Theoretical Uncertainties and Inaccuracies

Robert Thorne

February 23, 2008

University College London

Royal Society University Research Fellow

Theoretical Errors - various sources, e.g.

- higher orders (NNLO, NNNLO)

- QED and electroweak (comparable to NNLO ? ($\alpha_s^3 \sim \alpha$, $\log^2(E^2/M^2_{W,Z})$ terms).

- small x ($\alpha_s^n \ln^{n-1}(1/x)$)

- large x ($\alpha_s^n \ln^{2n-1}(1-x)$)

- low Q^2 (higher twist)

All lead to systematic theoretical uncertainties, and there are attempts to deal with this (e.g. Forte in a few minutes).

However, there are still uncertainties, and errors, within framework of strict fixed-order perturbative, leading-twist QCD. Mainly associated with heavy quarks.

Schemes - 1. Fixed Flavour

Near threshold $Q^2 \sim m_H^2$ massive quarks not partons. Created in final state. Described using **Fixed Flavour Number Scheme** (FFNS).

$$F(x, Q^2) = C_k^{FF}(Q^2/m_H^2) \otimes f_k^{n_f}(Q^2)$$

Does not sum $\alpha_S^n \ln^n Q^2/m_H^2$ terms in perturbative expansion. Usually achieved by definition of heavy flavour parton distributions and solution of evolution equations.

Self-consistent, i.e. *correct*, but perhaps not best approach.

However FFNS partons sometimes needed because hard cross-sections only calculated with all heavy flavour generated in the final state.

The NLO ($\mathcal{O}(\alpha_S^2)$) coefficient functions for heavy flavour in DIS calculated in scheme where the coupling α_S is fixed at **3** flavours. Partons have to be defined in same way. (Could change both.)

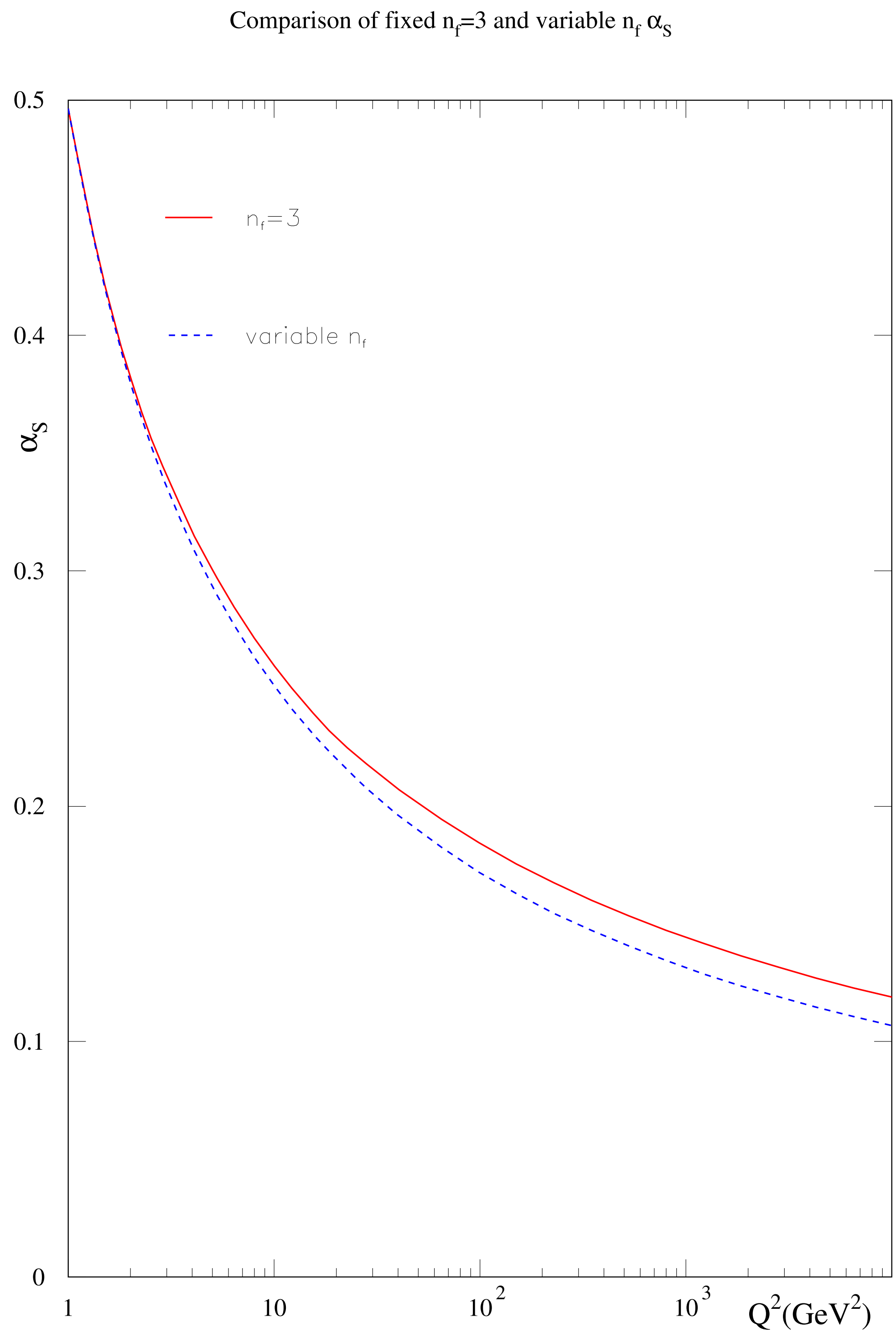
Often not the case (still).

(Various *correct* alternative definitions of α_S at fixed order. Best to match definition with parton.)

Compared to variable-flavour α_S the $n_f = 3$ version is either $\sim 12\%$ smaller at $\mu^2 = M_Z^2$ or if identical at this high scale, hugely bigger at low μ^2.

Cannot really determine $\alpha_S(M_Z^2)$ from a FFNS fit.

It is a $n_f = 3$ definition of $\alpha_S(M_Z^2)$ – simply not the same quantity as usual $n_f = 5$ definition of $\alpha_S(M_Z^2)$.

