

# Three-jet production with off-shell matrix elements and saturation

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based on

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# PLAN

- Motivation
- Framework
  - “hybrid” high energy factorization
  - off-shell matrix elements
  - unintegrated gluon density
  - comments on factorization breaking
- Results: three jet production in p-p and p-Pb
  - forward-central rapidity region
  - purely forward jets
- Summary and future plans

# Motivation

Studies of small  $x$  "unintegrated" gluon densities in more exclusive observables

Different evolution scenarios

- BFKL
- KMS (Kwieciński-Martin-Staśto) = BFKL + sub-leading corrections
- BK (Balitsky-Kovchegov)  $\Rightarrow$  saturation
- KS (Kutak-Staśto) = BK + sub-leading corrections
- CCFM
- KGBJS (Kutak-Golec-Biernat-Jadach-Skrzypek) = CCFM + nonlinear term

Some of them may be applied to heavy ions.

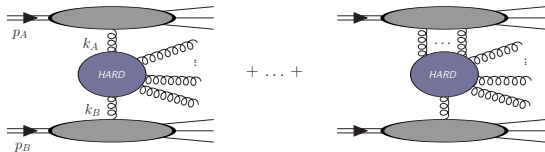
- relevant observables  $\Rightarrow$  forward jets
- why three jets?
  - $\rightarrow$  even more exclusive
  - $\rightarrow$  test high energy factorization for larger multiplicities (how important are MPIs?)
  - $\rightarrow$  test tools we have developed
  - $\rightarrow$  give (crude) predictions for LHC

# Framework

# Framework

"Hybrid" high energy factorization formula<sup>1,2</sup>

$$d\sigma_{AB \rightarrow X} = \sum_b \int \frac{d^2 k_{TA}}{\pi} \int \frac{dx_A}{x_A} \int dx_B \mathcal{F}(x_A, k_{TA}) f_{b/B}(x_B, \mu) d\sigma_{g^* b \rightarrow X}(x_A, x_B, k_{TA})$$



$$k_A = x_A p_A + k_{TA}, \quad k_B = x_B p_B, \quad x_A \ll x_B$$

- collinear PDFs  $f_{b/B}(x_B, \mu)$
- unintegrated gluon PDF  $\mathcal{F}(x_A, k_{TA})$
- off-shell gauge invariant tree-level matrix element resides in  $d\sigma_{g^* b \rightarrow X}$

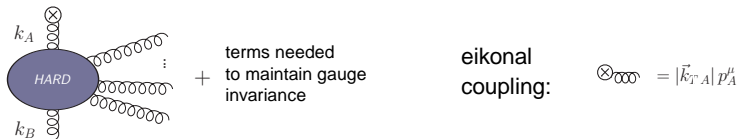
Implemented in MC codes: C++ code `LxJet` (dijets, trijets), fortran code of A. van Hameren (any process) – OSCARS (Off-shell Currents And Related Stuff)

<sup>1</sup> S. Catani, M. Ciafaloni, F. Hautmann, Nucl.Phys. B366 (1991) 135-188

<sup>2</sup> M. Deak, F. Hautmann, H. Jung, K. Kutak, JHEP 0909 (2009) 121

# Framework: off-shell amplitudes

## Definition of the off-shell high-energy amplitudes



- basic approach: Lipatov's effective action and resulting Feynman rules<sup>1</sup>
- one can also find the lacking contributions without extending QCD action (suitable for automatic calculation)
  - by embedding in a larger non-physical process; also for two off-shell legs and off-shell quarks<sup>2</sup> (see A. van Hameren's talk)
  - using the Slavnov-Taylor identities (only one off-shell gluon)<sup>3</sup>
  - using matrix elements of straight infinite Wilson lines (arbitrary number of off-shell gluons)

<sup>1</sup> E. Antonov, L. Lipatov, E. Kuraev, I. Cherednikov, Nucl.Phys. B721 (2005) 111-135

<sup>2</sup> A. van Hameren, P. Kotko, K. Kutak, JHEP 1301 (2013) 078; A. van Hameren, K. Kutak, T. Salva, arXiv:1308.2861

