Gluon saturation
and the determination of PDFs

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Gluon saturation
- Saturation, rescatterings
- Multiple scatterings
- Color Glass Condensate
- How to see saturation?

Saturation in DIS
- DIS amplitude
- Total cross-section
- Dilute limit

pA collisions
- Classical color field
- Gluon production
- QQbar production
- Breaking of factorization
- Other multiple scatterings

Summary

1. **Gluon saturation**
   - Saturation, rescatterings
   - Multiple scatterings
   - Color Glass Condensate
   - How to see saturation?
2. **Saturation in DIS**
   - DIS amplitude
   - Total cross-section
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3. **Case study: $Q\overline{Q}$ production in pA collisions**
   - Classical color field
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Criterion for gluon recombination

Gribov, Levin, Ryskin (1983)

Number of gluons per unit area :

$$\rho \sim \frac{xG_A(x,Q^2)}{\pi R_A^2}$$

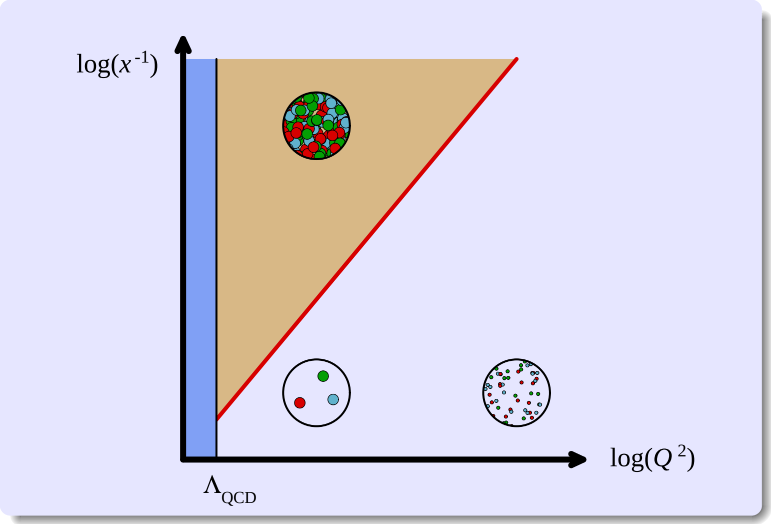
Recombination cross-section :

$$\sigma_{gg\to g} \sim \frac{\alpha_s}{Q^2}$$

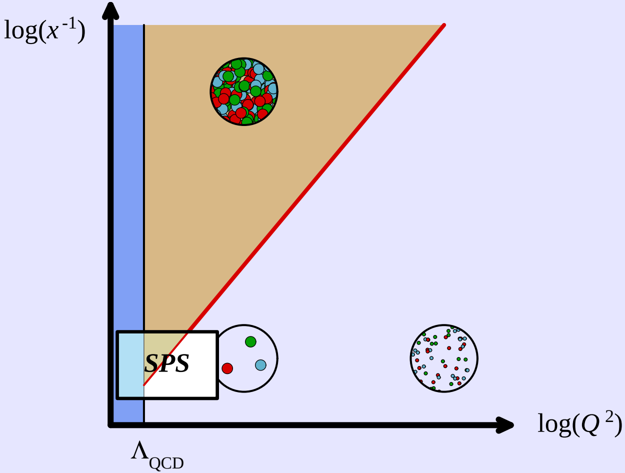
Recombination happens if $\rho\sigma_{gg\to g} \gtrsim 1$, i.e. $Q^2 \lesssim Q_s^2$, with :

$$Q_s^2 \quad\sim\quad \frac{\alpha_s xG_A(x,Q_s^2)}{\pi R_A^2} \quad\sim\quad A^{1/3}\frac{1}{x^{0.3}}$$

