Parton-level amplitudes for high-energy scattering

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in collaboration with
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Narodowe Centrum
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LIDER

High–energy factorization

Gribov, Levin, Ryskin 1983
Catani, Ciafaloni, Hautmann 1991

$$\sigma_{h_1,h_2\to QQ} = \int d^2k_{1\perp}\,\frac{dx_1}{x_1}\,\mathcal{F}(x_1,k_{1\perp})\,d^2k_{2\perp}\,\frac{dx_2}{x_2}\,\mathcal{F}(x_2,k_{1\perp})\,\hat{\sigma}_{gg}\left(\frac{m^2}{x_1x_2s},\frac{k_{1\perp}}{m},\frac{k_{2\perp}}{m}\right)$$

- to be applied in the 3-scale regime $s \gg m^2 \gg \Lambda^2_{QCD}$
- reduces to collinear factorization for $s \gg m^2 \gg k_\perp^2$,
  but holds also for $s \gg m^2 \sim k_\perp^2$
- *unintegrated pdf* $\mathcal{F}$ may satisfy BFKL-eqn, CCFM-eqn, BK-eqn, KGBJS-eqn, ...
- typically associated with small-x physics
- relevant for forward physics, saturation physics, heavy-ion physics...
- $k_\perp$ gives a handle on the size of the proton
- allows for higher-order kinematical effects at leading order

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Catani, Ciafaloni, Hautmann 1991

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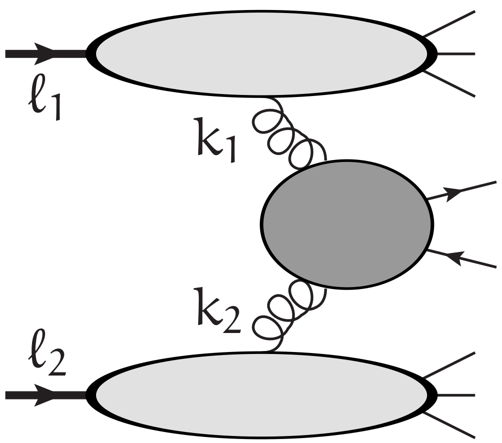

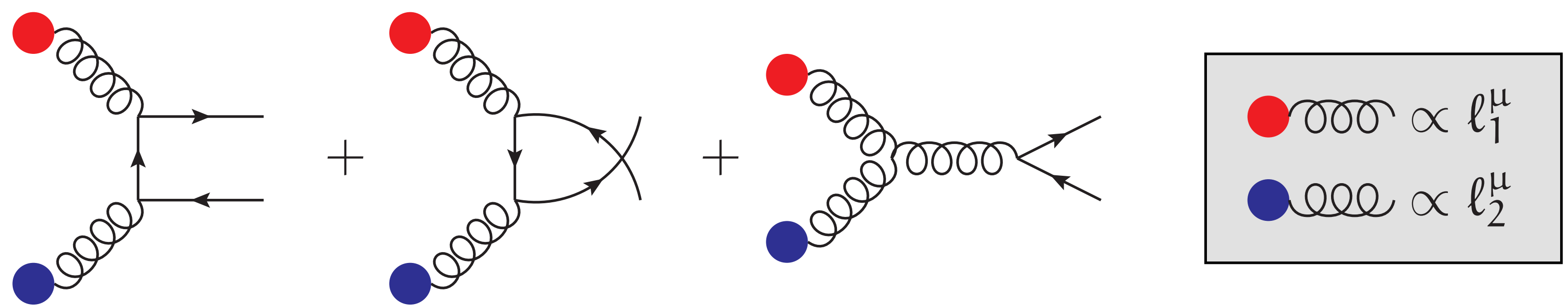

Imposing high-energy kinematics,

$$k_1^\mu = x_1\ell_1^\mu + k_{1\perp}^\mu \quad , \quad k_2^\mu = x_2\ell_2^\mu + k_{2\perp}^\mu \qquad \text{with} \quad \ell_{1,2}\cdot k_{1\perp,2\perp} = 0 \quad ,$$

the amplitude for $g^*g^* \to Q\bar{Q}$ is gauge invariant.

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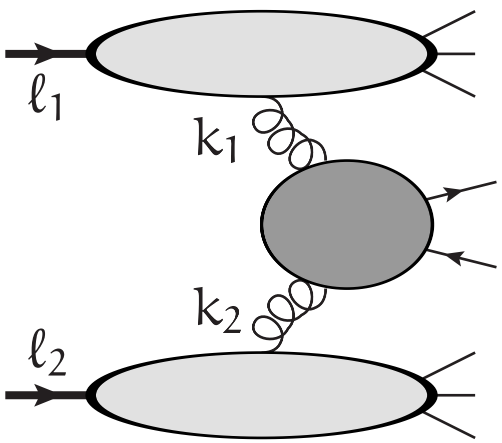

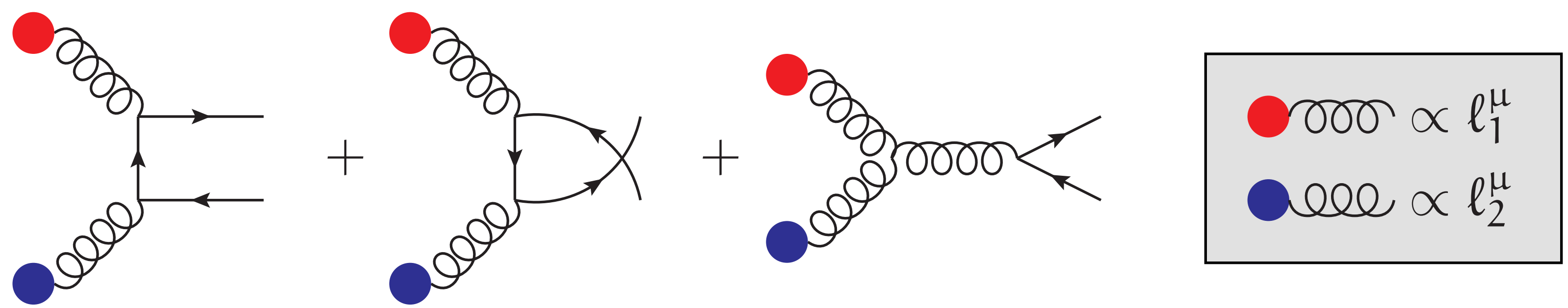

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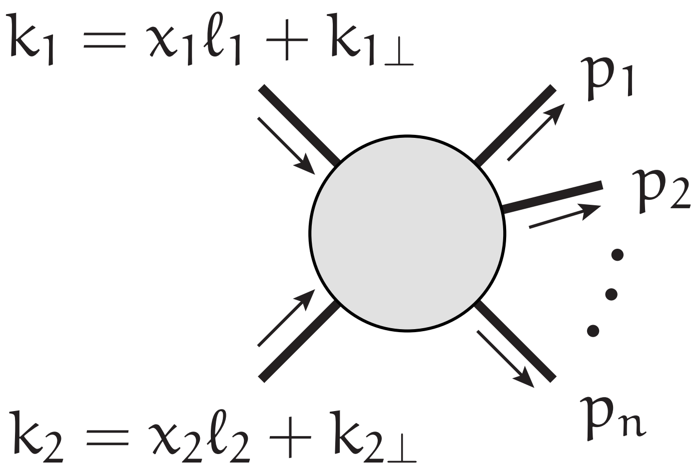
Can this be generalized to arbitrary processes?

Matrix elements

The issue:

High-energy factorization requires matrix elements for parton-level scattering process with off-shell initial states



where ℓ_1, ℓ_2 are light-like momenta associated with the scattering hadrons, and $k_{1\perp}, k_{2\perp}$ are perpendicular to both ℓ_1 and ℓ_2.

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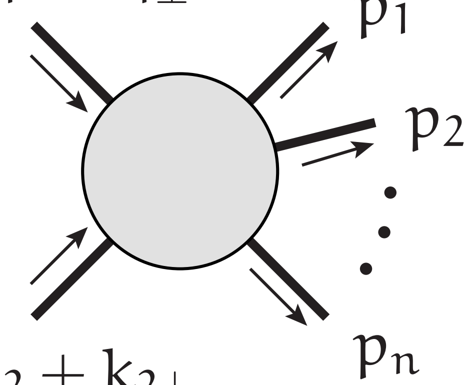
$$k_1 = x_1\ell_1 + k_{1\perp}$$



$$k_2 = x_2\ell_2 + k_{2\perp}$$

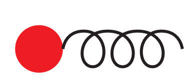
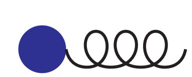
where ℓ_1, ℓ_2 are light-like momenta associated with the scattering hadrons, and $k_{1\perp}, k_{2\perp}$ are perpendicular to both ℓ_1 and ℓ_2.

Matrix elements, squared and summed over final-state spins, may be calculated using spin amplitudes.

Amplitudes must be gauge invariant

- must be calculable in any gauge
- must satisfy Ward identities.
- must preferably be practical.

We cannot just take a prescription to calculate on-shell matrix elements and keep initial-state momenta off-shell, because we won't have gauge invariance. Using projectors

 $\propto \ell_1^\mu$  $\propto \ell_2^\mu$

won't be enough.

Lipatov's effective action

Lipatov 1995, Antonov, Lipatov, Kuraev,Cherednikov 2005

Effective action in terms of quarks $\psi, \bar{\psi}$ gluons v_μ and reggeized gluons $A_\pm$.

$$\mathcal{L} = \mathcal{L}_{\mathrm{QCD}} + \mathcal{L}_{\mathrm{ind}}$$

$$\mathcal{L}_{\mathrm{QCD}} = i\bar{\psi}\not{D}\psi + \frac{1}{2}\mathrm{Tr}\,G_{\mu\nu}^2 \qquad D_\mu = \partial_\mu + g v_\mu \qquad G_{\mu\nu} = \frac{1}{g}[D_\mu, D_\nu]$$

$$\begin{aligned}\mathcal{L}_{\mathrm{ind}} = -\mathrm{Tr}\Bigg\{ &\frac{1}{g}\partial_+ \left[\mathcal{P}\exp\left(-\frac{g}{2}\int_{-\infty}^{x^+} v_+(y)\,dy^+\right)\right] \cdot \partial_\sigma^2 A_-(x) \\ &+ \frac{1}{g}\partial_- \left[\mathcal{P}\exp\left(-\frac{g}{2}\int_{-\infty}^{x^-} v_-(y)\,dy^-\right)\right] \cdot \partial_\sigma^2 A_+(x) \Bigg\}\end{aligned}$$

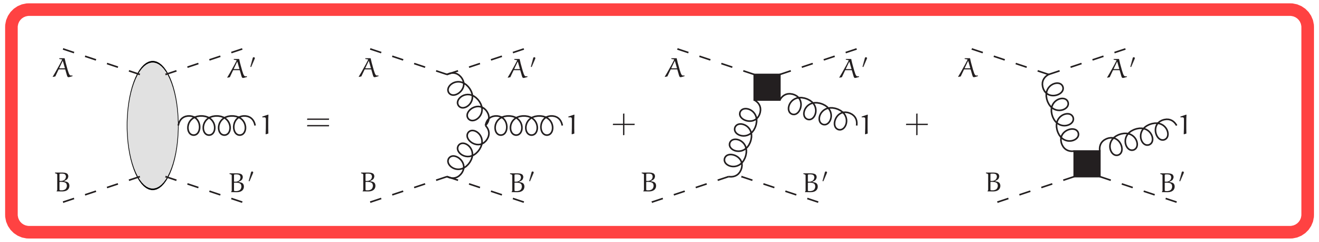
$$k_\pm = \frac{1}{E}(\ell_\mu^\pm)k^\mu \qquad (\ell^-)^2 = (\ell^+)^2 = 0 \qquad \ell^+ \cdot \ell^- = 2E^2$$

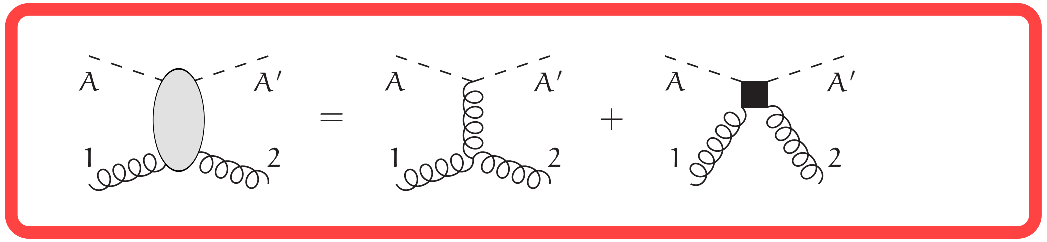
Reggeized gluon $\Longrightarrow$ gluon with momentum $x_\pm \ell^\pm + k_\perp$.