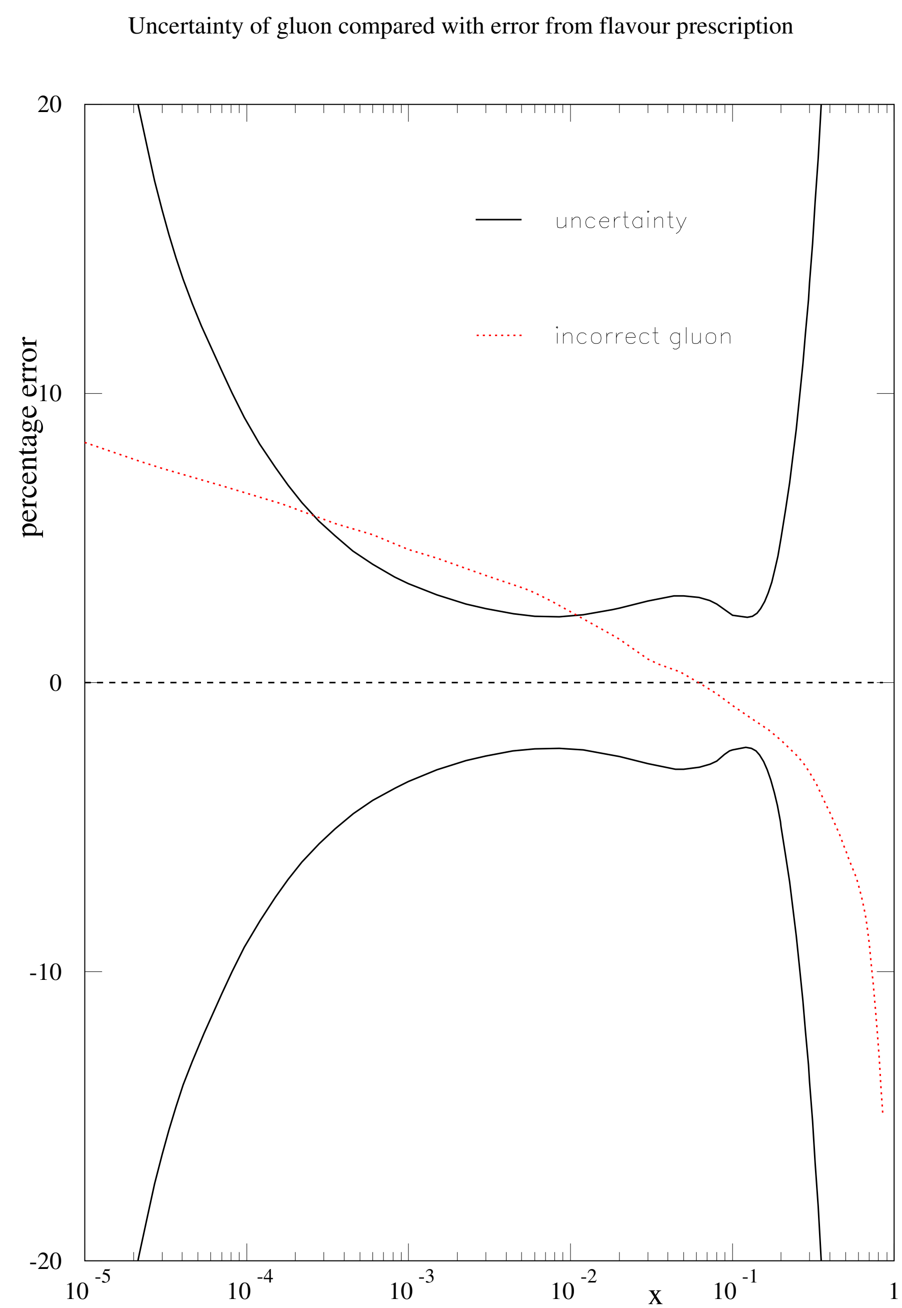


The error made in using the wrong coupling is quite significant.

Coupling too big $\longrightarrow$ evolution too quick.

Compare incorrect and correct gluons at $Q^2 = 100\mathrm{GeV}^2$. Error can be bigger than uncertainty.

Conclusion that size of change can be comparable to uncertainty confirmed by Alekhin in context of refit.



FFNS not defined at NNLO – $\alpha_S^3 C^{FF,3}_{2,Hg}$ unknown. Ordering always given by

LO $\frac{\alpha_S}{4\pi} C^{FF,1}_{2,Hg} \otimes g^{n_f}$

NLO $\left(\frac{\alpha_S}{4\pi}\right)^2 (C^{FF,2}_{2,Hg} \otimes g^{n_f} + C^{FF,2}_{2,Hq} \otimes \Sigma^{n_f})$

i.e. $F_2^H(x,Q^2) \neq 0$ at LO (it can be 30% of total $F_2(x,Q^2)$ after all), and at LO

$$\frac{d\, F_2^H(x,Q^2)}{d\, \ln Q^2} \rightarrow \alpha_S/(2\pi) P^0_{qg} \otimes g(x,Q^2)$$

and at NLO

$$\frac{d\, F_2^H(x,Q^2)}{d\, \ln Q^2} \rightarrow (\alpha_S/(2\pi))^2 P^1_{qg} \otimes g(x,Q^2).$$

$C^{FF,2}_{2,Hg}$ contains no information on P^2_{qg} and so $\alpha_S^2 C^{FF,2}_{2,Hg} \otimes g^{n_f}$ cannot represent the NNLO evolution of $F_2(x,Q^2)$.

This is important because unknown $\alpha_S^3 C^{FF,3}_{2,Hg}$ is not just $\mathcal{O}(\alpha_S^3)$, it is $\mathcal{O}(\alpha_S^3 \ln^3(Q^2/m_H^2))$.

Approximations could be made and the correct $Q^2/m_H^2 \rightarrow \infty$ limit found.

2. Variable Flavour - Zero-Mass

High scales $Q^2 \gg m_H^2$ massless partons – behave like up, down, strange. Sum $\ln(Q^2/m_H^2)$ terms via evolution. (ZM-VFNS) ignores $\mathcal{O}(m_H^2/Q^2)$ corrections.

$$F(x,Q^2) = C_j^{ZMVF} \otimes f_j^{n_f+1}(Q^2).$$

Partons in different number regions related to each other perturbatively.

$$f_k^{n_f+1}(Q^2) = A_{jk}(Q^2/m_H^2) \otimes f_k^{n_f}(Q^2),$$

Perturbative matrix elements $A_{jk}(Q^2/m_H^2)$ containing $\ln(Q^2/m_H^2)$ terms relate $f_k^{n_f}(Q^2)$ and $f_k^{n_f+1}(Q^2)$ $\rightarrow$ correct evolution for both.

At LO, i.e. zeroth order in α_S, relationship trivial $q(g)_k^{n_f+1}(Q^2) \equiv q(g)_k^{n_f+1}(Q^2)$.

At NLO, i.e. first order in α_S

$$(h+\bar{h})(Q^2) = \frac{\alpha_S}{4\pi} P_{qg}^0 \otimes g^{n_f}(Q^2) \ln\left(\frac{Q^2}{m_H^2}\right), \quad g^{n_f+1}(Q^2) = \left(1 + \frac{\alpha_S}{6\pi} \ln\left(\frac{Q^2}{m_H^2}\right)\right) g^{n_f}(Q^2),$$

i.e. the heavy flavour evolves from zero at $Q^2 = m_H^2$ according to standard quark evolution, gluon loses corresponding momentum.

Consider ZM-VFNS *scheme* at NLO.

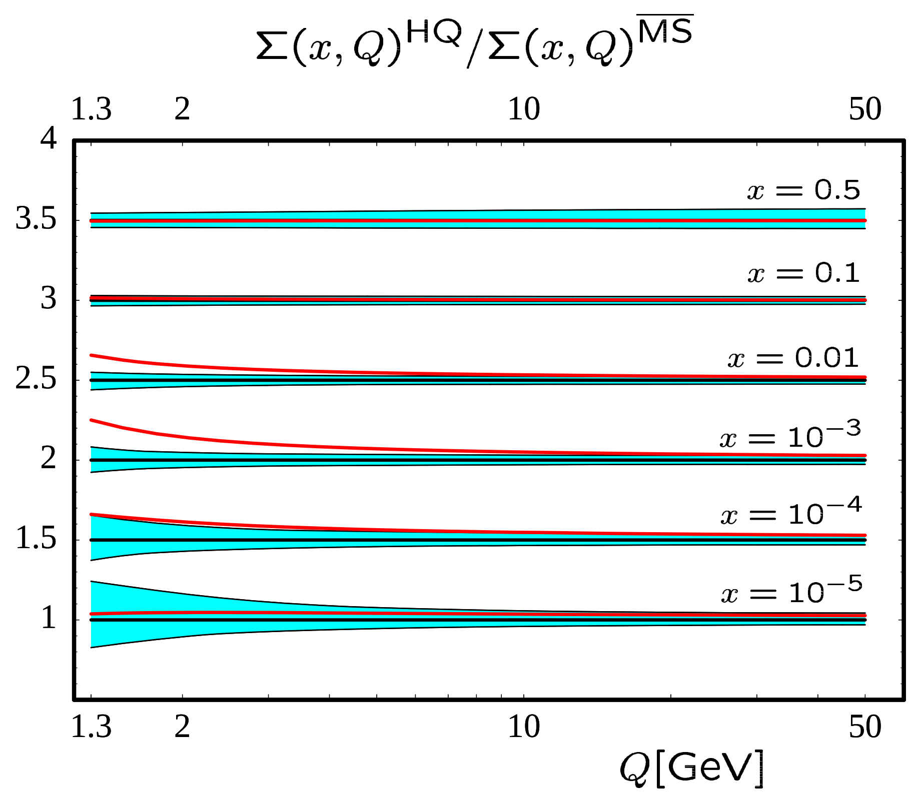
Terminology *scheme* misleading. Usually different way of arranging correct result.