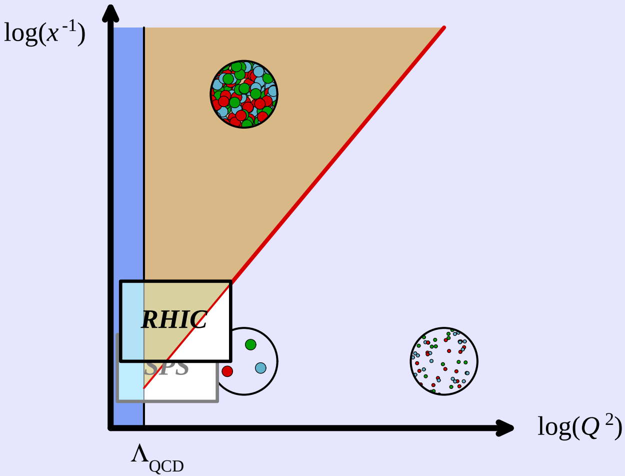
Saturation domain



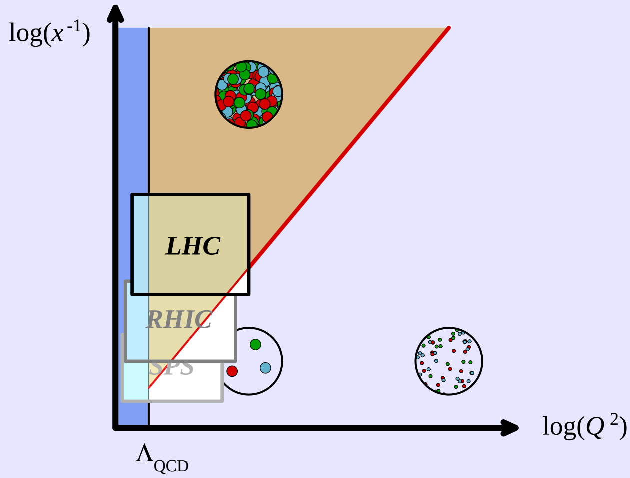
Saturation domain



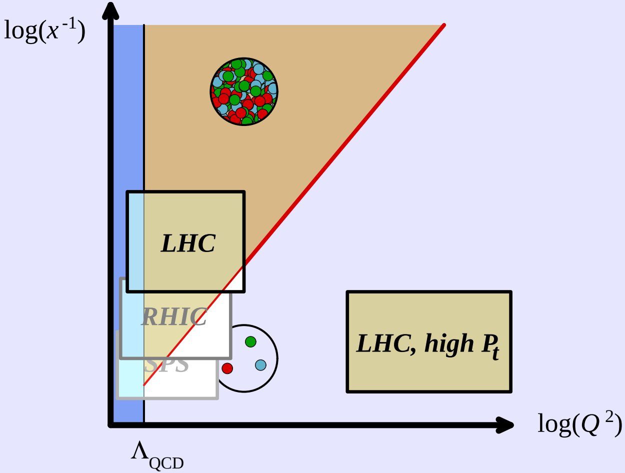
Saturation domain



Saturation domain



Saturation domain



Multiple scatterings

- The saturation criterion can also be seen as a condition for multiple scatterings

- The mean free path of a gluon in a nucleus is

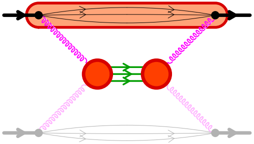
$$\lambda = \frac{1}{n\sigma_{gg\to g}} \quad , \quad n \sim \frac{xG_A(x,Q^2)}{\frac{4}{3}\pi R_A^3}$$

- Multiple scatterings are important if λ becomes smaller than the size of the nucleus, $\lambda \lesssim R_A$, i.e.

$$Q^2 \lesssim \alpha_s \frac{xG_A(x,Q^2)}{\pi R_A^2} \sim Q_s^2$$

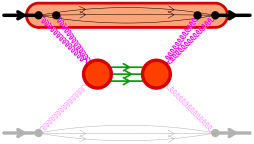
Multiple scatterings

- Single scattering :



▷ 2-point function in the projectile ▷ gluon number

- Multiple scatterings :



▷ 4-point function in the projectile ▷ higher correlations

Color Glass Condensate: degrees of freedom

- The fast partons (large x) are frozen by time dilation
  ▷ described as static color sources on the light-cone :

$$J^\mu = \delta^{\mu +}\delta(x^-)\rho(\vec{\boldsymbol{x}}_\perp) \qquad\qquad (x^- \equiv (t-z)/\sqrt{2})$$

- Slow partons (small x) cannot be considered static over the time-scales of the collision process ▷ they must be treated as the usual gauge fields

  Since they are radiated by the fast partons, they must be coupled to the current J^μ by a term : $A_\mu J^\mu$

- The color sources ρ are random, and described by a distribution functional $W_Y[\rho]$, with Y the rapidity that separates “soft” and “hard”