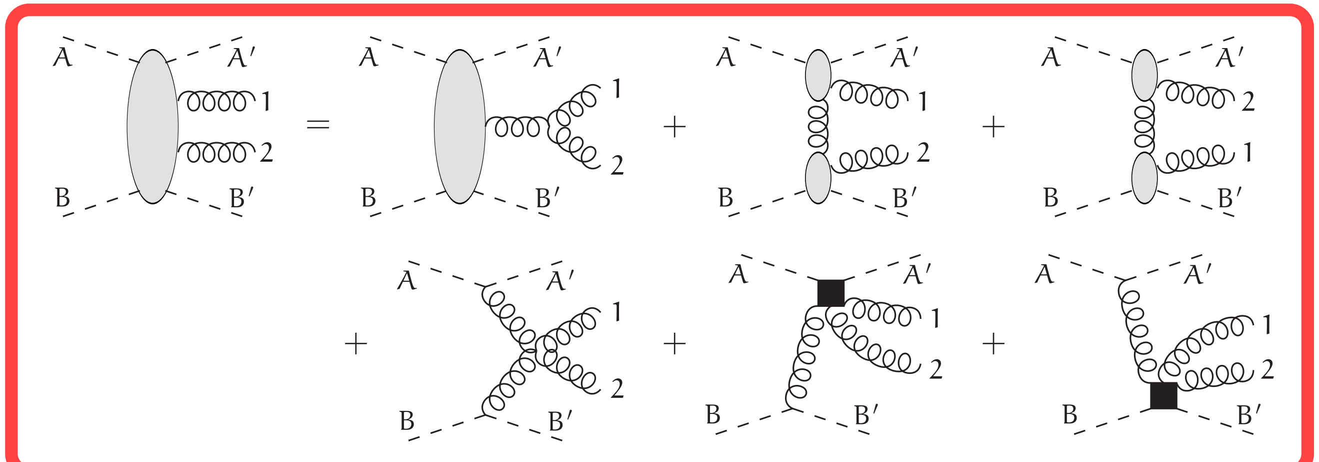
Effective action $\Longrightarrow$ vertices of arbitrary order.

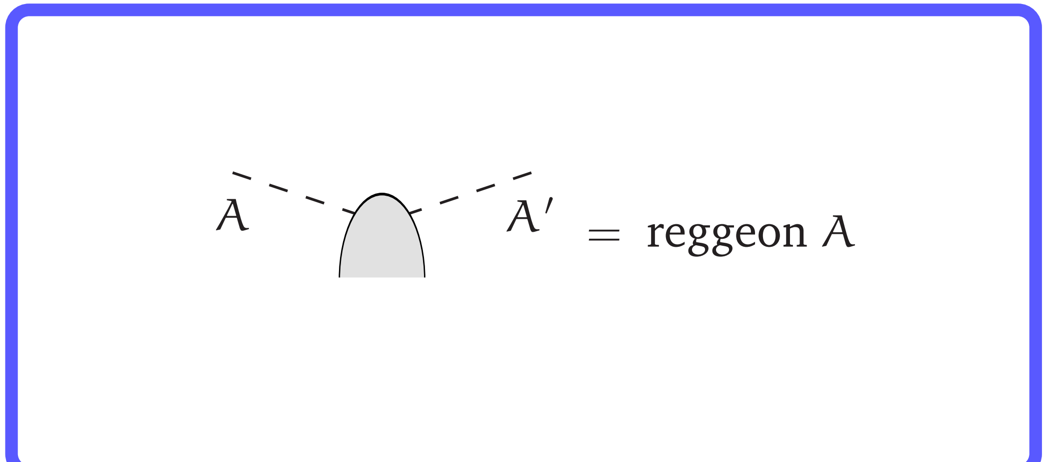
Reggeon–gluon vertices

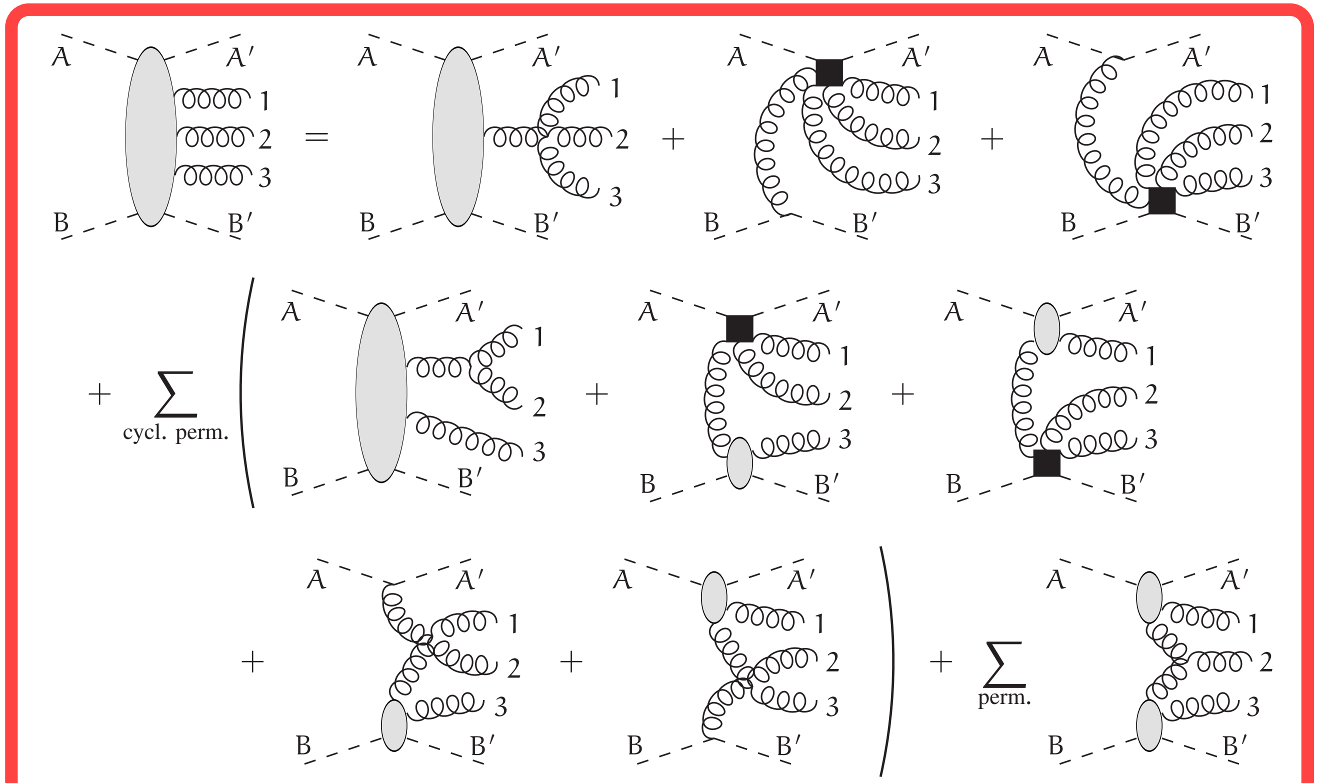
Antonov, Lipatov, Kuraev, Cherednikov
2005











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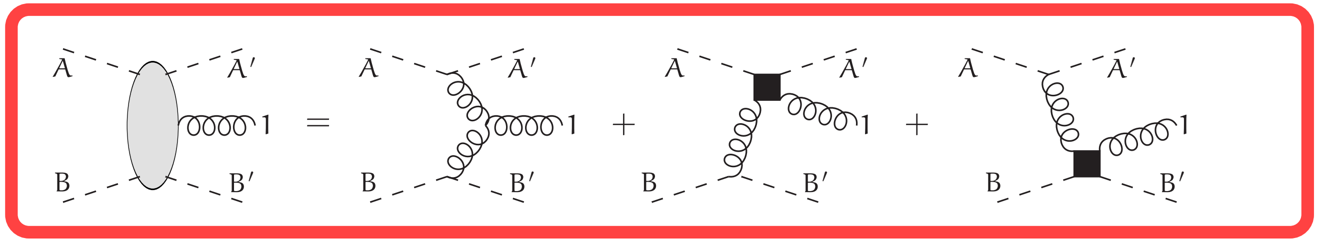
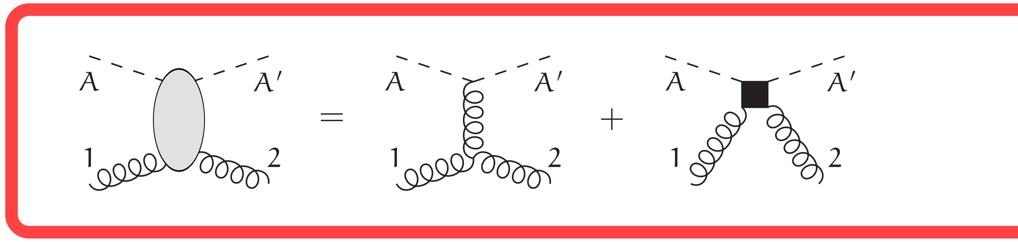
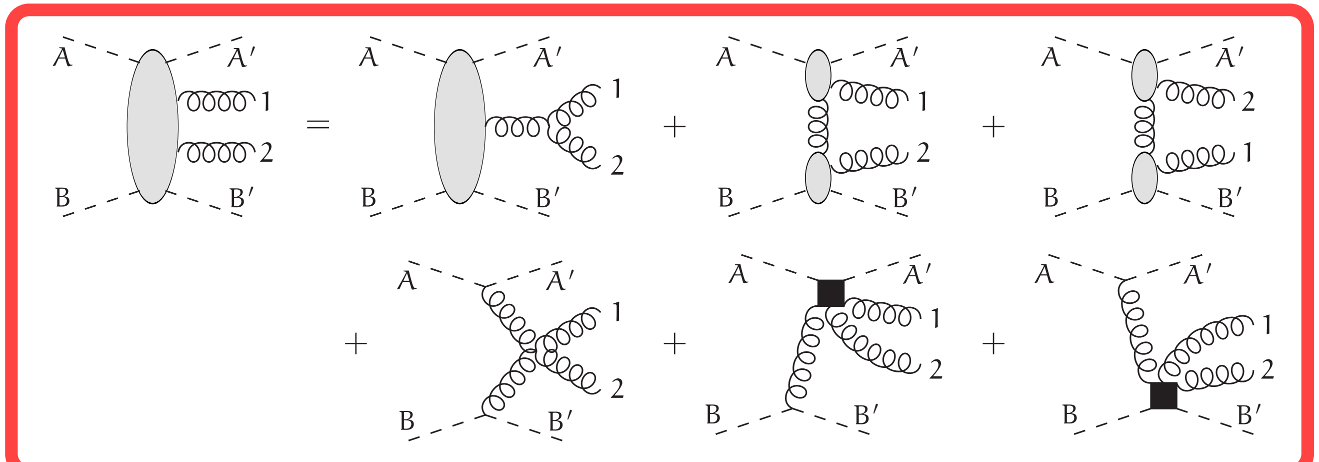
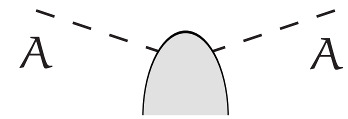
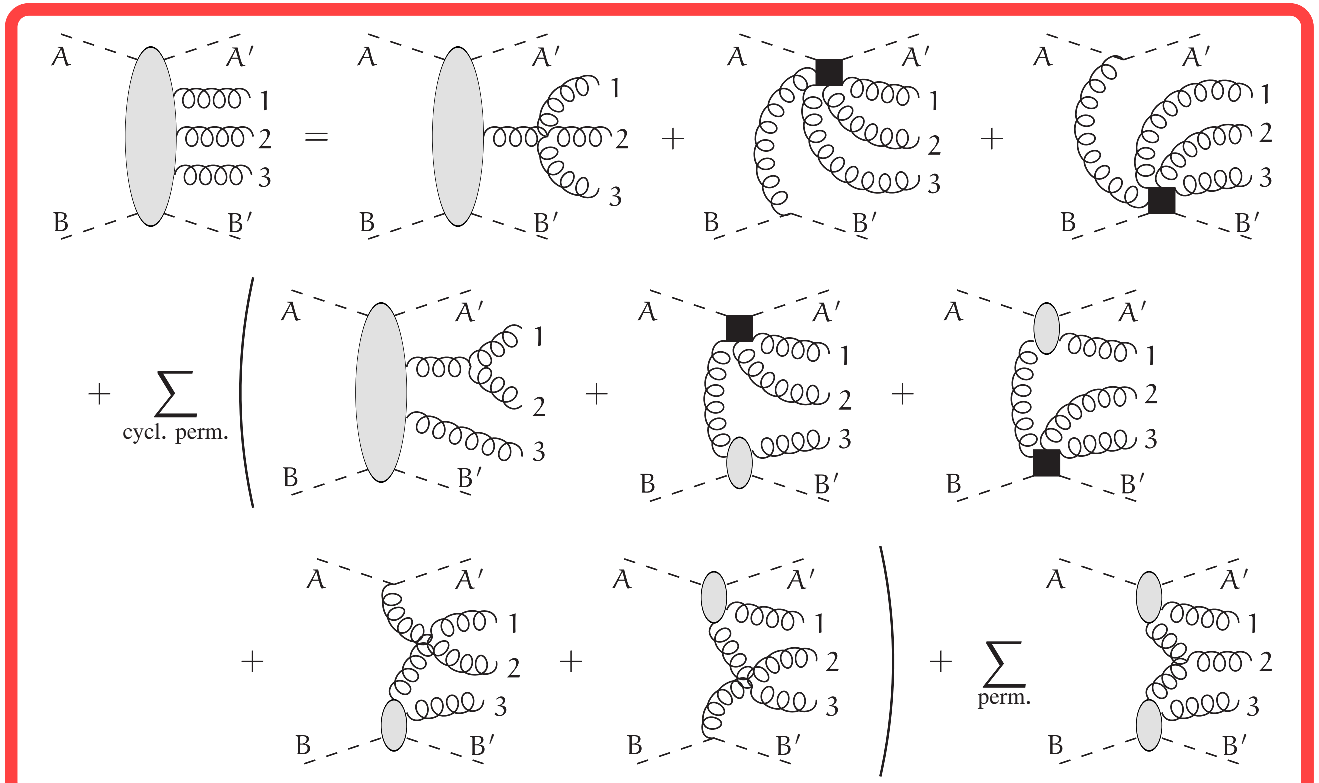


