Finite-mass corrections to NNLO Higgs production

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in collaboration with:
R.D.Ball, V.Del Duca, S.Forte and A.Vicini, **arXiv:0801.2544** [hep-ph]

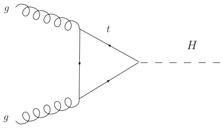
Outline

Searching the Higgs boson at the LHC

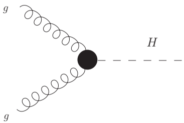
- The Higgs boson is the missing particle of the SM
- Its discovery is the main reason the LHC has been built for
- The main production channel is gluon-gluon fusion via a top loop
- We study finite-top-mass effects on the inclusive cross section

The state of the art (QCD)

- NLO (large contribution) [Spira et al. 1995]



- The dominant contribution does not resolve the effective ggH vertex: $m_t \to \infty$ limit (one less loop)
- NLO [Spira et al. 1991; Dawson, 1991]
- NNLO [Anastasiou and Melnikov, 2002; Harlander and Kilgore, 2002; Ravindran, Smith and van Neerven, 2003]



The high energy behaviour

- The heavy top approximation fails in the limit $\hat{s} \longrightarrow \infty$:

$$\hat{\sigma} \underset{\hat{s}\to\infty}{\sim} \begin{cases} \alpha_s^2 \sum_{k=1}^{\infty} \alpha_s^k \ln^{2k-1}\left(\frac{\hat{s}}{m_H^2}\right) & \text{pointlike: } m_t \longrightarrow \infty \\ \alpha_s^2 \sum_{k=1}^{\infty} \alpha_s^k \ln^{k-1}\left(\frac{\hat{s}}{m_H^2}\right) & \text{resolved: finite } m_t \end{cases}$$

- In the pointlike approximation there are double logs [Hautmann, 2002]
- In the finite m_t case the high energy behaviour is softened by a form factor
- Using high energy resummation techniques we compute the coefficients of the single logs
- We use the high energy behaviour of the exact cross section to improve the infinite m_t results

Some definitions

- We write the partonic cross section as:

$$\hat{\sigma}_{gg}\left(\alpha_s;\tau;y_t,m_H^2\right) = \sigma_0(y_t)C(\alpha_s(m_H^2),\tau,y_t)$$

$$C(\alpha_s(m_H^2),\tau,y_t) = \delta(1-\tau)+\frac{\alpha_s(m_H^2)}{\pi}C^{(1)}(\tau,y_t) + \left(\frac{\alpha_s(m_H^2)}{\pi}\right)^2 C^{(2)}(\tau,y_t)+\dots$$

- $\sigma_0\delta(1-\tau)$ is the LO cross section
- $\tau=\frac{m_H^2}{\hat{s}}$ and $y_t=\frac{m_t^2}{m_H^2}$
- We consider Mellin moments of the coefficient function:

$$C(\alpha_s(m_H^2),N,y_t)=\int_0^1 d\tau\,\tau^{N-1}C(\alpha_s(m_H^2),\tau,y_t)$$

- The leading contribution at small τ is given order by order in α_s by the highest pole in N

High energy factorisation

Leading small N singularities: *high energy factorisation theorem*:

- LO $gg \to H$ cross section σ_{off} with off-shell gluons with virtualities ξ_1, ξ_2
- Mellin moments:

$$h(N, M_1, M_2) = M_1 M_2 \int_0^\infty d\xi_1 \int_0^\infty d\xi_2 \, \xi_1^{M_1-1} \xi_2^{M_2-1} \int_0^1 d\tau \tau^{N-1} \sigma_{off}$$

- Leading small N singularities of the coefficient function:

$$h(0, \gamma^+(N), \gamma^+(N))$$

- $h(0, M_1, M_2)$ cannot be computed analytically because of the complexity of σ_{off}
- Expansion in M_1, M_2 and numerical evaluation of the integrals
- Coefficients computed up to the 4^{th} order in M_i (N^4LO) in view of resummation

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Results

- Remember that:

$$C(\alpha_s(m_H^2), N, y_t) = 1 + \frac{\alpha_s(m_H^2)}{\pi} C^{(1)}(N, y_t) + \left(\frac{\alpha_s(m_H^2)}{\pi}\right)^2 C^{(2)}(N, y_t) + \dots$$

