

HERA & LHC Workshop — CERN, May 2008

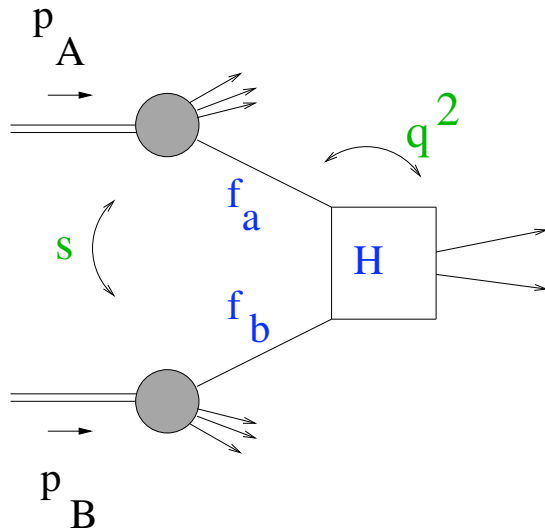
Multi-jet production and unintegrated parton distributions

F. Hautmann

- I.** Motivation: multiparticle production in high-energy hadron collisions
- II.** General issues on unintegrated parton distributions
- III.** Jet correlations in small- x final states

I. INTRODUCTION

Multi-scale hard processes at high-energy hadron colliders



$$\sigma = \int f_{a/A} \otimes f_{b/B} \otimes H_{ab}$$

phase space opening up for large \sqrt{s}



- large number of events with **multiple** hard scales: q_1^2, \dots, q_n^2
- potentially large corrections to all orders in α_s , $\sim \ln^k(q_i^2/q_j^2)$
- parton distributions probed near kinematic boundaries $x \rightarrow 0$, $1 - x \rightarrow 0$

▷ Part of the effects are universal

↪ ex.: high-order corrections in renormalization group evolution

$$\mu \frac{d}{d\mu} f = \gamma \otimes f$$

$$\gamma \simeq \gamma^{(LO)} (1 + c_1 \alpha_s + \dots + c_{n+m} \alpha_s^m (\alpha_s L)^n + \dots) , L = \text{“large log”}$$

▷ Part of them are not universal (final-state correlations, exclusive variables, ...)

↪ yet can be summed by techniques that generalize RG factorization

- pdf's **unintegrated** in both \parallel and \perp components

Examples:

- Sudakov processes
- small-x physics
- reconstruction of fully exclusive final states by Monte-Carlos

- perturbative calculations at fixed order (NLO, multi-leg)

Multi-jet final states:

- parton-shower event simulation (higher orders, hadronization)

Multiple

Finite transverse-momentum tail in initial-state radiative processes



important for $x \rightarrow 0$ (high-energy region)

hard scales

$x \rightarrow 1$ (Sudakov region)



- included partially, order-by-order, in perturbative calculations (higher loops)
- amounts to corrections to the angular ordering in standard parton showers

(HERWIG, PYTHIA)

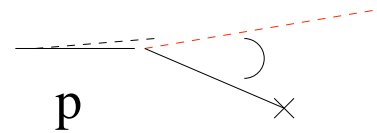
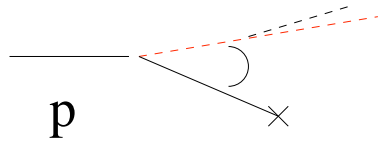
◇ inclusive jet cross sections: NLO (+ standard shower) probably sufficient

◇ correlations, exclusive final-state structure?

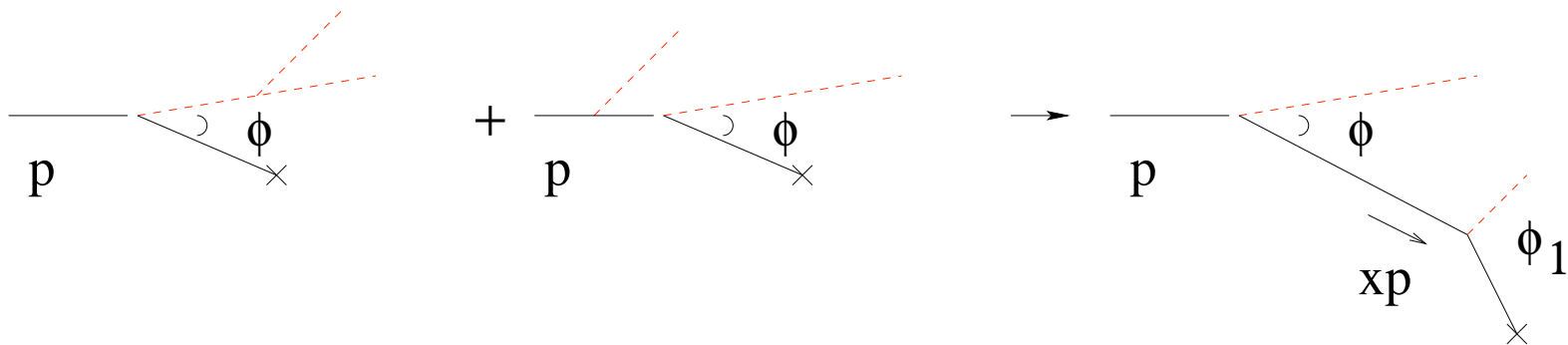
◇ Multiple QCD radiation \rightarrow parton shower:

▷ HERWIG: ordering in decay angles in space-like shower
($\approx k_{\perp}$ -ordering for $x \ll 1$)

- collinear (incoherent) emission



- soft gluon coherence ($x \sim 1$)



▷ Coherent radiation at $x \ll 1 \rightarrow$ corrections to angular ordering:

MC based on k_{\perp} -dependent unintegrated pdfs and MEs

Outline

i) General characterizations of u-pdf's

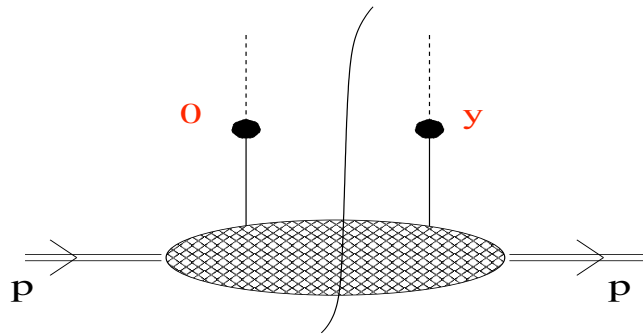
- ▷ gauge-invariant operator matrix elements
- ▷ physical definition for $x \rightarrow 0$ (high energy)
- ▷ lightcone divergences and regularization methods

ii) Applications to $x \ll 1$ parton showers and jets

- ▷ effects of initial-state radiation on multi-jet production
- ▷ angular correlations and momentum correlations

