PDFs uncertainties in the global fit

(S.Alekhin, IHEP, Protvino)

Qualitative estimate of the PDFs uncertainties

- **quarks:** u-quarks distributions are know with the precision of O(1%) due to to wealth of the precise fixed-target DIS data on the proton target; d-quarks ones are known worse due to limited statistics of the global deuteron sample.

- **anti-quarks:** At small x the total sea distribution is know with the precision of O(1%) due to the HERA data; at large x it can be constrained from combination of the DIS and fixed-target Drell-Yan data with precision better than 20% at $x \lesssim 0.7$, driven by the Drell-Yan ones.

- **gluons:** At small x can be constrained with the precision O(1%) from the slope of F_2 measured at HERA; at large x – with the precision $x \sim 30\%$ due to the jet data.

Sources of uncertainties in PDFs

- Random errors in the data – Gaussian distributed, propagation is well understood.

- Systematic shift – formally cannot be considered as a random, however the only practical way, used essentially everywhere, is statistical (Bayesian) treatment of these errors too.

  (d'Agostini 95)

- Hidden systematics – sometimes appears as a discrepancy between the data sets.

- Theoretical uncertainties – includes known unknowns and unknown unknowns in the theory.

Statistical treatment of the systematical errors

$$y_i = f_i + \sigma_i \mu_i + s_i \lambda$$

The covariance (Hessian) estimator

$$\chi^2 = \sum_{i,j} (f_i - y_i) E_{ij} (f_j - y_j),$$

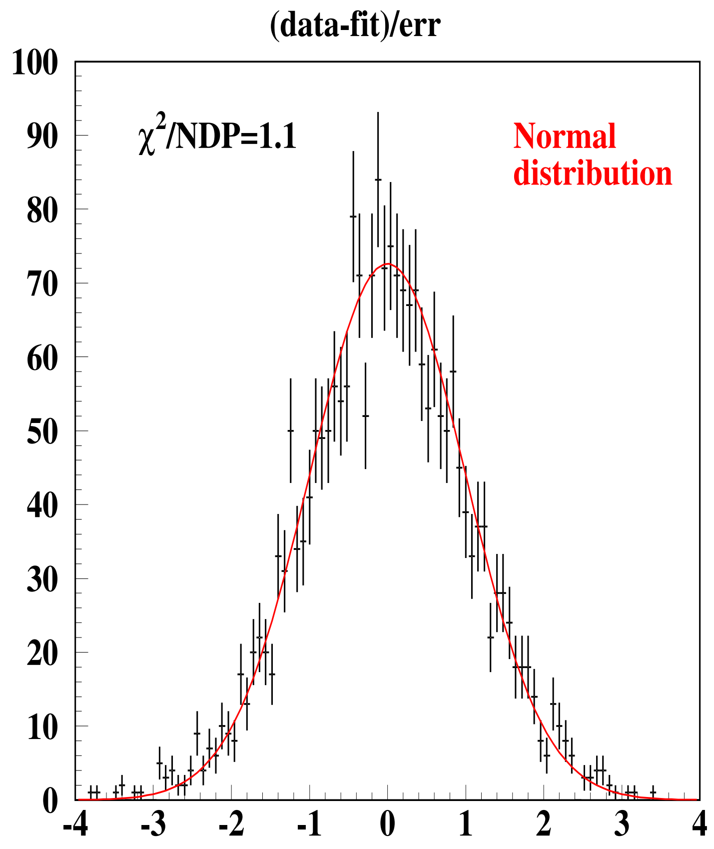
$$E_{ij} = \left(\delta_{ij}\sigma_i\sigma_j + s_i s_j\right)^{-1}$$

is efficient and asymptotically unbiased. Even if $\mu,\ \lambda$ do not obey Gaussian distribution the distribution of fitted parameters approaches the Gaussian one due to the Central Limit Theorem of statistics.

(Radescu, this meeting)
(Guffanti, this meeting)

The pulls distribution for the DIS fit of PDFs

(sa 02)



