

Input Parametrization and Neural Networks

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On behalf of the NNPDF Collaboration:

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Initial scale parametrization of PDFs

Standard Approach

- Introduce a simple functional form with enough (but not too many) free parameters

$$q(x, Q^2) = x^\alpha (1 - x)^\beta P(x; \lambda_1, \dots, \lambda_n).$$

- Fit parameters minimizing χ^2 .



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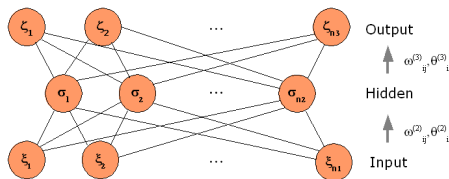
- Fit parameters minimizing χ^2 .

Open problem:

- **Theoretical bias** due to the chosen **parametrization** is difficult to assess.



Why use Neural Networks?



- Neural Networks are **non-linear** statistical tools.
- Any continuous function can be approximated with neural network with one internal layer and non-linear neuron activation function.
- **Efficient minimization algorithms** for complex parameter spaces.
- They provide a parametrization which is **redundant and robust** against variations.



Neural Networks

... just another basis of functions

Multilayer feed-forward networks

- Each neuron receives input from neurons in preceding layer and feeds output to neurons in subsequent layer
- Activation determined by **weights** and **thresholds**

$$\xi_i = g \left(\sum_j \omega_{ij} \xi_j - \theta_i \right)$$

- Sigmoid activation function

$$g(x) = \frac{1}{1 + e^{-\beta x}}$$



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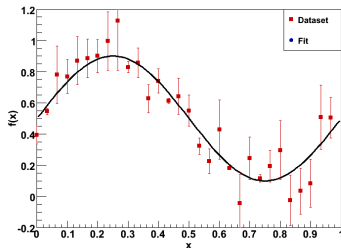
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A 1-2-1 NN:

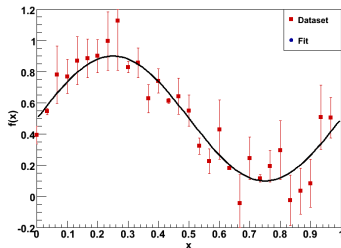
$$\xi_1^{(3)}(\xi_1^{(1)}) = \frac{1}{1 + e^{\theta_1^{(3)} - \frac{\omega_{11}^{(2)}}{1 + e^{\theta_1^{(2)} - \xi_1^{(1)} \omega_{11}^{(1)}} - \frac{\omega_{12}^{(2)}}{1 + e^{\theta_2^{(2)} - \xi_1^{(1)} \omega_{21}^{(1)}}}}$$



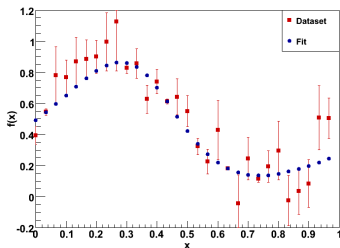
Neural Network vs. Polynomial form



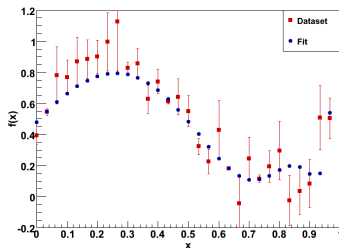
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Neural Net Fit ($\chi^2 = 1$)



Polynomial form Fit ($\chi^2 = 1$)



NNPDF Singlet fit: Datasets

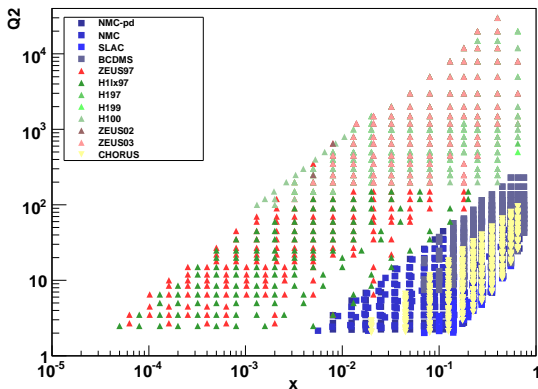
Datasets included in the analysis

Name	N_c	N_n	N_N	Data points	Target
NMC_pd	5	0	0	260	F_2^d / F_2^p
NMC	12	0	4	288	F_2^p
SLAC	0	1	1	211 (211)	$F_2^{p(d)}$
BCDMS	3	9	1	351 (254)	$F_2^{p(d)}$
ZEUS97	10	0	1	240	$\tilde{\sigma}^{NC,+}$
H1x97	5	0	1	133	$\tilde{\sigma}^{NC,+}$
H197	6	0	1	130 (25)	$\tilde{\sigma}^{NC(CC),+}$
H199	6	0	1	126 (27)	$\tilde{\sigma}^{NC(CC),-}$
H100	7	0	1	147 (28)	$\tilde{\sigma}^{NC(CC),+}$
ZEUS02	6	0	1	92	$\tilde{\sigma}^{NC,-}$
ZEUS03	8	0	1	90	$\tilde{\sigma}^{NC,+}$
CHORUS	13	0	2	607 (607)	$\tilde{\sigma}^{\nu(\bar{\nu})}$



NNPDF Singlet fit: Datasets

Kinematical coverage



NNPDF Singlet fit: Neural Network set up

- 5 Neural Networks [g , Σ , Isotriplet, $(u_v + d_v)$, $(\bar{d} - \bar{u})$]
- Network Architecture: 2-3-2-1 (exception Sea Asymmetry: 2-3-1)
- 93 free parameters



Preliminary results: 25 replicas

