hep-ph/0507244, 0603030, 0606169, 0611204

Small-x Resummation: the Thorne-White Approach

Chris White, Nikhef

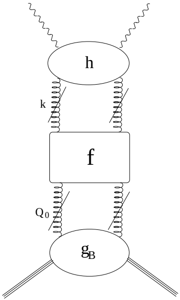
HERA-LHC meeting - 27th May 2008

Overview

What effect do small x resummations have on a global parton fit?

- Solution of BFKL solution at NLL order with running coupling.
- Implementation of impact factors for light and heavy flavours.
- Global fit with resummed splitting and coefficient functions.
- Comparison with toy calculation at NLO and NLL orders.

The High Energy Problem



- Coefficient and splitting functions for the proton structure functions unstable at low x due to terms $\sim x^{-1}\bar{\alpha}_S^n \log^m(1/x)$, $m \leq n-1$.
- Divergence due to t-channel gluon exchange at LL order, with some quark mixing at NLL order.
- Must resum the gluon 4-point function by solving the BFKL equation.
- Relate gluon to structure functions using the k_T factorisation formula (Collins & Ellis; Catani, Ciafaloni & Hautmann).

Running coupling solution of BFKL equation

- Mellin moments:

$$f(\gamma, N) = \int_0^\infty (k^2)^{-\gamma-1} \int_0^1 dx x^N f(x, k^2)$$

- Substitute LO running coupling into BFKL equation (Collins & Kwiecinski):

$$\frac{d^2 f(\gamma, N)}{d\gamma^2} = \frac{d^2 f_I(\gamma, Q_0^2)}{d\gamma^2} - \frac{1}{\bar{\beta}_0 N} \frac{d(\chi_0(\gamma) f(\gamma, N))}{d\gamma} + \frac{\pi}{3\bar{\beta}_0^2 N} \chi_1(\gamma) f(\gamma, N),$$

  with $\bar{\beta}_0 = 3/(\pi\beta_0)$.
- Solve with ansatz:

$$f(N, \gamma) = \exp\left(-\frac{X_1(\gamma)}{\bar{\beta}_0 N}\right) \int_\gamma^\infty A(\tilde{\gamma}) \exp\left(\frac{X_1(\tilde{\gamma})}{\bar{\beta}_0 N}\right) d\tilde{\gamma}$$

  (Ciafaloni & Colferai).

- Can shift lower limit $\gamma \to 0$ up to power-suppressed corrections (Thorne).
- Gluon factorises:

$$\mathcal{G}(N,t) = \mathcal{G}_E(N,t)\mathcal{G}_I(Q_0^2,N)$$

($t = \log Q^2/\Lambda^2$).

- Perturbative piece:

$$\mathcal{G}_E^1(N,t) = \frac{1}{2\pi\imath}\int_{1/2-\imath\infty}^{1/2+\imath\infty} \frac{f^{\beta_0}}{\gamma} \exp\left[\gamma t - X_1(\gamma,N)/(\bar{\beta}_0 N)\right] d\gamma$$

with:

$$X_1(\gamma,N) = \int_{\frac{1}{2}}^{\gamma} \left[\chi_0(\tilde{\gamma}) + N\frac{\chi_1(\tilde{\gamma})}{\chi_0(\tilde{\gamma})}\right] d\tilde{\gamma}.$$

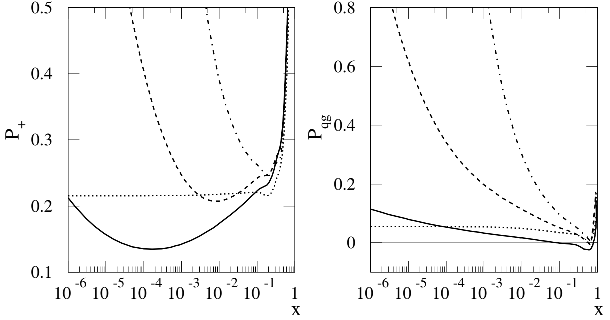
- Similarly, get structure functions:

$$\mathcal{F}_E^1(N,t) = \frac{1}{2\pi\imath}\int_{1/2-\imath\infty}^{1/2+\imath\infty} \frac{h(\gamma,N)f^{\beta_0}}{\gamma} \exp\left[\gamma t - X_1(\gamma,N)/(\bar{\beta}_0 N)\right] d\gamma$$

