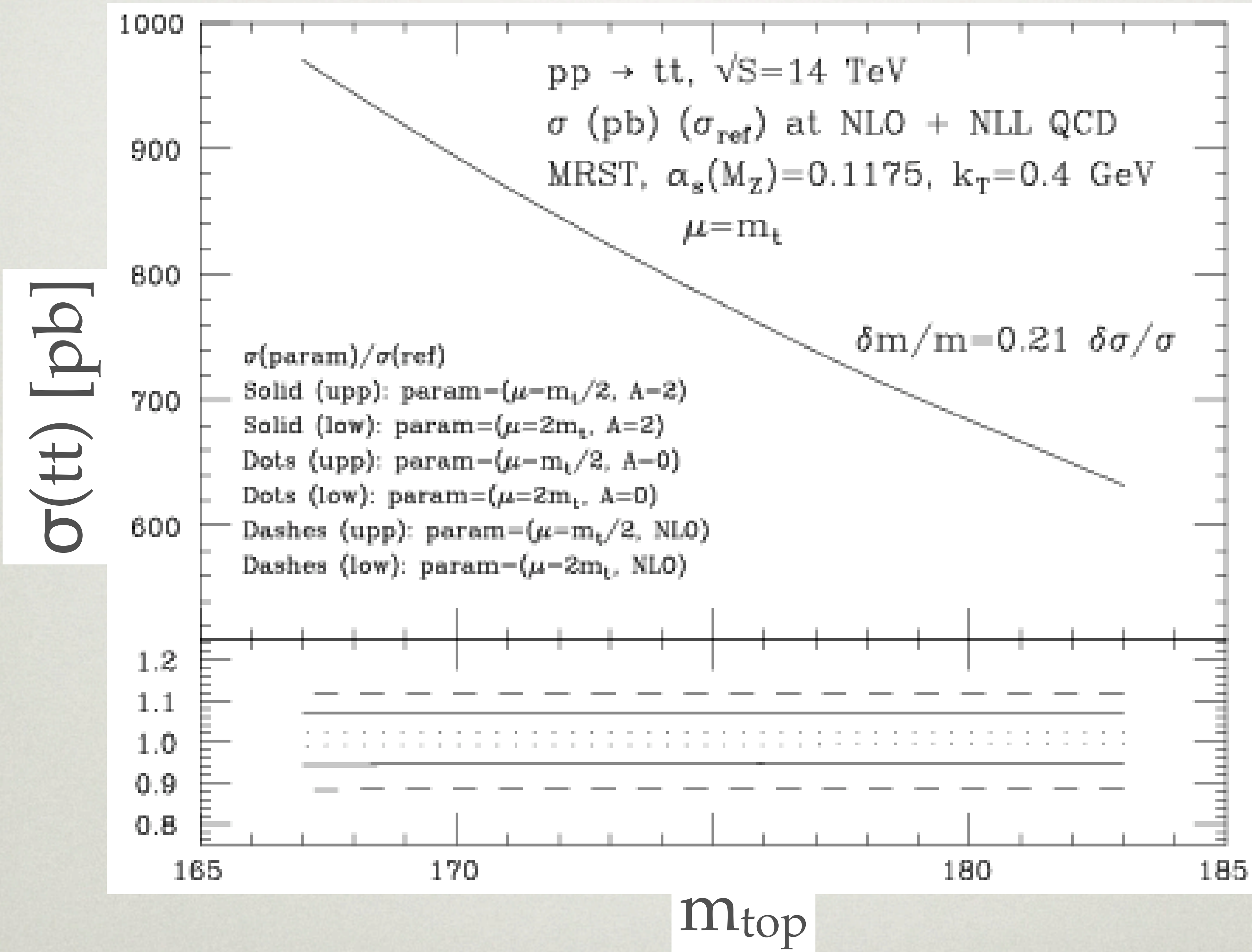
$t\bar{t}$ Cross section in $p\overset{(-)}{p}$ collisions