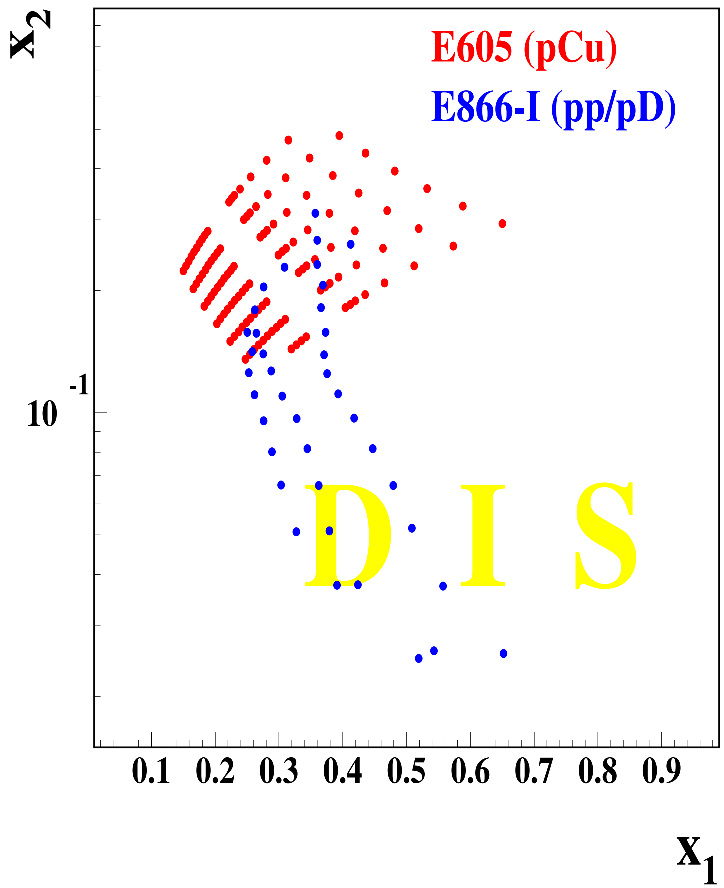
For the fit of PDFs to the global DIS data distribution of the errors, including systematic ones, does obey the Gaussian one. One can evidently apply criterion $\Delta\chi^2 = 1$ for the errors estimation.

Update of the global DIS fit

(sa-Kulagin-Petti 07)

- the fixed target Drell-Yan data by FNAL-E-605 (p Cu) and FNAL-E-866 (pp/pD)
- the charged-lepton DIS data down to $Q^2 = 1$ GeV^2
- inclusive νN data by CHORUS
- data on dimuon production in the νN interactions by NuTeV and CCFR