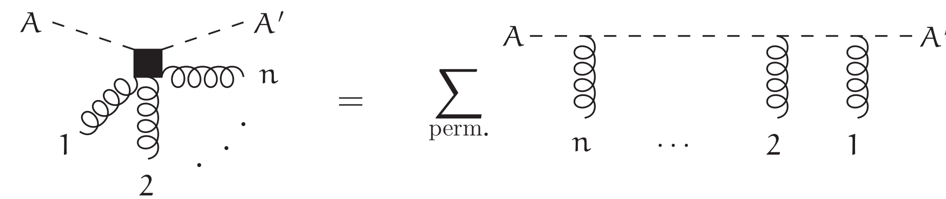


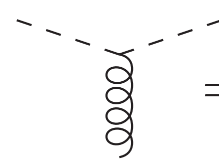




= reggeon A







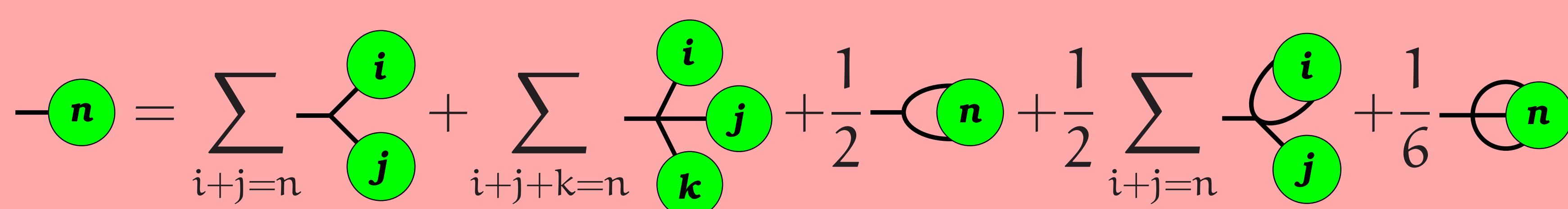
$= \ell_1^\mu T^a$

$- - - - - = \dfrac{i}{\ell_1 \cdot p}$

Dyson–Schwinger recurrence

$$\left(i(\Box - m^2)\frac{\delta}{\delta J(x)} + \frac{g}{2}\frac{\delta^2}{\delta J(x)^2} - \frac{i\lambda}{6}\frac{\delta^3}{\delta J(x)^3}\right) Z[J] + J(x)Z[J] = 0$$

$$Z[J] = \exp\left(\sum_n \int dx_1 dx_2 \cdots dx_n G_n(x_1, x_2, \ldots, x_n)\, J(x_1)J(x_2)\cdots J(x_n)\right)$$



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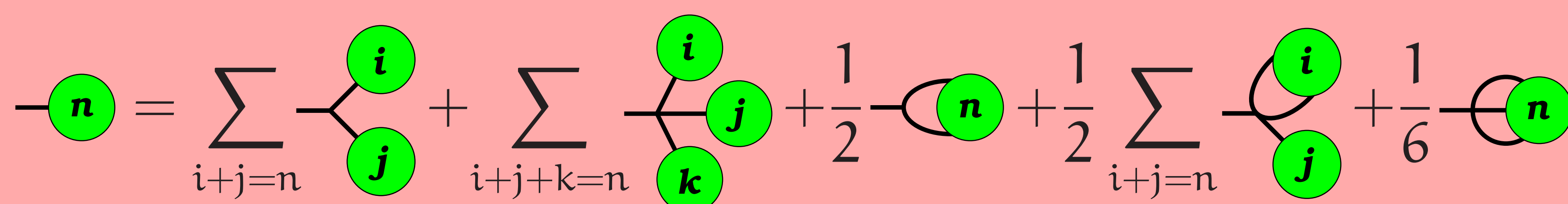
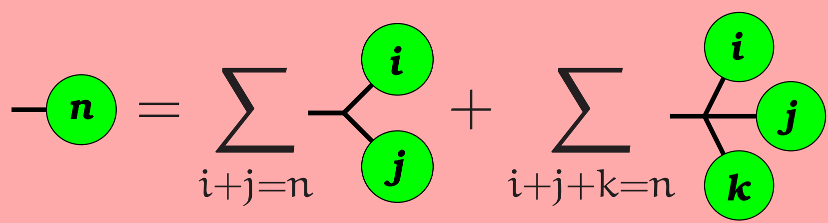


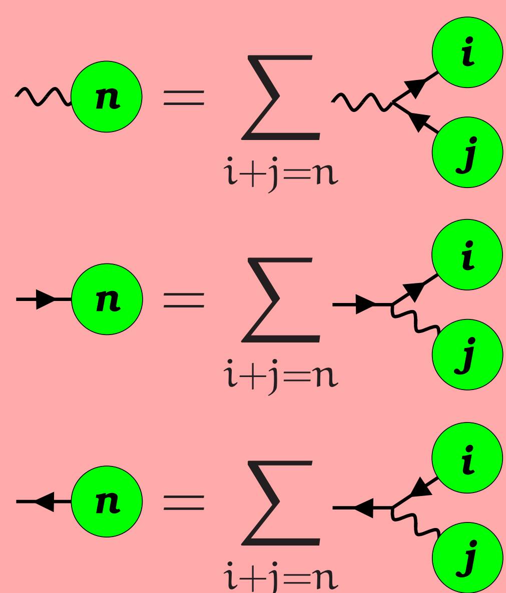
ALPGEN, HELAC, O'MEGA, COMIX, CAMORRA, RECOLA, ...

Recursive calculation of amplitudes

Tree-level amplitudes can be directly calculated using the Dyson-Schwinger recursive relations.
Theories with four-point vertices:



Theories with more types of currents:



Currents may have several components.

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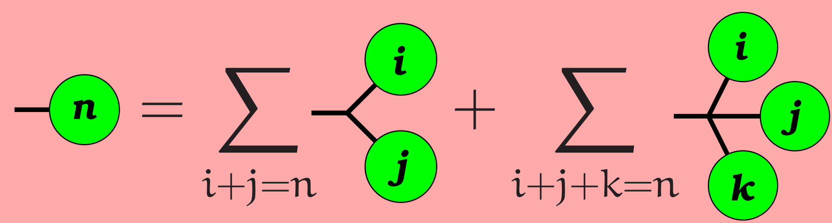

