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Pyr@te

Generation of non-SUSY two-loop RGEs: Automation

arXiv:1309.7030

Florian Lyonnet

In collaboration with Ingo Schienbein, Florian Staub, Akın Wingerter

Laboratoire de Physique Subatomique et de Cosmologie
Université Joseph Fourier, Grenoble

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Motivations

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- Generate the **Renormalization Group Equations** for non-supersymmetic theories @ 2-loop

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  - $B_s \to \mu^+\mu^-, \ b \to s\gamma, \ldots$
  - collider experiments
  - direct DM detection experiments

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- No evidence of SUSY so far :
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  - collider experiments
  - direct DM detection experiments
- Systematic studies of non-SUSY models require the RGEs
- E.g. improved effective potential in minimal extensions of the SM, DM, inflation, ...

- RGEs for general gauge theories known for a long time:
  - *I. Jack and H. Osborn, Nucl.Phys.B207 (1982), J.Phys.A16 (1983), Nucl.Phys.B249 (1985)*
  - *M. Machacek and M. T. Vaughn, 1983 Nucl.Phys.B222*
  - *M. Luo et al. Phys.Rev. D67 (2003) 065019*

- Calculation of beta functions "by hand" is time consuming and prone to error $\Rightarrow$ Difficult to use in practice.
- Full set of 2-loop RGEs known only for few specific cases:
  from *C. Cheung et al. JHEP 1207 (2012), A. Wingerter Phys.Rev. D84 (2011)*
  - SM + Neutrinos
  - SM + chiral fourth generation
  - SM + real singlet scalar
  - SM + real triplet scalar
  - SM + complex doublet scalar
  - ...

SUSY

- SARAH *Comp. Phys. Com. 185 (2014) 1773-1790* (spectrum generator generator)
- SUSYNO *Comput.Phys.Commun. 183 (2012) 2298-2306*

NON-SUSY

- **PyR@TE** cross checked with the beta version of SARAH 4.0.

Outline

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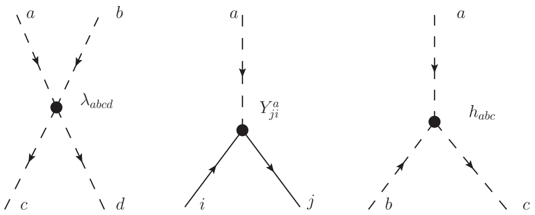
Renormalization Group Equations

- Renormalization scale μ
  $\Rightarrow g_{10}, \alpha_{S0}, \lambda_0 \cdots \Rightarrow \tilde{g}_1(\mu), \tilde{\alpha}_S(\mu), \tilde{\lambda}(\mu)$.
- RGEs : ensure the invariance of the observables.
  - e.g. : $\mu \frac{d}{d\mu} \tilde{\alpha}_S(\mu) = \beta_{\alpha_S}$



- β functions depend on the theory i.e. **particles and gauge groups**.
- Can be approximated in perturbation theory.

Definition

- Take a general gauge field theory

$G_1 \times G_2 \times \cdots \times G_n$ direct product of simple groups

$$
\begin{aligned}
\mathcal{L} \supset \quad & - \quad N_a Y^a_{jk} \psi_j \xi \psi_k \phi_a + h.c. \Rightarrow \beta^a_{jk} \\
& - \quad N_\lambda \lambda_{abcd} \phi_a \phi_b \phi_c \phi_d \Rightarrow \beta_{abcd} \\
& - \quad N_{mf} (mf)_{jk} \psi_j \xi \psi_k + h.c. \Rightarrow (\beta_{mf})_{jk} \\
& - \quad N_{mab} m^2_{ab} \phi_a \phi_b \Rightarrow \beta_{ab} \\
& - \quad N_h \phi_a \phi_b \phi_c \Rightarrow \beta_{abc},
\end{aligned}
$$

$\Rightarrow$ 6 types of beta functions to calculate:

- $\beta(g) \Rightarrow$ gauge couplings
- $\beta^a_{jk} \Rightarrow$ yukawas
- $\beta_{abcd} \Rightarrow$ quartic couplings
- $\beta_{ab} \Rightarrow$ scalar mass
- $(\beta_{mf})_{jk} \Rightarrow$ fermion mass
- $\beta_{abc} \Rightarrow$ trilinear couplings