In this case simply an error of $\mathcal{O}(m_H^2/Q^2)$. Incorrect compared to general VFNS.

Can't see why it is useful. At high scales often in massless limit for c and b. VFNS reduces to this limit.

Partons obtained from fitting in region where $\mathcal{O}(m_H^2/Q^2)$. Ignoring these terms $\rightarrow$ incorrect partons at both high and low Q^2.

Difference between approaches for CTEQ compared to (conservative) uncertainties.



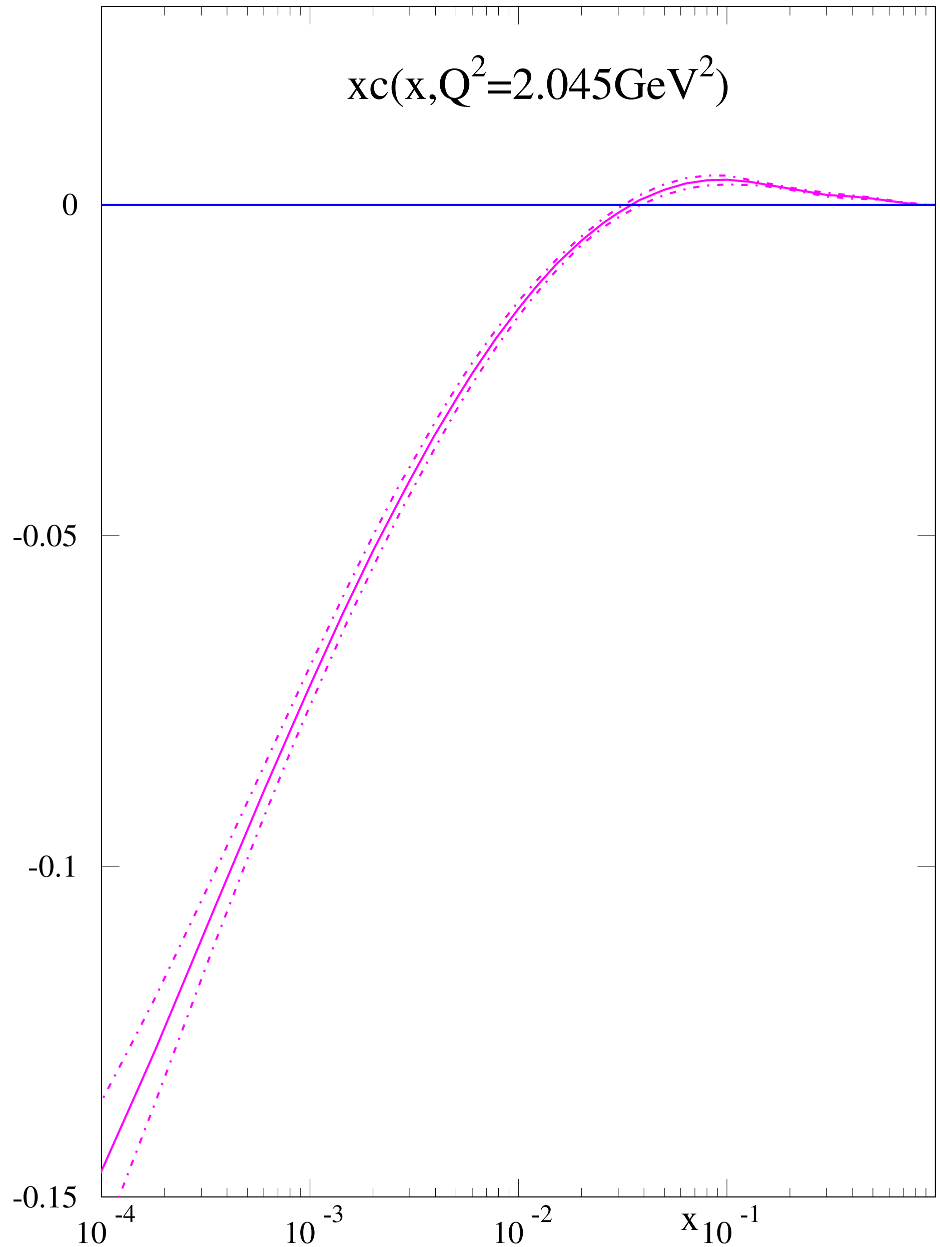
Moreover at NNLO $A_{ij}^{2,0}$ is generally nonzero. No longer any possibility of a smooth transition. In fact $A_{Hg}^{2,0}$ negative at small x.

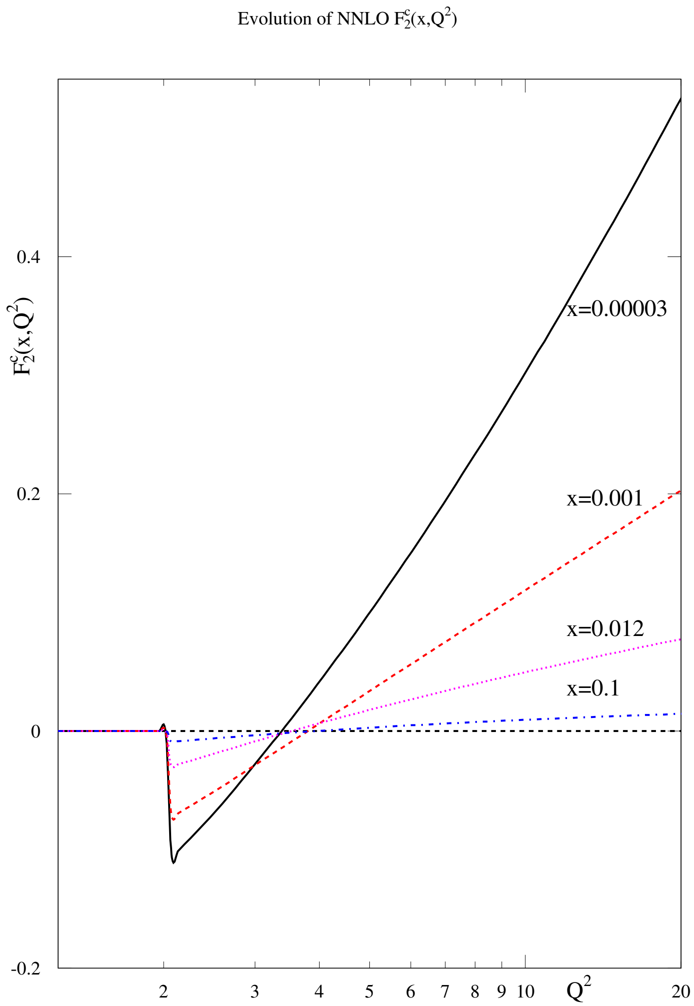
ZM-VFNS not really feasible at NNLO. Huge discontinuity in $F_2^c(x,Q^2)$. Significant in $F_2^{Tot}(x,Q^2)$.