Color Glass Condensate: evolution

Evolution equation (JIMWLK) :

$$\frac{\partial W_Y}{\partial Y} = \mathcal{H}\ W_Y$$

$$\mathcal{H} = \frac{1}{2} \int\limits_{\vec{x}_\perp, \vec{y}_\perp} \frac{\delta}{\delta \widetilde{\mathcal{A}}^+(\epsilon, \vec{y}_\perp)} \eta(\vec{x}_\perp, \vec{y}_\perp) \frac{\delta}{\delta \widetilde{\mathcal{A}}^+(\epsilon, \vec{x}_\perp)}$$

where $-\partial_\perp^2\, \widetilde{\mathcal{A}}^+(\epsilon, \vec{x}_\perp) = \rho(\epsilon, \vec{x}_\perp)$

- $\eta(\vec{x}_\perp, \vec{y}_\perp)$ is a non-linear functional of ρ
- This evolution equation resums all the powers of $\alpha_s \ln(1/x)$ and of $Q_s/p_\perp$ that arise in loop corrections
- This equation simplifies into the BFKL equation when the source ρ is small (one can expand η in powers of ρ)

CGC and hadron-hadron collisions

- For hadron-hadron collisions, there are two strong sources that contribute to the color current :

$$J^\mu \equiv \delta^{\mu+}\delta(x^-)\,\rho_1(\vec{\boldsymbol{x}}_\perp) + \delta^{\mu-}\delta(x^+)\,\rho_2(\vec{\boldsymbol{x}}_\perp)$$

Average over the sources ρ_1, ρ_2 :

$$\langle\mathcal{O}\rangle_Y = \int [D\rho_1]\,[D\rho_2]\,W_{Y_{\rm beam}-Y}\,[\rho_1]\,W_{Y+Y_{\rm beam}}\,[\rho_2]\,\mathcal{O}[\rho_1,\rho_2]$$

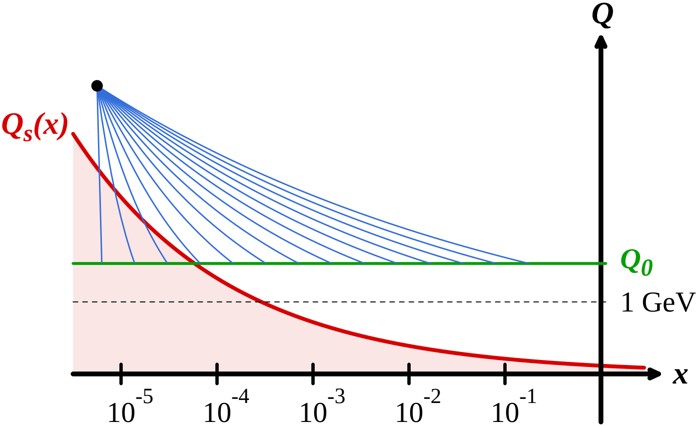
(FG, Lappi, Venugopalan (2008))

Effects of saturation

- Geometrical scaling : in DIS, $F_2(x, Q)$ becomes a function of $Q/Q_s(x)$
- Anomalous nuclear size dependence : requires that the same measurement be performed with several different nuclear targets
- Anomalies in factorization : e.g. the gluon distribution extracted from one quantity does not fit when applied to another quantity
- Conversely, could saturation effects masquerade as effects that hide into the standard gluon distribution function?

Effects of saturation

- Even if the observable under consideration involves parton distributions only outside of the saturation domain, the DGLAP evolution may have been contaminated by saturation physics :



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DIS amplitude

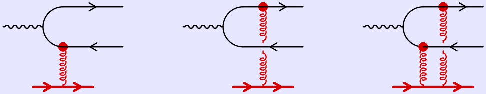
- Differential photon-target cross-section :

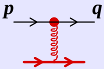
$$d\sigma_{\gamma^* T} = \frac{d^3\boldsymbol{k}}{(2\pi)^2 2E_{\boldsymbol{k}}}\frac{d^3\boldsymbol{p}}{(2\pi)^3 2E_{\boldsymbol{p}}}\;\frac{1}{2q^-}2\pi\delta(q^- - k^- - p^-)$$
$$\times \left\langle \mathcal{M}^\mu(\boldsymbol{q}|\boldsymbol{k},\boldsymbol{p})\mathcal{M}^{\nu *}(\boldsymbol{q}|\boldsymbol{k},\boldsymbol{p})\right\rangle \epsilon_\mu(Q)\epsilon^*_\nu(Q)\;,$$

  - $\boldsymbol{k},\boldsymbol{p}$: momenta of the quark and antiquark
  - $\boldsymbol{q}$: momentum of the virtual photon

