Correlation analysis of PDF uncertainties

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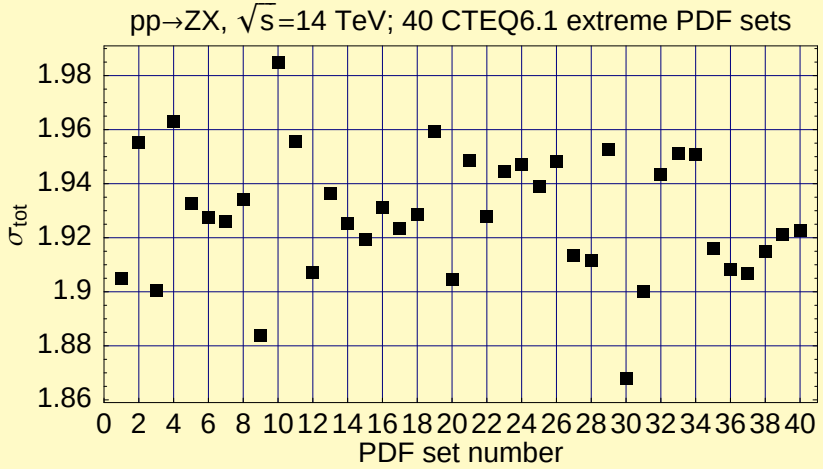
An inefficient application of the error analysis

Compute σ_W for 40 (now 44) extreme PDF eigensets

Find eigenparameter(s) producing largest variation(s), such as #9, 10, 30

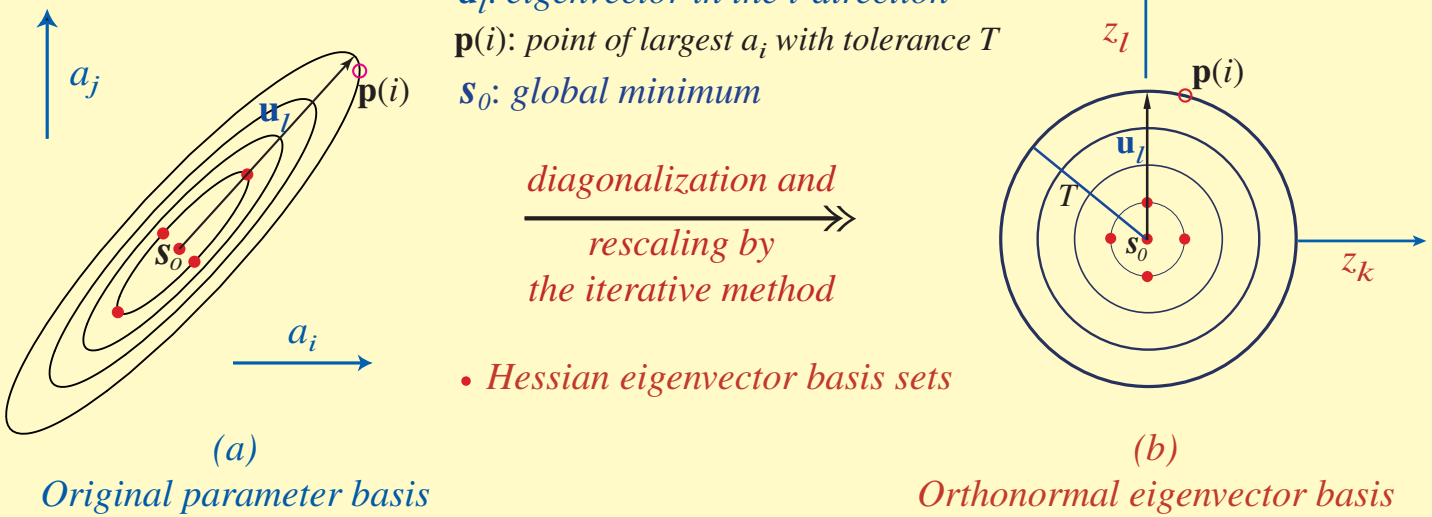


Check that the same eigenparameters produce largest variations in σ_Z

It is not obvious how to relate abstract eigenparameters to physical PDF's $u(x)$, $d(x)$, etc.

Tolerance hypersphere in the PDF space

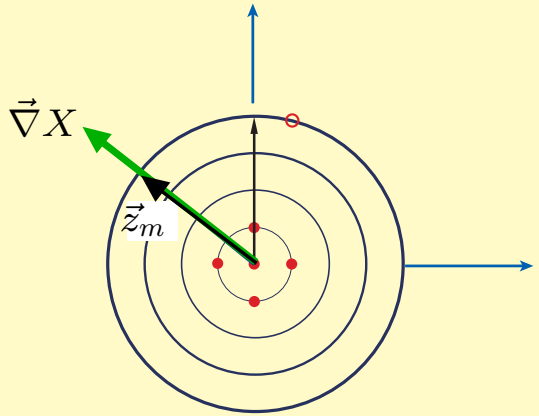
2-dim (i,j) rendition of N-dim (22) PDF parameter space