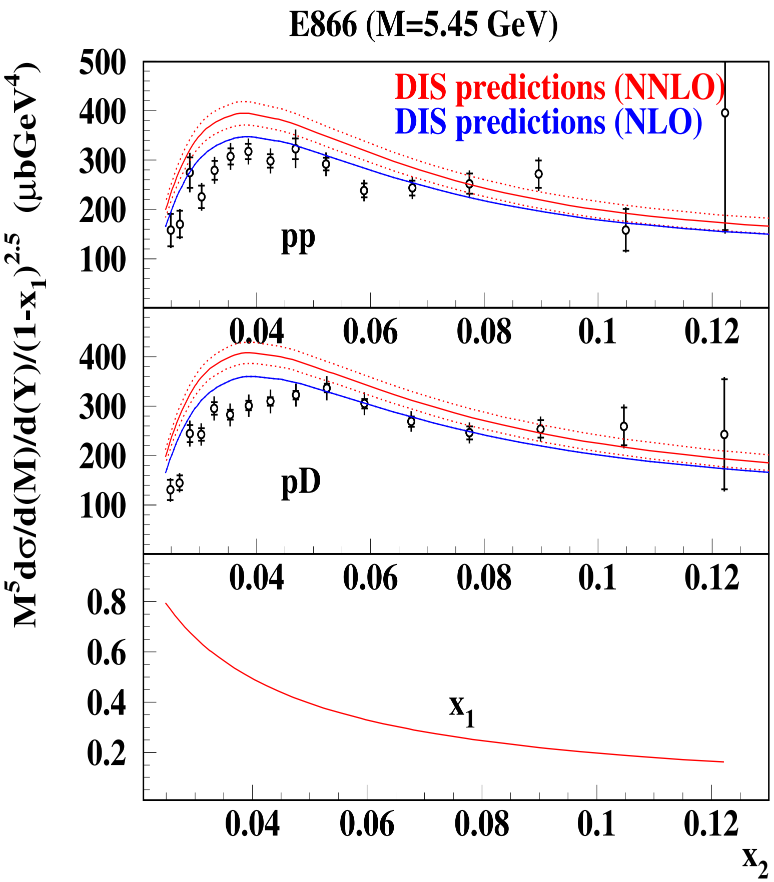
The Drell-Yan data kinematics



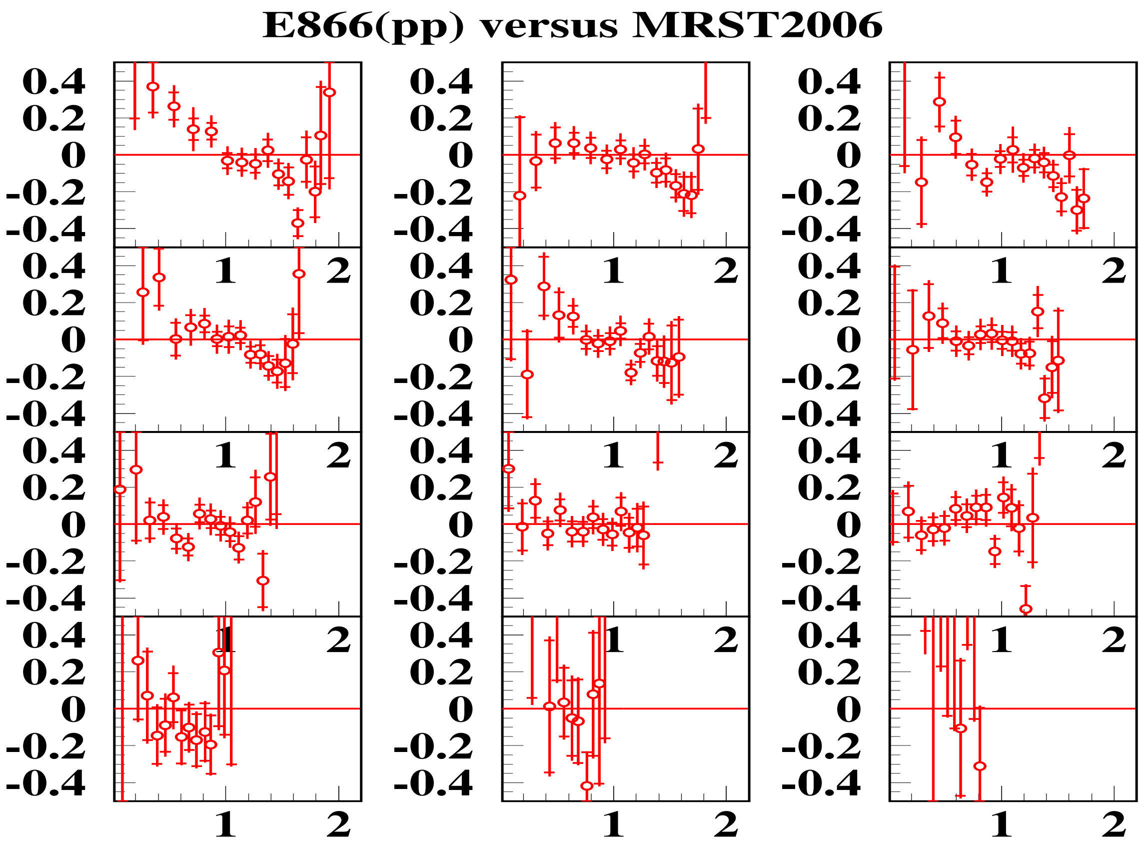
The Drell-Yan data are supplementary to the DIS ones providing constraint on the sea distribution at large x.

The Drell-Yan E866 data in the global fit

(sa-Melnikov-Petriello 06)

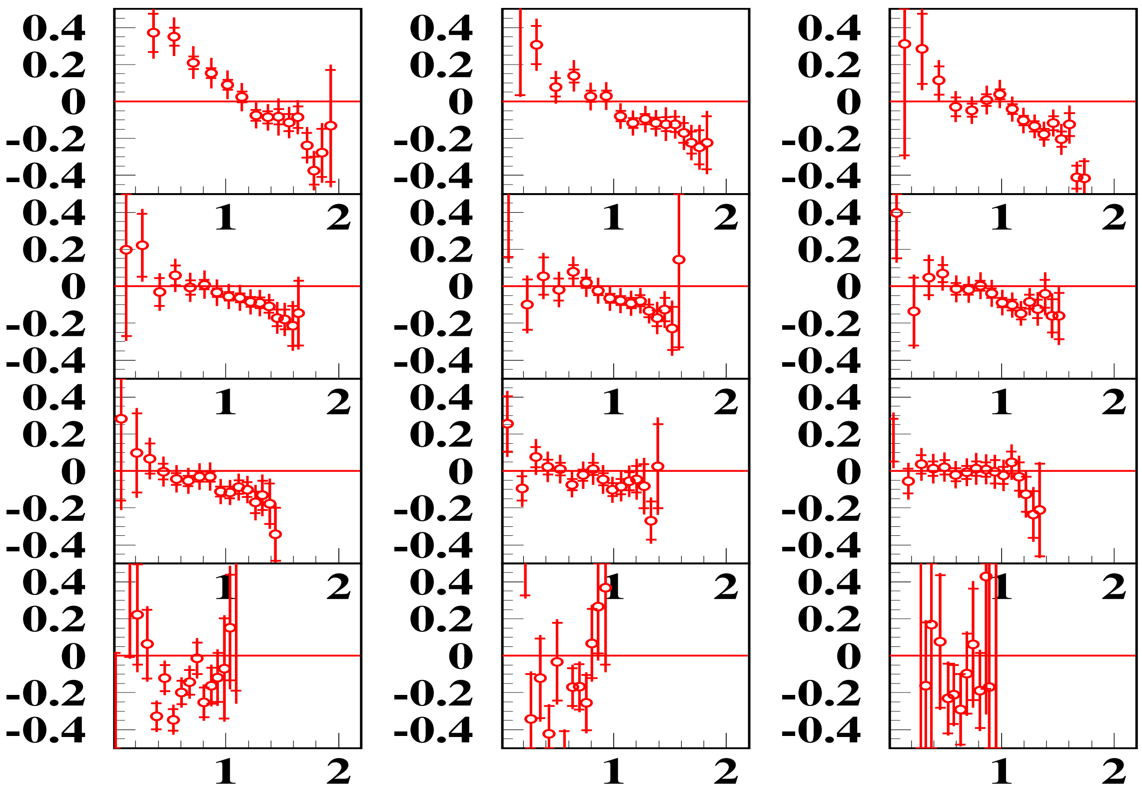


The absolute cross section DY data by E-866 at low $/mu/mu$ masses are incompatible with the global DIS data and cannot be included in the fit.