Scattering amplitude :

$$\mathcal{M}^\mu(\boldsymbol{q}|\boldsymbol{k},\boldsymbol{p}) =$$





$$= 2\pi\,\delta(p^- - q^-)\,\gamma^-\int d^2\vec{\boldsymbol{x}}_\perp\; e^{i(\vec{\boldsymbol{q}}_\perp - \vec{\boldsymbol{p}}_\perp)\cdot\vec{\boldsymbol{x}}_\perp}\left[U(\vec{\boldsymbol{x}}_\perp) - 1\right]$$

DIS amplitude

- The sum of the three terms simplifies considerably :

$$\mathcal{M}^\mu(\boldsymbol{q}|\boldsymbol{k},\boldsymbol{p}) = \frac{i}{2}\int\frac{d^2\vec{\boldsymbol{l}}_\perp}{(2\pi)^2}\int d^2\vec{\boldsymbol{x}}_{1\perp}d^2\vec{\boldsymbol{x}}_{2\perp}\left[\overline{u}(\vec{\boldsymbol{k}})\,\Gamma^\mu\,v(\vec{\boldsymbol{p}})\right]$$
$$\times\, e^{i\vec{\boldsymbol{l}}_\perp\cdot\vec{\boldsymbol{x}}_{1\perp}}\, e^{i(\vec{\boldsymbol{p}}_\perp+\vec{\boldsymbol{k}}_\perp-\vec{\boldsymbol{q}}_\perp-\vec{\boldsymbol{l}}_\perp)\cdot\vec{\boldsymbol{x}}_{2\perp}}\left[U(\vec{\boldsymbol{x}}_{1\perp})U^\dagger(\vec{\boldsymbol{x}}_{2\perp})-1\right]$$

with

$$\Gamma^\mu \equiv \frac{\gamma^-(\not{K}-\not{l}+m)\gamma^\mu(\not{K}-\not{Q}-\not{l}+m)\gamma^-}{p^-[(\vec{\boldsymbol{k}}_\perp-\vec{\boldsymbol{l}}_\perp)^2+m^2-2k^-q^+]+k^-[(\vec{\boldsymbol{k}}_\perp-\vec{\boldsymbol{q}}_\perp-\vec{\boldsymbol{l}}_\perp)^2+m^2]}$$

- By inserting this into the DIS cross-section, we see that the differential cross-section (with two resolved jets in the final state) depends on the correlator of four Wilson lines

  ▷ in principle, cannot be absorbed into the usual gluon distribution

Total cross-section

- If we integrate out the final quark and antiquark, two of the Wilson lines cancel and we get :

$$\sigma_{\gamma^* T} = \int_0^1 dz \int d^2\vec{\boldsymbol{r}}_\perp \left|\psi(\boldsymbol{q}|z,\vec{\boldsymbol{r}}_\perp)\right|^2 \sigma_{\rm dipole}(\vec{\boldsymbol{r}}_\perp)$$

with

$$\sigma_{\rm dipole}(\vec{\boldsymbol{r}}_\perp) \equiv \frac{2}{N_c}\int d^2\vec{\boldsymbol{X}}_\perp\, {\rm Tr}\left\langle 1 - U\Big(\vec{\boldsymbol{X}}_\perp + \frac{\vec{\boldsymbol{r}}_\perp}{2}\Big)U^\dagger\Big(\vec{\boldsymbol{X}}_\perp - \frac{\vec{\boldsymbol{r}}_\perp}{2}\Big)\right\rangle$$

and

$$\left|\psi(\boldsymbol{q}|z,\vec{\boldsymbol{r}}_\perp)\right|^2 \equiv \frac{N_c\, \epsilon_\mu(Q)\epsilon^*_\nu(Q)}{64\pi(q^-)^2 z(1-z)} \int \frac{d^2\vec{\boldsymbol{l}}_\perp}{(2\pi)^2}\frac{d^2\vec{\boldsymbol{l}}'_\perp}{(2\pi)^2}\, e^{i(\vec{\boldsymbol{l}}_\perp - \vec{\boldsymbol{l}}'_\perp)\cdot\vec{\boldsymbol{r}}_\perp}$$
$$\times\, {\rm Tr}\Big((\not{k} + m)\Gamma^\mu(\not{p} - m)\Gamma^{\nu\prime}\Big)$$

- Note : in the inclusive F_2, all the saturation effects enter via a 2-point function ▷ quite difficult to disentangle from non-saturation effects