- If impact factors known, can disentangle all resummed coefficient and splitting functions (within a particular factorisation scheme).
- However, NLL impact factors $h(\gamma, N)$ not known. Work in progress (Bartels, Colferai, Gieseke & Kyrieleis).
- Instead LL factors with exact gluon kinematics have been calculated (Bialas, Navelet & Peschanski; White, Peschanski & Thorne).
- These provide a very good estimate to the full NLL impact factors (White & Thorne).
- Can use these to calculate all the NLL resummed coefficient and splitting functions in the DIS scheme.
- Finally, combine resummed results with NLO DGLAP:

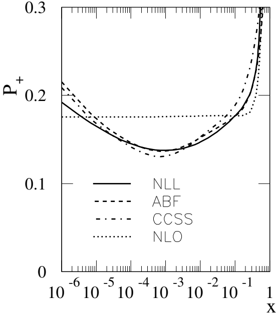
$$P^{tot.} = P^{NLL} + P^{NLO} - \left[P^{NLL(0)} + P^{NLL(1)} \right]$$

Results for Splitting Functions



- Results shown at $n_f = 4$, $t = 6$.
- Running coupling suppresses low x divergence.
- NLL kernel and impact factor effects lead to even more suppression.
- Main feature is a dip below the NLO DGLAP result.

Comparison with Alternative Approaches



- Can compare P_{gg} with ABF and CCSS groups (not latest results)...
- Dip common to all approaches at NLL.

Implementation

- Have calculated all splitting functions (using approx. impact factors).
- Also have resummed heavy flavour coefficient functions and matrix elements.
- These are needed for a consistent (general mass variable flavour) scheme at NLL order.
- Have defined such a scheme (DIS(χ)) up to NLL+NLO order.
- Heavy flavours are important!
- Have all necessary ingredients for a global parton fit...

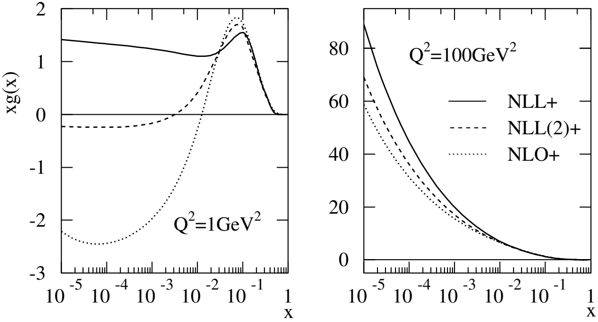
A Global Parton Fit

- Have undertaken a global parton fit at NLL + NLO order in the DIS scheme.
- Resummed fit gives an overall fit quality $\chi^2 = 2249$ for 2181 data points.
- Compare NLO DIS scheme $\chi^2 = 2352$ and $\overline{\text{MS}}$ scheme $\chi^2 = 2307$.
- A previous LL resummed fit gave $\chi^2 = 2336$, with significant momentum conservation violation.
- Resummation seems to decrease tension between data sets.

Effects of Resummation in a Global Fit

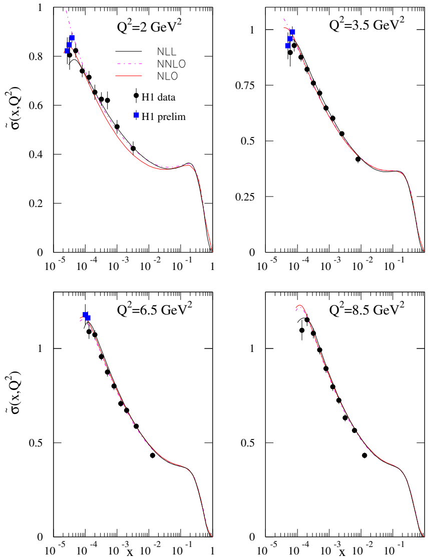
- Recall the NLL splitting functions dip below their NLO counterparts.
- Allows increased gluon at both large and small x.
- Thus can improve fit to Tevatron jet data without compromising fit the HERA data.
- Gluon is qualitatively different - leads to more stable F_L.
- Correct turnover in reduced cross-section at high y.

Gluon Distribution



- Gluons differ for $x \lesssim 10^{-2}$.
- NLL resummed gluon positive and growing at small x!
- Not true at fixed order.
- Positive gluon avoids negative structure functions.

Reduced Cross-Section



- Turnover required by data at low x (high y) - NLO fails.
- Resummation helps! Interesting to compare with NNLO.