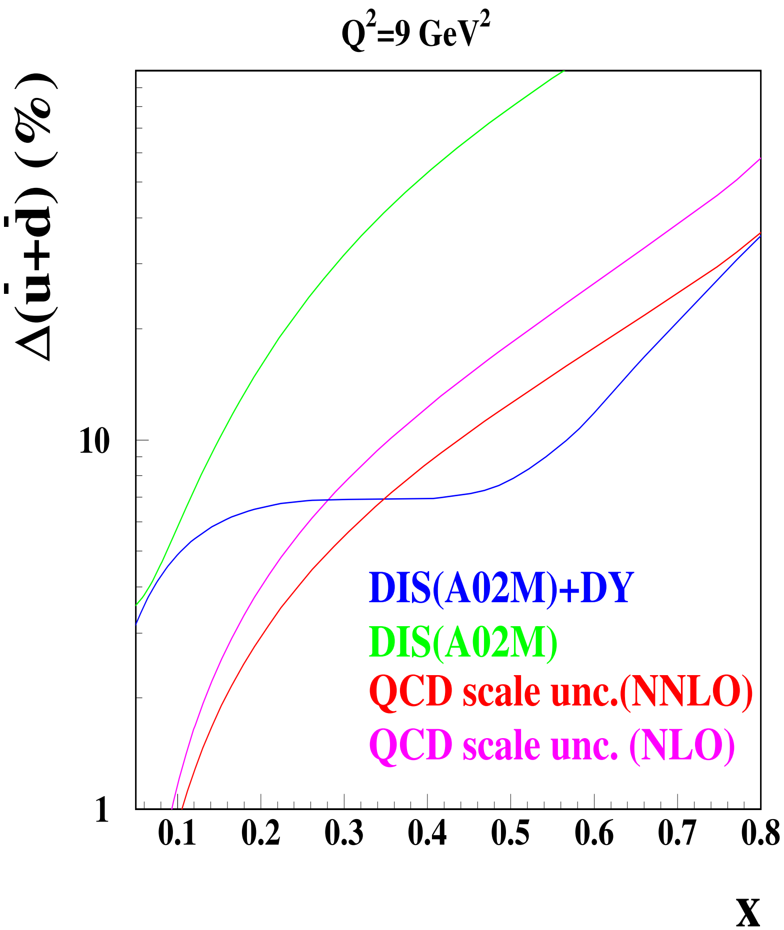
$\chi^2 = 322(209)/184$ with(out) account of the errors correlation.

E866(pD) versus MRST2006



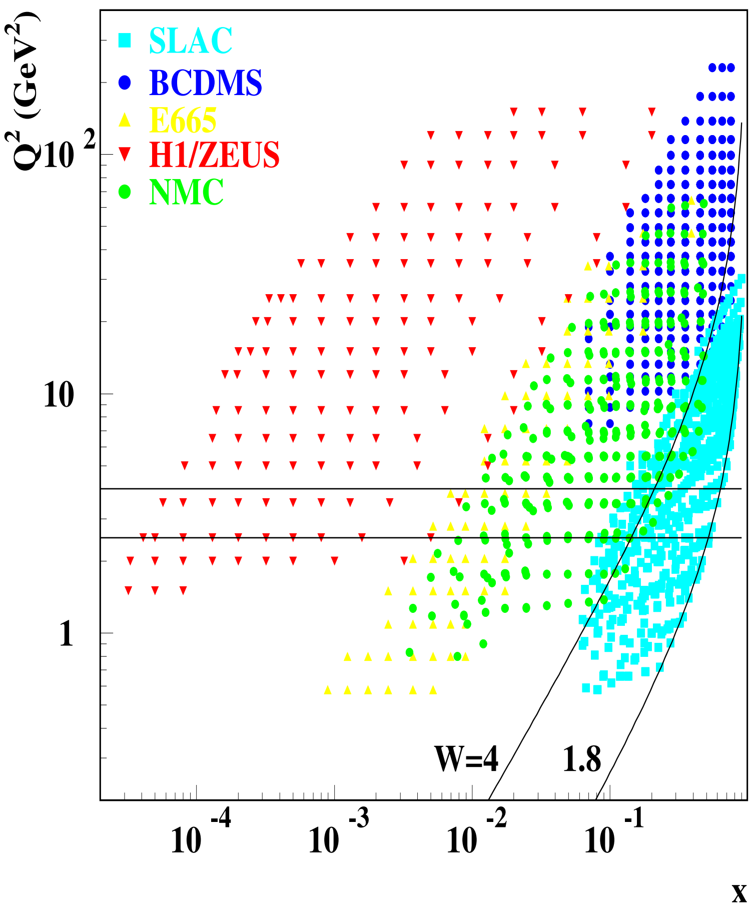
Impact of the DY data on the sea distribution