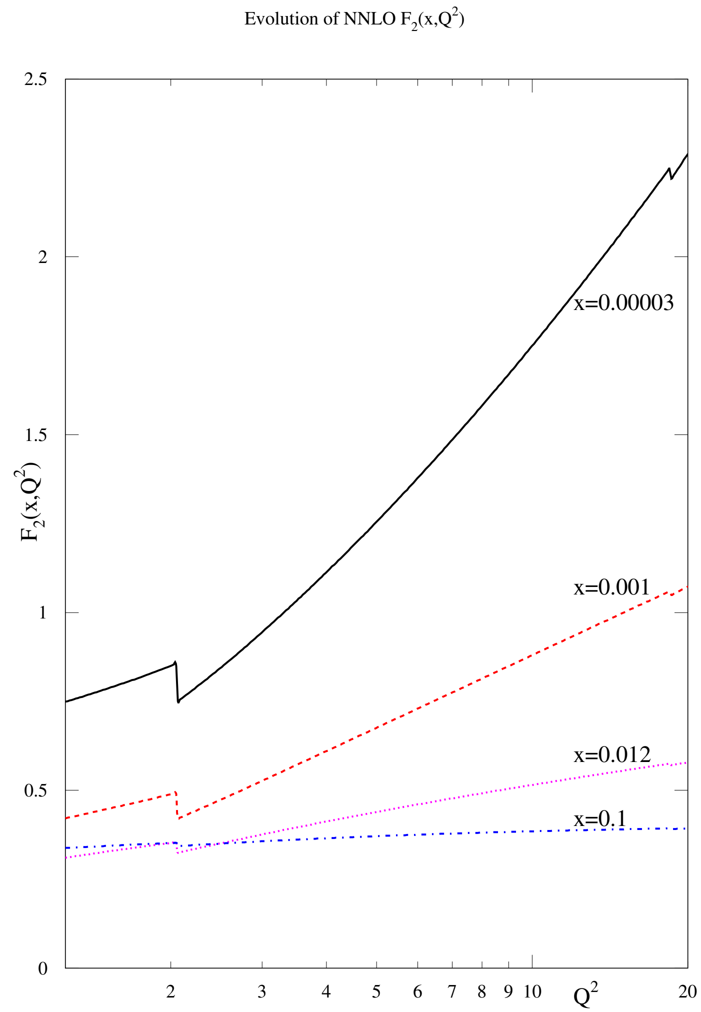
Heavy flavour no longer turns on from zero at $\mu^2 = m_c^2$

$$(c+\bar{c})(x,m_c^2) = A_{Hg}^2(m_c^2) \otimes g(m_c^2)$$

In practice turns on from negative value, (for general gluon).







At NNLO partons discontinuous at transition points (NLO $\overline{MS}$-scheme lucky). $c(x,Q^2)$ at m_c^2 very negative – nothing to do with negative $g(x,Q^2)$.

Need a general mass **Variable Flavour Number Scheme** (VFNS) taking one from the two well-defined limits of $Q^2 \leq m_H^2$ and $Q^2 \gg m_H^2$.

General-mass variable Flavour number scheme.

The VFNS can be defined by demanding equivalence of the n_f (FFNS) and n_f+1-flavour descriptions at all orders,

$$F^H(x,Q^2) = C_k^{FF}(Q^2/m_H^2) \otimes f_k^{n_f}(Q^2) = C_j^{VF}(Q^2/m_H^2) \otimes f_j^{n_f+1}(Q^2)$$
$$\equiv C_j^{VF}(Q^2/m_H^2) \otimes A_{jk}(Q^2/m_H^2) \otimes f_k^{n_f}(Q^2).$$

Hence, the VFNS coefficient functions satisfy

$$C_k^{FF}(Q^2/m_H^2) = C_j^{VF}(Q^2/m_H^2) \otimes A_{jk}(Q^2/m_H^2),$$

which at $\mathcal{O}(\alpha_S)$ gives

$$C_{2,g}^{FF,1}(Q^2/m_H^2) = C_{2,HH}^{VF,0}(Q^2/m_H^2) \otimes P_{qg}^0 \ln(Q^2/m_H^2) + C_{2,g}^{VF,1}(Q^2/m_H^2),$$

The VFNS coefficient functions tend to the massless limits as $Q^2/m_H^2 \to \infty$.

Can swap $\mathcal{O}(m_H^2/Q^2)$ terms between $C_{2,HH}^{VF,0}(Q^2/m_H^2)$ and $C_{2,g}^{VF,1}(Q^2/m_H^2)$.

Variety of different definitions of VFNS. However, some convergence. Most variations perfectly good.

Some sensible requirements, e.g. each coefficient function satisfies threshold $W^2 > 4m_H^2$ (for neutral current). This is not always applied.

Original ACOT prescription violated this requirement. TR variable flavour number scheme (TR-VFNS) imposed it in fairly complicated manner.

Most recently Tung, Kretzer, Schmidt proposed simpler ACOT(χ) prescription which is now used in updated RT prescription. Some differences in ordering and latter applies details at NNLO (where some approximation used – only at low Q^2).

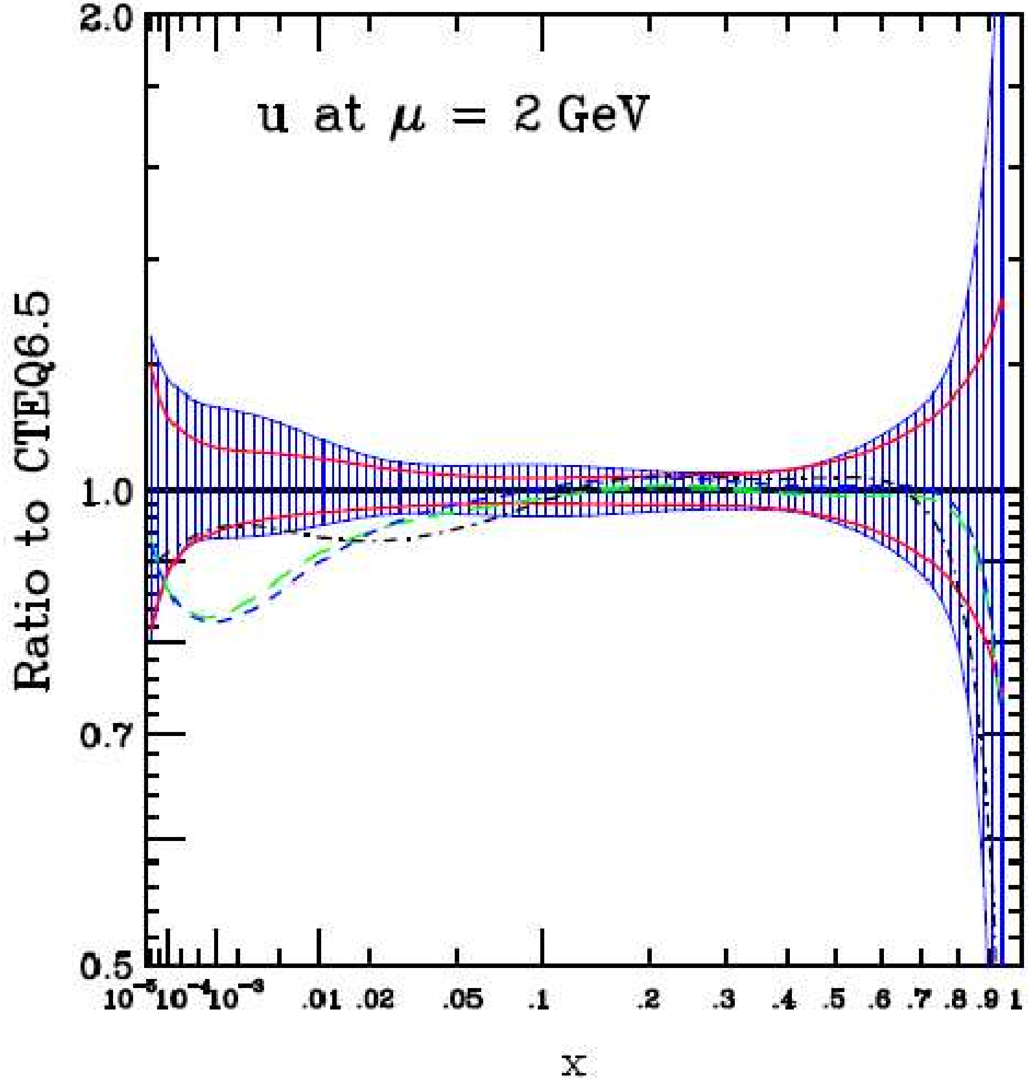
Any sensible GM-VFNS a *correct* way to extract partons by fitting to structure function data.

When making predictions if VFNS coefficient functions not known error of $\mathcal{O}(m_H^2/Q^2)$ from using GM-VFNS. No worse than permanent error from using ZM-VFNS. Can also input at least general kinematic requirements.