DIS amplitude in the dilute limit

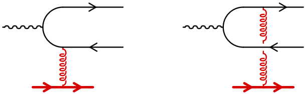
- In the dilute limit, one can expand the Wilson lines :

$$\mathcal{M}^\mu(\boldsymbol{q}|\boldsymbol{k},\boldsymbol{p}) = \frac{i}{2}\int\frac{d^2\vec{\boldsymbol{l}}_\perp}{(2\pi)^2}\int d^2\vec{\boldsymbol{x}}_{1\perp}d^2\vec{\boldsymbol{x}}_{2\perp}\;\Big[\overline{u}(\vec{\boldsymbol{k}})\;\Gamma^\mu\;v(\vec{\boldsymbol{p}})\Big]$$

$$\times\; e^{i\vec{\boldsymbol{l}}_\perp\cdot\vec{\boldsymbol{x}}_{1\perp}}\,e^{i(\vec{\boldsymbol{p}}_\perp+\vec{\boldsymbol{k}}_\perp-\vec{\boldsymbol{q}}_\perp-\vec{\boldsymbol{l}}_\perp)\cdot\vec{\boldsymbol{x}}_{2\perp}}$$

$$\times\;\Big[ig\int dx^-\mathcal{A}^+(x^-,\vec{\boldsymbol{x}}_{1\perp})-\mathcal{A}^+(x^-,\vec{\boldsymbol{x}}_{2\perp})\Big]$$

$$\mathcal{M}^\mu(\boldsymbol{q}|\boldsymbol{k},\boldsymbol{p}) =$$



- In this limit, the cross-section involves only 2-point correlations of the color field, which can be rewritten in terms of the usual gluon distribution

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Classical color field
Gluon production
QQbar production
Breaking of factorization
Other multiple scatterings

Classical color field

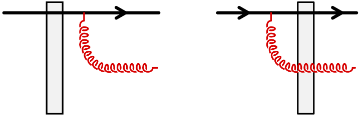
Blaizot, FG, Venugopalan (2004)

- Assume that $\rho_{\rm p}$ is a weak source, ρ_A is a strong source $\Rightarrow$ compute the pair production amplitude to first order in $\rho_{\rm p}$ and to all orders in ρ_A

Yang-Mills equations:

$$[D_\mu, F^{\mu\nu}] = J^\nu \quad , \qquad [D_\nu, J^\nu] = 0$$
$$J^\nu\big|_{\rm lowest\ order} = \delta^{\nu+}\delta(x^-)\rho_{\rm p}(\boldsymbol{x}_\perp) + \delta^{\nu-}\delta(x^+)\rho_A(\boldsymbol{x}_\perp)$$
$$\partial_\mu A^\mu = 0$$

- (Very sketchy) diagrammatic interpretation:



Classical color field

- Solution in the Lorenz gauge $(\partial_\mu A^\mu = 0)$:

$$A^\mu(k) = \frac{ig}{k^2} \int_{\vec{\boldsymbol{k}}_{1\perp}} \Big\{ C_U^\mu \Big[U(\vec{\boldsymbol{k}}_{2\perp}) - (2\pi)^2 \delta(\vec{\boldsymbol{k}}_{2\perp}) \Big] + C_V^\mu \Big[V(\vec{\boldsymbol{k}}_{2\perp}) - (2\pi)^2 \delta(\vec{\boldsymbol{k}}_{2\perp}) \Big] \Big\} \frac{\rho_{\mathrm{p}}(\vec{\boldsymbol{k}}_{1\perp})}{\boldsymbol{k}_{1\perp}^2}$$

$$C_U^- \equiv -\boldsymbol{k}_{1\perp}^2 / k^+ \,,\; C_U^+ \equiv (\boldsymbol{k}_{2\perp}^2 - \boldsymbol{k}_\perp^2)/k^- \,,\; C_U^i \equiv -2\boldsymbol{k}_1^i$$