(sa-Melnikov-Petriello 06)



- Experimental errors in the sea is < 20 % at $x \lesssim 0.7$.
- The errors in PDFs due to variation of the DY scales are comparable to the experimental ones (*the corrections of* $O(\alpha_s^2)$ *by Anastasiou-Dixon-Melnikov-Petriello are crucial at this point*).

Low-Q DIS data in the global fit



The low-Q DIS data are necessary in order to outline the region of validity of the QCD-factorization. The high-twist terms extraction is crucial for this kinematics.

The HO pQCD corrections in DIS

Splitting Functions (up to $O(\alpha_s^3)$):

(Moch-Vermasseren-Vogt 04)

Massless quarks coefficient functions (up to $O(\alpha_s^3)$)

(Zijlstra-van Neerven 91-92)

(Kazakov-Kotikov 92)

(Vermasseren-Moch-Vogt 05)

Heavy quarks coefficient functions (up to $O(\alpha_s^2)$):

(Laenen-Riemersam-Smith-van Neerven 92-93)

Non-pQCD corrections

- The DIS structure functions are calculated using OPE

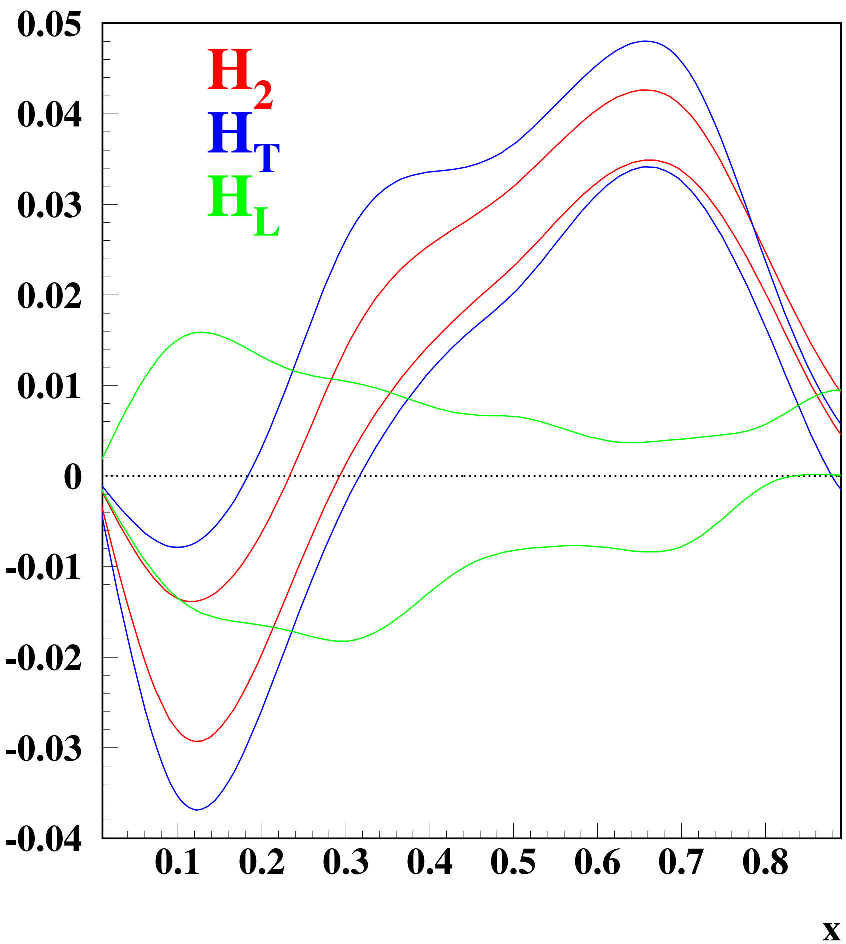
$$F_{2,\mathrm{T}}(x,Q) = F_{2,\mathrm{T}}^{\mathrm{LT}}(x,Q) + \frac{H_{2,\mathrm{T}}^{(2)}(x)}{Q^2} + \left(\frac{H_{2,\mathrm{T}}^{(4)}(x)}{Q^4}\right)$$

  The leading-twist terms (entirely dominant at $Q^2 \gtrsim 10$ GeV^2). The twist-4 terms (contributes at $Q^2 \lesssim 10$ GeV^2) and the twist-6 terms (might contribute at $Q^2 \lesssim 3$ GeV^2) – no QCD evolution.

- The target-mass correction by Georgi-Politzer.