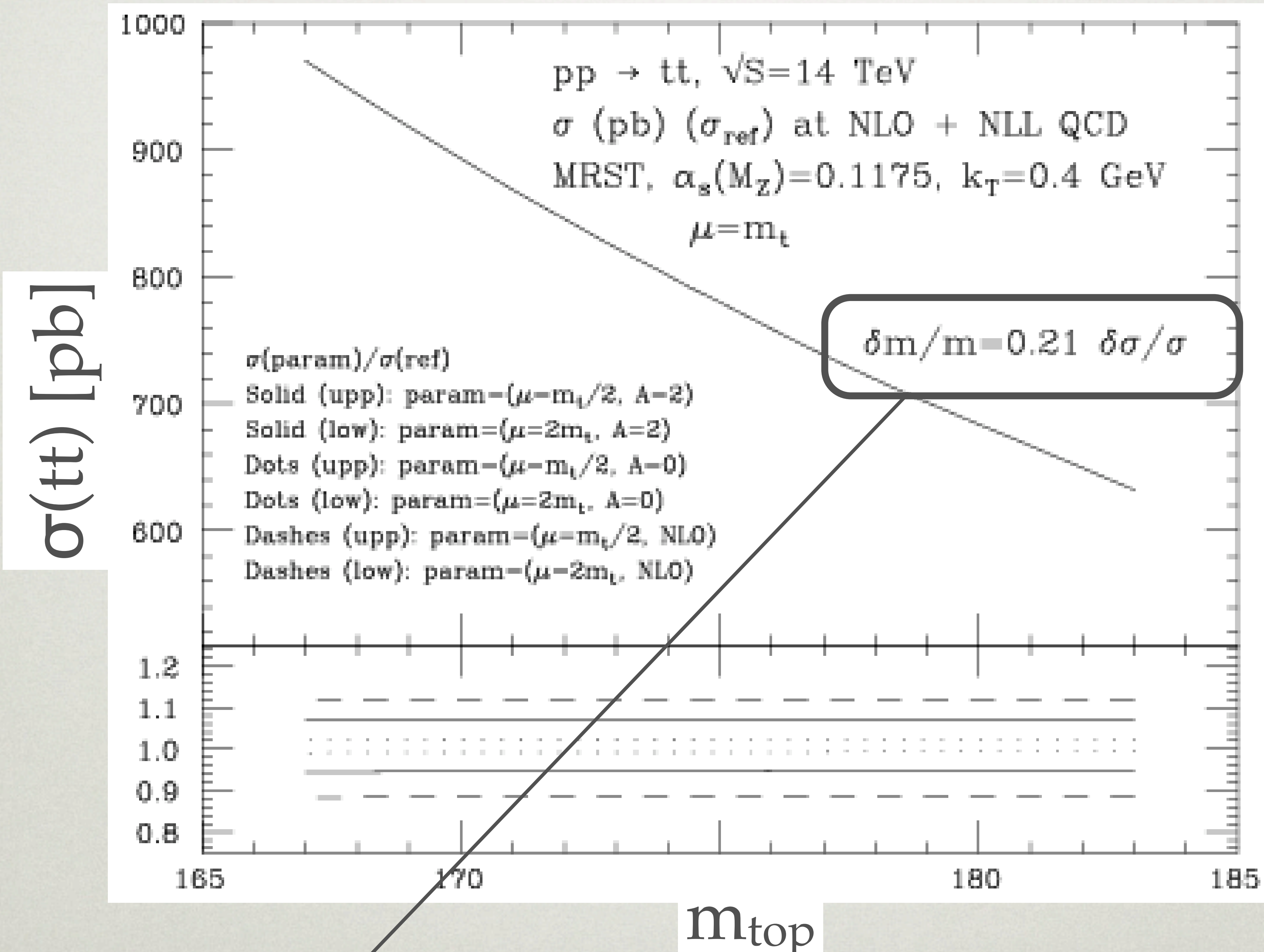
Michelangelo MANGANO
CERN, TH/PH

σ(tt) vs M_{top}



pp → tt, √S=14 TeV
σ (pb) (σ_{ref}) at NLO + NLL QCD
MRST, $\alpha_s(M_Z)$=0.1175, k_T=0.4 GeV
$\mu=m_t$
δm/m=0.21 δσ/σ
σ(param)/σ(ref)
Solid (upp): param=($\mu=m_t/2$, A=2)
Solid (low): param=($\mu=2m_t$, A=2)
Dots (upp): param=($\mu=m_t/2$, A=0)
Dots (low): param=($\mu=2m_t$, A=0)
Dashes (upp): param=($\mu=m_t/2$, NLO)
Dashes (low): param=($\mu=2m_t$, NLO)
1000
900
800
700
600
1.2
1.1
1.0
0.9
0.8
165
170
180
185
σ(tt) [pb]
m_{top}

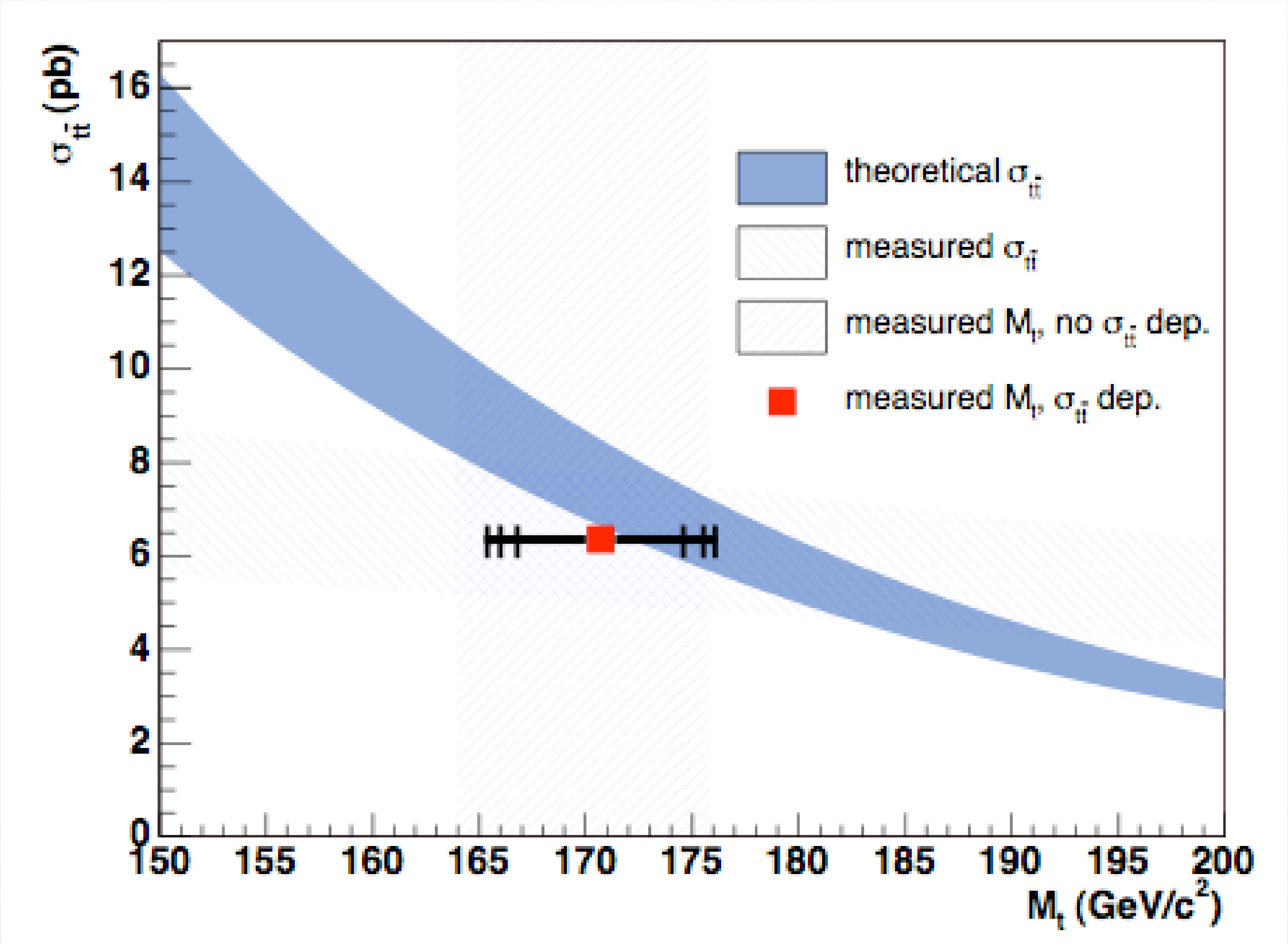
σ(tt) vs M_{top}



Δσ/σ= ± 5% ⇔ Δm/m= ± 1% ≲ 2 GeV, comparable to Δm_{direct}

Example: CDF

CDF, "Cross-section constrained top quark mass measurement from dilepton events at the Tevatron", Phys.Rev.Lett.100:062005,2008.



$$m_{top} = 178.3\,^{+10.1}_{-8.0}\ (\text{exp})\ ^{+4.0}_{-5.8}\ (\text{Theory})\ \text{GeV}$$

Where do we stand

RECENT PROGRESS AT NLO

- **σ(tt):**
  - **Approx. NNLO:** Moch and Uwer, arXiv/08041476
    - Scale-dep, Coulomb and all powers of logβ_t at 2-loops
  - **NLL resum, update:** Cacciari, Frixione, Mangano, Nason, Ridolfi, arXiv/08042800
  - Kidonakis, Vogt, arXiv/0805.3844
- Spin correlations at NLO

W. Bernreuther, A. Brandenburg, Z. G. Si and P. Uwer, Phys. Rev. Lett. **87** (2001) 242002 [arXiv:hep-ph/0107086].

W. Bernreuther, A. Brandenburg, Z. G. Si and P. Uwer, Nucl. Phys. B **690** (2004) 81 [arXiv:hep-ph/0403035].

- NLO corrections to σ(tt+jet)

S. Dittmaier, P. Uwer and S. Weinzierl, Phys. Rev. Lett. 98 (2007) 262002

- EW effects in σ(tt)

Electroweak effects in top-quark pair production at hadron colliders.
J.H. Kuhn, A. Scharf (Karlsruhe U., TTP) , P. Uwer (CERN) . CERN-PH-TH-
Published in **Eur.Phys.J.C51:37-53,2007**.

17) **Weak corrections to gluon-induced top-antitop hadro-production.**
S. Moretti, M.R. Nolten, D.A. Ross (Southampton U.) . SHEP-06-04, Mar 2006.
Published in **Phys.Lett.B639:513-519,2006, Erratum-ibid.B660:607-609,2008**

6) **NLO QED contributions to top-pair production at hadron collider.**
Wolfgang Hollik, Monika Kollar (Munich, Max Planck Inst.) . MPP-2007-106,
Published in **Phys.Rev.D77:014008,2008**.