<sup>3</sup> A. van Hameren, P. Kotko, K. Kutak, JHEP 1212 (2012) 029

# Framework: off-shell amplitudes

Color ordered result for  $g^* g \rightarrow g \dots g$

$$\tilde{\mathcal{A}}(\varepsilon_1, \dots, \varepsilon_N) = -|\vec{k}_{TA}| \left[ k_{TA} \cdot J(\varepsilon_1, \dots, \varepsilon_N) + \left( \frac{-g}{\sqrt{2}} \right)^N \frac{\varepsilon_1 \cdot p_A \dots \varepsilon_N \cdot p_A}{k_1 \cdot p_A (k_1 - k_2) \cdot p_A \dots (k_1 - \dots - k_{N-1}) \cdot p_A} \right]$$

where

$$J^\mu(\varepsilon_1, \dots, \varepsilon_N) = \frac{-i}{k_{1N}^2} \left( g_\nu^\mu - \frac{k_{1N}^\mu p_{A,\nu} + k_{1N,\nu} p_A^\mu}{k_{1N} \cdot p_A} \right) \left\{ \sum_{i=1}^{N-1} V_3^{\nu\alpha\beta}(k_{1i}, k_{(i+1)N}) J_\alpha(\varepsilon_1, \dots, \varepsilon_i) J_\beta(\varepsilon_{i+1}, \dots, \varepsilon_N) + \sum_{i=1}^{N-2} \sum_{j=i+1}^{N-1} V_4^{\nu\alpha\beta\gamma} J_\alpha(\varepsilon_1, \dots, \varepsilon_i) J_\beta(\varepsilon_{i+1}, \dots, \varepsilon_j) J_\gamma(\varepsilon_{j+1}, \dots, \varepsilon_N) \right\}$$

where  $k_{ij} = k_i + k_{i+1} + \dots + k_j$ ,  $V_3$  and  $V_4$  are three and four-gluon vertices.

The **red piece** was obtained using the Slavnov-Taylor identities and correspond to bremsstrahlung from the straight infinite Wilson line along  $p_A$  (in axial gauge).

<sup>1</sup> A. van Hameren, P. Kotko, K. Kutak, JHEP 1212 (2012) 029

# Framework: unintegrated gluon densities

- in the high-energy factorization originally BFKL gluon evolution was used  
⇒ why not to try to include more subtle effects relevant to small  $x$ ?
- **nonlinear evolution with saturation**<sup>1,2</sup> fitted to HERA data<sup>3</sup>

$$\begin{aligned} \mathcal{F}(x, k_T^2) = & \mathcal{F}_0(x, k_T^2) + \frac{\alpha_s N_c}{\pi} \int_x^1 \frac{dz}{z} \int_{k_{T0}^2}^{\infty} \frac{dq_T^2}{q_T^2} \left\{ \frac{q_T^2 \mathcal{F}\left(\frac{x}{z}, q_T^2\right) \theta\left(\frac{k_T^2}{z} - q_T^2\right) - k_T^2 \mathcal{F}\left(\frac{x}{z}, k_T^2\right)}{|q_T^2 - k_T^2|} + \frac{k_T^2 \mathcal{F}\left(\frac{x}{z}, k_T^2\right)}{\sqrt{4q_T^4 + k_T^4}} \right\} \\ & + \frac{\alpha_s}{2\pi k_T^2} \int_x^1 dz \left\{ \left( P_{gg}(z) - \frac{2N_c}{z} \right) \int_{k_{T0}^2}^{k_T^2} dq_T^2 \mathcal{F}\left(\frac{x}{z}, q_T^2\right) + z P_{gq}(z) \Sigma\left(\frac{x}{z}, k_T^2\right) \right\} \\ & - \frac{2\alpha_s^2}{R^2} \left\{ \left[ \int_{k_T^2}^{\infty} \frac{dq_T^2}{q_T^2} \mathcal{F}(x, q_T^2) \right]^2 + \mathcal{F}(x, k_T^2) \int_{k_T^2}^{\infty} \frac{dq_T^2}{q_T^2} \ln\left(\frac{q_T^2}{k_T^2}\right) \mathcal{F}(x, q_T^2) \right\} \end{aligned}$$