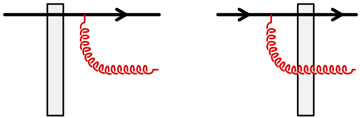
$$C_V^- \equiv 2k^- \,,\; C_V^+ \equiv -2k^+ + 2\boldsymbol{k}_\perp^2/k^- \,,\; C_V^i \equiv 2\boldsymbol{k}^i$$

$$\text{Note}: \quad C_U^\mu + C_V^\mu/2 = L^\mu \quad \text{(Lipatov's effective vertex)}$$

$$U(\vec{\boldsymbol{k}}_{2\perp}) \equiv \int_{\vec{\boldsymbol{x}}_\perp} e^{-i\vec{\boldsymbol{k}}_{2\perp}\cdot\vec{\boldsymbol{x}}_\perp} \; \mathrm{T}\, e^{ig\int_{z^-} \mathcal{A}_{\mathrm{A}}^+(z^-,\vec{\boldsymbol{x}}_\perp)}$$

$$V(\vec{\boldsymbol{k}}_{2\perp}) \equiv \int_{\vec{\boldsymbol{x}}_\perp} e^{-i\vec{\boldsymbol{k}}_{2\perp}\cdot\vec{\boldsymbol{x}}_\perp} \; \mathrm{T}\, e^{i\frac{g}{2}\int_{z^-} \mathcal{A}_{\mathrm{A}}^-(z^-,\vec{\boldsymbol{x}}_\perp)}$$

Inclusive gluon spectrum



- Amplitude :

$$\mathcal{M}_{\rm g}^{(\lambda)}(\vec{\boldsymbol{k}}) = k^2\,\epsilon_\mu^{(\lambda)}(\vec{\boldsymbol{k}})\,\mathcal{A}^\mu(k)$$

Gluon spectrum :

$$\frac{d\sigma_{\rm g}}{d^2\vec{\boldsymbol{k}}_\perp dy} \sim \frac{\alpha_s}{\boldsymbol{k}_\perp^2}\int\frac{d^2\boldsymbol{p}_\perp}{(2\pi)^2}\;\phi_A(\boldsymbol{p}_\perp)\;\phi_{\rm p}(\boldsymbol{k}_\perp-\boldsymbol{p}_\perp)$$

Gluon spectrum

- Despite saturation, the gluon cross-section depends only on a 2-point function of the target :

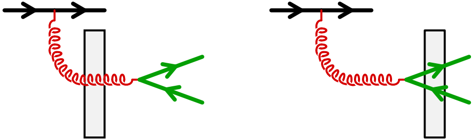
$$\phi_A(\vec{\boldsymbol{p}}_\perp) \propto \int_{\vec{\boldsymbol{x}}_\perp,\vec{\boldsymbol{y}}_\perp} e^{i\vec{\boldsymbol{p}}_\perp\cdot(\vec{\boldsymbol{x}}_\perp-\vec{\boldsymbol{y}}_\perp)}\ \mathrm{tr}\Big\langle U(\vec{\boldsymbol{x}}_\perp)U^\dagger(\vec{\boldsymbol{y}}_\perp)\Big\rangle$$

- If one looks only at this process, saturation effects may be there and be absorbed into the standard gluon distribution

- One should look at other processes, that may involve different target correlators

QQbar production amplitude

- The $Q\overline{Q}$ pair can be produced before or after the collision with the target :



$$\mathcal{M}_{q\bar q} = g^2 \int\limits_{\vec{k}_{1\perp},\vec{k}_\perp} \frac{\rho_{p,a}(\vec{k}_{1\perp})}{k_{1\perp}^2} \int\limits_{\vec{x}_\perp,\vec{y}_\perp} e^{i\vec{k}_\perp\cdot\vec{x}_\perp} e^{i(\vec{p}_\perp+\vec{q}_\perp-\vec{k}_\perp-\vec{k}_{1\perp})\cdot\vec{y}_\perp}$$

$$\times\bar{u}(\vec{q})\Big\{[\widetilde{U}(\vec{x}_\perp)t^a\widetilde{U}^\dagger(\vec{y}_\perp)]T_{q\bar q}(\vec{k}_\perp) + [t^b U_{ba}(\vec{x}_\perp)]\not{L}\Big\}v(\vec{p})$$

with

$$T_{q\bar q}(\vec{k}_\perp) \equiv \frac{\gamma^+(\not{q}-\not{k}+m)\gamma^-(\not{q}-\not{k}-\not{k}_1+m)\gamma^+}{2p^+[(\vec{q}_\perp-\vec{k}_\perp)^2+m^2]+2q^+[(\vec{q}_\perp-\vec{k}_\perp-\vec{k}_{1\perp})^2+m^2]}$$