II. General issues on u-pdf's

Example 1: Ordinary (integrated) pdf



$$\mathbf{p} = (p^+, m^2 / 2 p^+, \mathbf{0}_\perp)$$

$$f_q(x, \mu) = \int \frac{dy^-}{2\pi} e^{-ixp^+ y^-} \tilde{f}(y)$$

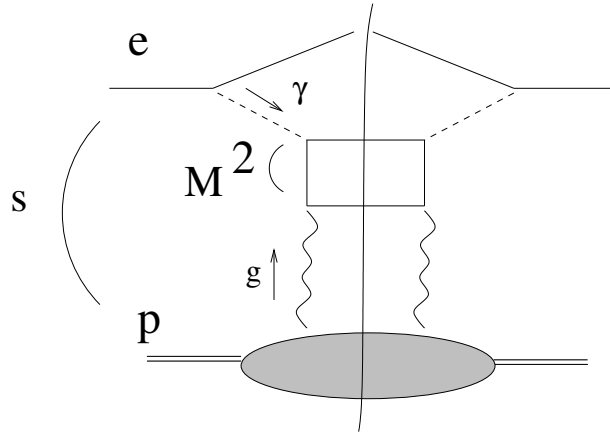
correlation of quark fields at lightlike separation $y = (0, y^-, 0_\perp)$:

$$\tilde{f}(y) = \langle P | \bar{\psi}(y) V_y^\dagger(n) \gamma^+ V_0(n) \psi(0) | P \rangle ,$$

$$V_y(n) = \mathcal{P} \exp \left(ig_s \int_0^\infty d\tau n \cdot A(y + \tau n) \right) \quad \text{eikonal line in direction } n = (0, 1, 0_\perp)$$

- μ -dependence from renormalization of operator product
 - gauge-invariant Wilson line matrix elements

Example 2: Unintegrated pdf from physical cross section (high energy limit). E.g.:



◇ single gluon polarization dominates $s \gg M^2 \gg \Lambda_{\text{QCD}}^2$

↳ gauge invariance rescued (despite gluon off-shell)

◇ but to define u-pdf gauge-invariantly over the whole phase space is more difficult

Example 3: Generalize matrix element to non-lightlike distances

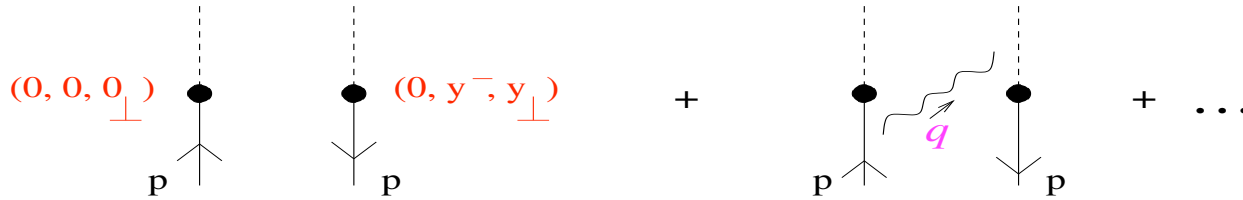
$$\tilde{f}(y) = \langle P | \bar{\psi}(y) V_y^\dagger(n) \gamma^+ V_0(n) \psi(0) | P \rangle \quad , \quad y = (0, y^-, y_\perp)$$

- works at tree level [Mulders, 2002; Belitsky et al., 2003; Collins, 2004]
- subtler at level of radiative corrections ↳
- spectator interactions possibly non-decoupling

[Mulders, Bomhof, Collins, Qiu, Brodsky, ...]

◇ Suppose a gluon is absorbed or emitted by eikonal line:

$$n = (0, 1, 0_{\perp})$$



$$f_{(1)} = P_R(x, k_{\perp}) - \delta(1-x) \delta(k_{\perp}) \int dx' dk'_{\perp} P_R(x', k'_{\perp})$$

where
$$P_R = \frac{\alpha_s C_F}{\pi^2} \left[\frac{1}{1-x} \frac{1}{k_{\perp}^2 + \rho^2} + \{\text{regular at } x \rightarrow 1\} \right]$$
 $\rho = \text{IR regulator}$

↑
endpoint singularity ($q^+ \rightarrow 0, \forall k_{\perp}$)

◇ Physical observables:

$$\begin{aligned} \mathcal{O} &= \int dx dk_{\perp} f_{(1)}(x, k_{\perp}) \varphi(x, k_{\perp}) \\ &= \int dx dk_{\perp} [\varphi(x, k_{\perp}) - \varphi(1, 0_{\perp})] P_R(x, k_{\perp}) \end{aligned}$$

inclusive case: φ independent of $k_{\perp} \Rightarrow 1/(1-x)_+$ from real + virtual

general case: endpoint divergences (incomplete KLN cancellation)

- Distributions at fixed k_{\perp} are no longer protected by KLN mechanism against uncancelled lightcone divergences
- Only after supplying matrix element with a regularization prescription is distribution well defined.
- Note: regularization of endpoint divergences also affects distributions integrated over k_{\perp} and UV subtractions

[H, hep-ph/0702196]

$$\text{Ex. : } \int dk_{\perp} f(x, k_{\perp}, \mu) \Theta(\mu - k_{\perp}) \stackrel{?}{=} f^{\overline{\text{MS}}}(x, \mu)$$

= holds **only at tree level**: full relation involves coefficient function R

$$\int^{\mu} dk_{\perp} f(x, k_{\perp}, \mu) = R(x) \otimes f^{\overline{\text{MS}}}(x, \mu)$$

◇ R calculable as a power series in α_s , $R(x) = \delta(1 - x) + \sum_k r_k \alpha_s^k$

- Applications: Cut-off regularization vs. Subtractive regularization

CUT-OFF REGULARIZATION

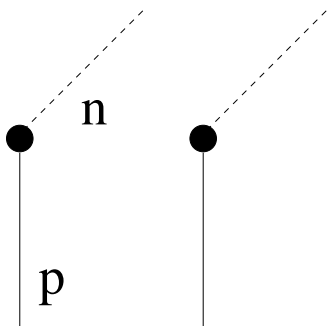
▷ cut-off in Monte-Carlo generators using u-pdf's

CASCADE www.desy.de/~jung/cascade

SMALLX Marchesini & Webber, 90's

LDCMC www.thep.lu.se/~leif/ariadne

▷ cut-off from gauge link in non-lightlike direction n :



$$\eta = (\mathbf{p} \cdot \mathbf{n})^2 / n^2$$

Collins, Rogers & Stasto, arXiv:0708.2833

Ji, Ma & Yuan, 2005, 2006

earlier work from 80's and 90's

finite $\eta \Rightarrow$ singularity is cut off at $1 - x \gtrsim \sqrt{k_{\perp}/4\eta}$

• Note: lightcone limits $y^2 \rightarrow 0$ and $n^2 \rightarrow 0$ do not commute \Rightarrow

$$\Rightarrow \int dk_{\perp} f(x, k_{\perp}, \mu, \eta) = F(x, \mu, \eta) \neq \text{ordinary pdf}$$

UPDF'S WITH SUBTRACTIVE REGULARIZATION

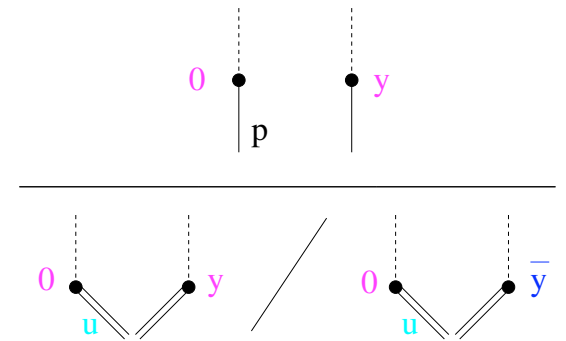
- Endpoint divergences $x \rightarrow 1$ from incomplete KLN cancellation

Subtractive method: more systematic than cut-off. Widely used in NLO calculations.