- We compute:

$$C^{(1)}(N, y_t) = \mathcal{C}^{(1)}(y_t) \frac{C_A}{N} [1 + O(N)]$$
$$C^{(2)}(N, y_t) = \mathcal{C}^{(2)}(y_t) \frac{C_A^2}{N^2} [1 + O(N)]$$

- Upon Mellin inversion:

$$C^{(1)}(\tau, y_t) = \mathcal{C}^{(1)}(y_t) C_A + O(\tau)$$
$$C^{(2)}(\tau, y_t) = -\mathcal{C}^{(2)}(y_t) C_A^2 \ln \tau + O(\tau^0)$$

- $\mathcal{C}^{(1)}(y_t)$ agrees with the small τ limit of the full NLO computation
- The NNLO coefficient $\mathcal{C}^{(2)}(y_t)$ is our main result.

Matching

- Our computation only determines the leading small τ behaviour of the coefficient function $C(\tau, y_t)$:

$$C(\tau \to 0, y_t)$$

- It has to be matched with the $m_t \to \infty$ result at large τ

$$C^{\text{app.}}(\tau, y_t) \approx C(\tau, \infty) + \left[C(\tau \to 0, y_t) - \lim_{\tau \to 0} C(\tau, \infty) \right] \, T(\tau, \tau_0)$$

- Step function $T(\tau, \tau_0) = \theta(\tau_0 - \tau)$
- Smooth function $T(\tau, \tau_0) = \frac{1}{2} \left[1 + tanh \left(\frac{\tau_0 - \tau}{\omega} \right) \right]$
- How do we choose τ_0 ?

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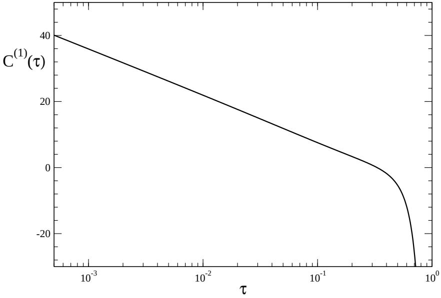
NLO with $m_H = 130 GeV$

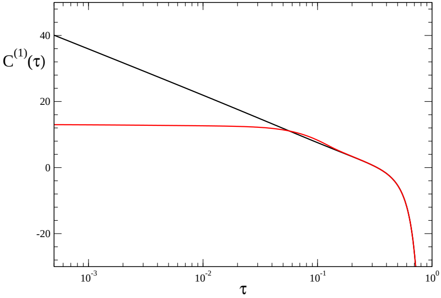
- $m_t \to \infty$ case $C^{(1)}(\tau, \infty)$
- Spurious logarithmic growth at small τ



The full NLO computation is known, so we can use it as a testing ground

NLO with $m_H = 130 GeV$

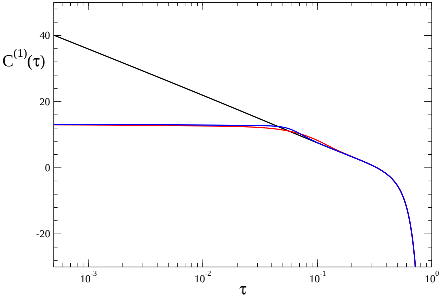
- Exact case $C^{(1)}(\tau, y_t)$, with $m_t = 170.9 GeV$
- Correct small τ behaviour



Matching point at the intersection of $m_t \to \infty$ with the asymptotic curve

NLO with $m_H = 130 GeV$

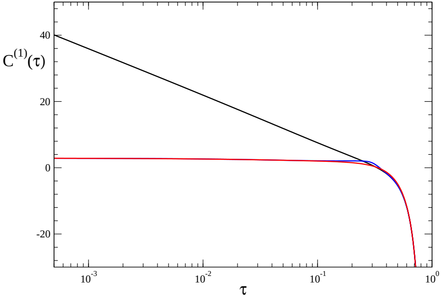
$$C^{(1),\mathrm{app.}}(\tau, y_t) \approx C^{(1)}(\tau, \infty) + \left[\mathcal{C}^{(1)}(y_t) C_A - \lim_{\tau \to 0} C^{(1)}(\tau, \infty)\right] T(\tau, \tau_0)$$