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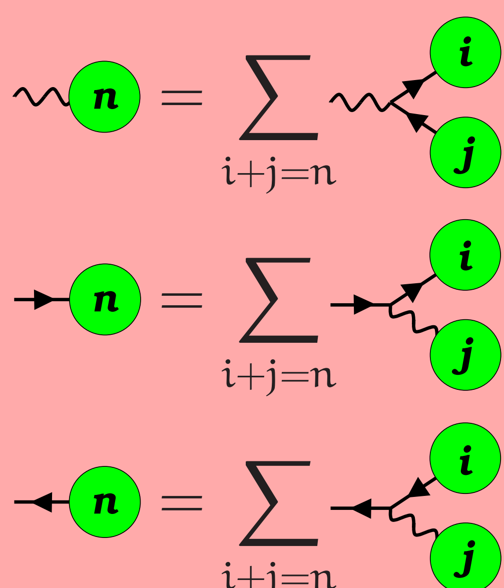
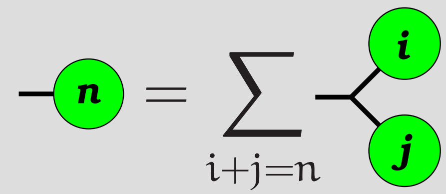
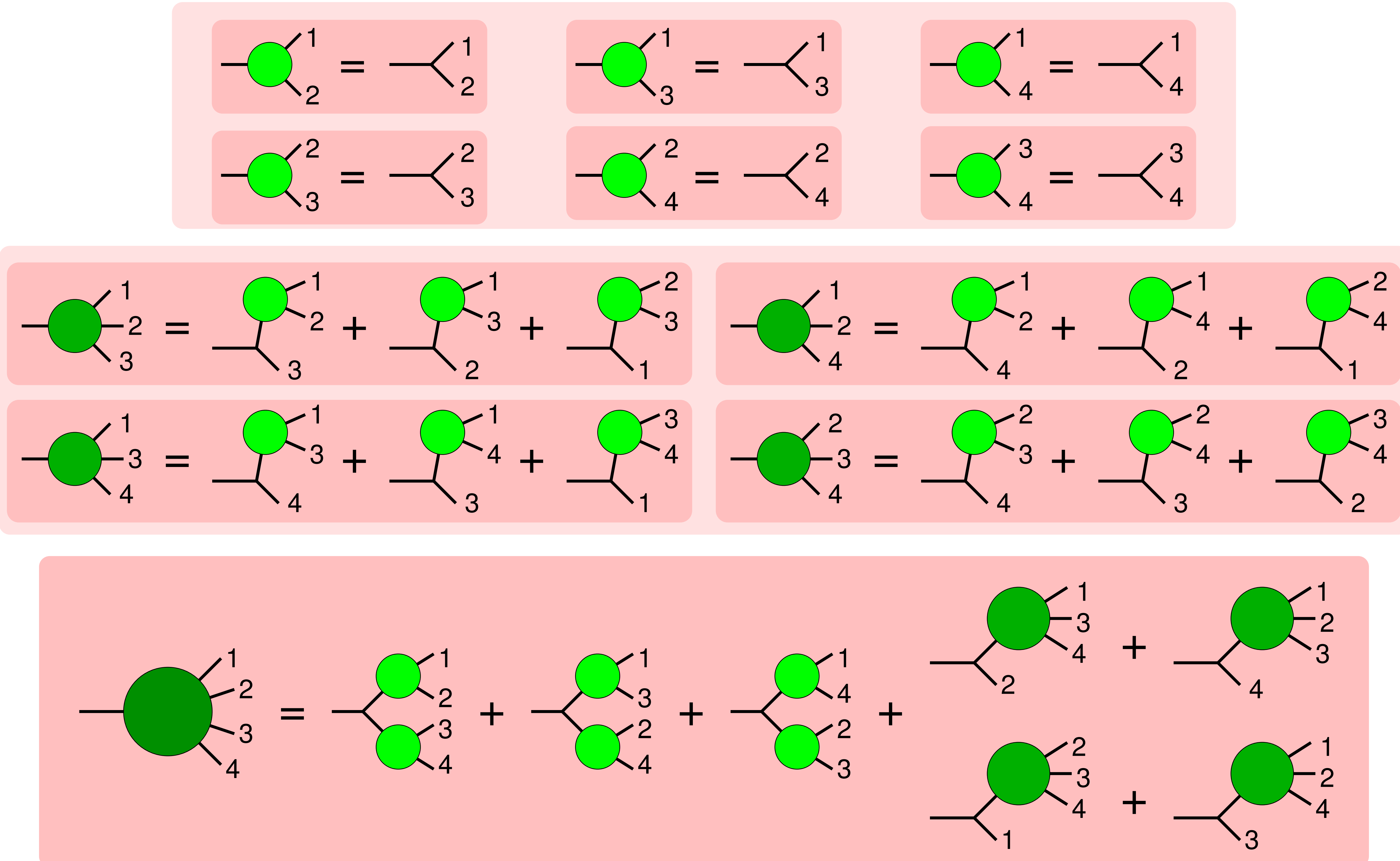


Currents may have several components.

- distinguishable external lines correspond to on-shell particles
  $\Longrightarrow$ polarization vectors, spinors, 1
- sum of momenta of on-shell lines is equal to momentum of off-shell line
- vertices directly from Feynman rules in momentum space
- off-shell line carries propagator from Feynman rules, in any gauge
- on-shell $(n+1)$-leg amplitude
  - from current with n on-shell legs
  - by omitting the final propagator
  - and contracting with pol.vec. or spinor instead

Recursive computation





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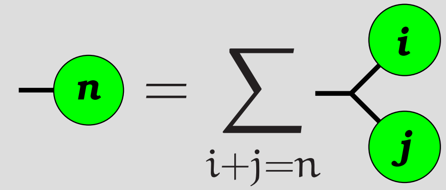
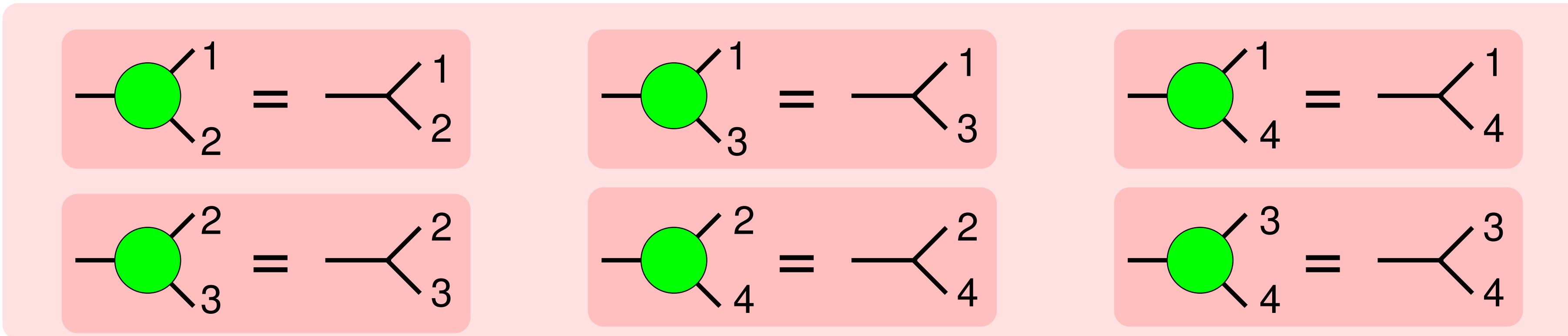
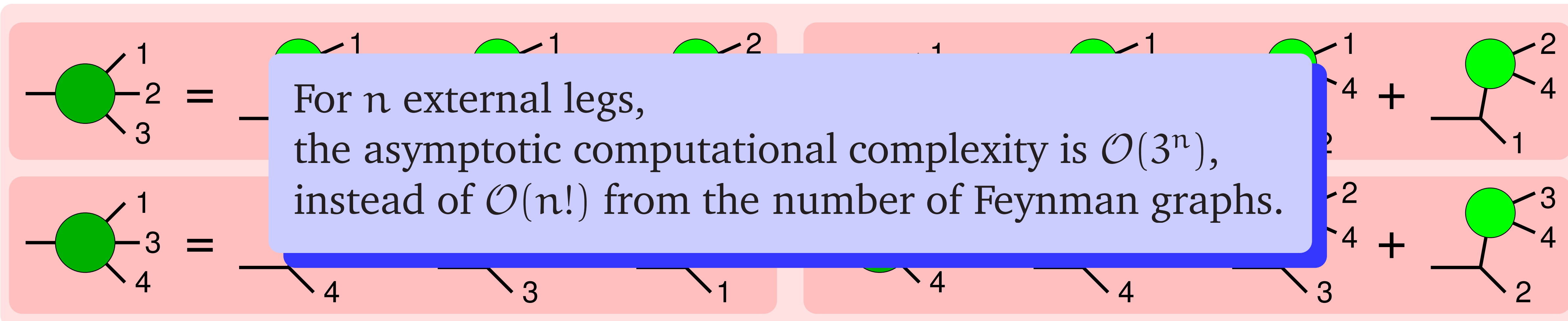
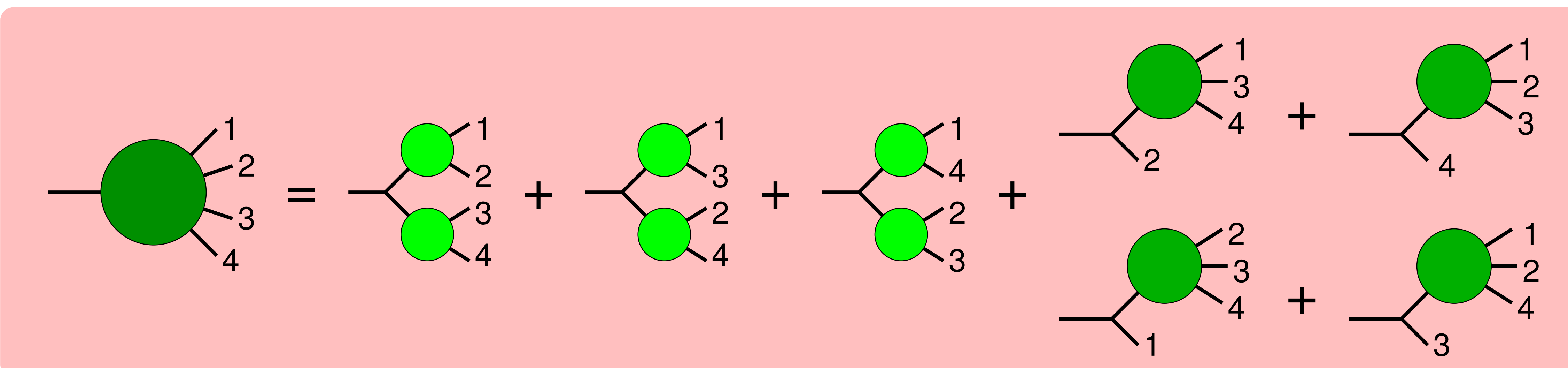






For n external legs,
the asymptotic computational complexity is $\mathcal{O}(3^n)$,
instead of $\mathcal{O}(n!)$ from the number of Feynman graphs.



DS skeleton for 0 –> e+ e– e+ e– A

```
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same momentum, different particle

$$\overline{\Psi}_{11} =+ \overline{\Psi}_3 \not{A}_4 (-ie) \frac{i}{\not{p}_{12} - m}$$

$$\Psi_{12} =+ \frac{i}{-\not{p}_7 - m}(-ie)\not{A}_8 \Psi_1$$

$$A^{\mu}_{13} =+ \frac{-i}{p^2_{13}}(-ie)\overline{\Psi}_{11}\gamma^{\mu}\Psi_1$$

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fermi sign

$$(-1)^{\chi(p,q)} \quad , \quad \chi(p,q) = \sum_{i=n}^{2} \hat{p}_i \sum_{j=1}^{i-1} \hat{q}_j$$

$\hat{p}_i = 1$ if external particle i is a fermion and is present in p, else $\hat{p}_i = 0$

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 4: -1,   8[   6 A ] <--    3[   4 E-]    2[   2 E+]
 5: -1,   9[   6 Z ] <--    3[   4 E-]    2[   2 E+]
 6:  1,  10[  10 E+] <--    4[   8 A ]    2[   2 E+]
 7:  1,  11[  12 E-] <--    4[   8 A ]    3[   4 E-]
 8: -1,  12[   7 E+] <--    2[   2 E+]    5[   5 A ]
 9: -1,  12[   7 E+] <--    2[   2 E+]    6[   5 Z ]
10:  1,  12[   7 E+] <--    1[   1 E+]    8[   6 A ]
11:  1,  12[
12: -1,  13[
13: -1,  14[
14: -1,  13[
15: -1,  14[
16: -1,  15[  14 A ] <--    3[   4 E-]   10[  10 E+]
17: -1,  16[  14 Z ] <--    3[   4 E-]   10[  10 E+]
18: -1,  15[  14 A ] <--    2[   2 E+]   11[  12 E-]
19: -1,  16[  14 Z ] <--    2[   2 E+]   11[  12 E-]
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24:  1,  17[  15 E+] <--    4[   8 A ]   12[   7 E+]
25: -1,  17[  15 E+] <--    2[   2 E+]   13[  13 A ]
26: -1,  17[  15 E+] <--    2[   2 E+]   14[  13 Z ]
27:  1,  17[  15 E+] <--    1[   1 E+]   15[  14 A ]
28:  1,  17[  15 E+] <--    1[   1 E+]   16[  14 Z ].
```

same momentum, different particle

$$\overline{\Psi}_{11} =+ \overline{\Psi}_3 \not{A}_4 (-ie) \frac{i}{\not{p}_{12} - m}$$

$$\Psi_{12} =+ \frac{i}{-\not{p}_7 - m} (-ie) \not{A}_8 \Psi_1$$

Ψ_1

Effective action approach does not combine optimally with the recursive DS approach.