Formulation suitable for eikonal-operator matrix elements: Collins & H, 2001.

- gauge link still evaluated at n lightlike, but multiplied by “subtraction factors”

$$\tilde{f}^{(\text{subtr})}(y^-, y_\perp) = \frac{\overbrace{\langle P | \bar{\psi}(y) V_y^\dagger(n) \gamma^+ V_0(n) \psi(0) | P \rangle}^{\text{original matrix element}}}{\underbrace{\langle 0 | V_y(u) V_y^\dagger(n) V_0(n) V_0^\dagger(u) | 0 \rangle / \langle 0 | V_{\bar{y}}(u) V_{\bar{y}}^\dagger(n) V_0(n) V_0^\dagger(u) | 0 \rangle}_{\text{counterterms}}}$$



$\bar{y} = (0, y^-, 0_\perp)$; $u =$ auxiliary non-lightlike eikonal $(u^+, u^-, 0_\perp)$

H, arXiv:0708.1319

◇ u serves to regularize the endpoint; drops out of distribution integrated over k_\perp

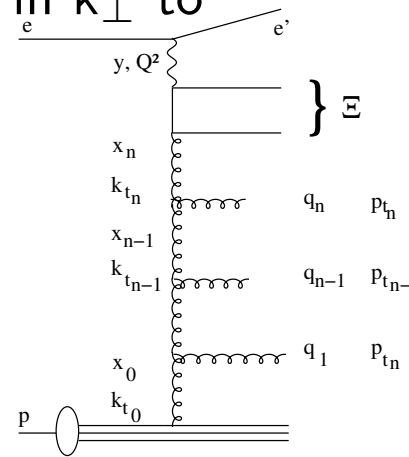
III. Jet correlations in small-x final states

◇ All MC's based on u-pdf's rely on factorization in k_{\perp} to

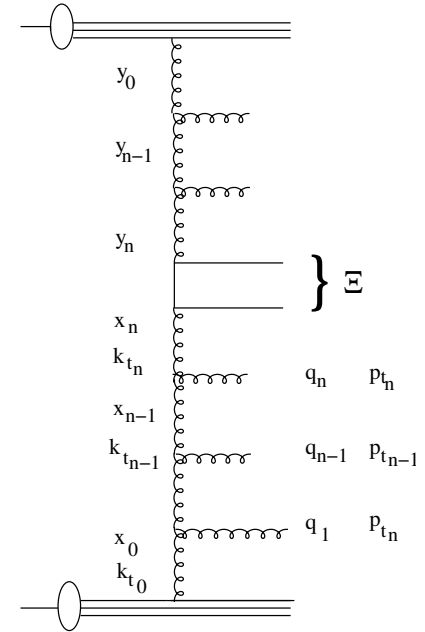
a) generate hard-scattering event (hard ME)

b) couple it to initial-state shower

◇ differ by detailed model for initial state



(a)



(b)

[reviewed in J.R. Andersen et al. (2006); B. Andersson et al. (2002)]

- all implement correct $\alpha_s^n x^{-1} \ln^{n-1} x$ behavior for spacelike evolution at $x \ll 1$ to all orders in α_s

$$\exp \int (dk^2/k^2) \gamma(\alpha_s(k^2))$$

- resum non-universal $\alpha_s^k \ln^k(s/p_{\perp}^2)$ (in certain cases)
- non-leading contributions possibly important for final states

Implementations:

Höche, Krauss and Teubner, arXiv:0705.4577	(KMR)
Golec, Jadach, Placzek, Stephens, Skrzypek, hep-ph/0703317	(CCFM)
LDCMC	Lönnblad & Sjö Dahl, 2005; Gustafson, Lönnblad & Miu, 2002 (LDC)
CASCADE	Jung, 2004, 2002; Jung and Salam, 2001 (CCFM)
SMALLX	Marchesini & Webber, 1992 (CCFM)

Advantages over standard Monte-Carlo like PYTHIA or HERWIG:

- better treatment of high-energy logarithmic effects
- likely more suitable for simulating underlying event's k_{\perp}

Current limitations:

- radiative terms associated to $x \sim 1$ not automatically included
- procedure to correct for this not yet systematic
 - ↔ e.g.: LO-DGLAP in Höche et al
- quark contributions in initial state included partially
 - ↔ see also: k_{\perp} kernel for sea-quark evolution [Catani & H]
- limited knowledge of u-pdf's [Jung et al., arXiv:0706.3793; J. R. Andersen et al., 2006]

Basic ingredients in the CASCADE Monte-Carlo

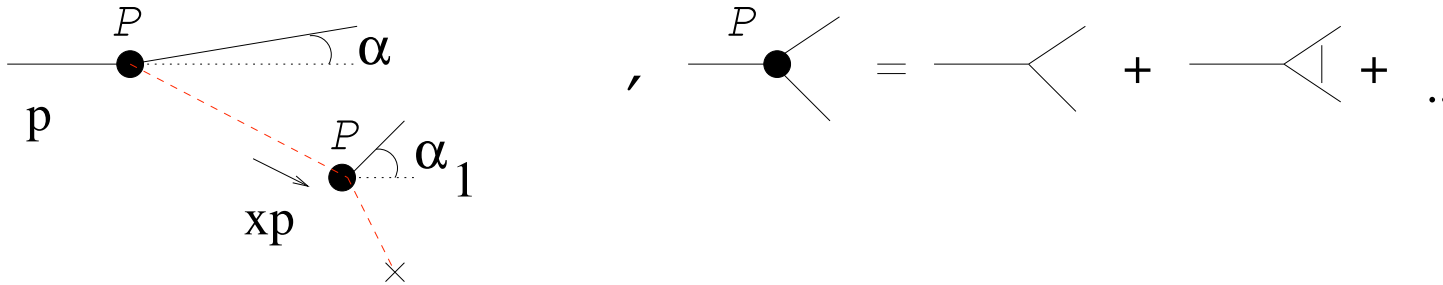
- ME by perturbative computation; u-pdf fit from data

Jung, Salam, Ciafaloni,

Catani, Hautmann, Marchesini, Webber, ...

• branching eq. : $\mathcal{A}(x, k_T, \mu) = \mathcal{A}_0(x, k_T, \mu) + \int \frac{dz}{z} \int \frac{dq^2}{q^2} \Theta(\mu - zq)$

$$\times \underbrace{\Delta(\mu, zq)}_{\text{Sudakov}} \underbrace{\mathcal{P}(z, q, k_T)}_{\text{unintegr. splitting}} \mathcal{A}\left(\frac{x}{z}, k_T + (1-z)q, q\right)$$



(left) Coherent radiation in the space-like parton shower for $x \ll 1$;

(right) the unintegrated splitting function \mathcal{P} , including small- x virtual corrections.