An ellipse $\chi^2 - \chi^2_{min} \leq T^2$ in space of N physical PDF parameters $\{a_i\}$ is mapped onto a hypersphere of radius T in space of N orthonormal PDF parameters $\{z_i\}$

Tolerance hypersphere in the PDF space

2-dim (i,j) rendition of N-dim (22) PDF parameter space



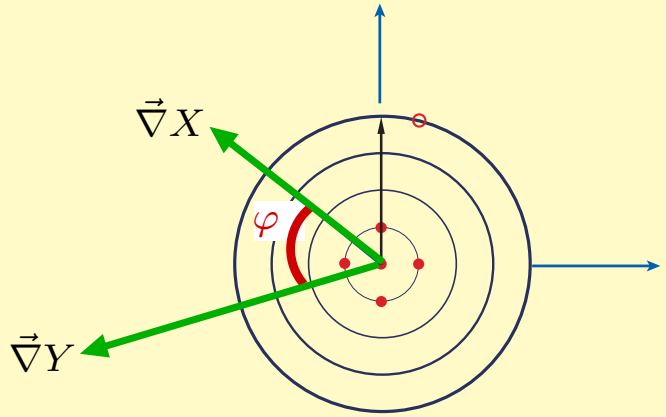
(b)
Orthonormal eigenvector basis

PDF error for a physical observable X is given by

$$\Delta X = \vec{\nabla} X \cdot \vec{z}_m = \left|\vec{\nabla} X\right| = \frac{1}{2}\sqrt{\sum_{i=1}^{N}\left(X_i^{(+)} - X_i^{(-)}\right)^2}$$

Tolerance hypersphere in the PDF space

2-dim (i,j) rendition of N-dim (22) PDF parameter space



(b)
Orthonormal eigenvector basis

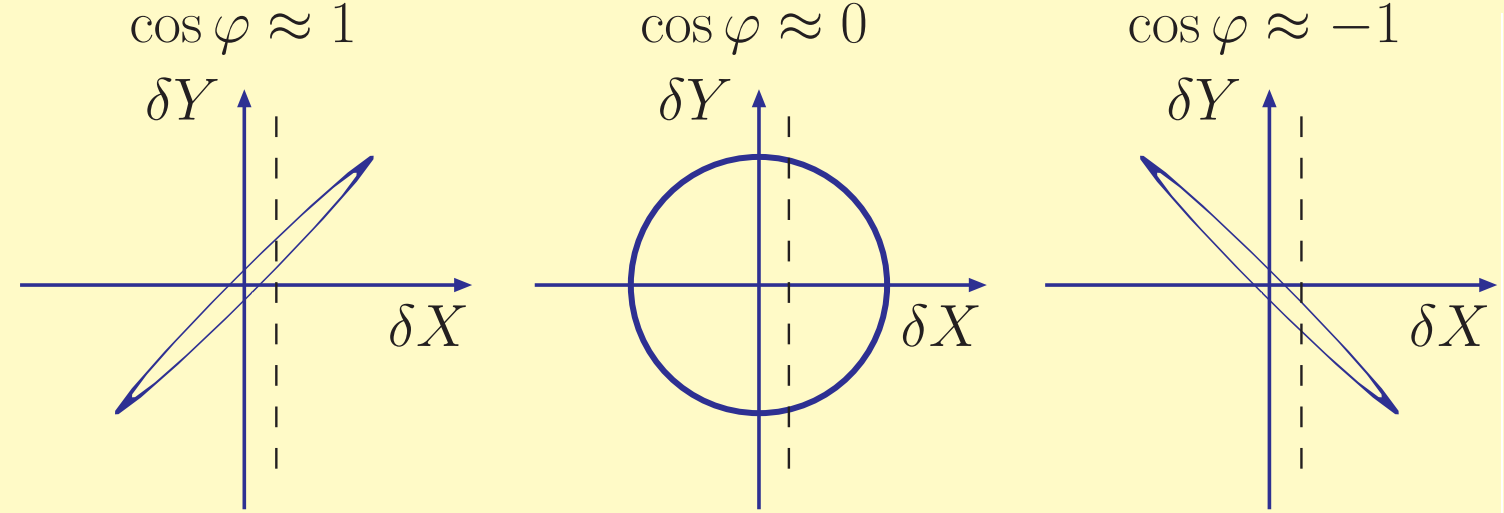
Correlation cosine for observables X and Y:

$$\cos\varphi = \frac{\vec{\nabla}X \cdot \vec{\nabla}Y}{\Delta X \Delta Y} = \frac{1}{4\Delta X\,\Delta Y} \sum_{i=1}^{N} \left(X_i^{(+)} - X_i^{(-)}\right)\left(Y_i^{(+)} - Y_i^{(-)}\right)$$

Role of the correlation angle

Determines the parametric form of the $X - Y$ correlation ellipse

$$X = X_0 + \Delta X \cos\theta$$
$$Y = Y_0 + \Delta Y \cos(\theta + \varphi)$$



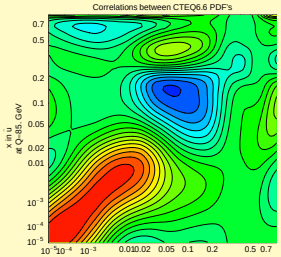
$\cos\varphi \approx \pm 1$: Measurement of X imposes tight constraints on Y