16) **Weak interaction corrections to hadronic top quark pair production.**
Werner Bernreuther, Michael Fuecker (Aachen, Tech. Hochsch.) , Zong-Guo Si (Shandong U.)
Published in **Phys.Rev.D74:113005,2006**.

FOR A RECENT REVIEW, SEE ALSO: BERNREUTHER, ARXIV/08051333

Moch&Uwer vs Cacciari et al: LHC

$m_{top} = 171$ GeV

CTEQ6.5

M&U $\sigma = 918\ ^{-9(1.0\%)}_{-39(4.2\%)}\ \text{(scales)}\ ^{+30(3.3\%)}_{-30(3.3\%)}\ \text{(PDFs) pb}$

C&al $\sigma = 908\ ^{+82(9.0\%)}_{-85(9.3\%)}\ \text{(scales)}\ ^{+30(3.3\%)}_{-29(3.2\%)}\ \text{(PDFs) pb}$

MRSTW-06

M&U $\sigma = 969\ ^{-13(1.3\%)}_{-39(4.0\%)}\ \text{(scales)}\ ^{+11(1.1\%)}_{-11(1.1\%)}\ \text{(PDFs) pb}$

C&al $\sigma = 961\ ^{+89(9.2\%)}_{-91(9.4\%)}\ \text{(scales)}\ ^{+11(1.1\%)}_{-12(1.2\%)}\ \text{(PDFs) pb}$

- Central values within 1%
- PDF uncertainty results agree, and confirm that δ_{PDF} is underestimated
- NNLL scale uncertainty smaller than NLL?

Moch&Uwer vs Cacciari et al: TeV

$m_{top} = 171$ GeV

CTEQ6.5

M&U $\sigma = 7.93\ ^{+0.06(1.0\%)}_{-0.28(3.5\%)}\ \text{(scales)}\ ^{+0.44(5.5\%)}_{-0.45(5.5\%)}\ \text{(PDFs) pb}$

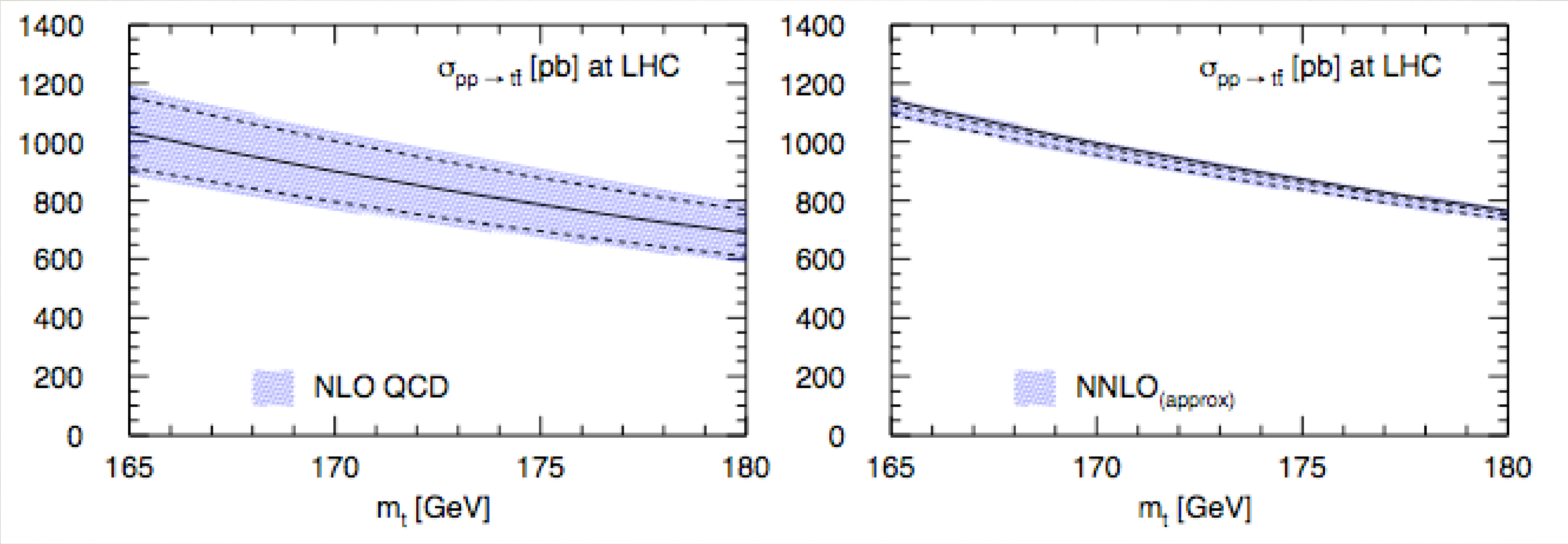
C&al $\sigma = 7.61\ ^{+0.38(5.1\%)}_{-0.80(10.9\%)}\ \text{(scales)}\ ^{+0.49(6.6\%)}_{-0.34(4.6\%)}\ \text{(PDFs) pb}$

MRSTW-06

M&U $\sigma = 8.23\ ^{+0.08(1.0\%)}_{-0.33(4.0\%)}\ \text{(scales)}\ ^{+0.21(2.6\%)}_{-0.23(2.8\%)}\ \text{(PDFs) pb}$

C&al $\sigma = 7.93\ ^{+0.34(4.3\%)}_{-0.56(7.1\%)}\ \text{(scales)}\ ^{+0.24(3.1\%)}_{-0.20(2.5\%)}\ \text{(PDFs) pb}\,.$

Scale dependence



Moch and Uwer, arXiv/08041476

Scale dependence criteria:

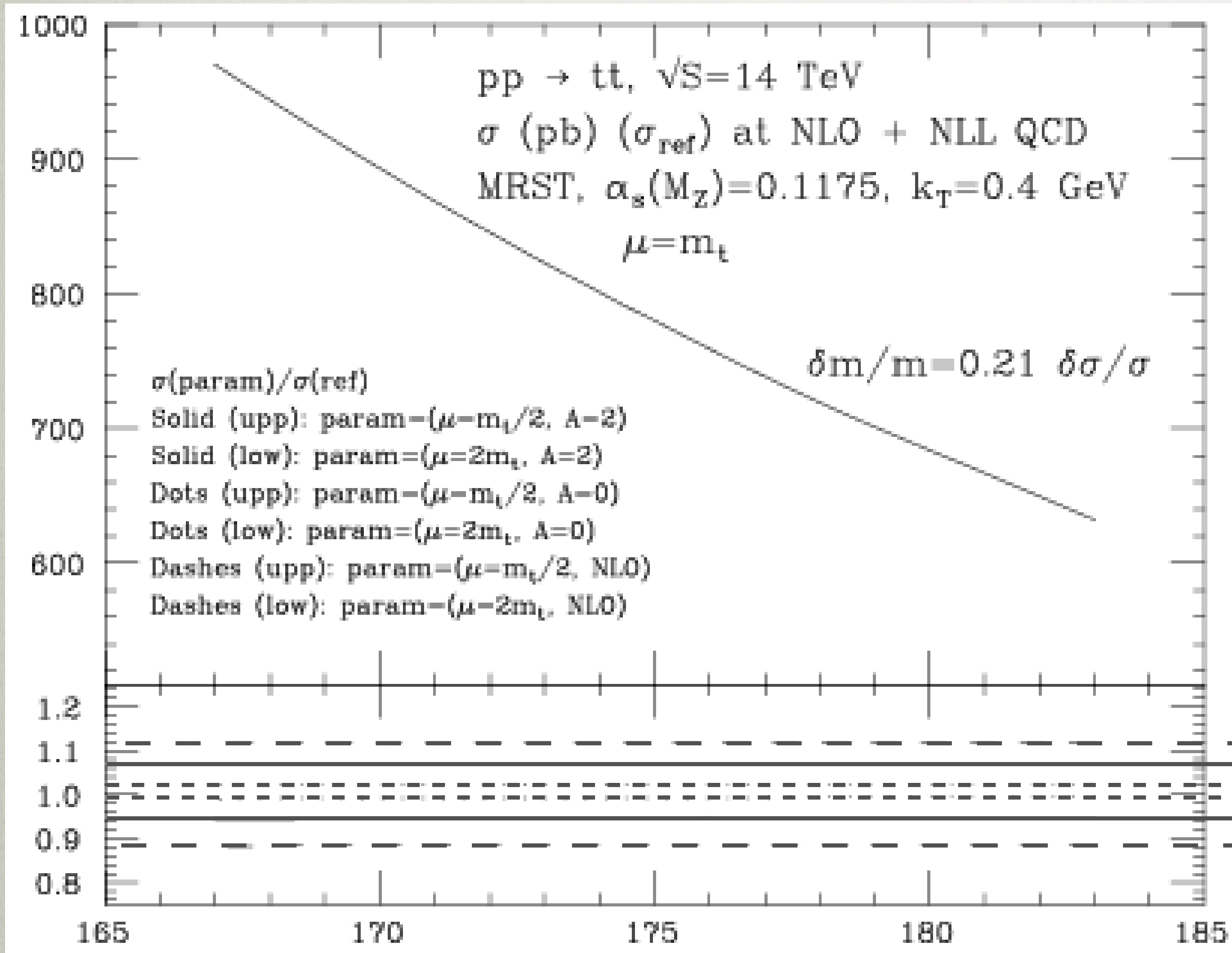
Moch and Uwer:

$\mu_R = \mu_F = \mu$ $\mu_0/2 < \mu < 2\,\mu_0$ $\mu_0 = m_{top}$

Cacciari et al.

$\mu_R \neq \mu_F$ $1/2 < \mu_F / \mu_R < 2$ $\mu_0/2 < \mu_{R,F} < 2\,\mu_0$ $\mu_0 = m_{top}$

... ON THE OTHER HAND ...



Bonciani et al 1998

$\Delta_2 = \Delta_{NLO} \sim 12\%$

$\Delta_1 = \Delta_{NLL}$ [**A=2**, $\mu_R = \mu_F$] $\sim 7\%$ “Conservative” NLL scale uncertainty

$\Delta_0 = \Delta_{NLL}$ [**A=0**, $\mu_R = \mu_F$] $\sim 2\%$ “Aggressive” NLL scale uncertainty

NLL RESUMMATION

$$\sigma^{(\mathrm{res})}(\rho, m^2) = \frac{1}{2\pi i}\int_{C_{\mathrm{MP}}-i\infty}^{C_{\mathrm{MP}}+i\infty} dN\, \rho^{-N} \sigma_N^{(\mathrm{res})}(m^2)\,.$$

$$\sigma_N^{(res)}(m^2) = \sum_{ij=q\bar{q},gg} F_{i,N+1}(\mu^2)\, F_{j,N+1}(\mu^2) \left[\hat{\sigma}_{ij,N}^{(res)}(m^2, \alpha_s(\mu^2), \mu^2) - \left(\hat{\sigma}_{ij,N}^{(res)}(m^2, \alpha_s(\mu^2), \mu^2)\right)_{\alpha_s^3}\right] + \sigma_N^{(NLO)}(m^2)\ , \qquad (61)$$

$$\hat{\sigma}_{ij,N}^{(res)}(m^2, \alpha_s(\mu^2), \mu^2) = \frac{\alpha_s^2(\mu^2)}{m^2} \sum_{\mathrm{I}=1,8} f_{ij,\mathrm{I},N}^{(res)}(\alpha_s(\mu^2), \mu^2/m^2)\ ,$$

$$f_{ij,\mathrm{I},N}^{(res)}(\alpha_s(\mu^2), \mu^2/m^2) = f_{ij,\mathrm{I},N}^{(corr)}(\alpha_s(\mu^2), \mu^2/m^2)\, \Delta_{ij,\mathrm{I},N+1}\left(\alpha_s(\mu^2), \frac{\mu^2}{m^2}\right)$$

$$f_{gg,N}^{(1)} + \overline{f}_{gg,N}^{(1)} \ln\frac{\mu^2}{m^2} = \frac{1}{4\pi^2} f_{gg,N}^{(0)} \left\{2C_A \ln^2 N + \left[\frac{N_c^2-4}{N_c^2-2}C_A + 4C_A\gamma_E + 2C_A \ln\frac{\mu^2}{4m^2}\right]\ln N + C_{gg}(\mu^2/m^2) + \mathcal{O}\left(\frac{1}{N}\right)\right\} + f_{gg,\mathbf{1},N}^{(1),\mathrm{Coul}} + f_{gg,\mathbf{8},N}^{(1),\mathrm{Coul}}\ ,$$

$$C_{gg}\left(\frac{\mu^2}{m^2}\right) = \overline{C}_3\left(\frac{\mu^2}{m^2}\right) + 2C_A\left[\ln^2 2 + (\gamma_E - 2)\ln\frac{\mu^2}{4m^2} + \frac{\pi^2}{2} + \gamma_E(\gamma_E - 4)\right] + \frac{C_A(9N_c^2 - 20)}{N_c^2 - 2}(\gamma_E - 2 - \ln 2)\ .$$

parameter **A**:

$$C_{ij} \to C_{ij}\left(1 - \frac{A}{N+A-1}\right), \quad ij = q\bar{q}, gg\,.$$

- A~1/N
- Vanishing 1st moment
- Non-log terms at order α_s^4 and higher

A and $\mu_R \neq \mu_F$ systematics at NLL

A=2

$\mu_R \neq \mu_F$ $\quad 1/2 < \mu_F / \mu_R < 2 \quad \mu_0/2 < \mu_{R,F} < 2\,\mu_0 \quad \mu_0 = m_{top}$

$$\sigma = 908\ ^{+82(9.0\%)}_{-85(9.3\%)}\ \text{(scales) pb}$$

(Cacciari et al default scale dependence)

A=0

$\mu_R \neq \mu_F$ $\quad 1/2 < \mu_F / \mu_R < 2 \quad \mu_0/2 < \mu_{R,F} < 2\,\mu_0 \quad \mu_0 = m_{top}$