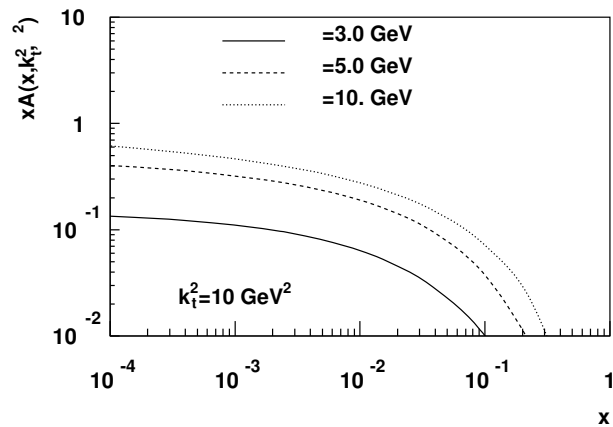
$$\alpha/x > \alpha_1 > \alpha \quad (\text{small-}x \text{ coherence region})$$

U-pdf fits \oplus evolution

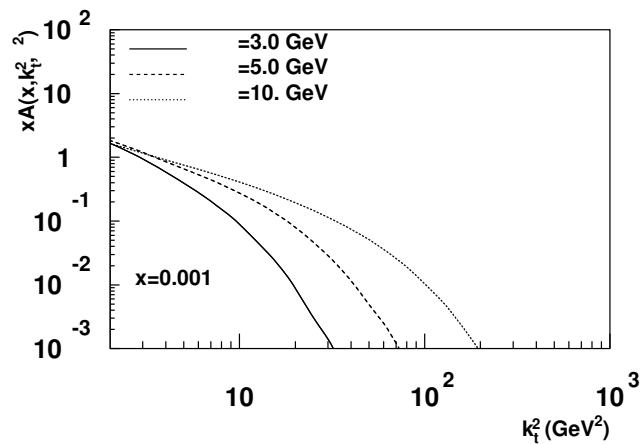
[Hansson & Jung, 2007]

[\leftrightarrow DIS, jets, heavy flavors]

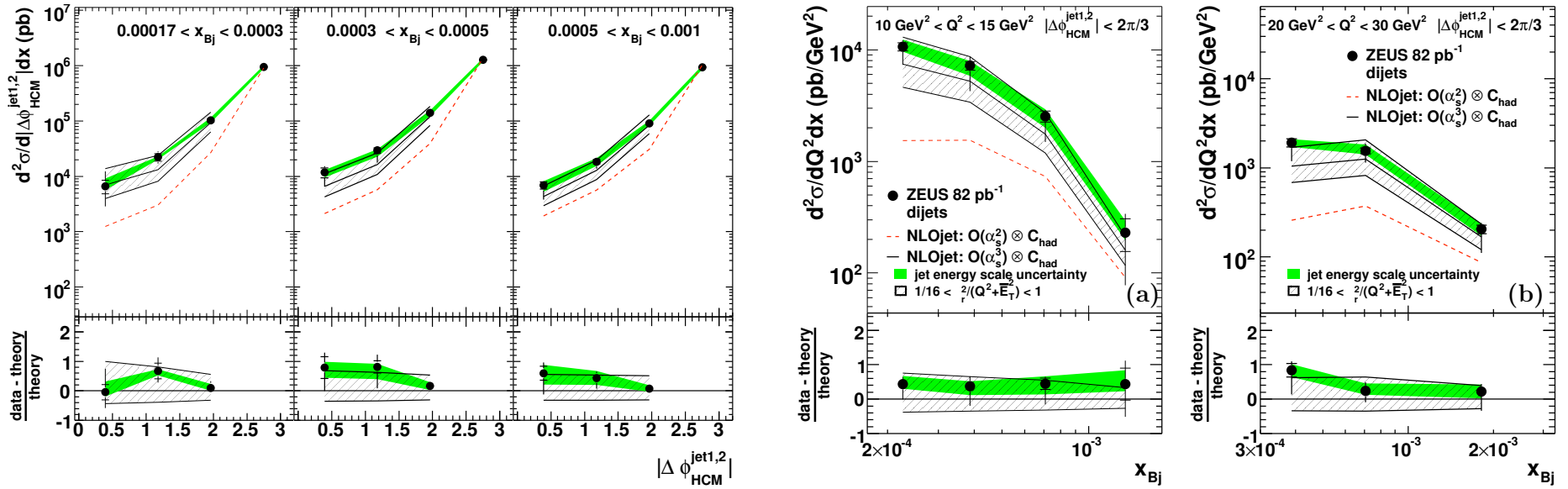
$$x\mathcal{A}_0(x, k_T, Q_0) = A x^{-B} (1-x)^C \exp\left[-(k_T - \lambda)^2/\nu^2\right]$$



$Q_0 = 1.1 \text{ GeV}$				
A	B	C	λ	ν
0.4695	0.025	4.0	1.5 GeV	1.06 GeV



MULTI-JET DIS PRODUCTION AND NLO RESULTS



(left) Azimuth dependence and (right) Bjorken- x dependence of di-jet distributions

- measured by ZEUS [arXiv:0705.1931]
- compared with NLO results [Z. Nagy and Z. Trocsanyi, PRL 87 (2001)]

$$Q^2 > 10 \text{ GeV}^2 \quad , \quad 10^{-4} < x < 10^{-2}$$

◇ large variation from order- α_s^2 to order- α_s^3 prediction as $\Delta\phi$ and x decrease
 \Rightarrow sizeable theory uncertainty at NLO (underestimated by “ μ error band”)

$$E_{T,HCM}^{\text{jet}-1} > 7 \text{ GeV} \quad , \quad E_{T,HCM}^{\text{jet}-2,3} > 5 \text{ GeV} \quad , \quad -1 < \eta_{lab} < 2.5$$

- Jet clustering and hadronization:

- ▷ moderate hadronization corrections from jet algorithm used by Zeus and H1
[arXiv:0705.1931 [hep-ex]; hep-ex/0310019]

- ▷ jet clustering free of non-global logarithms

- [Dasgupta et al., hep-ph/0610242]

- ▷ asymmetric jet cuts to avoid double logs in minimum p_T

- [Banfi and Dasgupta, hep-ph/0312108]

- ▷ nonperturbative corrections in inverse powers of Q moderate for $Q^2 > 10\text{GeV}^2$

- Radiative effects at higher order:

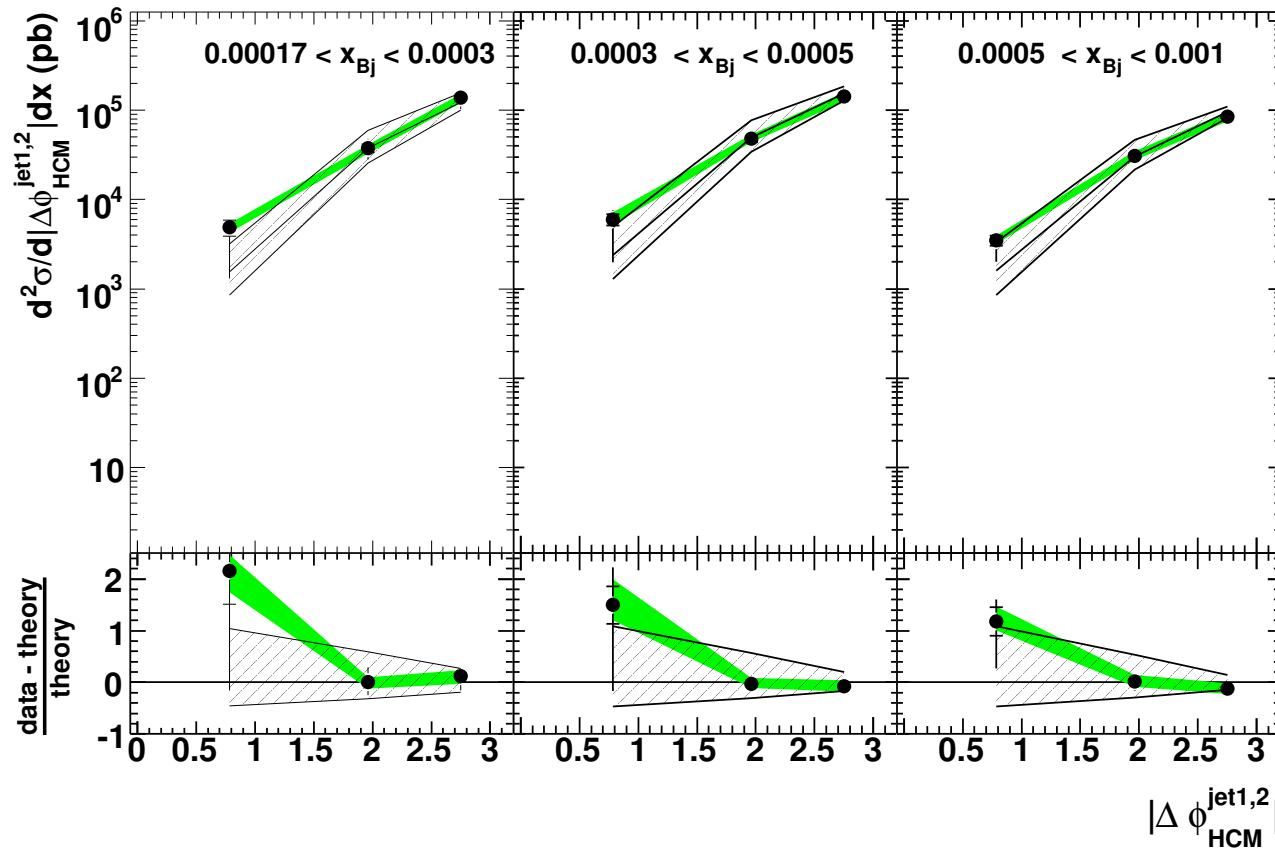
- ◇ fixed-order beyond NLO is outside present reach for multi-jets in ep and pp

- ◇ enhanced (soft/collinear) higher orders from near back-to-back region

- Y.Delenda et al., arXiv:0706.2172; arXiv:0804.3786; HERWIG

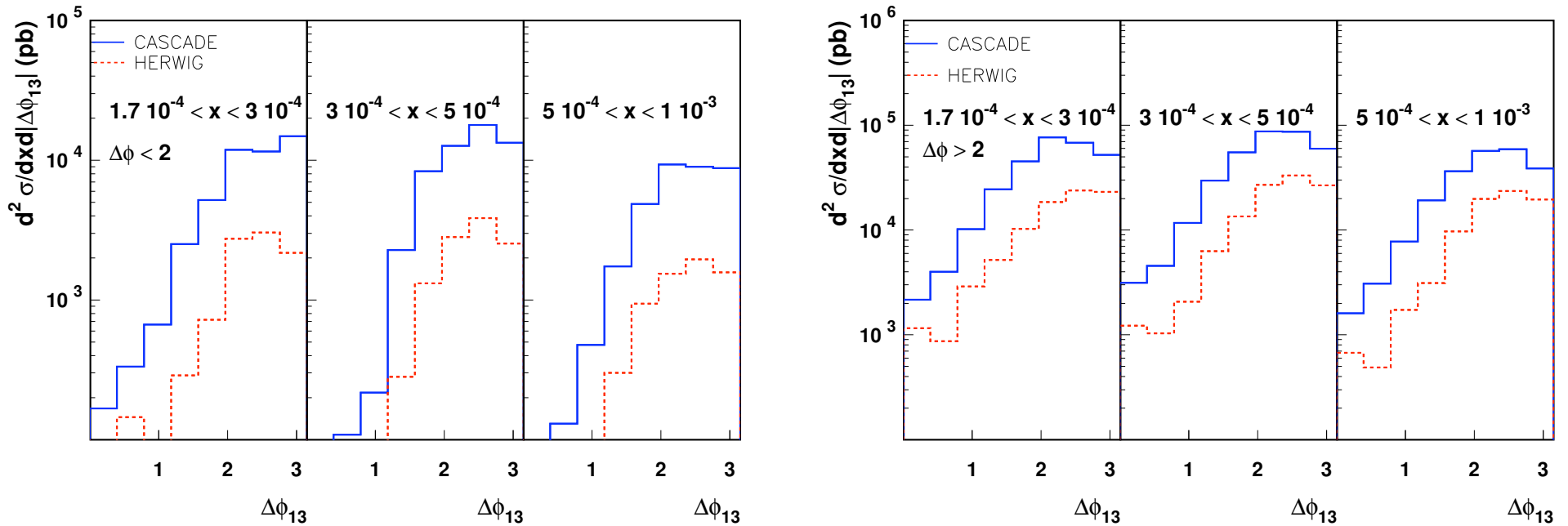
- ◇ largest effects seen at small $\Delta\phi$ (3 well-separated hard jets)

azimuthal distribution in 3-jet cross section [Zeus, 2007]



- besides angular correlations, sizeable NLO uncertainties in other associated distributions
- NLO results are much more stable for inclusive jet cross sections