fermi sign

$$(-1)^{\chi(p,q)} \quad , \quad \chi(p,q) = \sum_{i=n}^{2} \hat{p}_i \sum_{j=1}^{i-1} \hat{q}_j$$

$\hat{p}_i = 1$ if external particle i is a fermion and is present in p, else $\hat{p}_i = 0$

Prescription for g* g* –> X

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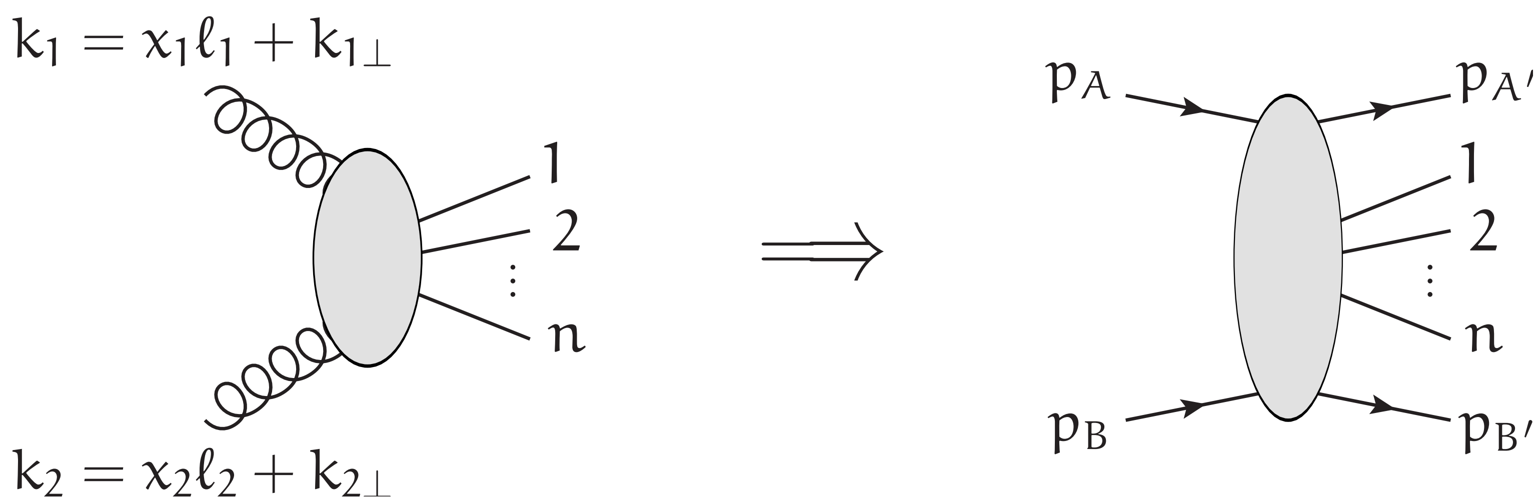
1. Consider the embedding $q_A q_B \to q_A q_B X$



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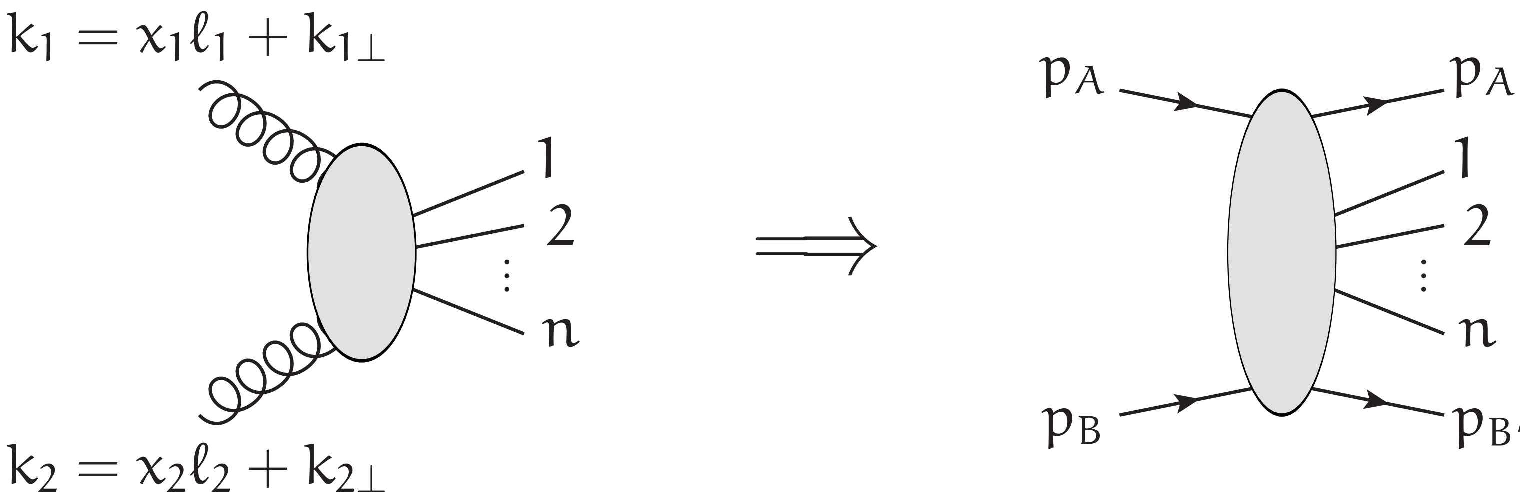

with momentum flow as if the momenta p_A, p_B of the initial-state quarks and $p_{A'}, p_{B'}$ of the final-state quarks are given by

$$p_A^\mu = k_1^\mu \quad , \quad p_B^\mu = k_2^\mu \quad , \quad p_{A'}^\mu = p_{B'}^\mu = 0 \ .$$

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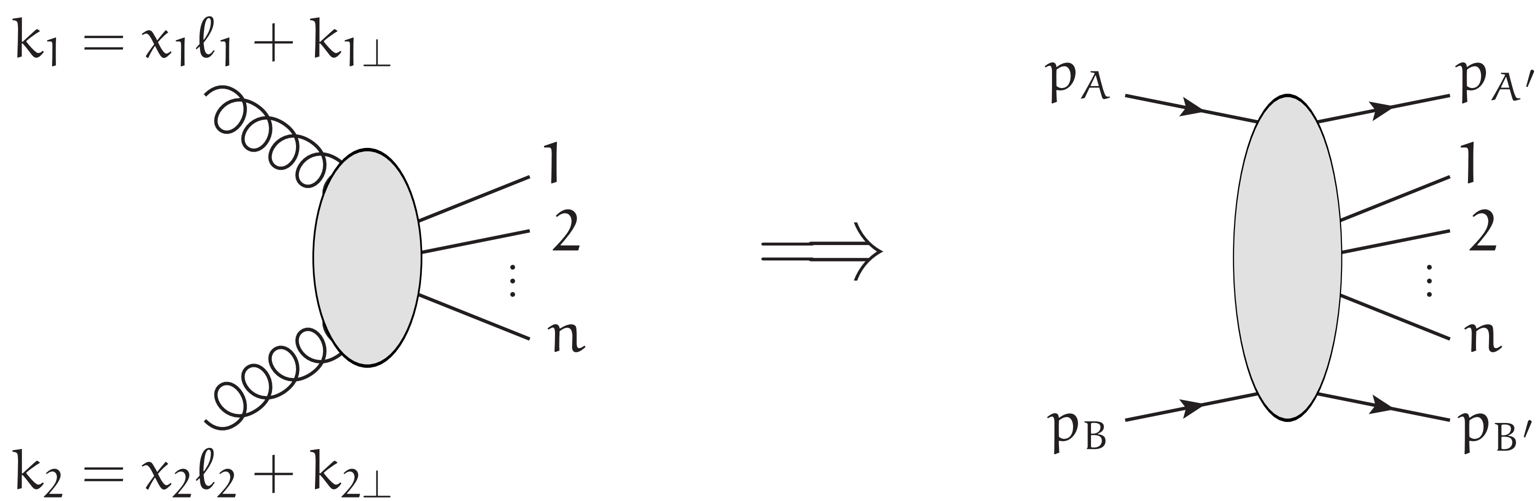

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2. Assign the spinors $|\ell_1]$, $\langle\ell_1|$ to the external A-quarks, and assign $i\not{\ell}_1/(2\ell_1\cdot p)$ instead of $i\not{p}/p^2$ to the propagators on the A-quark line.

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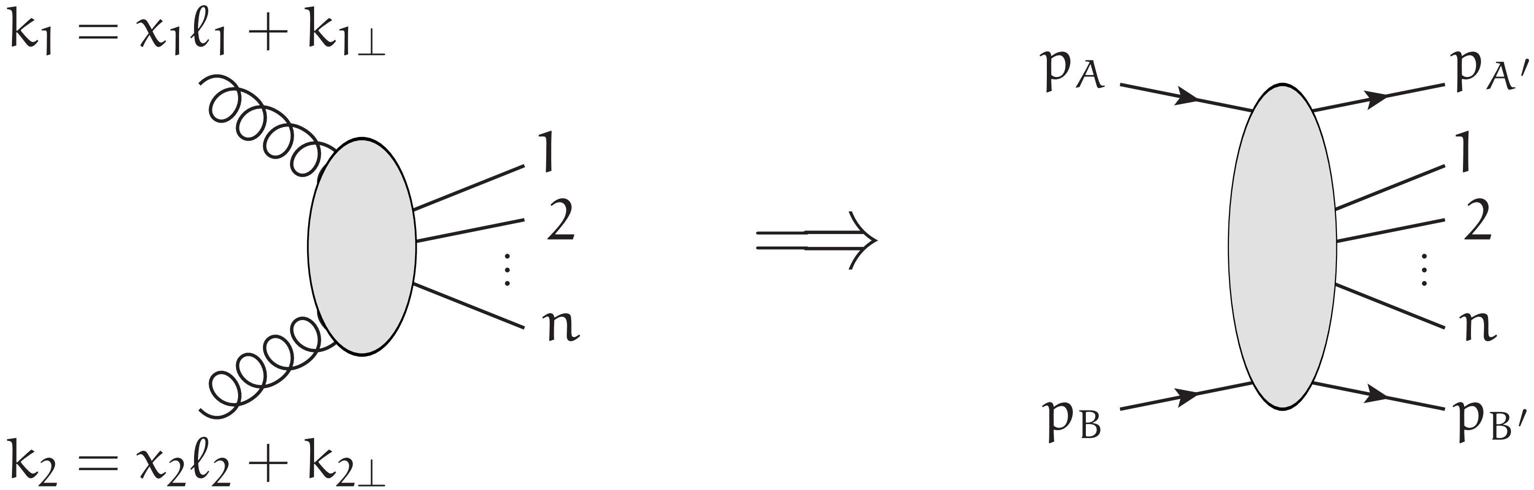