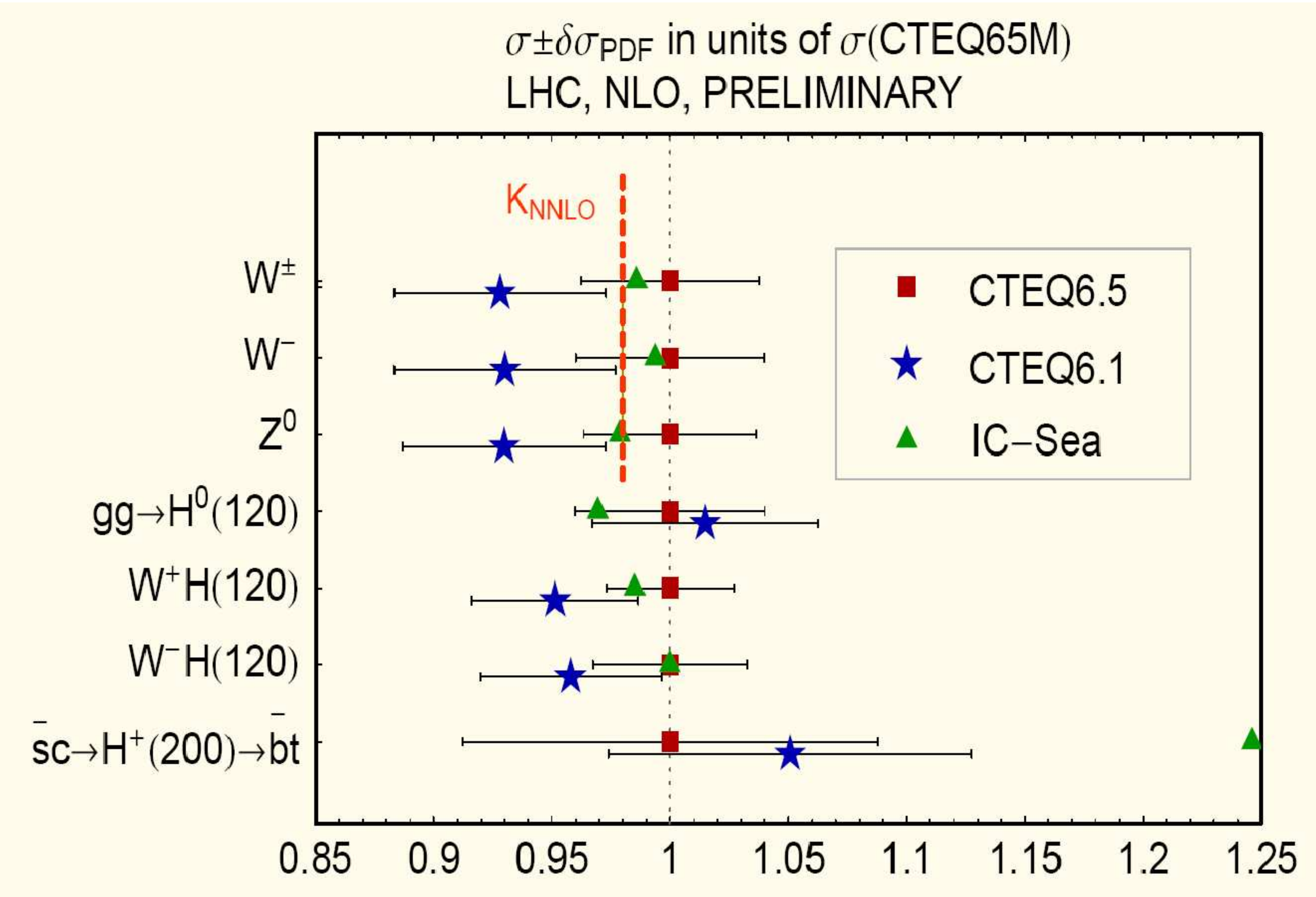
Importance of treating heavy flavour correctly illustrated by CTEQ6.5 up quark with uncertainties compared with previous versions, e.g. CTEQ6 in green.

MRST in dash-dot line. Reasonable agreement. Already used heavy flavour treatment in default sets.

CTEQ now also regard these as default partons.



Leads to large change in predictions using CTEQ partons at LHC



CTEQ6.5 predictions *correct*, CTEQ6.5 *incorrect*.

Importance of treating heavy flavour correctly illustrated at NNLO with MRST partons.

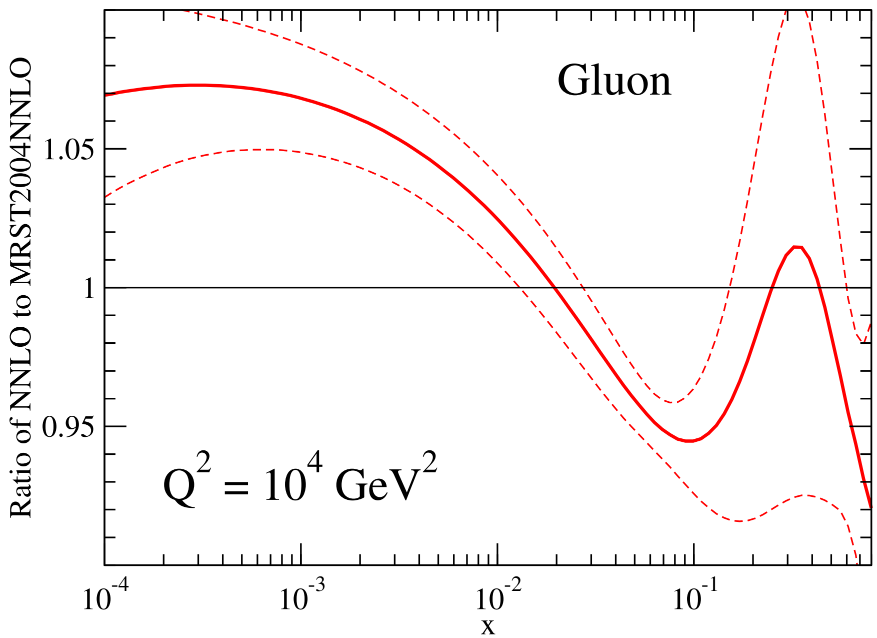
Previous approximate NNLO sets used (declared) approximate VFNS at flavour thresholds.

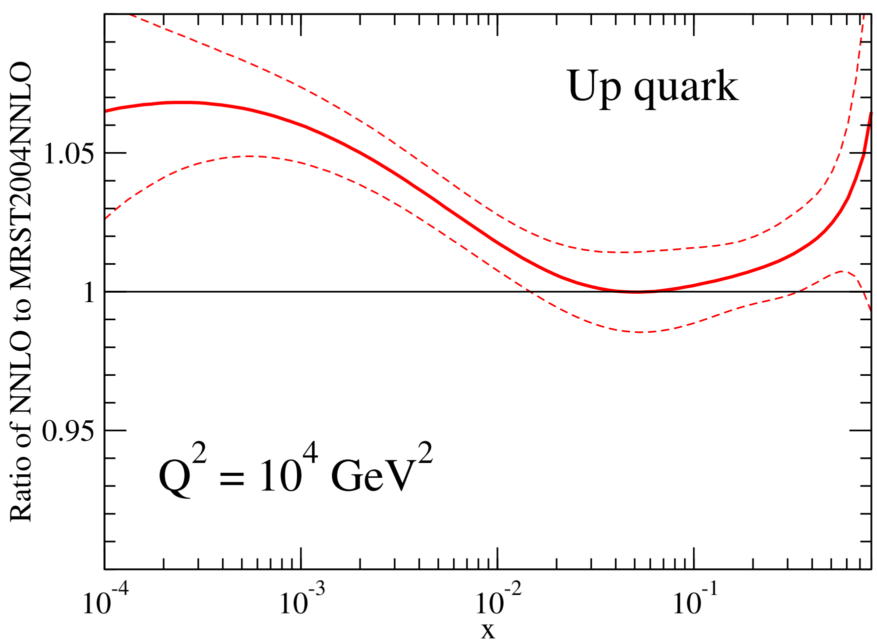
Full VFNS $\longrightarrow$ flatter evolution of charm

$\longrightarrow$ bigger gluon and more evolution of light sea.