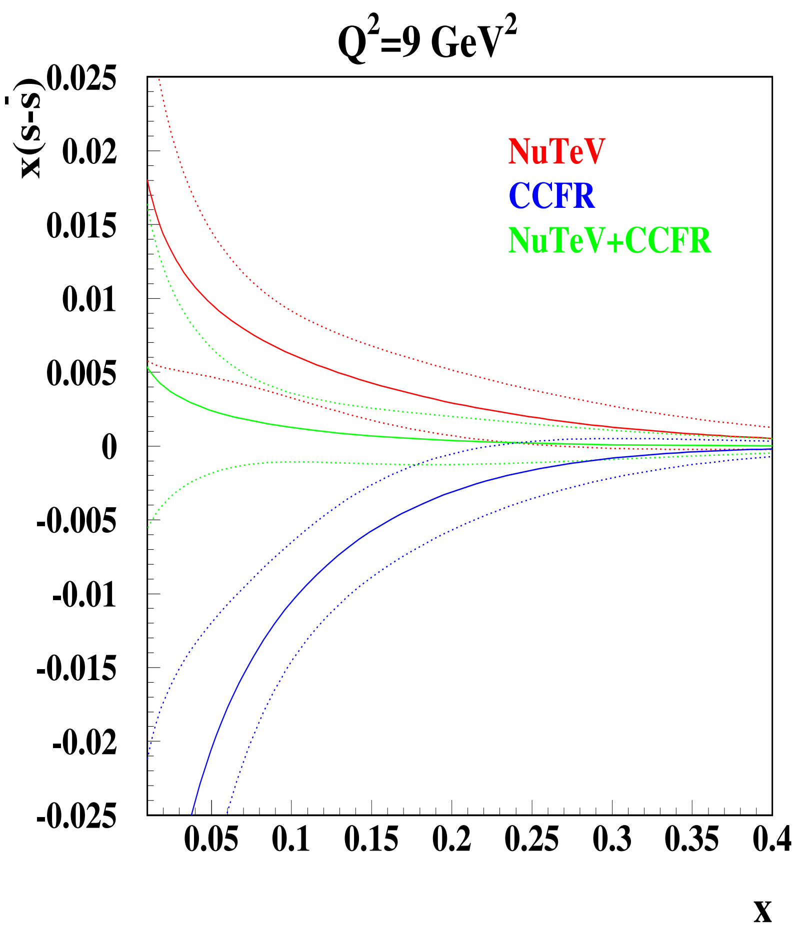
- The deuteron nuclear corrections by Kulagin-Petti.

The HT terms of the final fit



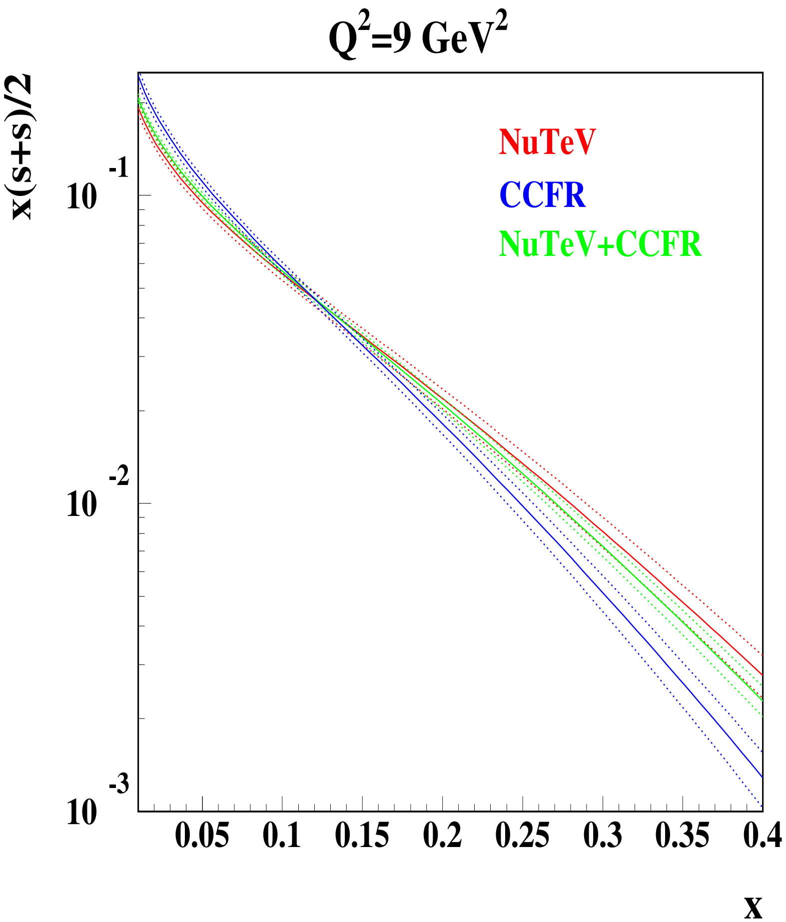
The HT terms in the charged-lepton F_2 and F_{T} averaged over proton and neutron are very similar within the errors; therefore the HT term in F_L is comparable to 0. The constraint $H_2^{lN} = H_{\mathrm{T}}^{lN}$ was further imposed everywhere.