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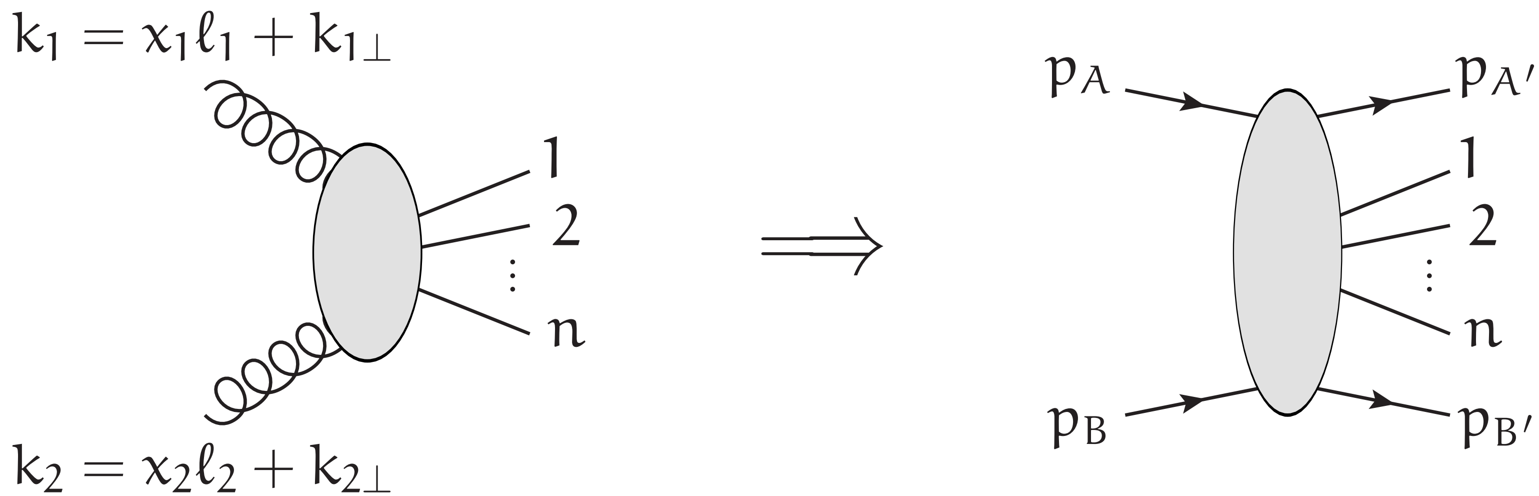

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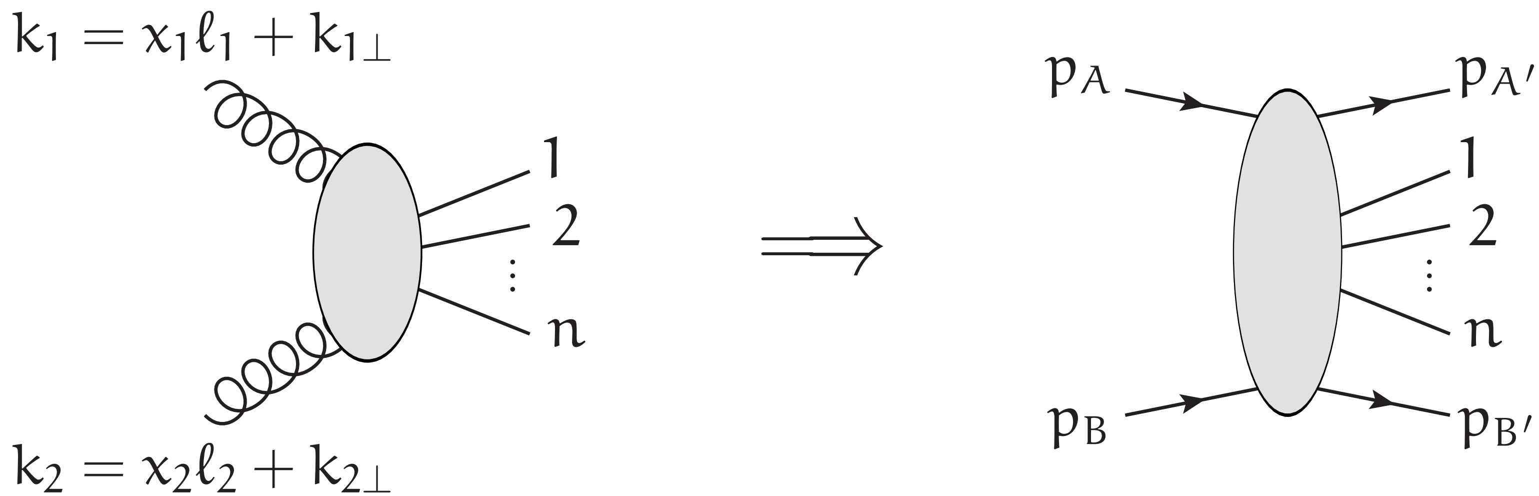

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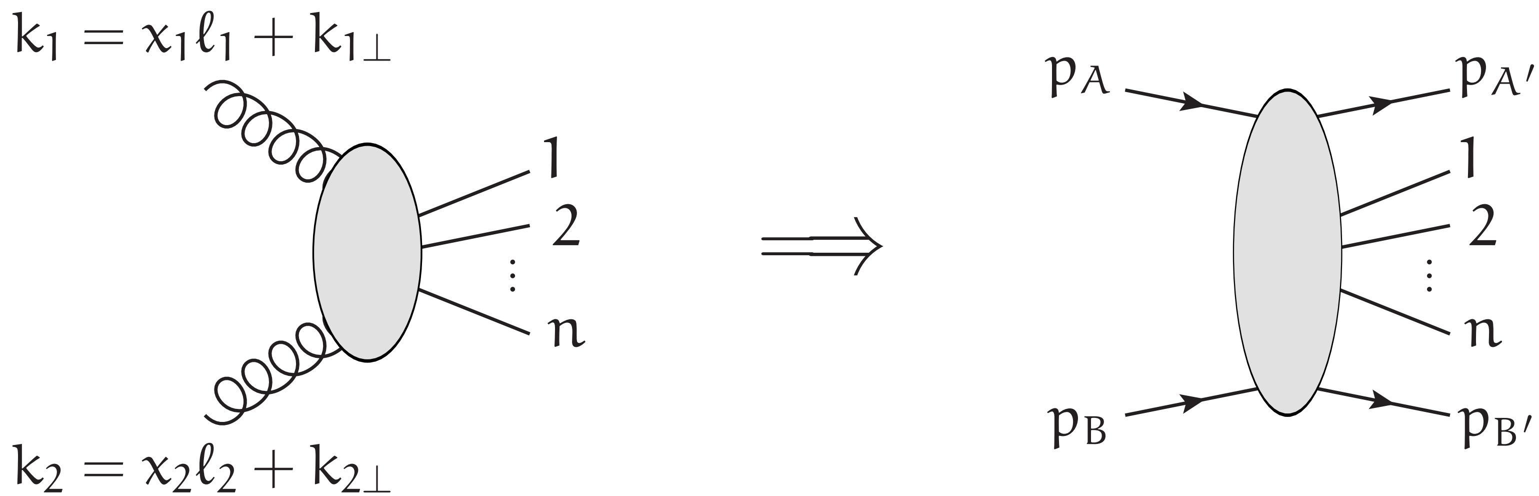

with … s and
$p_{A'}$, p…

In agreement with Lipatov's effective action!

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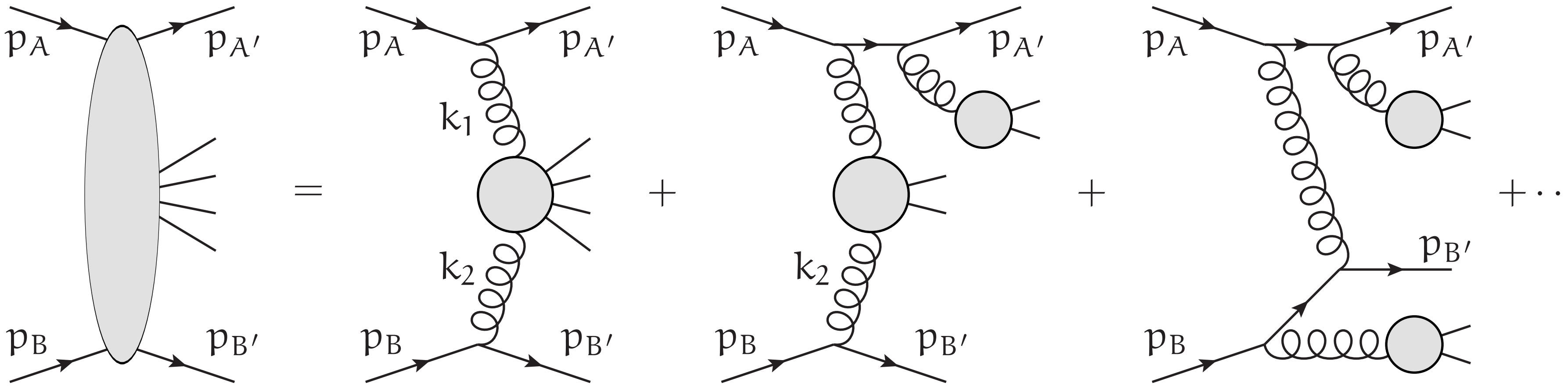
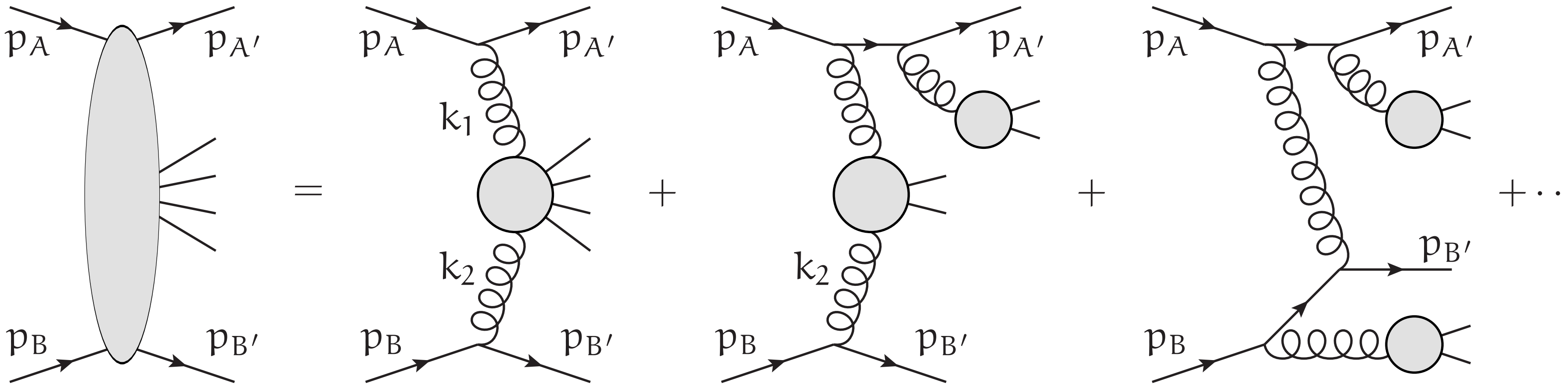
Derivation



The embedding with on-shell quarks ensures gauge invariance.

Derivation



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High-energy factorization dictates that

$$p_A - p_{A'} = x_1\ell_1 + k_{1\perp} \quad , \quad p_B - p_{B'} = x_2\ell_2 + k_{2\perp}$$

Now there is a freedom in the choice of the momenta $p_A, p_{A'}, p_B, p_{B'}$.