- Note: $\widetilde{U}$ is a Wilson line in the fundamental representation

Pair cross-section

Pair production cross-section:

$$
\begin{aligned}
\frac{d\sigma_{q\bar q}}{d^2\vec{\boldsymbol{p}}_\perp d^2\vec{\boldsymbol{q}}_\perp dy_p dy_q} &= \frac{\alpha_s^2 N}{8\pi^4 d_A}\int_{\vec{\boldsymbol{k}}_{1\perp},\vec{\boldsymbol{k}}_{2\perp}} \frac{\delta(\vec{\boldsymbol{p}}_\perp+\vec{\boldsymbol{q}}_\perp-\vec{\boldsymbol{k}}_{1\perp}-\vec{\boldsymbol{k}}_{2\perp})}{\boldsymbol{k}_{1\perp}^2\boldsymbol{k}_{2\perp}^2}\\
&\times\Big\{\int_{\vec{\boldsymbol{k}}_\perp,\vec{\boldsymbol{k}}'_\perp}\mathrm{tr}\Big[(\not q+m)T_{q\bar q}(\vec{\boldsymbol{k}}_\perp)(\not p-m)T^*_{q\bar q}(\vec{\boldsymbol{k}}'_\perp)\Big]\phi_A^{(4)}(\vec{\boldsymbol{k}}_{2\perp}|\vec{\boldsymbol{k}}_\perp,\vec{\boldsymbol{k}}'_\perp)\\
&\quad+\int_{\vec{\boldsymbol{k}}_\perp}\mathrm{tr}\Big[(\not q+m)T_{q\bar q}(\vec{\boldsymbol{k}}_\perp)(\not p-m)\not L^*+\mathrm{h.c.}\Big]\phi_A^{(3)}(\vec{\boldsymbol{k}}_{2\perp}|\vec{\boldsymbol{k}}_\perp)\\
&\quad+\mathrm{tr}\Big[(\not q+m)\not L(\not p-m)\not L^*\Big]\phi_A(\vec{\boldsymbol{k}}_{2\perp})\Big\}\phi_p(\vec{\boldsymbol{k}}_{1\perp})
\end{aligned}
$$

▷ factorization broken on the target side: one needs three different “distributions” in order to describe the target

Target correlators

- Target “gluon distributions”:

$$\phi_A(\vec{\boldsymbol{k}}_{2\perp}) \propto \int\limits_{\vec{\boldsymbol{x}}_\perp,\vec{\boldsymbol{y}}_\perp} e^{i\vec{\boldsymbol{k}}_{2\perp}\cdot(\vec{\boldsymbol{x}}_\perp-\vec{\boldsymbol{y}}_\perp)}\ \mathrm{tr}\left\langle U(\vec{\boldsymbol{x}}_\perp)U^\dagger(\vec{\boldsymbol{y}}_\perp)\right\rangle$$

(already encountered in the gluon spectrum)

$$\phi_A^{(3)}(\vec{\boldsymbol{k}}_{2\perp}|\vec{\boldsymbol{k}}_\perp) \propto \int\limits_{\vec{\boldsymbol{x}}_\perp,\vec{\boldsymbol{y}}_\perp,\vec{\boldsymbol{z}}_\perp} e^{i\left[\vec{\boldsymbol{k}}_\perp\cdot\vec{\boldsymbol{x}}_\perp+(\vec{\boldsymbol{k}}_{2\perp}-\vec{\boldsymbol{k}}_\perp)\cdot\vec{\boldsymbol{y}}_\perp-\vec{\boldsymbol{k}}_{2\perp}\cdot\vec{\boldsymbol{z}}_\perp\right]} \times\ \mathrm{tr}\left\langle \widetilde{U}(\vec{\boldsymbol{x}}_\perp)t^a\widetilde{U}^\dagger(\vec{\boldsymbol{y}}_\perp)t^bU_{ba}(\vec{\boldsymbol{z}}_\perp)\right\rangle$$

$$\phi_A^{(4)}(\vec{\boldsymbol{k}}_{2\perp}|\vec{\boldsymbol{k}}_\perp,\vec{\boldsymbol{k}}'_\perp) \propto \int\limits_{\vec{\boldsymbol{x}}_\perp,\vec{\boldsymbol{y}}_\perp,\vec{\boldsymbol{x}}'_\perp,\vec{\boldsymbol{y}}'_\perp} e^{i\left[\vec{\boldsymbol{k}}_\perp\cdot\vec{\boldsymbol{x}}_\perp-\vec{\boldsymbol{k}}'_\perp\cdot\vec{\boldsymbol{x}}'_\perp+(\vec{\boldsymbol{k}}_{2\perp}-\vec{\boldsymbol{k}}_\perp)\cdot\vec{\boldsymbol{y}}_\perp-(\vec{\boldsymbol{k}}_{2\perp}-\vec{\boldsymbol{k}}'_\perp)\cdot\vec{\boldsymbol{y}}'_\perp\right]} \times\ \mathrm{tr}\left\langle \widetilde{U}(\vec{\boldsymbol{x}}_\perp)t^a\widetilde{U}^\dagger(\vec{\boldsymbol{y}}_\perp)\widetilde{U}(\vec{\boldsymbol{y}}'_\perp)t^a\widetilde{U}(\vec{\boldsymbol{x}}'_\perp)\right\rangle$$