Results

- Known @two-loop in the $\overline{\mathrm{MS}}$ scheme:
- e.g. gauge coupling constant for unique gauge group factor :

$$\beta(g) = -\frac{g^3}{(4\pi)^2}\left\{\frac{11}{3}C_2(G) - \frac{4}{3}\kappa S_2(F) - \frac{1}{6}S_2(S) + 2\frac{\kappa}{(4\pi)^2}Y_4(F)\right\}$$
$$+\frac{g^5}{(4\pi)^4}\left\{\frac{34}{3}[C_2(G)]^2 - \kappa[4C_2(F) + \frac{20}{3}C_2(G)]S_2(F) - [2C_2(S) + \frac{1}{3}C_2(G)]S_2(S)\right\},$$

$$Y_4(F) = \frac{1}{d(G)}\,Tr\left(C_2(F)\,Y^a\,Y^{\dagger a}\right)$$

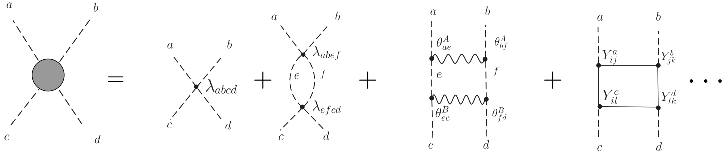
- Notation extremely compact, difficult to find the right multiplicity!

SUSY vs Non-SUSY RGEs

- Expressions more involved $\Rightarrow$ more time consuming
- No need for quartic coupling RGEs in SUSY
- One needs the explicit matrices of the representation for the scalars:
  - $D_\mu \phi_a = \partial_\mu \phi_a - ig\theta^A_{ab} V^A_\mu \phi_b$
- θ^A_{ab} assumed purely imaginary and antisymmetric in the calculation.
  For a field charged under $\mathrm{SU}(n) \Rightarrow$ Hermitian Basis

$$L_i = \begin{pmatrix} \mathrm{Im}(\tilde{L}_i) & i\mathrm{Re}(\tilde{L}_i) \\ -i\mathrm{Im}(\tilde{L}_i) & \mathrm{Re}(\tilde{L}_i) \end{pmatrix}$$

The Quartic Terms

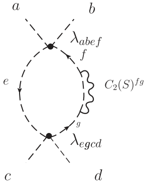


$$\sim \sum_{perm} \lambda_{abef}\lambda_{efcd}$$

$$\sim \sum_{perms,k,l} g^{2k} g^{2l} \{\theta^A, \theta^B\}_{ab} \{\theta^A, \theta^B\}_{cd}$$

$$\sim \sum_{perms} \sum_{i,j,k,l} Y^a_{ij} Y^{b\dagger}_{jk} Y^c_{kl} Y^{d\dagger}_{li}$$

$+$



$\cdots$

$$\sim \sum_{perm} g^2 C_2^{fg}(S) \lambda_{abef} \lambda_{cdeg}$$

Summary

What are the different ingredients needed ?

- C_2, S_2 for all the representations involved
- θ^A, t^A matrix representation for the scalars and fermions
- Contract the different terms in the Lagrangian into singlets :
  - CGCs, database built from Susyno arxiv: 1106.5016
- Replacement rules to go from single gauge group factor to product :
  - $G \to G_1 \times G_2 \times \cdots \times G_n$
  - e.g. $g^4 C_2(R) C_2(R') \to \sum_{k,l} g_k^2 g_l^2 C_2^k(R) C_2^l(R')$

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Main features

- Public code for any non-SUSY theories, RGEs at 2-loop .
- Version 1.1.0 is out : http://pyrate.hepforge.org
- Gauge Groups : $SU(n), n = 2, ..., 6$ (no kinetic mixing).
- shell and Interactive mode (IPython notebook)

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Output

- LateX
- Coupled system of beta functions $\Rightarrow$ Python, Mathematica
  - Wrote wrapper to solve the RGEs in Python, Mathematica (from $t_{min},\ t_{max}$)
  - Very easy using the IPython notebook ! (see examples on our webpage)
- Lately, export of the beta functions to C++ (can be solved with the same Python wrapper)

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Validation

- Collaborator F. Staub implemented same RGEs in SARAH 4 $\Rightarrow$ independent cross check.
- Many one loop results including 2HDM from D.R.T. Jones JHEP 0908 (2009) 069
- All the models from C. Cheung et al. JHEP 1207 (2012) 105
- Cross checking the beta functions that are not in the SM e.g.
  - SM + one real scalar field $\Rightarrow$ Trilinear term
  - SM + $t^{'}$ vector like quark $\Rightarrow$ Fermion mass term

Future developments :