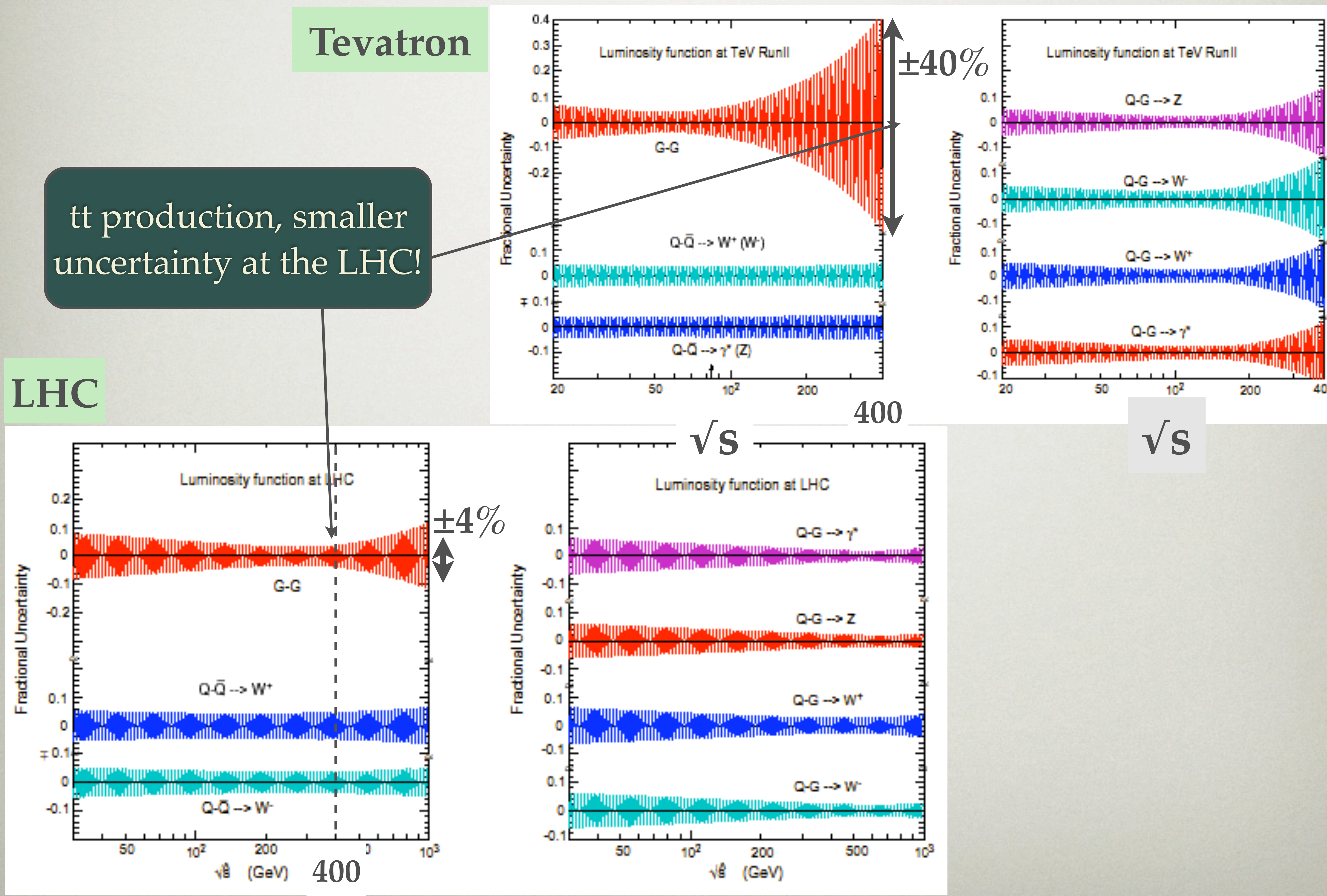
$$\sigma = 945\ ^{+95(10\%)}_{-85(9.0\%)}\ \text{(scales) pb}$$

A=0

$\mu_R = \mu_F$ $\quad \mu_0/2 < \mu < 2\,\mu_0 \quad \mu_0 = m_{top}$

$$\sigma = 945\ ^{+19(2\%)}_{-7(0.7\%)}\ \text{(scales) pb}$$

PDF Uncertainties



PDF Uncertainties

m=171	gg	qq	tot
σ_{TeV}(pb)	1.1	6.3	7.3
%	15%	85%	
σ_{LHC}(pb)	804	84	898
%	90%	9%	

Tevatron

$$\delta_{PDF} \sim \%[\sigma_{gg}] \times 40\% \oplus \%[\sigma_{qq}] \times 4\% \sim \pm 7\%$$

LHC

$$\delta_{PDF} \sim \%[\sigma_{gg}] \times 4\% \oplus \%[\sigma_{qq}] \times 4\% \sim \pm 4\%$$

PDF Uncertainties

Tevatron

CTEQ6.5 $\sigma = 7.61\ ^{+0.38(5.1\%)}_{-0.80(10.9\%)}\ (\text{scales})\ ^{+0.49(6.6\%)}_{-0.34(4.6\%)}\ (\text{PDFs})\ \text{pb}$

MRSTW-06 $\sigma = 7.93\ ^{+0.34(4.3\%)}_{-0.56(7.1\%)}\ (\text{scales})\ ^{+0.24(3.1\%)}_{-0.20(2.5\%)}\ (\text{PDFs})\ \text{pb}\,.$