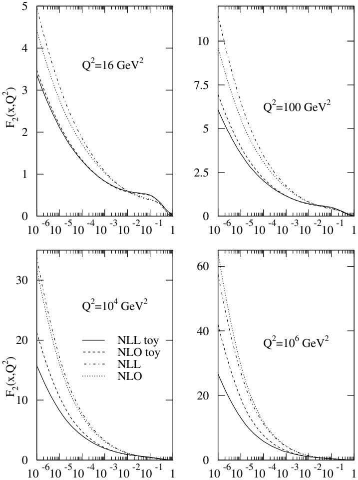
Toy Calculation

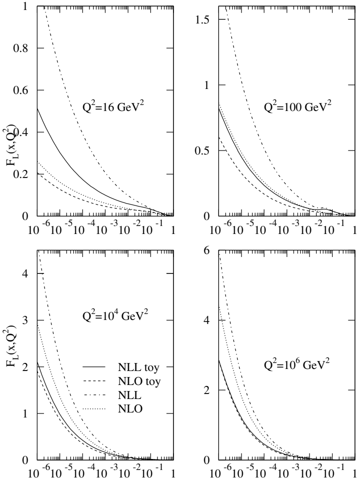
- Can examine the effects of NLL vs. NLO splitting / coefficient functions using toy partons ($Q_0^2 = 4\text{GeV}^2$):

$$xg(x) = r_s xS(x) = k_g x^{-0.18}(1-x)^5;$$
$$xq_v = k_q x^{0.5}(1-x)^4,$$

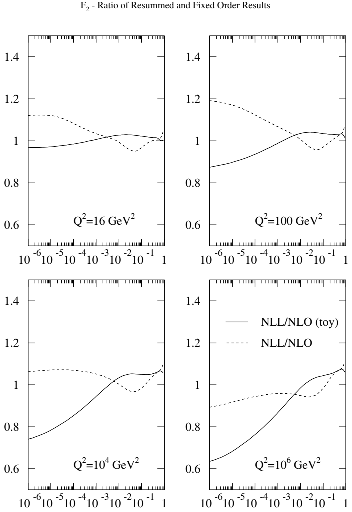
  with $r_s = 3$, and all other constants fixed by number / momentum sum rules.
- In practice, partons reshuffle themselves in a global fit.
- Thus will compare toy calculation with results obtained using fitted NLL / NLO partons.

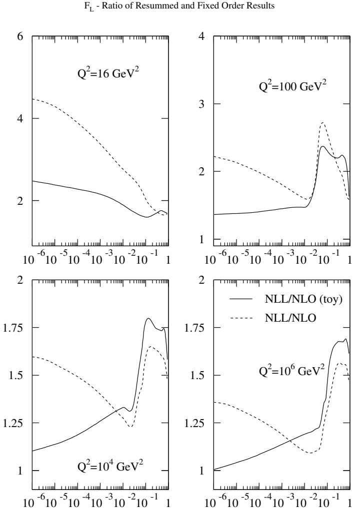
Results for F_2 and F_L



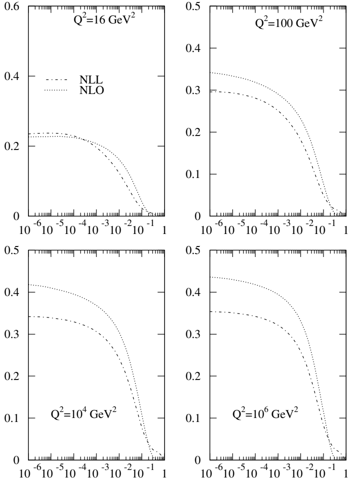


Ratios: NLL/NLO



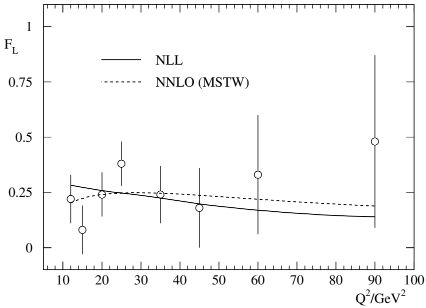


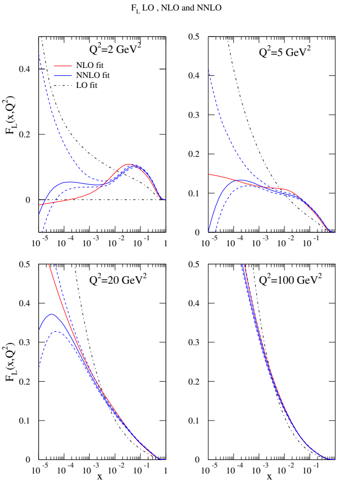
Charm contribution to F_2



- Clearly important at small x.
- Thus, consistent treatment of heavy flavours necessary.