- includes kinematic constraints
- includes non-singular pieces of the splitting functions
- the coupling is running
- the parameter  $R$  has an interpretation of a target radius  
⇒ one may attempt to use it for nuclei<sup>3</sup>

<sup>1</sup> K. Kutak, J. Kwiecinski, Eur.Phys.J. C29, 521 (2003)

<sup>2</sup> K. Kutak, A. Stasto, Eur.Phys.J. C41, 343 (2005)

<sup>3</sup> K. Kutak, S. Sapeta, Phys.Rev. D86, 094043 (2012)



## Framework: comments

- there is no TMD factorization for hadron-hadron collisions
- at small  $x$  there is a generalized TMD factorization for dijet production in the small momentum imbalance limit<sup>1</sup>
- for very large densities (very small  $x$ ) an appropriate approach is the Color Glass Condensate, but there are problems for jets with larger  $p_T$
- in the “hybrid” factorization there is just one unintegrated gluon density
  - there is a chance that the factorization might be present (needs more study)

<sup>1</sup>F. Dominguez, C. Marquet, B. Xiao, Feng Yuan, Phys.Rev. D83 (2011) 105005

# Results

# Results: process definition and cuts

Basic setup:

- p-p and p-Pb collisions
- CM energy 5 TeV and 7 TeV
- $p_{T1} > p_{T2} > p_{T3} > p_{T\text{cut}}$
- anti- $k_T$  clustering with  $R = 0.5$
- collinear PDF  $\Rightarrow$  CTEQ10 NLO set, scale choice  $\mu = a(E_1 + E_2 + E_3)$ , where the variation of  $a$  gives the (large) theoretical uncertainty
- calculations are made and cross-checked using LxJet and OSCARS

Two scenarios:

## ① central-forward jets

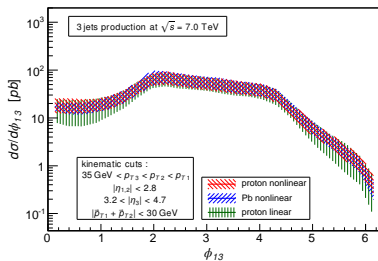
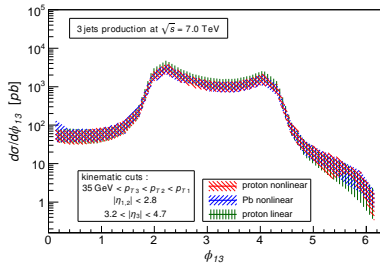
- $\rightarrow$  two leading jets are in the central region with  $|\eta_{1,2}| < 2.8$
- $\rightarrow$  the softest jet is in the forward region  $3.2 < \eta_3 < 4.7$
- $\rightarrow p_{T\text{cut}} = 35 \text{ GeV}$
- $\rightarrow$  we may restrict additional cuts on the central jets, e.g.  $|\vec{p}_{T1} + \vec{p}_{T2}| < D_{\text{cut}}$

## ② purely forward jets

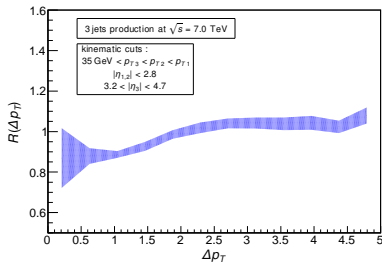
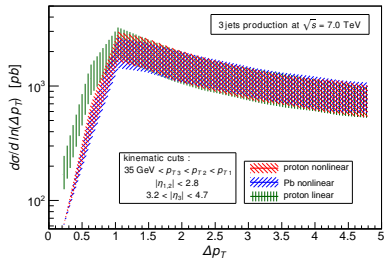
- $\rightarrow$  all the jets are in the forward region  $3.2 < \eta_{1,2,3} < 4.9$
- $\rightarrow p_{T\text{cut}} = 20 \text{ GeV}$

# Results: forward-central three jet production

- decorrelations ( $\phi_{13} = |\phi_1 - \phi_3|$ )