MRST–CTEQ = 0.32 ± 0.45 pb

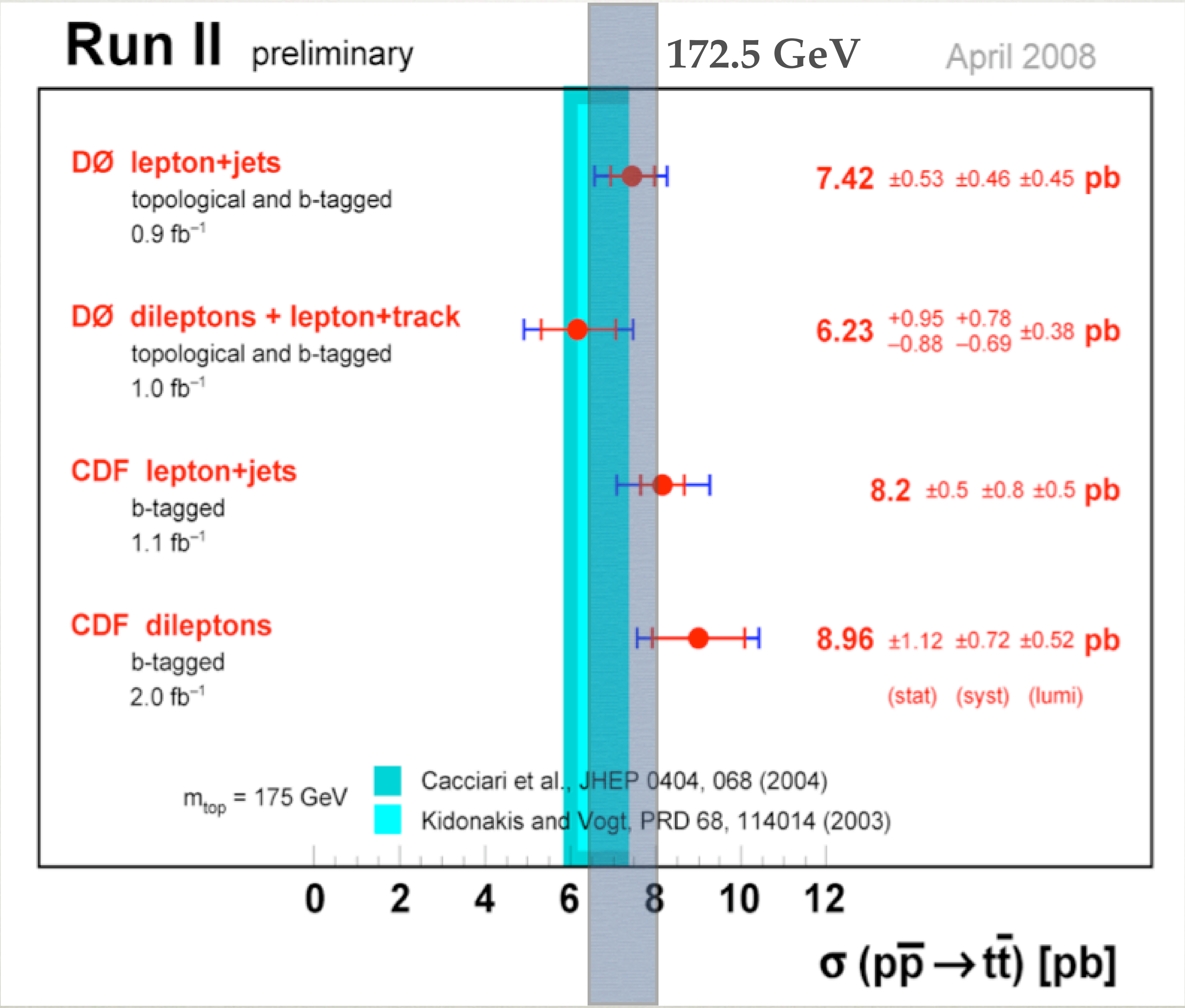
LHC

CTEQ6.5 $\sigma = 908\ ^{+82(9.0\%)}_{-85(9.3\%)}\ (\text{scales})\ ^{+30(3.3\%)}_{-29(3.2\%)}\ (\text{PDFs})\ \text{pb}$

MRSTW-06 $\sigma = 961\ ^{+89(9.2\%)}_{-91(9.4\%)}\ (\text{scales})\ ^{+11(1.1\%)}_{-12(1.2\%)}\ (\text{PDFs})\ \text{pb}$

MRST–CTEQ = 53 ± 33 pb

Tevatron results



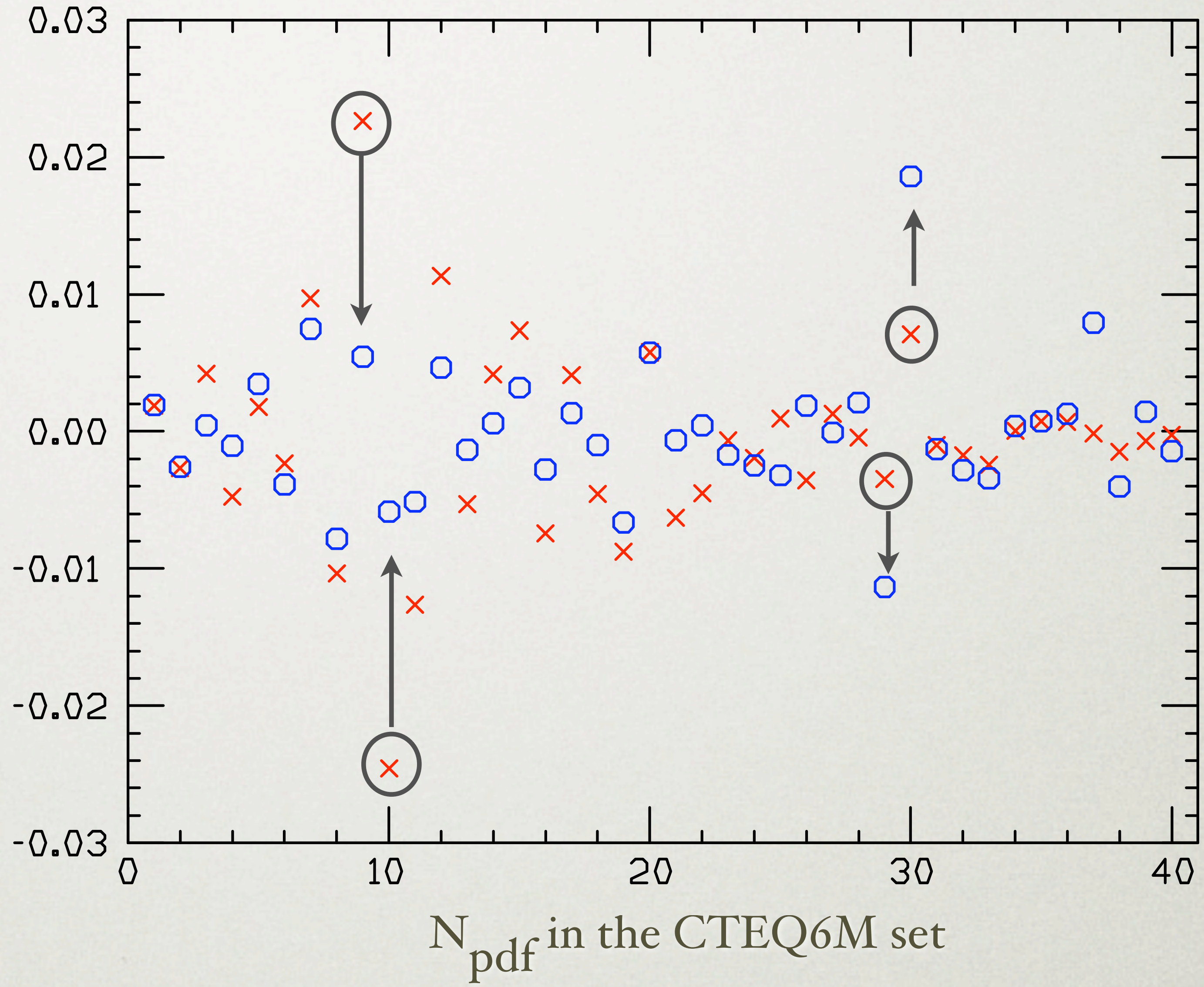
Exploiting PDF correlations?

Looking for PDF correlations with the inclusive jet sample:

σ_{jet}= rate of events with E_{Tjet}>175 GeV

X: $\dfrac{\sigma_{tt}(N_{pdf})}{\sigma_{tt}(6M)} - 1$

O: $\dfrac{\sigma_{tt}(N_{pdf})/\sigma_{jet}(N_{pdf})}{\sigma_{tt}(6M)/\sigma_{jet}(6M)} - 1$