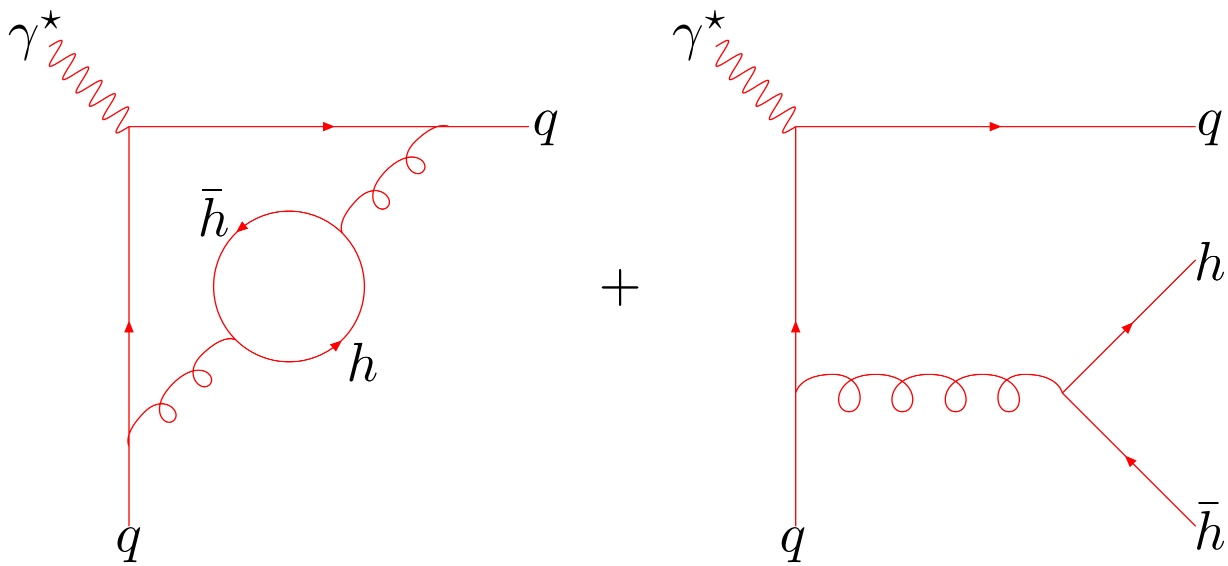
$\longrightarrow$ 6% increase in σ_W and σ_Z at the LHC. MRST04 predictions *incorrect*.

If change in details of VFNS this important ignoring NNLO heavy flavour a bit worrying?





At NNLO also get contribution due to heavy flavours away from photon vertex.



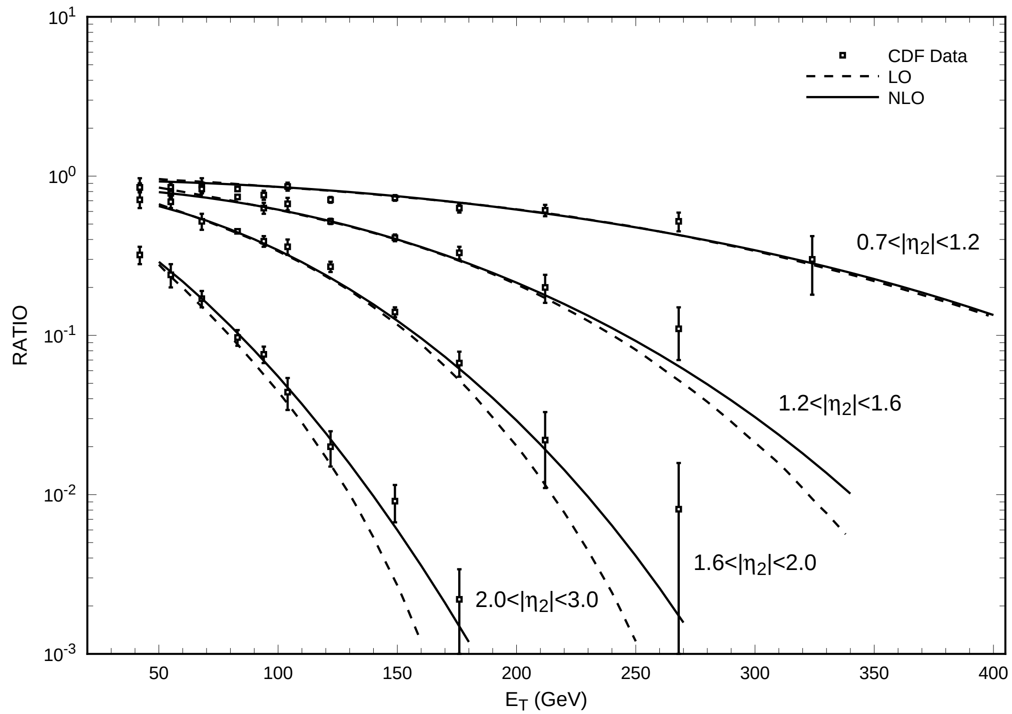
Strictly, left-hand type diagram and soft parts of right-hand type diagram should be light flavour structure function, and hard part of right-hand type diagram contributes to $F_2^H(x, Q^2)$ (Chuvakin, Smith, van Neerven).

Can be implemented (depends on separation parameter), but each contribution small contribution of NNLO. At moment all in light flavours (MSTW) or heavy flavours (FFNS).

Errors in jets at NNLO.

Do not know NNLO corrections to jet production in $pp(\bar{p})$ collisions. But MSTW include Tevatron jets in fit. (Only compare (successfully) with HERA jets at NNLO.)

NLO corrections themselves not large, except at high rapidities. At central rapidities $\leq 10\%$. Similar to correlated errors.



NNLO estimates Kidonakis, Owens – threshold correction logarithms. Uncertainties related to given jet definition – non-global logarithms, but imply $3-4\%$ correction. Consistent with what is known from NLO. Smaller than systematics on data.

Conclusions.

One assumes that the uncertainty on fixed-order, leading-twist parton distributions cannot include all theoretical uncertainties, but it should be as correct as possible within its prescribed framework.

A theoretical inaccuracy is important if the *error* is comparable to (or even bigger than) the quoted uncertainty (due to accuracy of data).

This type of inaccuracy does frequently exist at present, mainly associated with *simplifications* regarding heavy flavours. Change total $F_2(x,Q^2)$ by few $\%$. Cannot see any reliable approach other than GM-VFNS at NNLO at present.