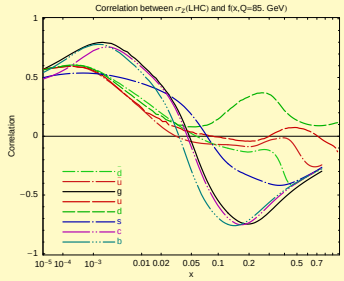
$\cos\varphi \approx 0$: Measurement of X imposes loose constraints on Y

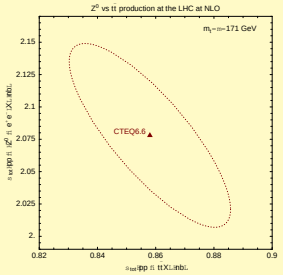
Types of correlations

X and Y can be

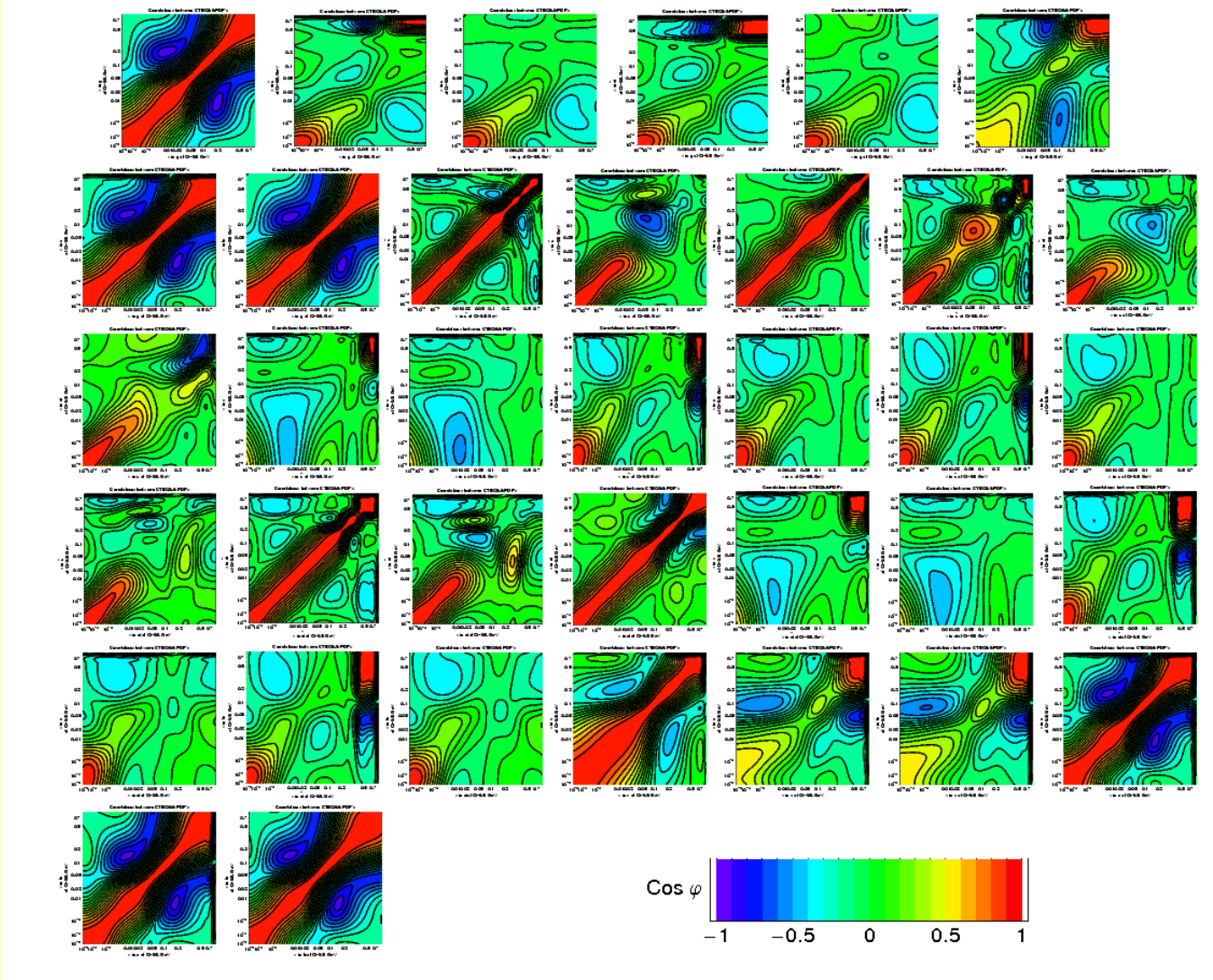
- two PDFs $f_1(x_1, Q_1)$ and $f_2(x_2, Q_2)$ (plotted as $\cos\varphi$ vs x_1 & x_2)
- a physical cross section σ and PDF $f(x, Q)$ (plotted as $\cos\varphi$ vs x)
- two cross sections σ_1 and σ_2