A correlation exists, but it is not perfect. Likely due to the fact that the initial state is not precisely the same:

$$\sigma_{gg}(tt) : \sigma_{qg}(tt) : \sigma_{qq}(tt) = 90\% : 1\% : 10\%$$

$$\sigma_{gg}(jet) : \sigma_{qg}(jet) : \sigma_{qq}(jet) = 45\% : 45\% : 10\%$$

Mass-dep$^{\mathrm{T}}$ parameterization

$$\sigma(m_t) = A + B(m_t - 171) + C(m_t - 171)^2 + D(m_t - 171)^3$$

Cacciari et al, arXiv/0804:2800

Tevatron, $p\bar{p}$ at $\sqrt{s} = 1960$ GeV		A (pb)	B (pb/GeV)	C (pb/GeV^2)	D (pb/GeV^3)
CTEQ6M	Central	7.59	-0.237	4.39×10^{-3}	-6.32×10^{-5}
	Scales+	7.89	-0.247	4.60×10^{-3}	-6.66×10^{-5}
	Scales−	7.07	-0.221	4.11×10^{-3}	-5.92×10^{-5}
	PDFs+	8.26	-0.260	4.86×10^{-3}	-7.02×10^{-5}
	PDFs−	7.12	-0.222	4.08×10^{-3}	-5.82×10^{-5}
CTEQ6.1	Central	7.77	-0.244	4.53×10^{-3}	-6.51×10^{-5}
	Scales+	8.08	-0.254	4.74×10^{-3}	-6.86×10^{-5}
	Scales−	7.23	-0.227	4.23×10^{-3}	-6.09×10^{-5}
	PDFs+	8.53	-0.269	5.04×10^{-3}	-7.27×10^{-5}
	PDFs−	7.20	-0.224	4.12×10^{-3}	-5.87×10^{-5}
CTEQ6.5	Central	7.61	-0.237	4.38×10^{-3}	-6.28×10^{-5}
	Scales+	7.90	-0.247	4.58×10^{-3}	-6.61×10^{-5}
	Scales−	7.08	-0.221	4.10×10^{-3}	-5.89×10^{-5}
	PDFs+	8.14	-0.256	4.78×10^{-3}	-6.91×10^{-5}
	PDFs−	7.24	-0.224	4.11×10^{-3}	-5.85×10^{-5}
MRST2001E	Central	7.66	-0.242	4.53×10^{-3}	-6.60×10^{-5}
	Scales+	7.97	-0.252	4.75×10^{-3}	-6.98×10^{-5}
	Scales−	7.13	-0.225	4.24×10^{-3}	-6.17×10^{-5}
	PDFs+	7.94	-0.252	4.75×10^{-3}	-6.95×10^{-5}
	PDFs−	7.44	-0.233	4.35×10^{-3}	-6.31×10^{-5}
MRST2004nlo	Central	7.99	-0.253	4.77×10^{-3}	-6.95×10^{-5}
MSTW2006nnlo	Central	7.93	-0.253	4.76×10^{-3}	-6.92×10^{-5}
	Scales+	8.27	-0.264	5.00×10^{-3}	-7.33×10^{-5}
	Scales−	7.37	-0.235	4.44×10^{-3}	-6.45×10^{-5}
	PDFs+	8.17	-0.261	4.93×10^{-3}	-7.19×10^{-5}
	PDFs−	7.73	-0.245	4.61×10^{-3}	-6.68×10^{-5}

LHC, pp at $\sqrt{s} = 14$ TeV		A (pb)	B (pb/GeV)	C (pb/GeV^2)	D (pb/GeV^3)
CTEQ6M	Central	933	-25.3	0.423	-5.60×10^{-3}
	Scales+	1018	-27.7	0.468	-6.22×10^{-3}
	Scales−	846	-22.8	0.379	-4.99×10^{-3}
	PDFs+	962	-25.8	0.432	-5.73×10^{-3}
	PDFs−	903	-24.6	0.413	-5.44×10^{-3}
CTEQ6.1	Central	934	-25.2	0.421	-5.56×10^{-3}
	Scales+	1019	-27.7	0.466	-6.19×10^{-3}
	Scales−	847	-22.7	0.377	-4.95×10^{-3}
	PDFs+	965	-25.8	0.430	-5.70×10^{-3}
	PDFs−	902	-24.5	0.411	-5.40×10^{-3}
CTEQ6.5	Central	908	-24.5	0.411	-5.46×10^{-3}
	Scales+	990	-26.9	0.455	-6.08×10^{-3}
	Scales−	823	-22.1	0.368	-4.87×10^{-3}
	PDFs+	938	-25.2	0.420	-5.57×10^{-3}
	PDFs−	879	-23.9	0.401	-5.29×10^{-3}
MRST2001E	Central	965	-25.9	0.429	-5.63×10^{-3}
	Scales+	1054	-28.4	0.475	-6.27×10^{-3}
	Scales−	874	-23.3	0.384	-5.00×10^{-3}
	PDFs+	981	-26.2	0.434	-5.68×10^{-3}
	PDFs−	954	-25.6	0.426	-5.57×10^{-3}
MRST2004nlo	Central	982	-26.3	0.436	-5.72×10^{-3}
MSTW2006nnlo	Central	961	-25.7	0.426	-5.58×10^{-3}
	Scales+	1050	-28.3	0.472	-6.21×10^{-3}
	Scales−	870	-23.1	0.381	-4.96×10^{-3}
	PDFs+	972	-25.9	0.428	-5.62×10^{-3}
	PDFs−	949	-25.4	0.422	-5.53×10^{-3}

LHC at 10 TeV

$$\sigma = 908\ {}^{+82(9.0\%)}_{-85(9.3\%)}\ (\text{scales})\ {}^{+30(3.3\%)}_{-29(3.2\%)}\ (\text{PDFs})\ \text{pb}$$

414 pb at 10 TeV

The future: Full NNLO

See presentation by Czakon,
hvq parallel session,
Tue@16:30

NNLO building blocks

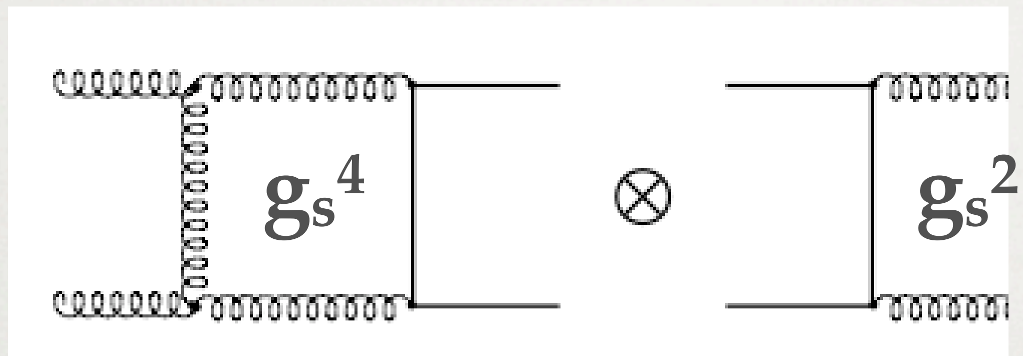
- **Real-emission diagrams, squared:**
  - tt gg, ttqg, ttqq ($g_s{}^8 = g_s{}^4 \times g_s{}^4$)
- **1-loop, O(ϵ, ϵ²):**
  - tt @ 1-loop, squared ($g_s{}^8 = g_s{}^{(2+2)} \times g_s{}^{(2+2)}$)
- **1-loop + real emission diagrams:**
  - ttg+ttq at 1-loop, interference with tree-level ttg+ttq ($g_s{}^8 = g_s{}^{(3+2)} \times g_s{}^3$)
- **2-loop**
  - tt at 2-loops, interference with tree-level tt ($g_s{}^8 = g_s{}^{(2+4)} \times g_s{}^2$)

1-LOOP, O(ϵ, ϵ²)

J.G. Korner, Z. Merebashvili and M. Rogal, Phys. Rev. D73 (2006) 034030.