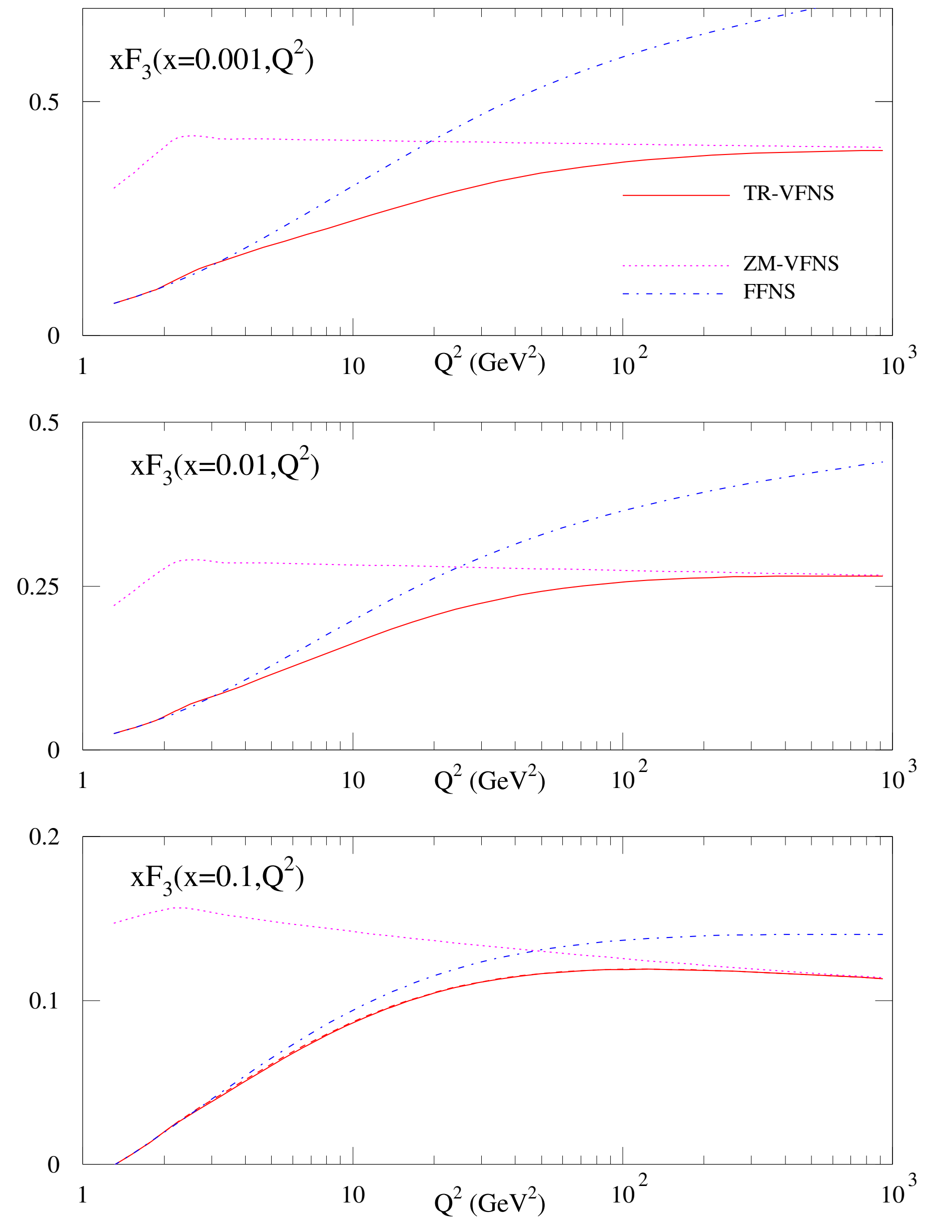
These inaccuracies should always be corrected if possible. *Errors* are not additional sources of uncertainty.

Slightly grey areas at NNLO where theoretical *errors* made for some quantities may well be less than *errors* made in leaving out data sets. Theoretical error should be smaller than experimental uncertainty.

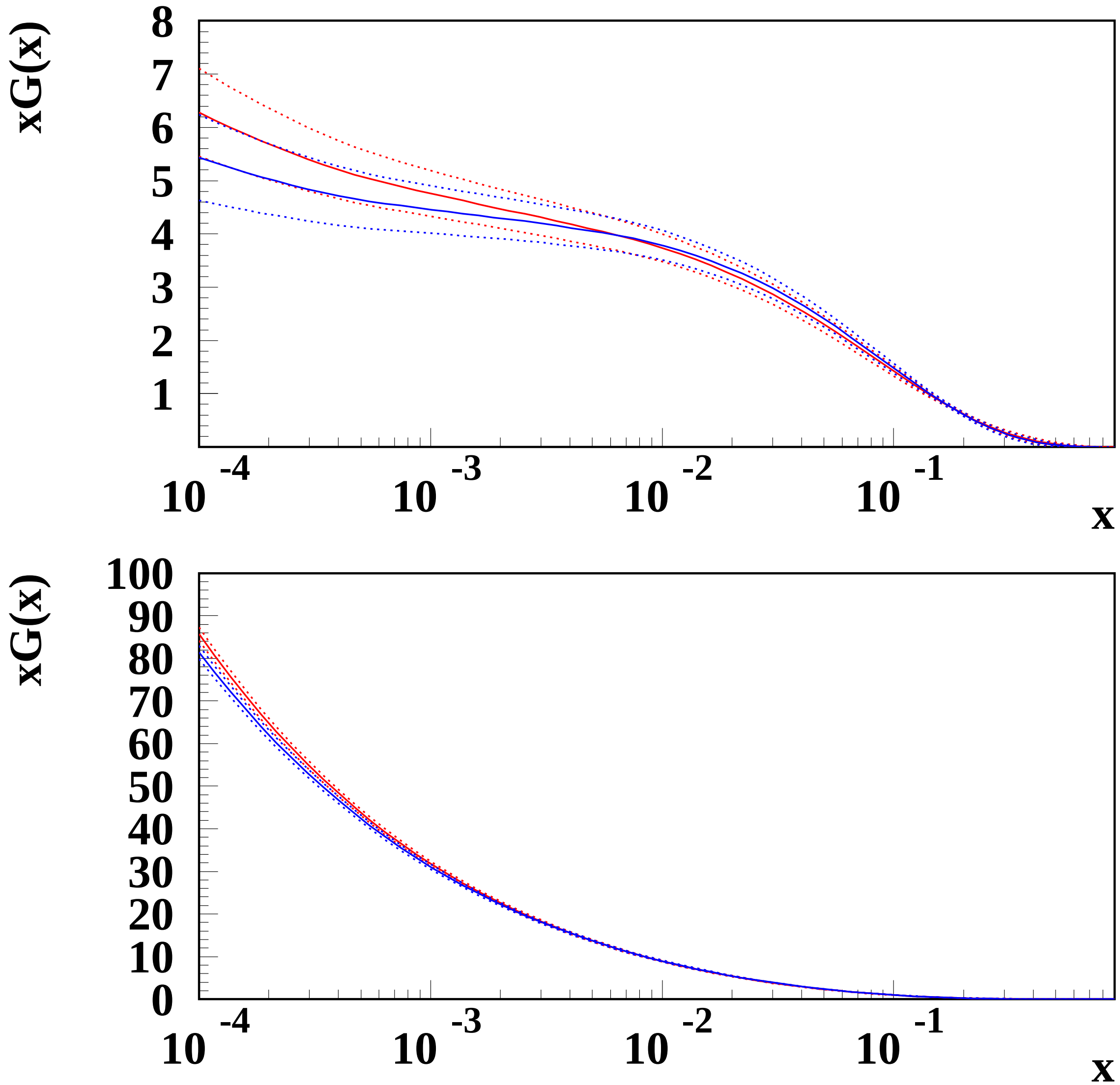
Not mentioned choices for form of parton distributions, i.e. $s(x,Q^2)$, $\bar{u}(x,Q^2)-\bar{d}(x,Q^2)$. Difficult to *prove* these are wrong, but some pretty *peculiar* features in fairly *standard* distributions. Dubious as *default* at very least.

Need a general **Variable Flavour Number Scheme** (VFNS) interpolating between the two well-defined limits of $Q^2 \le m_H^2$ and $Q^2 \gg m_H^2$.

Conclusion easily reached by looking at the extrapolation between the two simple kinematic regimes for xF_3, measured using neutrino scattering at NuTeV



Change in gluon when different flavour prescriptions for α_S are used in fit Alekhin.