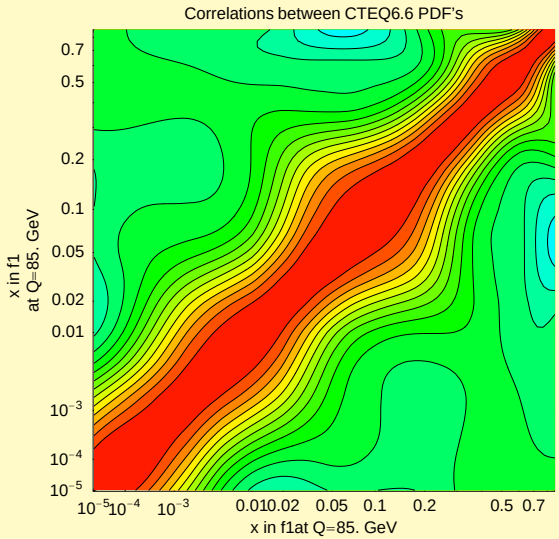
Correlations between $f_1(x_1, Q)$ and $f_2(x_2, Q)$ at $Q = 85$ GeV



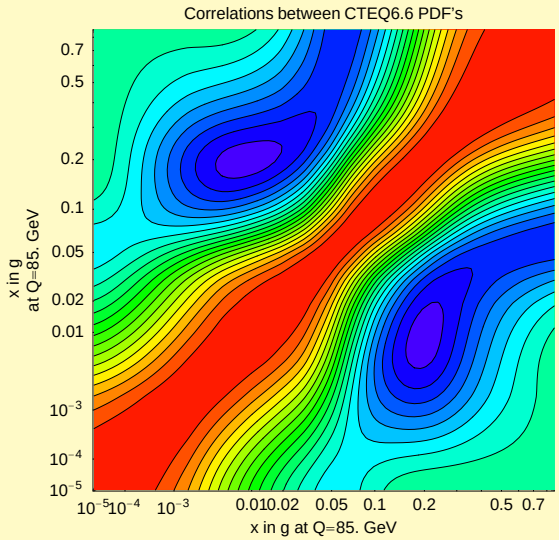
Figures from http://hep.pa.msu.edu/cteq/public/6.6/pdfcorrs/

Correlations between $f(x_1, Q)$ and $f(x_2, Q)$ at $Q = 85$ GeV

$u(x_1, Q)$ vs. $u(x_2, Q)$

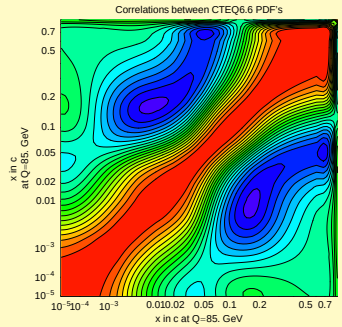


$g(x_1, Q)$ vs. $g(x_2, Q)$

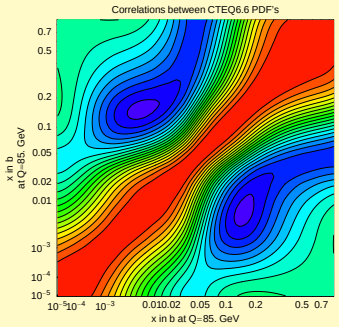


Correlation patterns look similar for g, c, b PDF's (no intrinsic charm here!)

