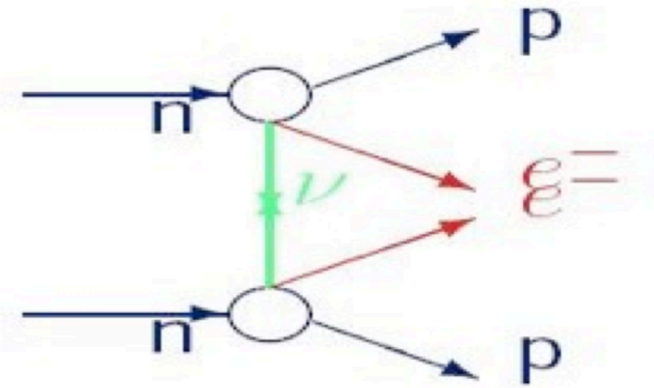
iii) **Strong CP problem**

iii) **Matter-antimatter asymmetry**

competing with SM at one-loop

# Neutrinoless double beta decay

Neutrinoless double beta decay, (A, Z) → (A, Z+2) + 2 e, will test the nature of neutrinos. It violates L by 2 units.




The half-life time depends on neutrino

$$\left[T^{1/2}_{0\nu}(0^+ \to 0^+)\right]^{-1} \propto \left|M_F - g_A^2 M_{GT}\right|^2 \left|<m>\right|^2$$

$$\boxed{\left|<m>\right| \equiv \left|m_1|U_{e1}|^2 + m_2|U_{e2}|^2 e^{i\alpha_{21}} + m_3|U_{e3}|^2 e^{i\alpha_{31}}\right|,}$$

Mixing angles (mostly known)

CPV phases (unknown)

courtesy S. Pascoli

e.g., **for 2 generations**, the mixing terms in $\mathbf{V}(\mathcal{Y}_\mathbf{E},\ \mathcal{Y}_\mathbf{\nu})$ is :

Leptons

$$\mathrm{Tr}(\mathcal{Y}_\mathrm{E}\ \mathcal{Y}_\mathrm{E}^+\ \mathcal{Y}_\nu\ \mathcal{Y}_\nu^+) \propto$$

$$(m_\mu^2 - m_e^2)\left[\cos 2\omega(m_{\nu_2} - m_{\nu_1})\cos 2\theta + 2\sin 2\omega\sqrt{m_{\nu_2}m_{\nu_1}}\, sin2\alpha \sin 2\theta\right]$$

This mixing term unphysical if either "up" or "down" fermions degenerate

Mixing physical even with degenerate neutrino masses, if Majorana phase non-trivial

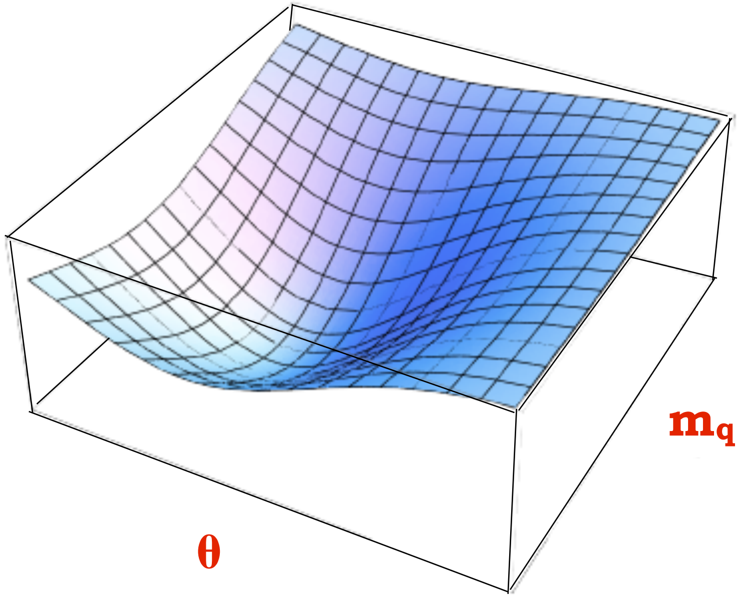
Quarks

$$\mathrm{Tr}(\mathcal{Y}_\mathrm{u}\ \mathcal{Y}_\mathrm{u}^+\ \mathcal{Y}_\mathrm{d}\ \mathcal{Y}_\mathrm{d}^+) \propto (m_c^2 - m_u^2)(m_s^2 - m_d^2)\cos 2\theta$$

e.g. for the case of two families:

$\mathrm{Tr}(\mathcal{Y}_u \mathcal{Y}_u^+ \mathcal{Y}_d \mathcal{Y}_d^+) \propto (m_c^2 - m_u^2)(m_s^2 - m_d^2) \cos 2\theta$

at the minimum: $\boxed{(m_c^2 - m_u^2)(m_s^2 - m_d^2) \sin 2\theta = 0}$




**-> NO MIXING**

**same conclusion for 3 families**

Berezhiani-Rossi; Anselm, Berezhiani; Alonso, Gavela, Merlo, Rigolin e.g., **for 2 generations**, the mixing terms in $\mathbf{V}(\mathcal{Y}_\mathbf{E}, \mathcal{Y}_\mathbf{\nu})$ is :

Minimisation (for non trivial $\sin 2\omega$)

$\mathrm{Tr}(\mathcal{Y}_E \mathcal{Y}_E^+ \mathcal{Y}_\nu \mathcal{Y}_\nu^+)$

* $$\sin 2\omega \sqrt{m_{\nu_2} m_{\nu_1}} \sin 2\theta \cos 2\alpha = 0 \longrightarrow \boxed{\alpha = \pi/4 \text{ or } 3\pi/4}$$

**Maximal Majorana phase**

* $$\mathrm{tg} 2\theta = \sin 2\alpha \frac{2\sqrt{m_{\nu_2} m_{\nu_1}}}{m_{\nu_2} - m_{\nu_1}} \ \mathbf{tgh}\ \mathbf{2\omega}$$

**Large angles correlated with degenerate masses**

For the past ~1.5 years, I worked mostly on Higgs physics:

- effective chiral Lagrangain for light dynamical Higgs
  http://arxiv.org/abs/arXiv:1212.3305; http://arxiv.org/abs/arXiv:1212.3307

- Disentangling an elementary from a dynamical Higgs
  http://arxiv.org/abs/arXiv:1311.1823 Ilaria Brivio in yesterday's session

- Lee Wick theories: elementary versus dynamical Higgs
  http://arxiv.org/abs/arXiv:1405.5412 (last week!)

- CP-violation versus Higgs physics → June 2014

- Comparison of specific strong groups (SO(5)/SO(4), SU(5), SU(3)) with low-energy effective Lag. → June 2014

- 1-loop renormalization of chiral Lag. with light Higgs → July 2014

→ Planck 2014