Strange sea asymmetry



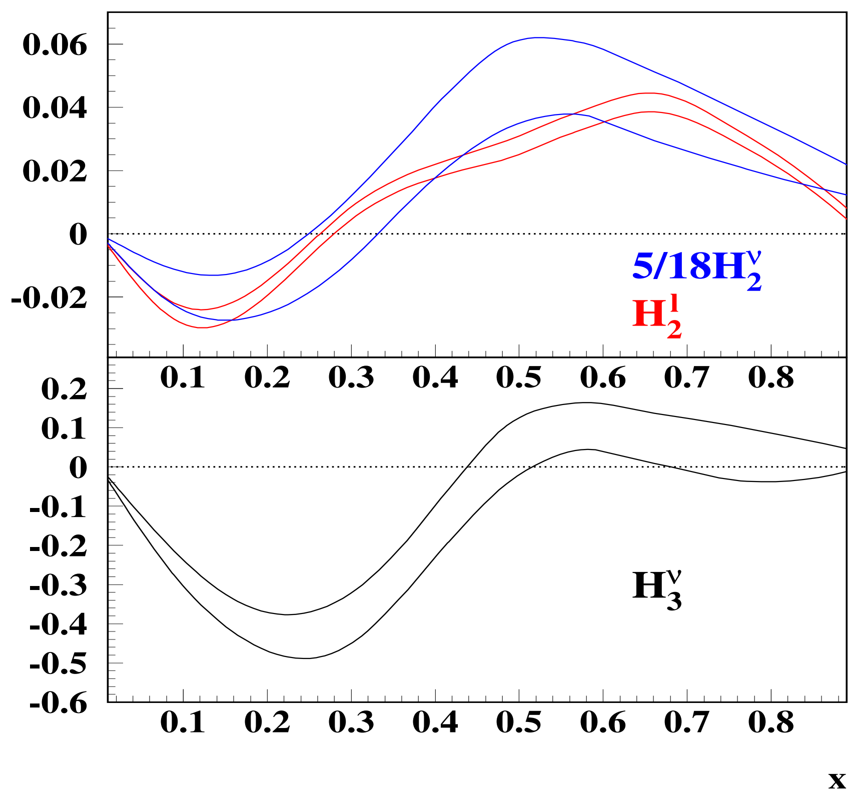
- The $O(\alpha_s)$ corrections by Gottschalk and EW corrections by Arbuzov-Bardin are taken into account.
- The NuTeV and CCFR data prefer asymmetry of different sign; averaging of both gives $S^- = -0.0005(6)$.
- The MSTW fit gives value $S^- = 0.0023(25)$, close to the NuTeV result $S^- = 0.0023(15)$.

Total strange sea



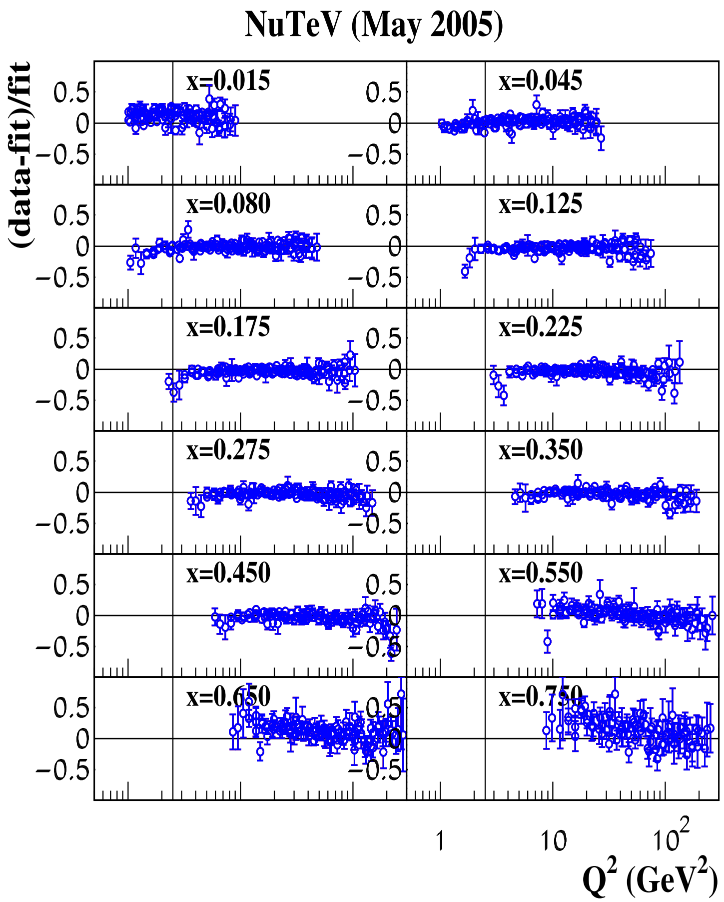
- The strange suppression factor value from the combined fit is 0.54 ± 0.02 at $Q^2 = 20$ GeV^2.
- The CCFR analysis of their own data gives this value about 0.4; due to unusually enhanced d-quark distribution defined from the inclusive sample.