AZIMUTHAL DISTRIBUTION OF THE THIRD JET

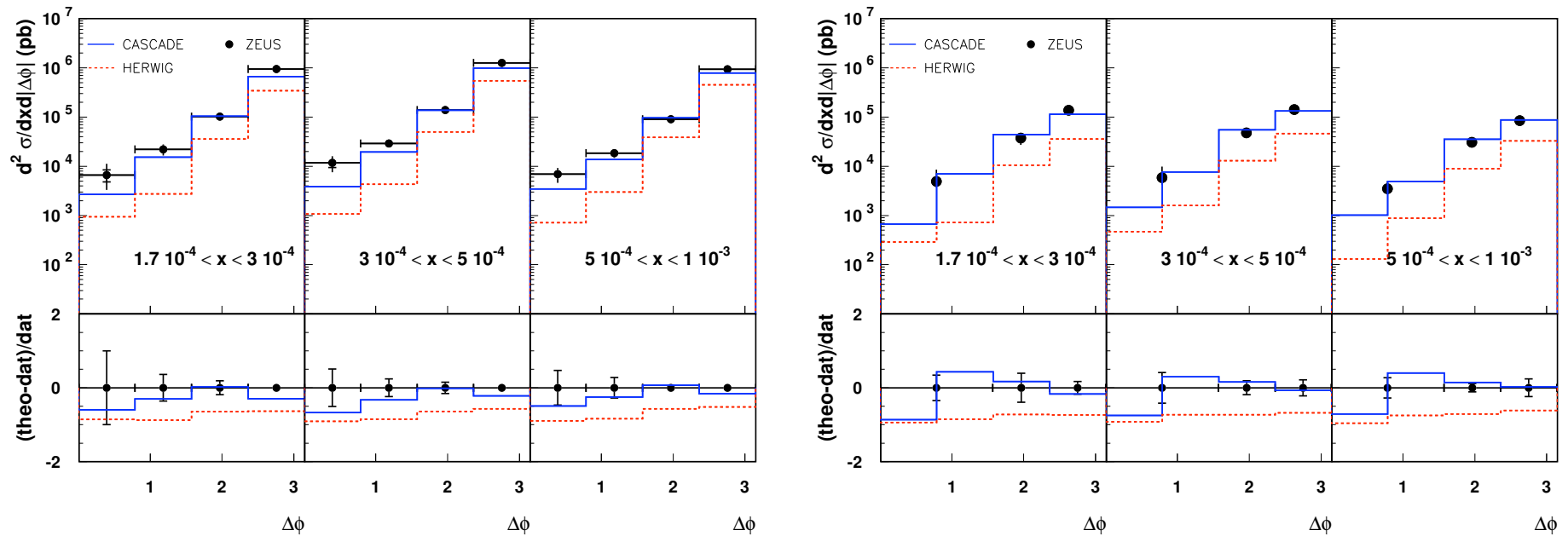


Cross section in the azimuthal angle between the hardest and the third jet for small (left) and large (right) azimuthal separations between the leading jets

Jung & H, arXiv:0712.0568 [hep-ph]

- small $\Delta\phi \Rightarrow$ non-negligible initial $k_{\perp} \Rightarrow$ larger corrections to collinear ordering
 - curves become closer at large $\Delta\phi$

Angular jet correlations from CASCADE and HERWIG compared with DIS data

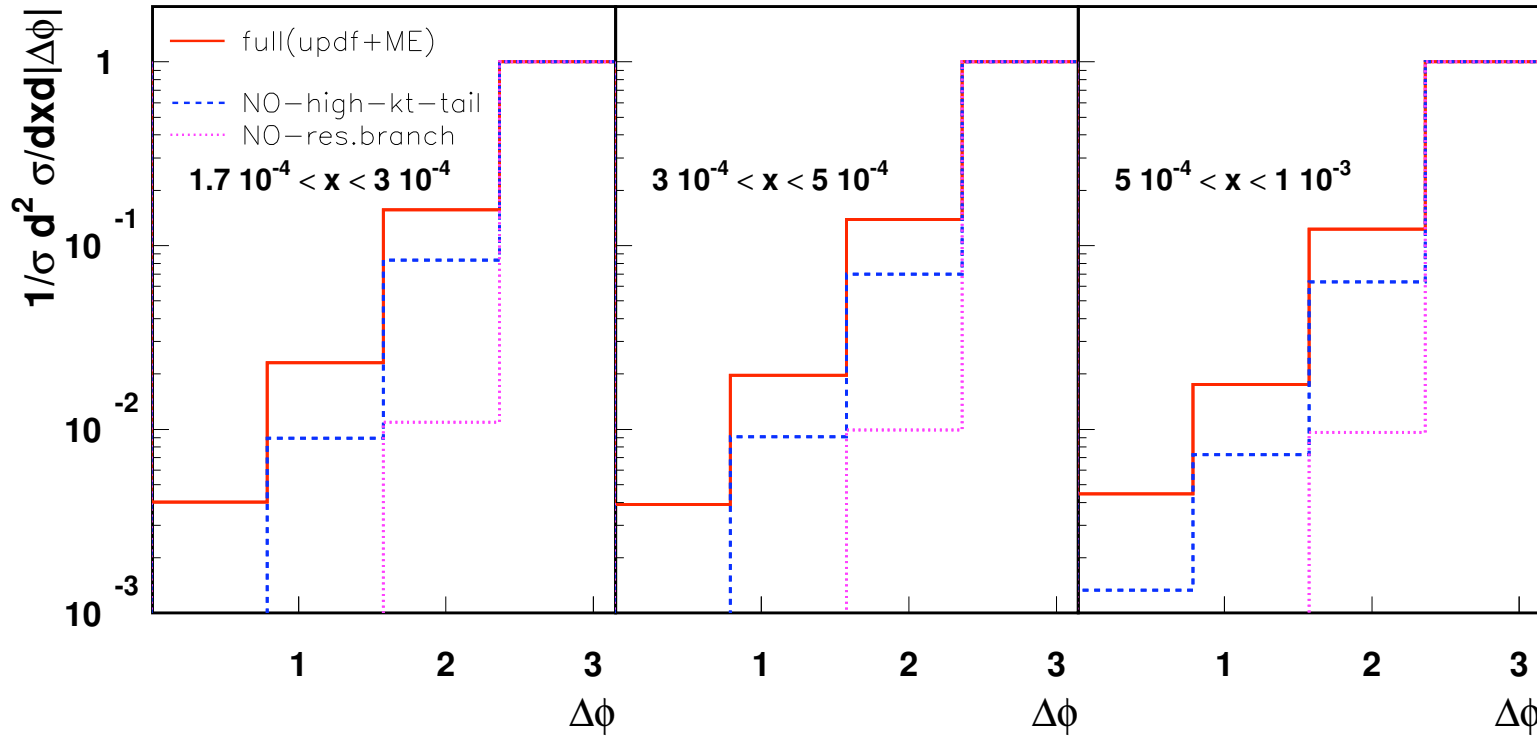


(left) di-jet cross section; (right) three-jet cross section

Jung & H, arXiv:0712.0568 [hep-ph]

- different shapes from the two MC
 - largest differences at small $\Delta\phi$
- good description of measurement by CASCADE

Normalize to the back-to-back cross section:



— updf \oplus ME

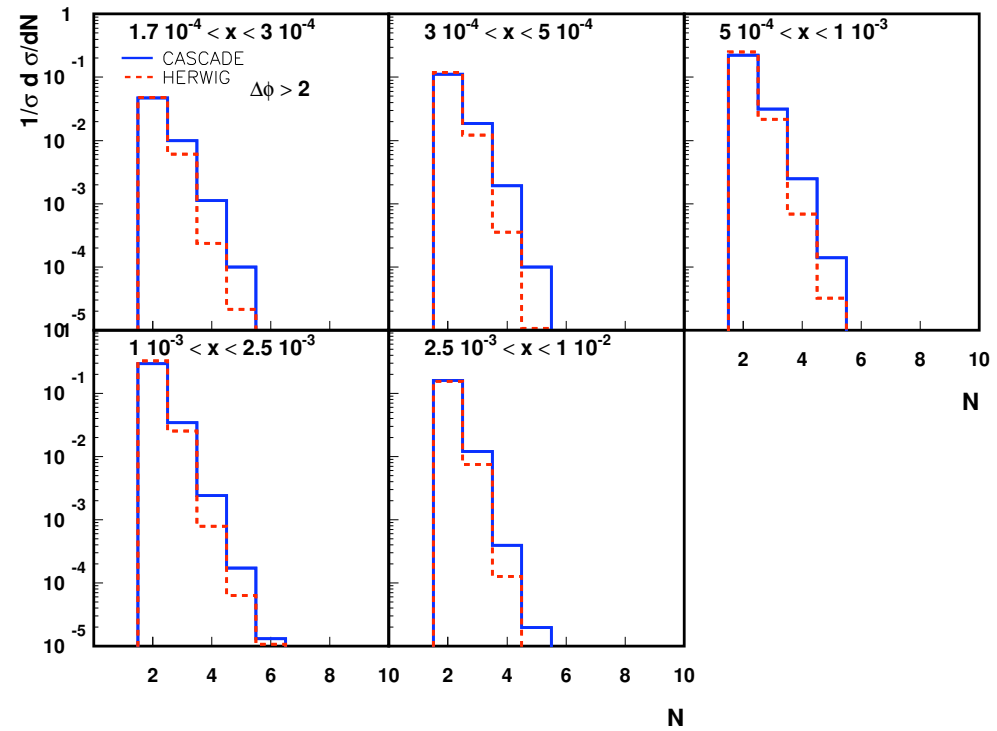
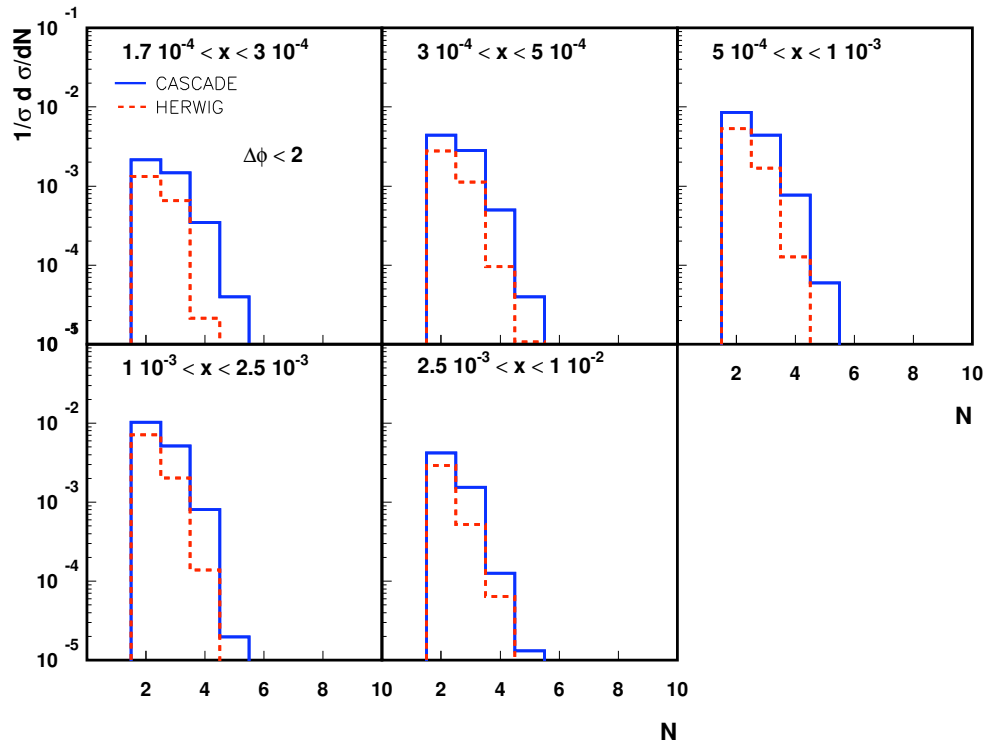
- - - updf \oplus ME_{collin.} : $\mathcal{M} \rightarrow \mathcal{M}_{collin.}(k_T) = \mathcal{M}(0_\perp) \Theta(\mu - k_T)$

..... no resolved branching : $\mathcal{A} \rightarrow \mathcal{A}_{no-res.}(x, k_T, \mu) = \mathcal{A}_0(x, k_T, Q_0) \Delta(\mu, Q_0)$

▷ high- k_\perp component in ME essential to describe correlation at small $\Delta\phi$

▷ k_\perp -dependence in u-pdf alone not sufficient

JET MULTIPLICITIES



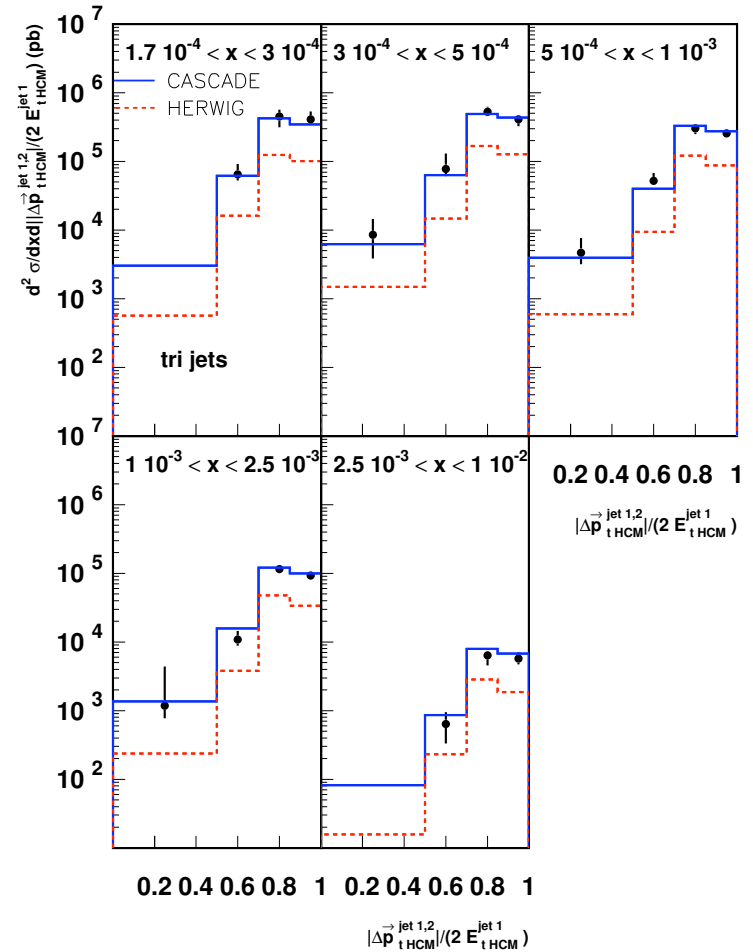
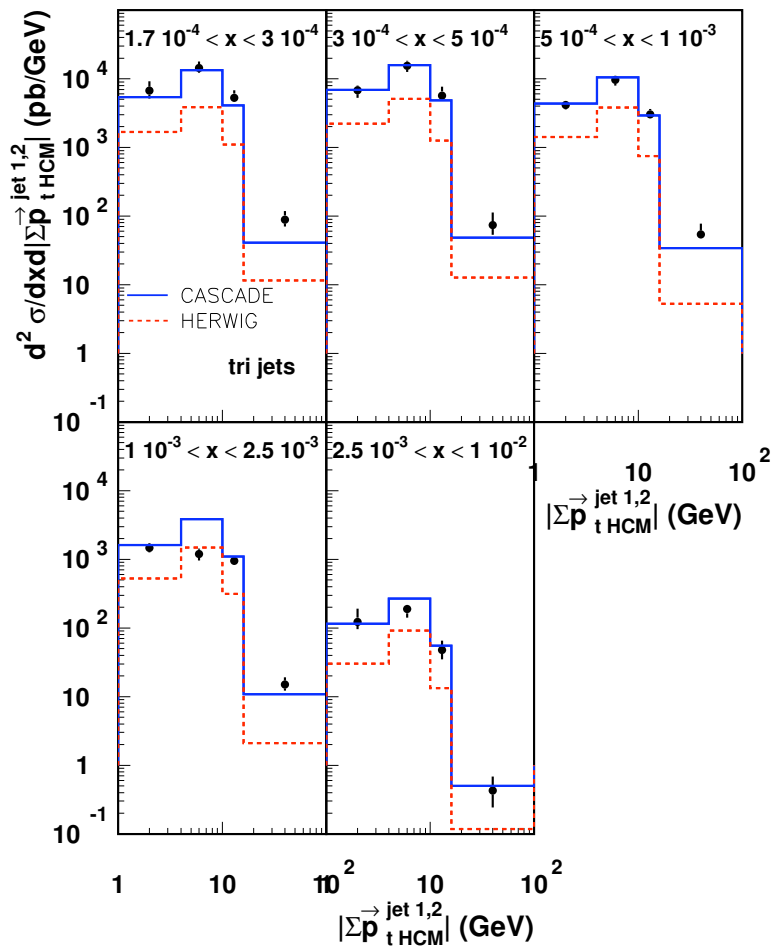
(left) $\Delta\phi < 2$; (right) $\Delta\phi > 2$