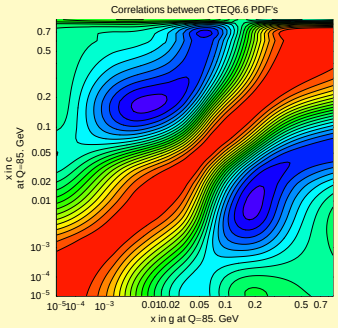
c vs. c



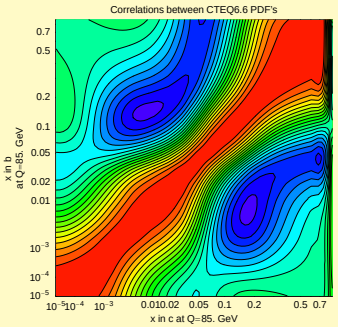
b vs. b



c vs. g

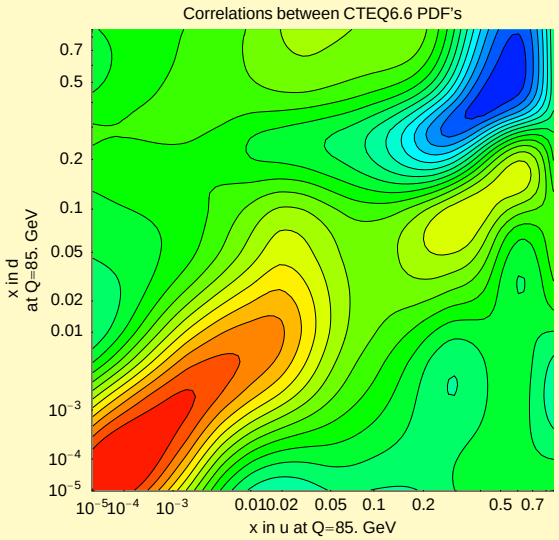


b vs. c

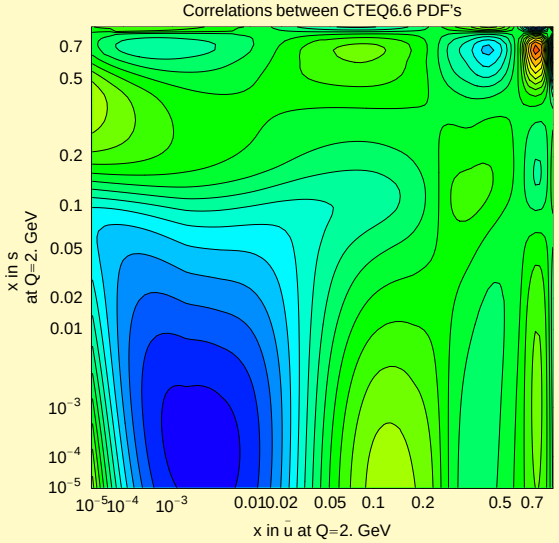


Correlations between $f_1(x_1, Q)$ and $f_2(x_2, Q)$ at $Q = 85$ GeV

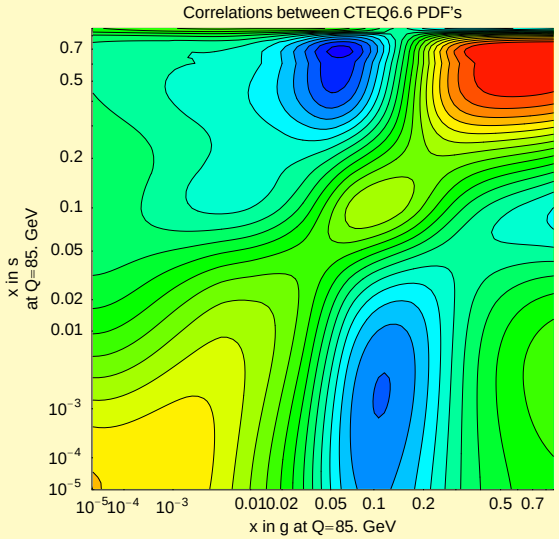
d vs u

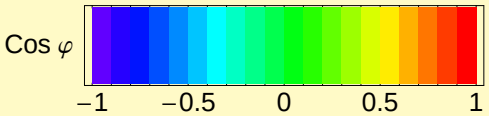


s vs $\bar{u}$ at Q=2 GeV



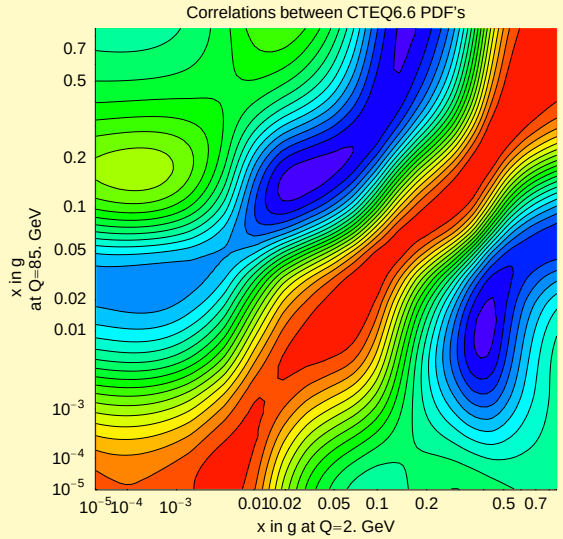
s vs. g





Sometimes there is a clear physics reason behind the correlation (e.g., sum rules or assumed Regge-like behavior); sometimes not

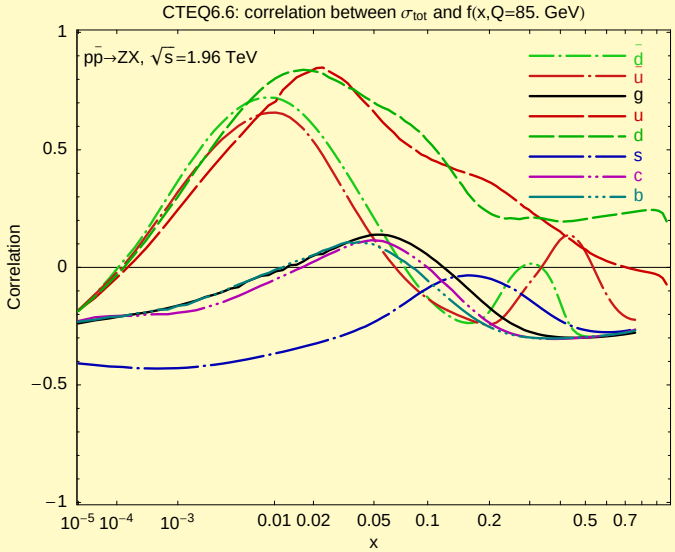
Correlations between $g(x_1, 2\ \text{GeV})$ and $g(x_2,\ 85\ \text{GeV})$