The HT terms in νN structure functions from the fit including the CHORUS data



- $H_2^{\nu N} = H_{\mathrm{T}}^{\nu N}$, motivated by the charged-leptons fit.
- $H_2^{\nu N}$ is in remarkable agreement to H_2^{lN} rescaled with the quarks charge.
- $\int H_3^{\nu N}(x)dx$ is -0.10 ± 0.03 GeV^2, in nice agreement to the early calculations by Braun-Kolesnichenko.

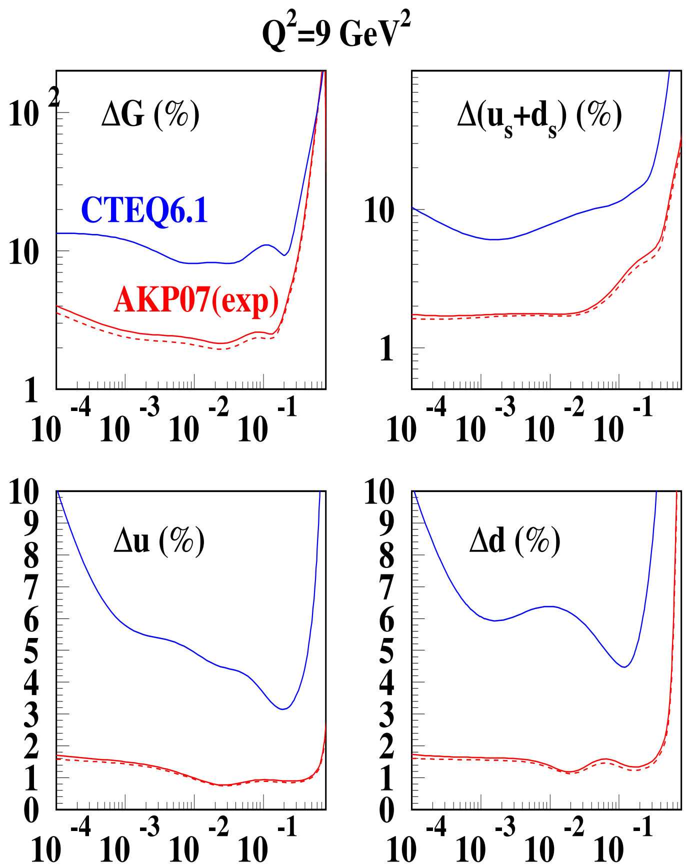
Status of the inclusive NuTeV data



The NuTeV data at $x \sim 0.01$ go above the charged-leptons fit. This discrepancy cannot be removed due to account of the $O(\alpha_s^3)$ corrections to the C-odd coefficient functions by Moch-Rogal-Vogt; modification of the nuclear corrections did not help too. At large x the NuTeV data also go above the the charged-leptons fit.

Experiment	NDP	χ^2/NDP	scale factor
SLAC-E-49A	214	0.51	–
SLAC-E-49B	389	1.30	1.15
SLAC-E-61	44	0.31	–
SLAC-E-87	218	0.99	–
SLAC-E-89A	148	1.45	1.21
SLAC-E-89B	216	1.10	1.05
SLAC-E-139	22	0.83	–
SLAC-E-140	46	1.22	1.11
BCDMS	605	1.16	1.08
NMC	578	1.39	1.19
E665	130	1.59	1.30
H1(96-97)	147	1.32	1.16
ZEUS(96-97)	161	1.45	1.22
E605	119	1.46	1.22
E866	39	1.34	1.16
NuTeV($\mu\mu$)	89	0.47	–
CCFR($\mu\mu$)	89	0.69	–
CHORUS	1084	1.23	1.11

The errors on PDFs



- Rescaling of the errors has marginal impact on the PDFs errors.
- The AKP07 PDFs errors ($\Delta\chi^2 = 1$) are in agreement to the qualitative estimates.
- The criterion $\Delta\chi^2 = 100$ is clearly inefficient.

Summary

The fit of the PDFs to the combined charged leptons DIS, fixed-target Drell-Yan, dimuon neutrino data by NuTeV and CCFR, and inclusive neutrino data by CHORUS demonstrates reasonable consistency of the data: $\chi^2/NDP = 5177/4338 = 1.2$; the rescaling factors on the errors are within 1.3. The errors on PDFs obtained with the standard criterion $\Delta\chi^2 = 1$ are in agreement with the qualitative estimates, while the criterion $\Delta\chi^2 = 100$ is clearly inefficient.