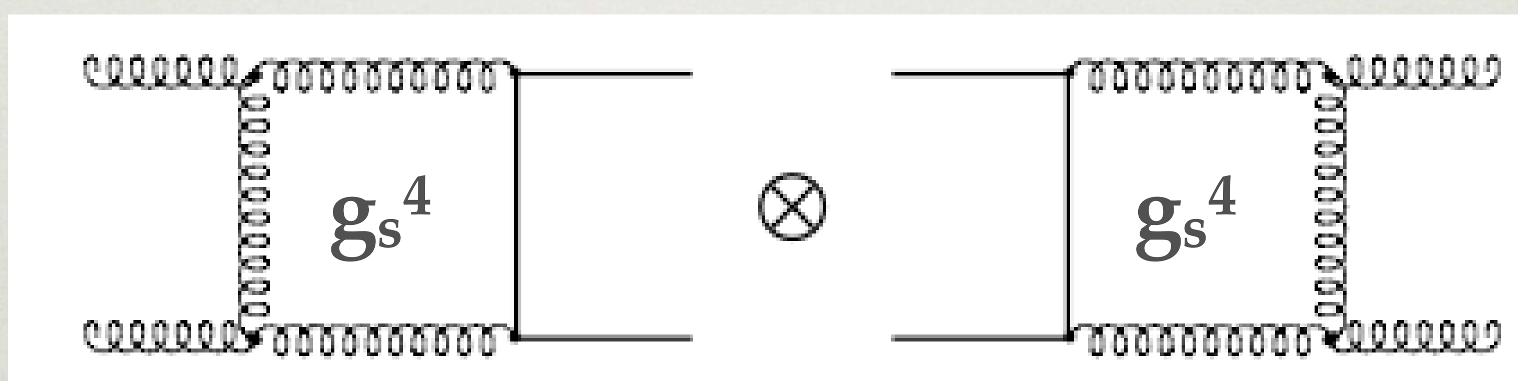
J.G. Korner, Z. Merebashvili and M. Rogal, arXiv:0802.0106 [hep-ph].

NLO



$A_{(-2)}/\epsilon^2 + A_{(-1)}/\epsilon + A_{(0)}$

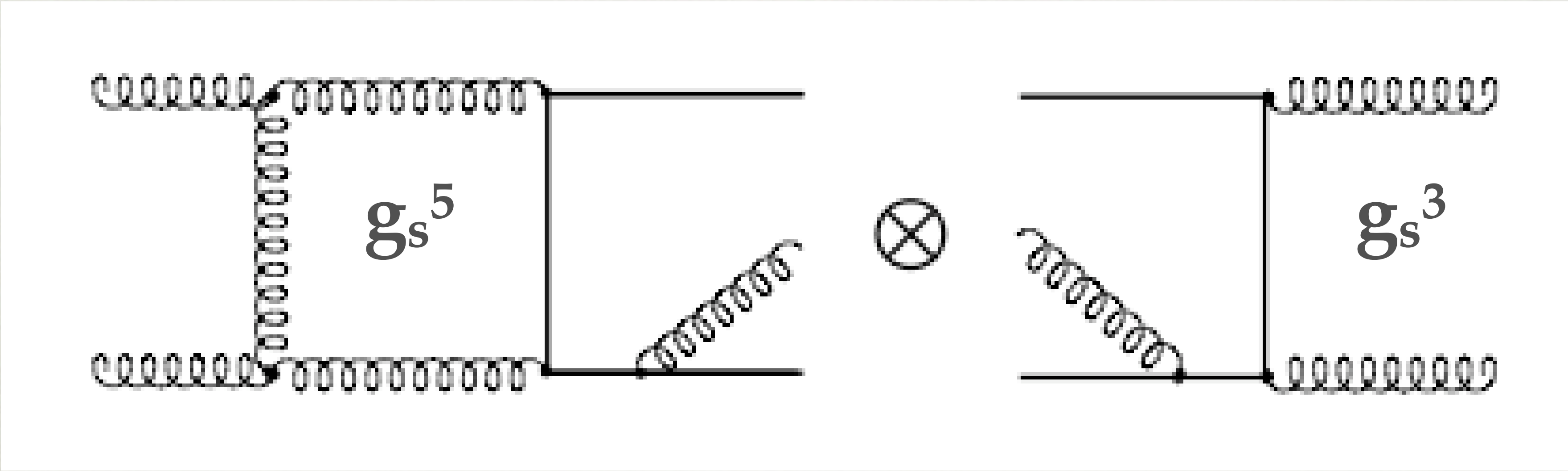
NNLO



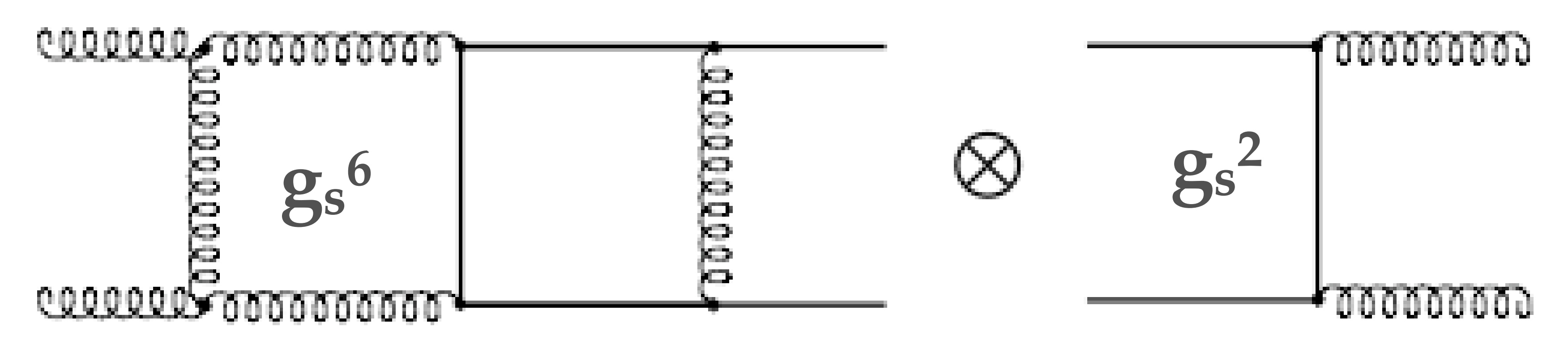
$A_{(-2)}/\epsilon^2 + A_{(-1)}/\epsilon + A_{(0)} + A_{(1)}\,\epsilon + A_{(2)}\epsilon^2$

Real-emission, 1-loop

S. Dittmaier, P. Uwer and S. Weinzierl, Phys. Rev. Lett. 98 (2007) 262002



2-LOOP



M. Czakon, A. Mitov and S. Moch, Phys. Lett. B651 (2007) 147.

M. Czakon, A. Mitov and S. Moch, arXiv:0707.4139 [hep-ph].

qq/gg->QQ at 2-loops, analytic, $\sqrt{S} \gg m_Q$

Czakon, arXiv/0803.1400

qq->QQ at 2-loops, numerical, $m_Q \neq 0$,

Czakon, in progress

gg->QQ at 2-loops, numerical, $m_Q \neq 0$,

Conclusions

- The LHC will challenge QCD in the determination of $\sigma(tt)$
- Estimates of the intrinsic theoretical accuracy vary from 2% (NLL or approx NNLO, $\mu_R = \mu_F$, A=0) to 9% (NLL, $\mu_R \neq \mu_F$, A=2)
- PDF uncertainty likely underestimated
- Current impressive progress towards full NNLO, but still some way from physical cross-sections