Very small discrepancy between approximate and exact results

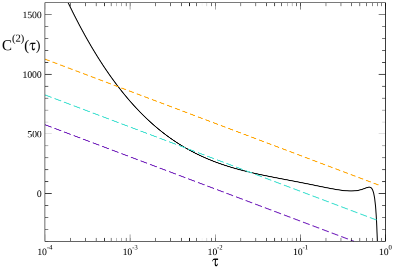
NLO with $m_H = 280 GeV$

For a heavier Higgs boson the spurious growth sets in at larger values of τ



- $m_t \longrightarrow \infty$
- exact
- approximate

NNLO with $m_H = 130 GeV$ (I)



$$C^{(2),\mathrm{app.}}(\tau, y_t) \approx C^{(2)}(\tau,\infty) + \left[-\mathcal{C}^{(2)}(y_t) C_A^2 \ln\tau + \mathcal{C}_0^{(2)}(y_t) - \lim_{\tau\to 0} C^{(2)}(\tau,\infty) \right] T(\tau,\tau_0)$$

- Spurious $\ln^3\tau$ growth to be replaced with $\ln\tau$
- The calculation fixes the slope but not the constant
- How do we choose τ_0 ? How do we deal with the constant ?

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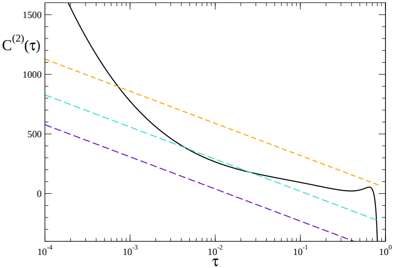

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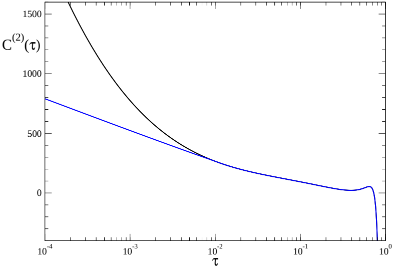
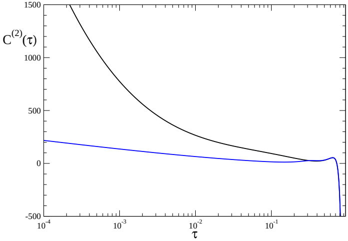
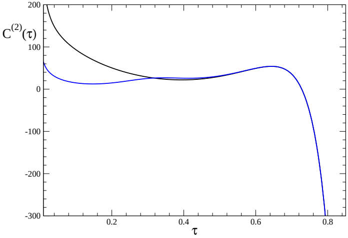
NNLO with $m_H = 130\,GeV$ (II)



- Two procedures to determine τ_0: same as NLO or matching the slopes
- The two methods give similar curves
- In both cases the constant is adjusted requiring continuity

NNLO with $m_H = 280\,GeV$





- $m_t \to \infty$ and improved partonic cross section for $m_H = 280\,GeV$

NNLO hadronic cross section

- Hadronic cross section $\sigma_{gg}(\tau_h; y_t, m_H^2)$ computed with MRST2002 gluon

$$\begin{aligned}
\kappa(\tau_h; y_t, m_H^2) &= \sigma_{gg}(\tau_h; y_t, m_H^2)/\sigma^0_{gg}(\tau_h; y_t, m_H^2) \\
\kappa(\tau_h; y_t, m_H^2) &= 1 + \frac{\alpha_s(m_H^2)}{\pi}\kappa^{\mathrm{NLO}}(\tau_h; y_t, m_H^2) + \\
&\quad + \left(\frac{\alpha_s(m_H^2)}{\pi}\right)^2 \kappa^{\mathrm{NNLO}}(\tau_h; y_t, m_H^2) + \ldots
\end{aligned}$$

- Small-τ finite-m_t effects on κ^{NNLO} are $\sim 2\%$ for $m_H \leq 280 GeV$
- This corresponds to a contribution to κ of the order of 0.5%

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Conclusions

- We have discussed the inclusive cross section for Higgs boson production via gluon-gluon fusion
- Thanks to high energy factorisation we have computed the leading high energy (small τ) singularities of the partonic cross section
- We have constructed an approximation to the cross section matching the exact small τ behaviour to the infinite m_t calculation
- The impact finite-m_t at small τ on the total κ factor is small $\sim 0.5\%$

Outlook

- A study of finite-m_t effects on the rapidity distribution is interesting and it does not look difficult
- The resummation of the small τ logarithms is work in progress (the first four orders have been computed)
- The combination of high energy ($\tau \longrightarrow 0$) and soft contributions ($\tau \longrightarrow 1$) will provide information about unknown higher order corrections