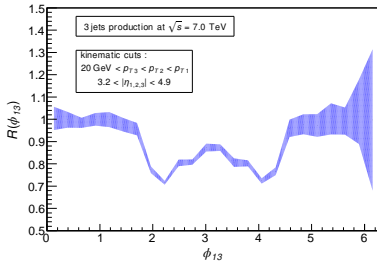
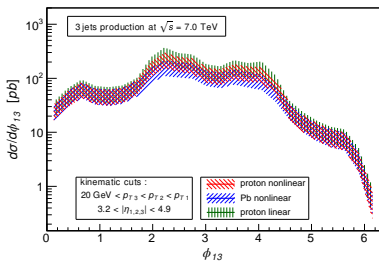


- unbalanced  $p_T$  of the jets

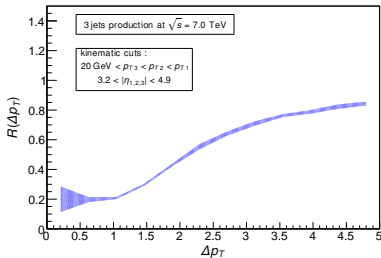
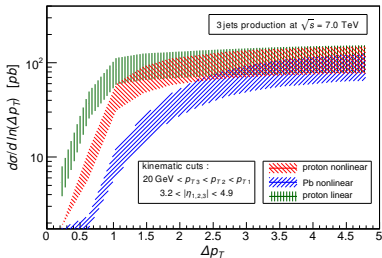


# Applications: forward three jet production

- decorrelations ( $\phi_{13} = |\phi_1 - \phi_3|$ )



- unbalanced  $p_T$  of the jets



# Summary and outline

- three jets at the LHC using off-shell gauge invariant tree-level matrix elements and gluon unintegrated densities relevant for larger  $p_T$ 
  - the forward-central setup with  $p_{T\text{cut}} = 35 \text{ GeV}$  does not discriminate between different evolutions, although additional restrictions on the central jets may do so
  - the purely forward jets with  $p_{T\text{cut}} = 20 \text{ GeV}$  are sensitive to saturation; in particular there is a strong suppression of the nuclear modification ratio
- calculations were performed using new MC codes: **LxJet**<sup>1</sup> (C++, ROOT, FOAM) and **OSCARS** (fortran code similar to HELAC)

## Future developments:

- final state parton shower (our programs – event generators)
- NLO jets within “hybrid” high energy factorization (possibility of use dipole subtraction method for massive Aivazis-Collins-Olness-Tung factorization<sup>2</sup>)
- nonlinear extension of CCFM evolution equation<sup>3</sup>
- MPIs

<sup>1</sup> <http://annapurna.ifj.edu.pl/~pkotko/LxJet.html>

<sup>2</sup> P. Kotko, W. Slominski, Phys.Rev. D86 (2012) 094008

<sup>3</sup> K. Kutak JHEP 1212 (2012) 033, solution currently investigated by D. Toton

Backup

# Small x and forward processes

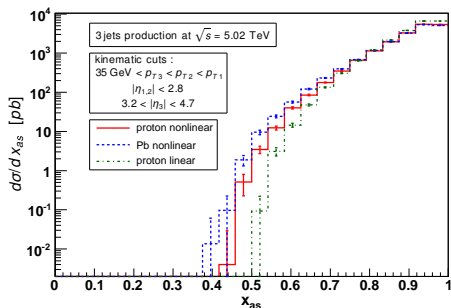
Forward processes (relevant for small x) correspond to **asymmetric configurations**

$$x_A = \sum_i \frac{|\vec{p}_{Ti}|}{\sqrt{S}} \exp(\eta_i)$$

$$x_B = \sum_i \frac{|\vec{p}_{Ti}|}{\sqrt{S}} \exp(-\eta_i)$$

$$x_{as} = |x_A - x_B| / (x_A + x_B)$$

central-forward three jet production



This accounts for a simplification:

- large fractions  $x_B \rightarrow$  collinear approach (with on-shell partons)
- small fractions  $x_A \rightarrow$  high energy factorization (with an off-shell gluon)