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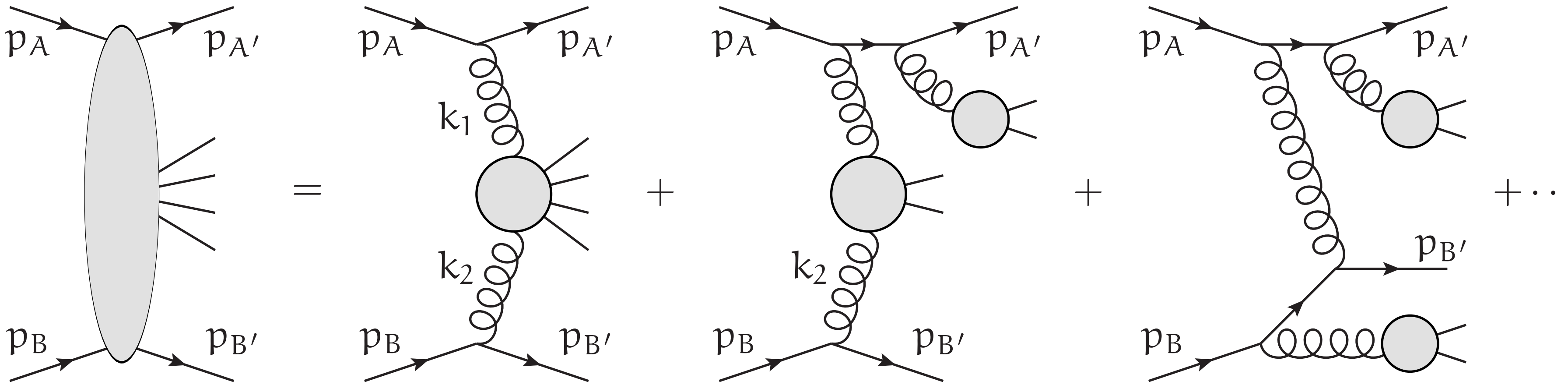

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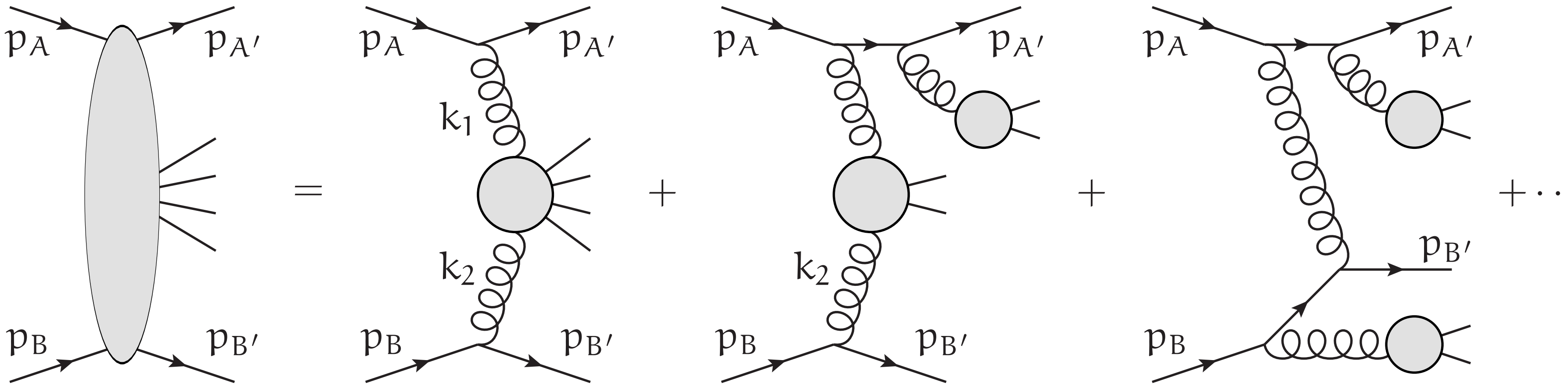
For every k^μ with $k^2 < 0$, there are an ℓ_k^μ and $k_\perp^\mu$ such that

$$\ell_k^2 = 0 \quad , \quad \ell_k \cdot k = 0 \quad , \quad \ell_k \cdot k_\perp = 0$$

$$\text{and} \quad k^\mu = x\ell_k^\mu + k_\perp^\mu$$

Derivation



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but this, together with on-shellness of $p_A, p_{A'}, p_B, p_{B'}$, is kinematically not possible.

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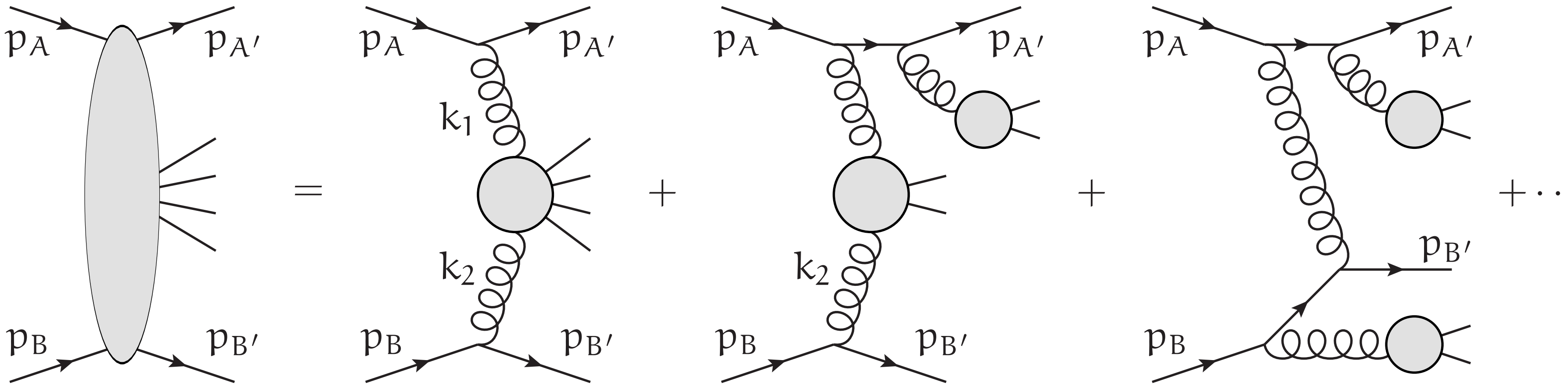

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Maybe it is possible in a certain limit...

Derivation: complex momenta

$$\ell_3^\mu = \tfrac{1}{2}\langle\ell_2|\gamma^\mu|\ell_1]$$
$$\ell_4^\mu = \tfrac{1}{2}\langle\ell_1|\gamma^\mu|\ell_2]$$

$$\ell_1^2 = \ell_2^2 = 0$$
$$\ell_3^2 = \ell_4^2 = 0$$
$$\ell_{1,2}\cdot\ell_{3,4} = 0$$

$$\ell_1\cdot\ell_2 = -\ell_3\cdot\ell_4$$

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$$p_A^\mu = (\Lambda + x_1)\ell_1^\mu - \frac{\ell_4 \cdot k_{1\perp}}{\ell_1 \cdot \ell_2}\,\ell_3^\mu \qquad p_{A'}^\mu = \Lambda \ell_1^\mu + \frac{\ell_3 \cdot k_{1\perp}}{\ell_1 \cdot \ell_2}\,\ell_4^\mu$$

$$p_B^\mu = (\Lambda + x_2)\ell_2^\mu - \frac{\ell_3 \cdot k_{2\perp}}{\ell_1 \cdot \ell_2}\,\ell_4^\mu \qquad p_{B'}^\mu = \Lambda \ell_2^\mu + \frac{\ell_4 \cdot k_{2\perp}}{\ell_1 \cdot \ell_2}\,\ell_3^\mu$$

Now we have both the high-energy limit and on-shellness:

$$p_A^\mu - p_{A'}^\mu = x_1 \ell_1^\mu + k_{1\perp}^\mu \qquad p_B^\mu - p_{B'}^\mu = x_2 \ell_2^\mu + k_{2\perp}^\mu$$

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for any value of the dimensionless parameter Λ.

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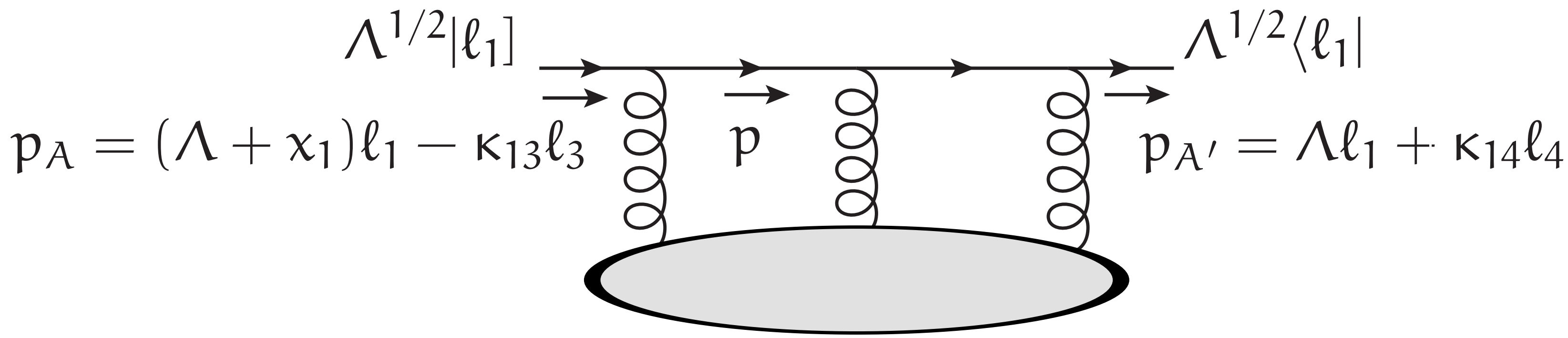
for any value of the dimensionless parameter Λ.

External spinors:

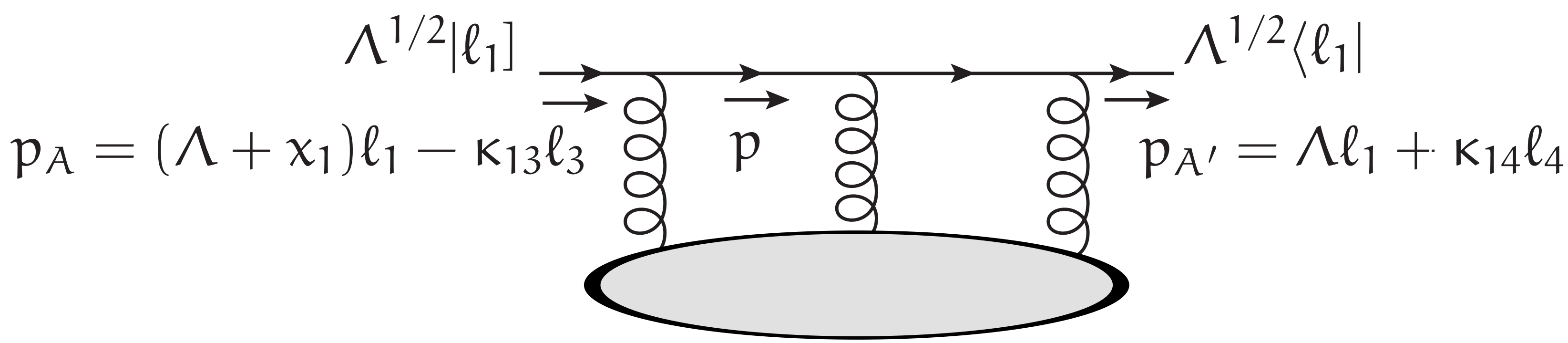
$$|p_A] = \frac{\Lambda + x_1 + \kappa_{13}}{\sqrt{|\Lambda + x_1 + \kappa_{13}|}}\,|\ell_1] \quad , \quad \langle p_{A'}| = \sqrt{|\Lambda - \kappa_{14}|}\,\langle \ell_1|$$

$$|p_B] = \frac{\Lambda + x_2 + \kappa_{24}}{\sqrt{|\Lambda + x_2 + \kappa_{24}|}}\,|\ell_2] \quad , \quad \langle p_{B'}| = \sqrt{|\Lambda - \kappa_{23}|}\,\langle \ell_2|$$

Derivation


$\Lambda^{1/2}|\ell_1]$
$\Lambda^{1/2}\langle\ell_1|$
$p_A = (\Lambda + x_1)\ell_1 - \kappa_{13}\ell_3$
p
$p_{A'} = \Lambda\ell_1 + \kappa_{14}\ell_4$

Derivation



Take the limit $\Lambda \to \infty$ to remove imaginary momentum components.