Results - F_L



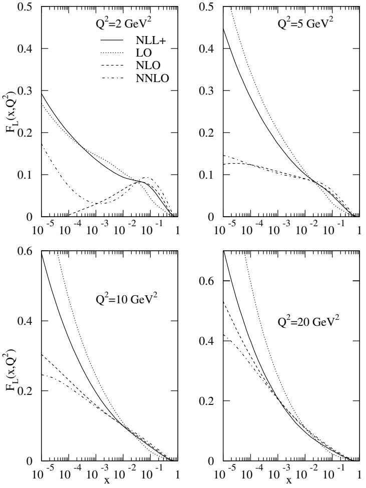


- Data currently unable to distinguish between predictions.
- But fixed order looks unstable.

Conclusions

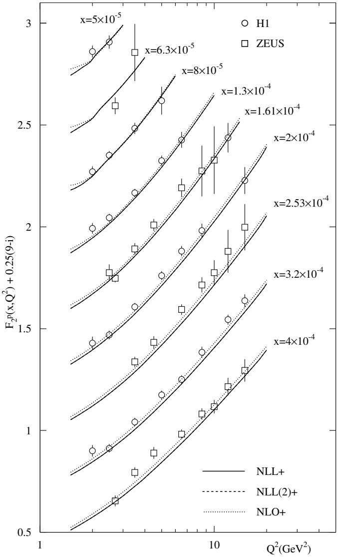
- Have undertaken a global parton fit including NLL BFKL resummations.
- Qualitative changes in gluon behaviour, and some quantitative improvement in the fit.
- Have to go to NLL in the resummation to achieve sensible results.
- Dip in evolution (also ABF, CCSS) important.
- So is correct treatment of heavy flavours.

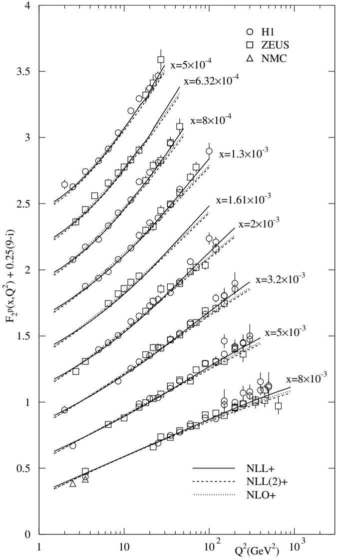
Longitudinal Structure Function



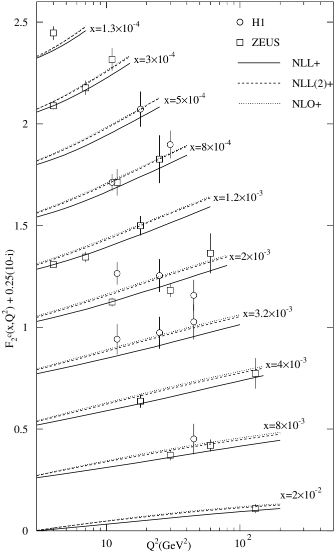
- Clearly see perturbative instability in fixed order results.
- This is cured by the resummation.

Results - F_2


x=5×10-5
x=6.3×10-5
x=8×10-5
x=1.3×10-4
x=1.61×10-4
x=2×10-4
x=2.53×10-4
x=3.2×10-4
x=4×10-4
H1
ZEUS
NLL+
NLL(2)+
NLO+
F2p(x,Q2) + 0.25(9-i)
Q2(GeV2)


x=5×10-4
x=6.32×10-4
x=8×10-4
x=1.3×10-3
x=1.61×10-3
x=2×10-3
x=3.2×10-3
x=5×10-3
x=8×10-3
H1
ZEUS
NMC
NLL+
NLL(2)+
NLO+
F2p(x,Q2) + 0.25(9-i)
Q2(GeV2)

Results - F_2^c



- Resummed fit performs better for small x data - note slope as Q^2 increases.
- Fit is also improved over the whole range of x.
- Resummed F_2^c at lower end of range allowed by data.