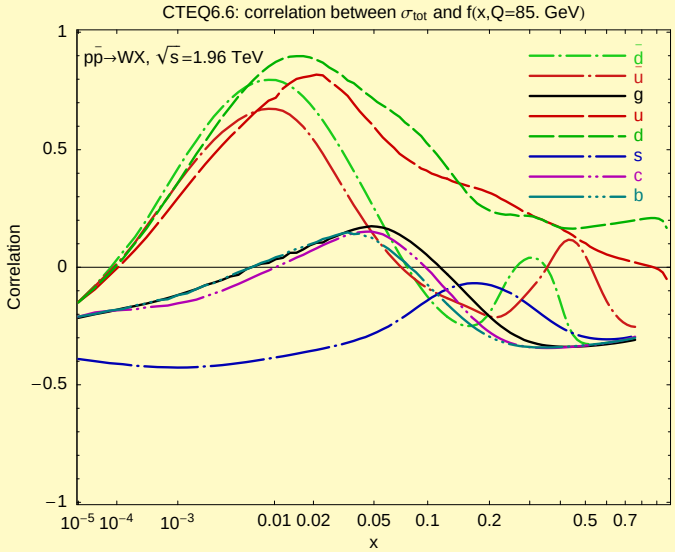


Gluons at $Q = 85$ GeV are correlated with gluons at $Q = 2$ GeV and larger x because of DGLAP evolution

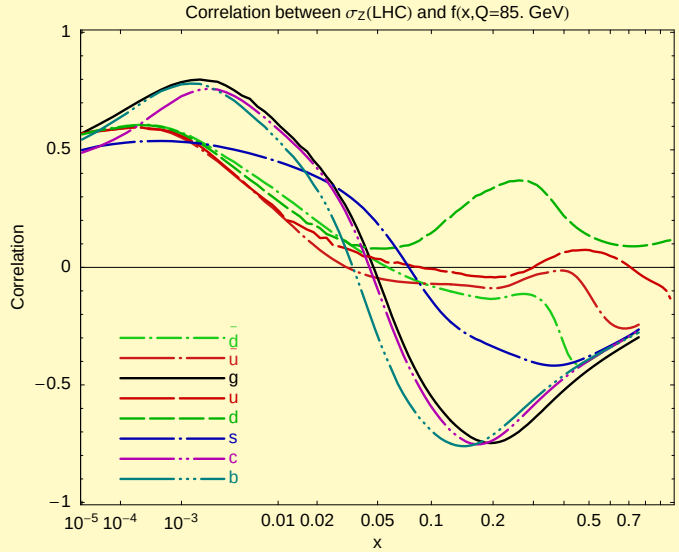
Correlations between W, Z cross sections and PDF's