[Jung & H, arXiv:0805.1049]

- larger contribution from high multiplicity in the MC with u-pdf

MOMENTUM CORRELATIONS

[Jung & H, arXiv:0805.1049]



- correlations in the transverse momentum imbalance between the leading jets

Summary on DIS 3-jet

▷ U-pdfs \oplus k_{\perp} -dependent hard MEs describe multi-jet measurements including correlations.

▷ Physical picture: • growth of k_{\perp} along spacelike jet
• finite-angle emission corrections

▷ Furthermore:

- Results similar to HERWIG if reduced to k_{\perp} -ordered phase space
- Similar to fixed NLO where corrections are not large

▷ Non-forward jets \Rightarrow results less dependent on details of u-pdf evolution models

IV. Further developments and conclusions

◇ Forward-region observables

- More sensitive to u-pdf evolution
- better understanding of pdf needed for target fragmentation region
[Trentadue et al, 2007]

◇ Gluon fusion processes:

- production of b, c — large NLO uncertainties at LHC energies
[MC@NLO; Nason et al.]
→ large corrections from $g \rightarrow b\bar{b}$ coupling to spacelike jet
- final states with Higgs
→ possibly 10 ÷ 20 % effects in p_t spectrum from $x \ll 1$ terms?
[Kulesza, Sterman & Vogelsang, 2004]

ISSUES AT HIGHER ORDER

- u-pdf defined gauge-invariantly for small x by high-energy factorization
 - general definitions including $x \sim 1$?

Collins, Rogers and Stasto, arXiv:0708.2833
H, PLB 655 (2007) [hep-ph/0702196]

- dijet back-to-back region

→ possible factorization-breaking from soft gluon exchange in pp?

Mulders, Bomhof, Collins,
Vogelsang, Qiu, Yuan, Pijlman, ...
2006-2008

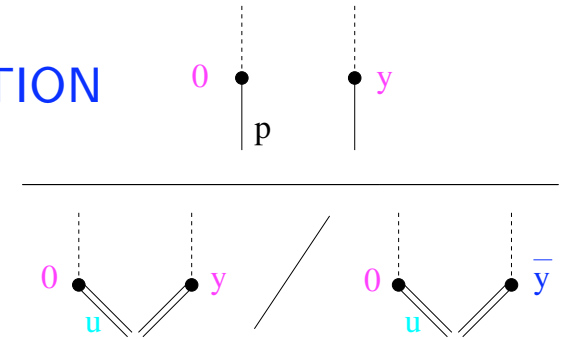
◇ appears at $N^3\text{LO}$ (2 soft, 1 collinear partons)

◇ does it survive destructive interference from multiple emission?

- Note: Coulomb/radiative mixing terms also appear to break coherence in di-jet cross sections with gap in rapidity

Forshaw, Kyrielleis & Seymour, JHEP 08 (2006)

MORE ON U-PDF'S WITH SUBTRACTIVE REGULARIZATION



One loop expansion:

$$f_{(1)}^{(\text{subtr})}(x, k_{\perp}) = P_R(x, k_{\perp}) - \delta(1-x) \delta(k_{\perp}) \int dx' dk'_{\perp} P_R(x', k'_{\perp}) \quad (\leftarrow \text{from numerator})$$

$$- W_R(x, k_{\perp}, \zeta) + \delta(k_{\perp}) \int dk'_{\perp} W_R(x, k'_{\perp}, \zeta) \quad (\leftarrow \text{from vev's})$$

with $P_R = \alpha_s C_F / \pi^2 \left\{ 1 / [(1-x) (k_{\perp}^2 + m^2(1-x)^2)] + \dots \right\} = \text{real emission prob.}$

$W_R = \alpha_s C_F / \pi^2 \left\{ 1 / [(1-x) (k_{\perp}^2 + 4\zeta(1-x)^2)] + \dots \right\} = \text{counterterm}$

- ζ -dependence cancels upon integration in k_{\perp} [$\zeta = (p^{+2}/2)u^-/u^+$]

$$\Rightarrow \mathcal{O} = \int dx dk_{\perp} f_{(1)}^{(\text{subtr})}(x, k_{\perp}) \varphi(x, k_{\perp})$$

$$= \int dx dk_{\perp} \{ P_R [\varphi(x, 0_{\perp}) - \varphi(1, 0_{\perp})] + (P_R - W_R) [\varphi(x, k_{\perp}) - \varphi(x, 0_{\perp})] \}$$

- first term: usual $1/(1-x)_+$ distribution
- second term: singularity in P_R cancelled by W_R

Note: it works because terms in $\delta(1-x)$ cancel between the two vev's,

$$-W_R(x, k_\perp, \zeta) + \delta(1-x) \delta(k_\perp) \int dx' dk'_\perp W_R$$

and

$$+\delta(k_\perp) \int dk'_\perp W_R(x, k'_\perp, \zeta) - \delta(1-x) \delta(k_\perp) \int dx' dk'_\perp W_R .$$

- virtual correction to gauge link does not depend on y_\perp

G. Korchemsky

V. Braun et al

- ▷ subtractions have (relatively) simple form in coordinate space
- ▷ operator representation valid to all orders
- ▷ one-loop counterterm gives extension for $k_\perp \neq 0$ of the plus-distribution regularization

Conclusions

- Branching methods based on k_{\perp} -dependent u-pdfs and MEs useful for simulation of high-energy parton showers
 - ▷ k_{\perp} shower Monte-Carlo gives good description of small- x multi-jet final states
- Extension of u-pdf's over whole phase space important to turn these Monte-Carlo's into general-purpose tools
 - ▷ special issues at $x \sim 1$ (and matching with small x) relevant for showering algorithm