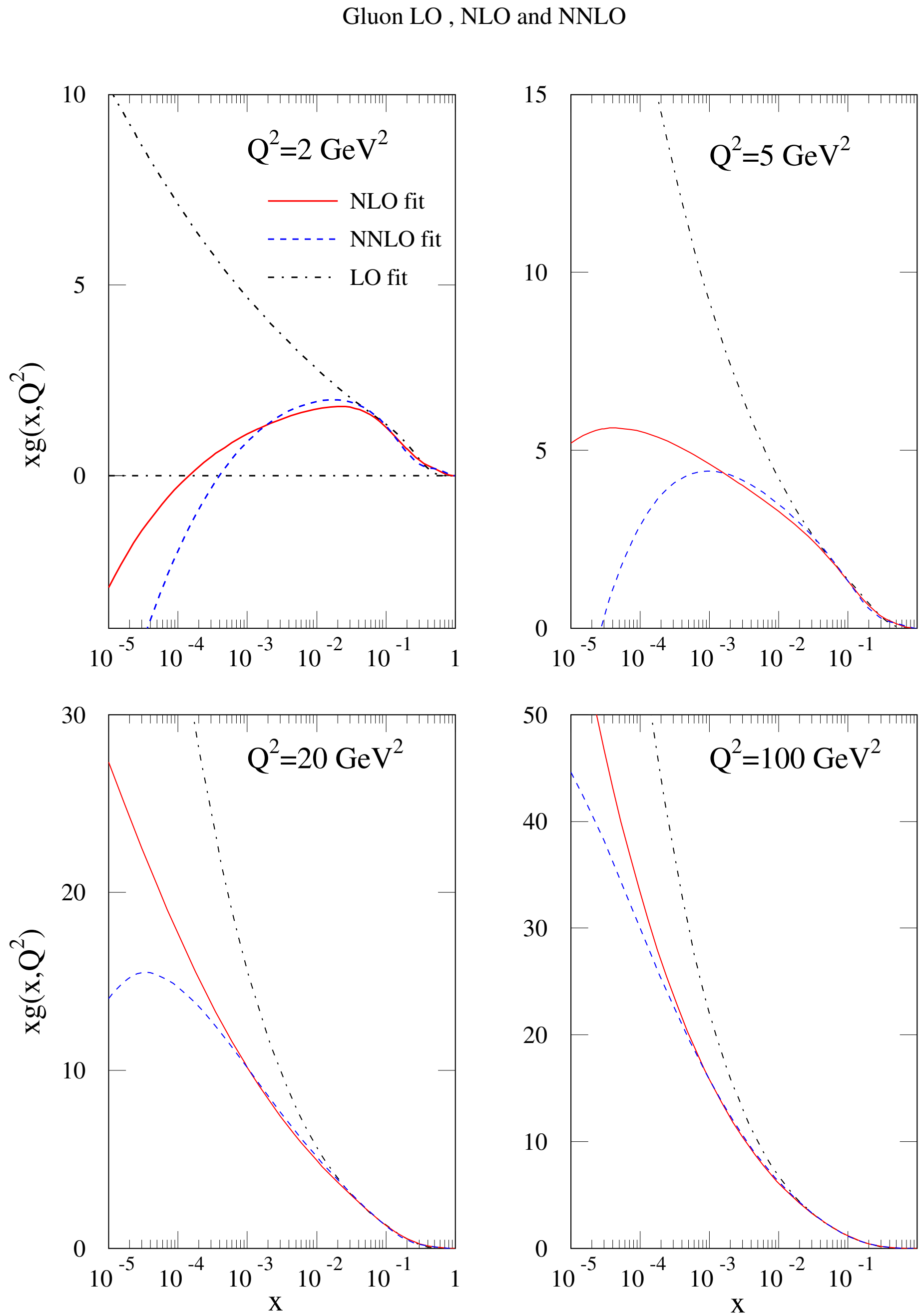


Well known that MRST gluon is negative at very small x and low Q^2. Same for MSTW though less at NLO.

Not guaranteed to be a problem since gluon is not directly related to physical quantity. At NNLO coefficient function restores positivity in $F_L(x,Q^2)$.

At NLO $F_L(x,Q^2)$ can be negative at low enough x and $Q^2 \rightarrow$ sign of required corrections (perturbative or nonperturbative) in this regime.

Parton *incorrect* probably because framework has become *incorrect*. From evolution will always happen at some x and Q^2.

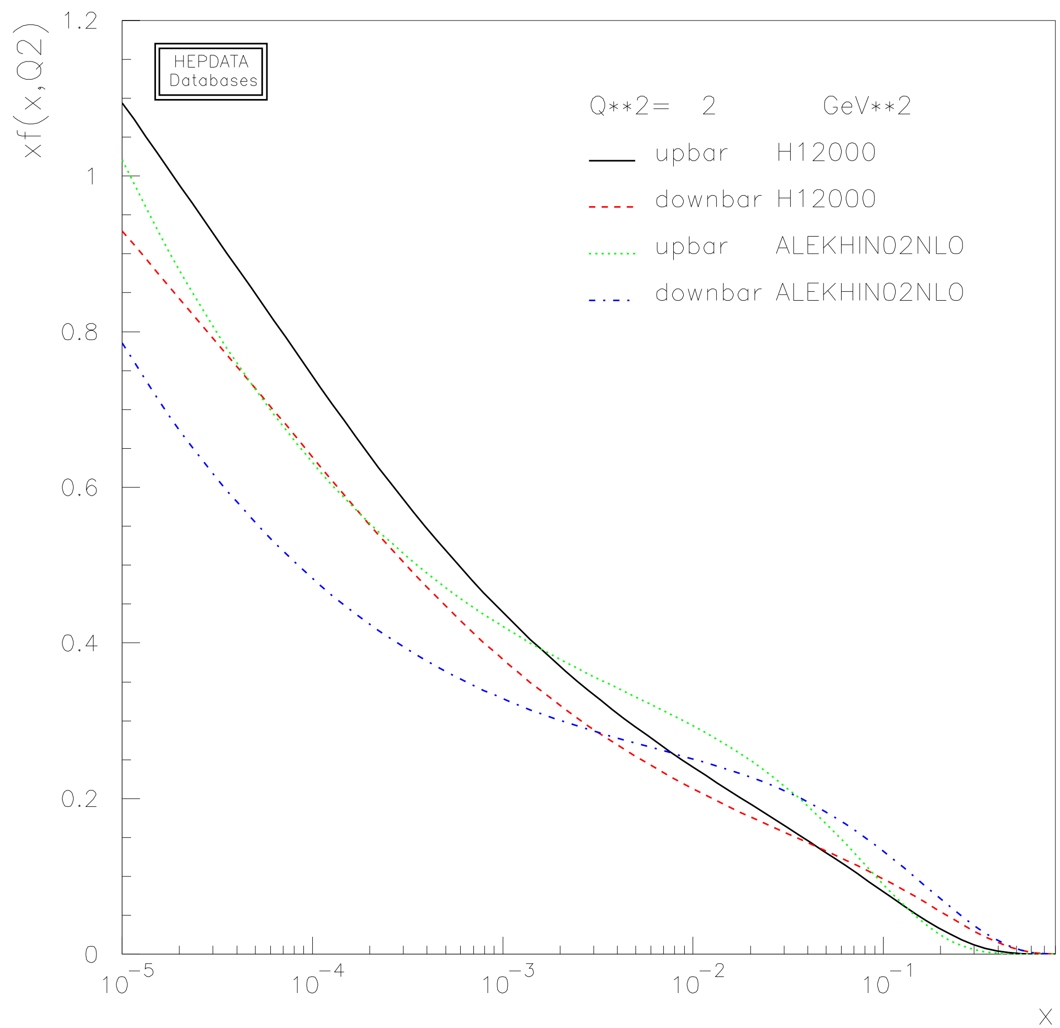


Various sets where $x\bar{u} - x\bar{d}$ remains large as $x \to 0$.

Difficult to prove this is not possible, but against all expectations from Regge physics.

Not aware of any theoretical prejudice for this assumption.

Not *incorrect*, but certainly should be remembered that *unconventional*.



Various sets where $s(x, Q^2)$ is a bit unusual in some sense.

Directly related to charm production in neutrino scattering. Higher order corrections may be significant at low Q^2 and very low x.

Direct constrains only down to $x = 0.001$ but certainly possible to strain limits of *acceptability* in this case if $s(x, Q^2)$ becomes negative.

At small x expect $s(x, Q^2) = \kappa d(x, Q^2)$, where $\kappa < 1$, due to mass suppression effects.

Alternatives difficult to prove *incorrect*, but again alternatives *unconventional*.