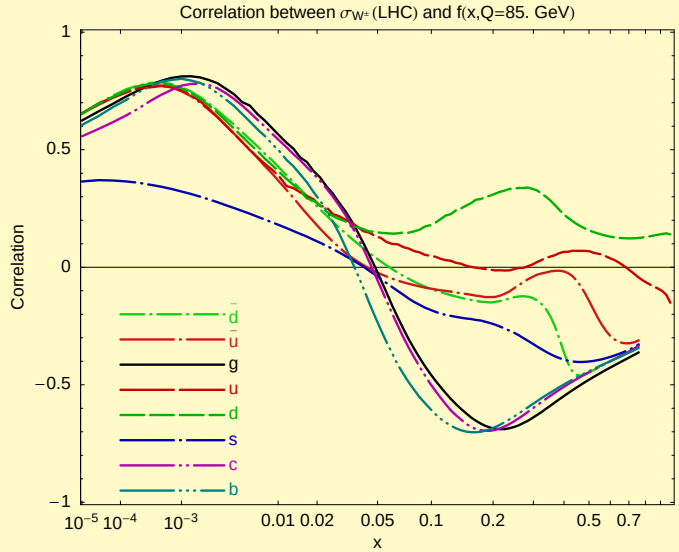
Tevatron Run-2



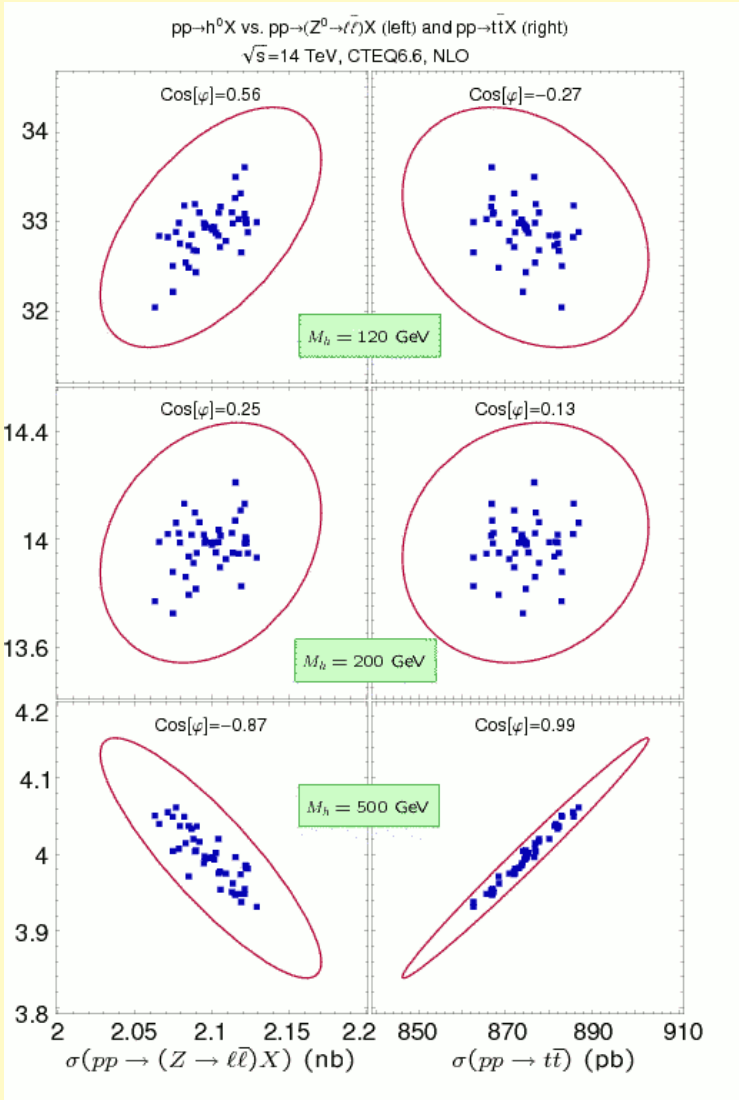


LHC

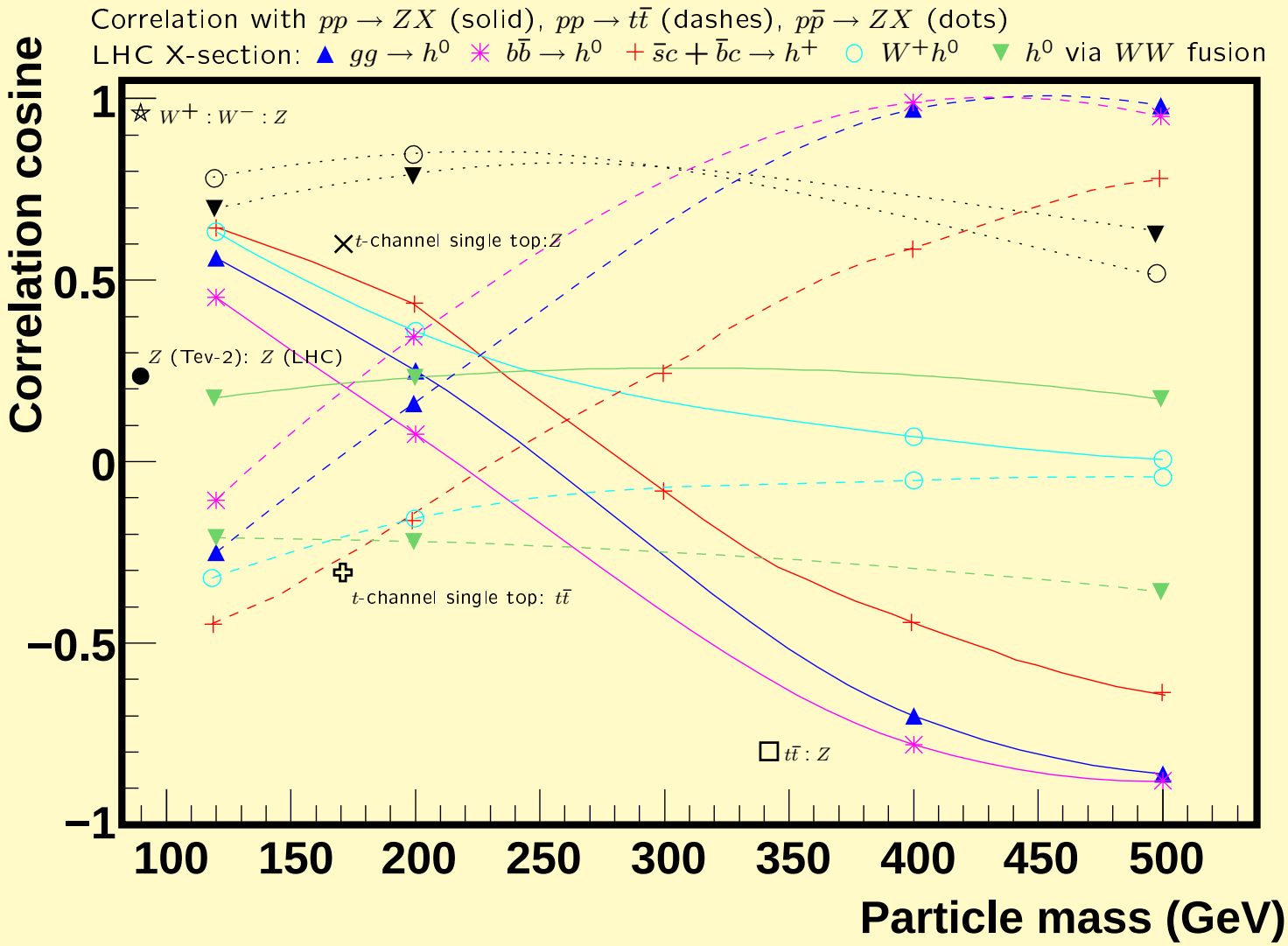




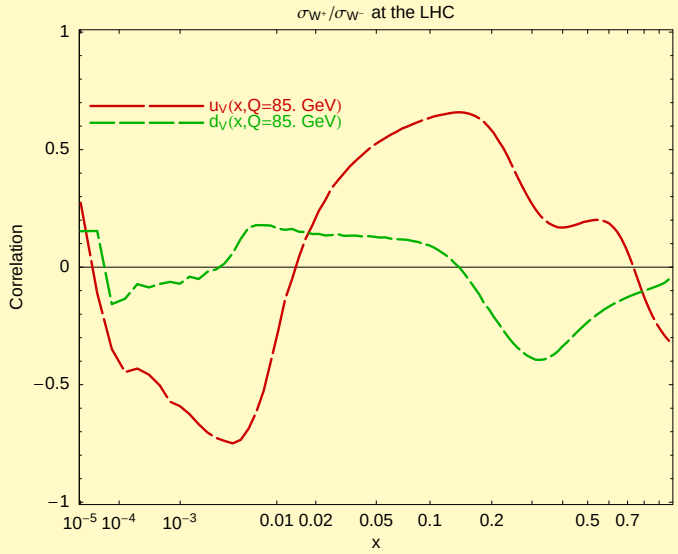
Correlations between $\sigma(gg \to H^0)$, σ_Z, $\sigma_{t\bar{t}}$



$\cos\varphi$ for various NLO Higgs production cross sections in SM and MSSM

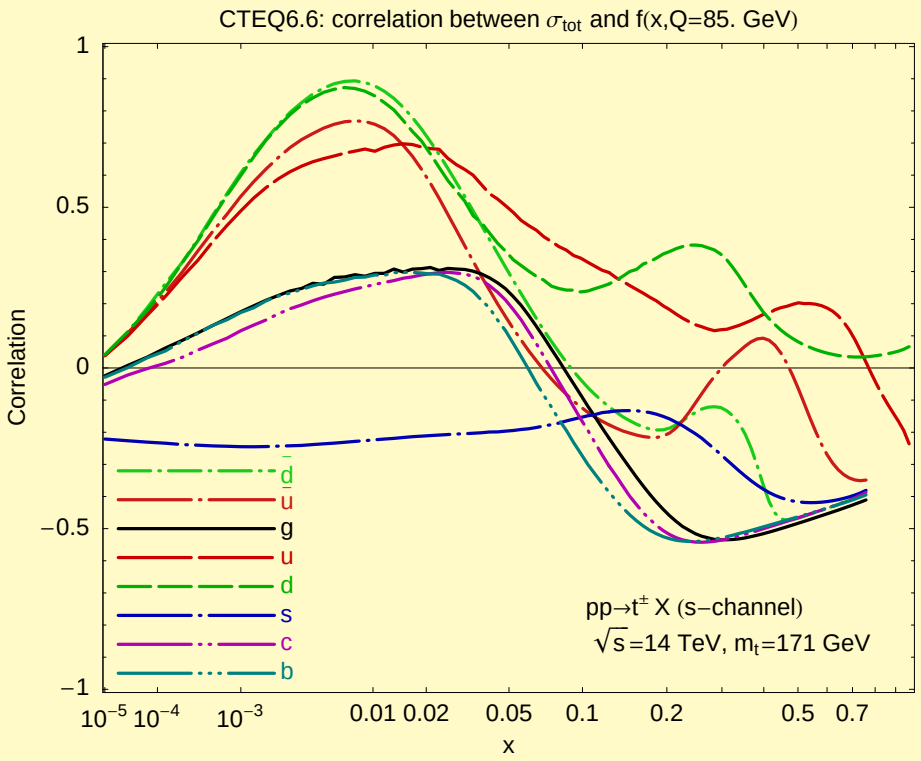


$\sigma(W^+)/\sigma(W^-)$

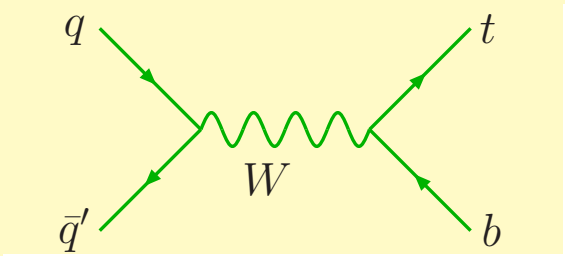


$\sigma(W^+)/\sigma(W^-) = 1.36 + 0.016$ (CTEQ6.6), 1.36 (MSTW'06NNLO), 1.35 (MRST'04NLO)

An example of a small correlation with the gluon



Single-top production (NLO)



- typical $x \sim 0.01$
- mostly correlated with *u*, *d* PDF's

PDF uncertainties in *W*, *Z total* cross sections are irrelevant for some quark scattering processes (single-top, *Z'*, ...)