- Auxiliary A-quark spinors give an overall factor Λ
- Further only the auxiliary quark line propagators are affected:

$$\frac{\not p}{p^2} = \frac{(\Lambda + x_p)\not\ell_1 + y_p\not\ell_2 + \not p_\perp}{2(\Lambda + x_p)\, y_p\, \ell_1 \cdot \ell_2 + p_\perp^2} \xrightarrow{\Lambda\to\infty} \frac{\not\ell_1}{2\, y_p\, \ell_1 \cdot \ell_2} = \frac{\not\ell_1}{2\ell_1 \cdot p} = \frac{\not\ell_1}{2\ell_1 \cdot (p - p_{A'})}$$

- Demand of correct collinear limit for $k_{1\perp} \to 0$ requires kinematical factor $x_1\sqrt{-k_\perp^2/2}$
- Completely analogously for B-quark line.

Off–shell initial–state quarks

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Effective action: Lipatov, Vyazovsky 2001 Nefedov, Saleev, Shipilova 2013

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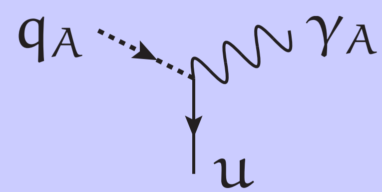
Our approach: embed the process

$u^* g \to u\,X \quad \Longrightarrow \quad q_A g \to \gamma_A u\,X$

auxiliary photon:



$= \gamma^\mu$

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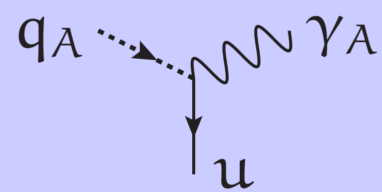

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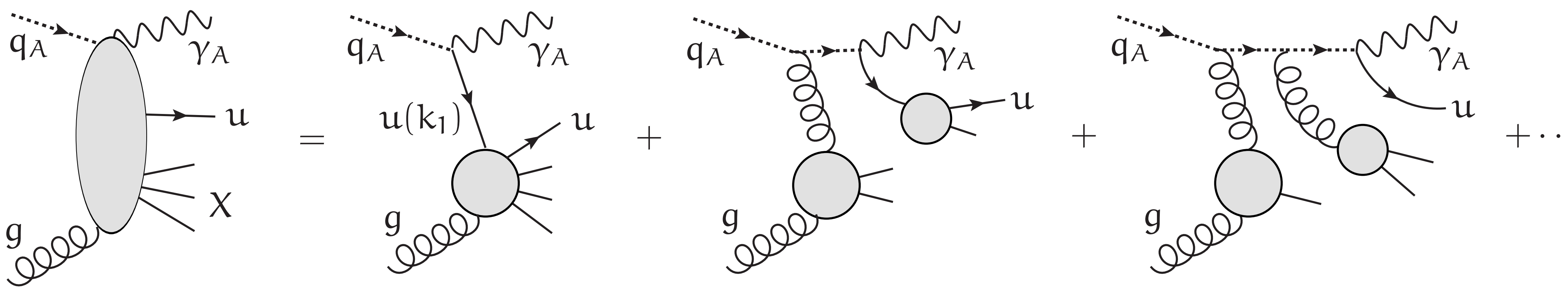

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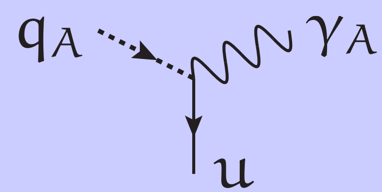
auxiliary photon: 

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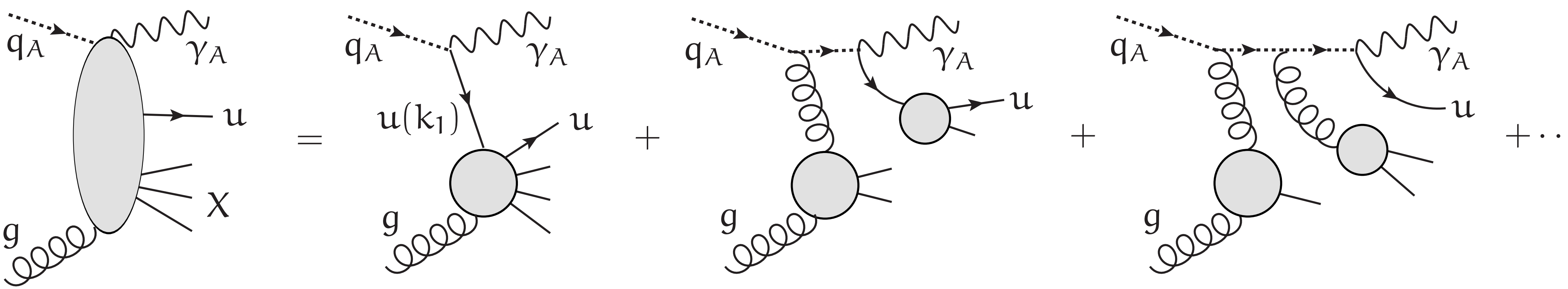

Use same momentum decomposition for $p_A, p_{A'}$ as for off-shell gluon.
What polarization vector for auxiliary photon?

$$\text{negative helicity } u\text{-quark} \;\to\; \langle p_u| \,, \quad q_A \;\leftarrow\; |\ell_1] \,, \quad \varepsilon^\mu_{A'} = \frac{\langle \ell_1|\gamma^\mu|\ell_2]}{\sqrt{2}\,[\ell_1|\ell_2]}$$

Analytic result for g* g –> g g

$$0 \to g^*(p_1 + k_T)\, g(p_2)\, g(p_3)\, g(p_4)$$

$$\mathcal{M}^{a_1 a_2 a_3 a_4}(1,2,3,4) = \frac{4g_s^2}{\sqrt{2}} \sum_{\text{non-cyclic}} \mathrm{Tr}(T^{a_1}T^{a_2}T^{a_3}T^{a_4})\,\mathcal{A}(1,2,3,4)$$

$$\mathcal{A}(2^-,3^-,4^-) = 0 \qquad \mathcal{A}(2^+,3^+,4^+) = 0$$

$$\mathcal{A}(2^-,3^-,4^+) = \frac{[3|\not{k}_T|1\rangle}{|k_T|[31]}\;\frac{[41]^4}{[12][23][34][41]} \qquad \mathcal{A}(2^+,3^+,4^-) = \frac{\langle 1|\not{k}_T|3]}{|k_T|\langle 13\rangle}\;\frac{\langle 41\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle}$$

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$$\left|\frac{[i|\not{k}_T|1\rangle}{|k_T|[i1]}\right| = \left|\frac{\langle 1|\not{k}_T|i]}{|k_T|\langle 1i\rangle}\right| = 1$$

Analytic result for u* g –> u g

$$0 \to g(p_1)\, g(p_2)\, u(p_u)\, \bar{u}^*(p_{\bar{u}} + k_T)$$

$$\mathcal{M}^{a_1,a_2}_{j_u,j_{\bar{u}}}(1,2,u,\bar{u}) = 2ig_S^2\Big[\big(T^{a_1}T^{a_2}\big)_{j_u,j_{\bar{u}}}\mathcal{A}(1,2,u,\bar{u}) + \big(T^{a_2}T^{a_1}\big)_{j_u,j_{\bar{u}}}\mathcal{A}(2,1,u,\bar{u})\Big]$$

$$\mathcal{A}(1^+,2^-,u^+,\bar{u}^+) = -\frac{[\bar{u}|\not{k}_T|1\rangle}{|k_T|\langle\bar{u}1\rangle}\,\frac{\langle\bar{u}1\rangle^3\langle u1\rangle}{\langle u1\rangle\langle 12\rangle\langle 2\bar{u}\rangle\langle\bar{u}u\rangle}$$

$$\mathcal{A}(1^-,2^+,u^+,\bar{u}^+) = -\frac{[\bar{u}|\not{k}_T|2\rangle}{|k_T|\langle\bar{u}2\rangle}\,\frac{\langle\bar{u}2\rangle^3\langle u2\rangle}{\langle u1\rangle\langle 12\rangle\langle 2\bar{u}\rangle\langle\bar{u}u\rangle}$$

$$\mathcal{A}(1^+,2^-,u^-,\bar{u}^-) = \frac{\langle\bar{u}|\not{k}_T|1]}{|k_T|[\bar{u}1]}\,\frac{[\bar{u}1]^3[u1]}{[u1][12][2\bar{u}][\bar{u}u]}$$

$$\mathcal{A}(1^-,2^+,u^-,\bar{u}^-) = \frac{\langle\bar{u}|\not{k}_T|2]}{|k_T|[\bar{u}2]}\,\frac{[\bar{u}2]^3[u2]}{[u1][12][2\bar{u}][\bar{u}u]}$$

Analytic result for u* g –> u g

$$0 \to g(p_1)\, g(p_2)\, u(p_u)\, \bar{u}^*(p_{\bar{u}} + k_T)$$

$$\mathcal{M}^{a_1,a_2}_{j_u,j_{\bar{u}}}(1,2,u,\bar{u}) = 2ig_S^2\Big[\big(T^{a_1}T^{a_2}\big)_{j_u,j_{\bar{u}}}\mathcal{A}(1,2,u,\bar{u}) + \big(T^{a_2}T^{a_1}\big)_{j_u,j_{\bar{u}}}\mathcal{A}(2,1,u,\bar{u})\Big]$$

$$\mathcal{A}(1^+,2^-,u^+,\bar{u}^+) = -\frac{[\bar{u}|\not{k}_T|1\rangle}{|k_T|\langle\bar{u}1\rangle}\,\frac{\langle\bar{u}1\rangle^3\langle u1\rangle}{\langle u1\rangle\langle 12\rangle\langle 2\bar{u}\rangle\langle\bar{u}u\rangle}$$

$$\mathcal{A}(1^-,2^+,u^+,\bar{u}^+) = -\frac{[\bar{u}|\not{k}_T|2\rangle}{|k_T|\langle\bar{u}2\rangle}\,\frac{\langle\bar{u}2\rangle^3\langle u2\rangle}{\langle u1\rangle\langle 12\rangle\langle 2\bar{u}\rangle\langle\bar{u}u\rangle}$$

$$\mathcal{A}(1^+,2^-,u^-,\bar{u}^-) = \frac{\langle\bar{u}|\not{k}_T|1]}{|k_T|[\bar{u}1]}\,\frac{[\bar{u}1]^3[u1]}{[u1][12][2\bar{u}][\bar{u}u]}$$

$$\mathcal{A}(1^-,2^+,u^-,\bar{u}^-) = \frac{\langle\bar{u}|\not{k}_T|2]}{|k_T|[\bar{u}2]}\,\frac{[\bar{u}2]^3[u2]}{[u1][12][2\bar{u}][\bar{u}u]}$$

$$\mathcal{A}(1^+,2^+,u^-,\bar{u}^-) = -|k_T|\,\frac{\langle\bar{u}u\rangle^3}{\langle u1\rangle\langle 12\rangle\langle 2\bar{u}\rangle\langle\bar{u}u\rangle}$$

$$\mathcal{A}(1^-,2^-,u^+,\bar{u}^+) = |k_T|\,\frac{[\bar{u}u]^3}{[u1][12][2\bar{u}][\bar{u}u]}$$

Summary

- We presented a prescription to evaluate tree-level scattering amplitudes with off-shell initial-state partons, that can be readily applied in automatic calculations for arbitrary final states.

- Has been implemented into
  - a `C++` program `LxJet` (Kotko) for $g^* x_b \longrightarrow x_1 x_2 x_3$ with x_i arbitrary partons. Uses the fact that almost all gauge contributions vanish in a suitable axial gauge.
  - a `Fortran` program (AvH) for arbitrary processes.

  to be public soon.

- Has been applied to a study of forward jets. More studies to follow.