- Extend the group part i.e. more groups e.g. $\mathrm{SO}(n)$, more irreps
- Index generation for scalars
- Multiple $U(1) \Rightarrow$ Kinetic mixing
  - R. Fonseca, M. Malinsky, F. Staub, arXiv:1308.1674
- Running of the vevs
  - Marcus Sperling, Dominik Stóckinger, Alexander Voigt, arXiv: 1305.1548
- Include available three loops results

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$\mathcal{G}_{221}$ models in a nutshell

- $\underbrace{\mathrm{SU}(2)_1}_{g_1} \times \underbrace{\mathrm{SU}(2)_2}_{g_2} \times \underbrace{\mathrm{U}(1)_X}_{g_X} \longrightarrow W', Z'$
- **BP-I**,**BP-II** $\Rightarrow$ only consider first breaking here i.e.
  - **Doublet (D)**: $\phi \sim (\mathbf{1}, \mathbf{2}, \frac{1}{2}) + H \sim (\mathbf{2}, \overline{\mathbf{2}}, 0)$
  - **Triplet (T)**: $\phi \sim (\mathbf{1}, \mathbf{3}, 1) + H \sim (\mathbf{2}, \overline{\mathbf{2}}, 0)$
- Left-right (LR), Lepto-phobic (LP), Fermio-phobic (FP) and Hadro-phobic (HP)$\in \mathcal{G}_{221}$
- We only focus on the scalar potential: **10** parameters

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\begin{aligned}
V =& \mu_\phi \left(\phi^\dagger \otimes \phi\right) + \mu_{H,1} \left(H \otimes H + h.c.\right) + \mu_{H,2} \left(H^\dagger \otimes H\right) \\
&+ \lambda_\phi \left(\phi^\dagger \otimes \phi\right)\left(\phi^\dagger \otimes \phi\right) + \lambda_{H,1} \left(H^\dagger \otimes H^\dagger\right)\left(H \otimes H\right) \\
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$\mathcal{G}_{221}$ RGEs

- We input V in PyR@TE and obtain the 10 beta functions, e.g.

$$\beta^{(2)}_{\lambda_i}(M) = \tilde{\beta}^{(2)}_{\lambda_i}(M) + \Delta_{\lambda_i} \, ,$$

with

$$\begin{aligned}\Delta^{\mathrm{D}}_{\lambda_\phi} &= -192\lambda^2_{H\phi,1}\lambda_{H\phi,2} - 160\lambda^2_{H\phi,1}\lambda_\phi + 96\lambda^2_{H\phi,1}g_1^2 \\ &+ 96\lambda^2_{H\phi,1}g_2^2 - 16\lambda^3_{H\phi,2} - 40\lambda^2_{H\phi,2}\lambda_\phi + 24\lambda^2_{H\phi,2}g_1^2 \\ &+ 24\lambda^2_{H\phi,2}g_2^2 + 15\lambda_{H\phi,2}g_2^4 - 312\lambda^3_\phi + 36\lambda^2_\phi g_1^2 + 108\lambda^2_\phi g_2^2 \, ,\end{aligned}$$

- Using the interactive mode we solve this system of 10 equations ...

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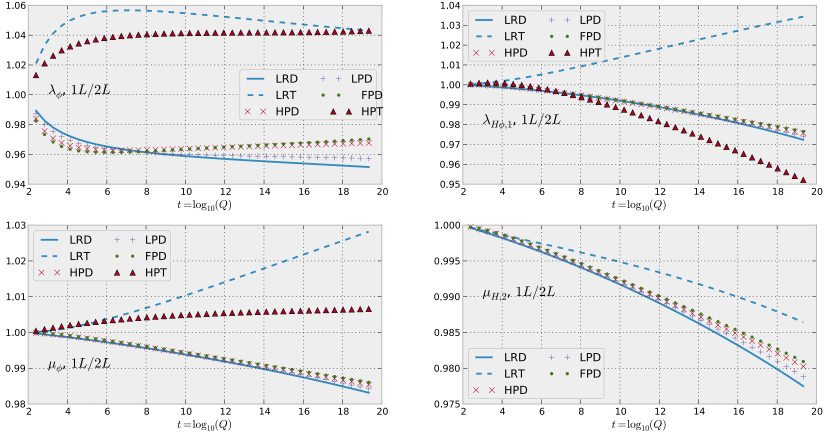

Conclusion and outlook

- For a more systematic study of non SUSY models RGEs are needed.
- We developed a tool that generates the RGEs @2-loop $\Rightarrow$ PyR@TE
- Two-loop effects could be sizeable and can now be included
- Have fun !