Limit of Kt factorization

Fujii, FG, Venugopalan (2005)

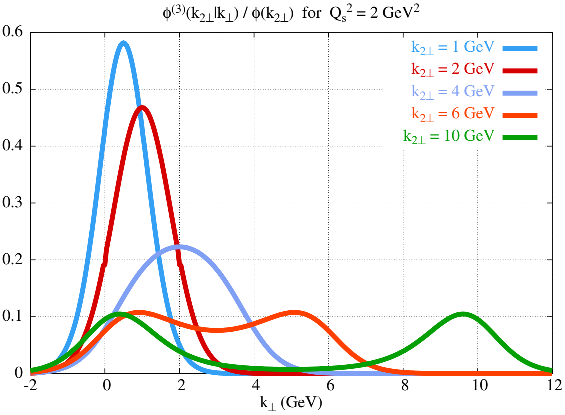
- The quark cross-section factorizes if the the 3-point and 2-point functions are related by:

$$\phi_A^{(3)}(\vec{\boldsymbol{k}}_{2\perp}|\vec{\boldsymbol{k}}_\perp) = (2\pi)^2 \frac{1}{2}\left[\delta(\vec{\boldsymbol{k}}_\perp) + \delta(\vec{\boldsymbol{k}}_\perp - \vec{\boldsymbol{k}}_{2\perp})\right]\phi_A(\vec{\boldsymbol{k}}_{2\perp})$$

- This relation would be satisfied if the $Q\overline{Q}$ pair interacts with the target in such a way that all the momentum exchanged goes to the quark or to the antiquark
- The ratio $\phi_A^{(3)}(\vec{\boldsymbol{k}}_{2\perp}|\vec{\boldsymbol{k}}_\perp)/\phi_A(\vec{\boldsymbol{k}}_{2\perp})$ should be close to the sum of two delta functions for factorization to be approximately valid

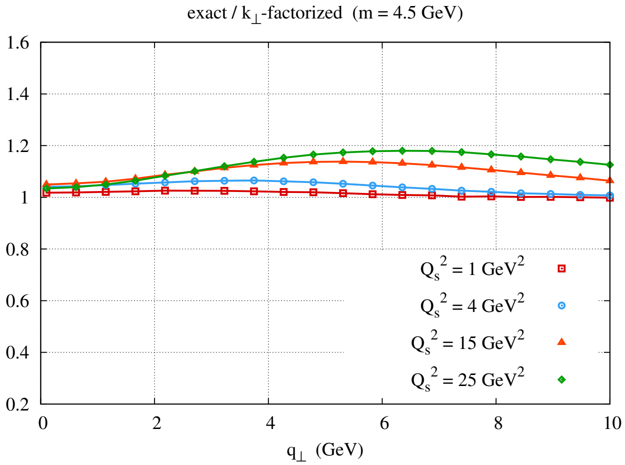
3-point correlator

- 3-point function/2-point function (in the MV model):



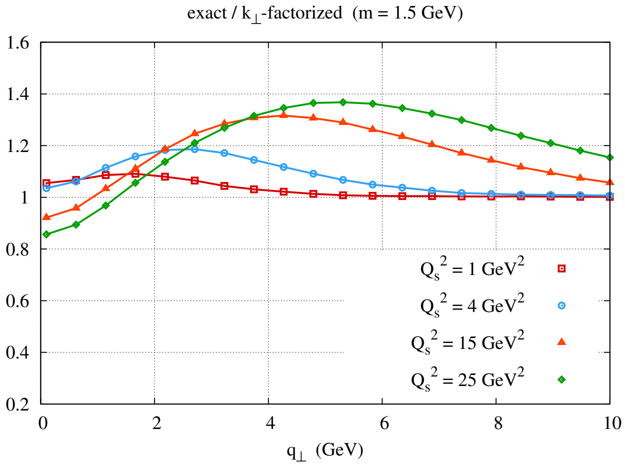
Factorization for b quarks

- Single *b*-quark cross-section :



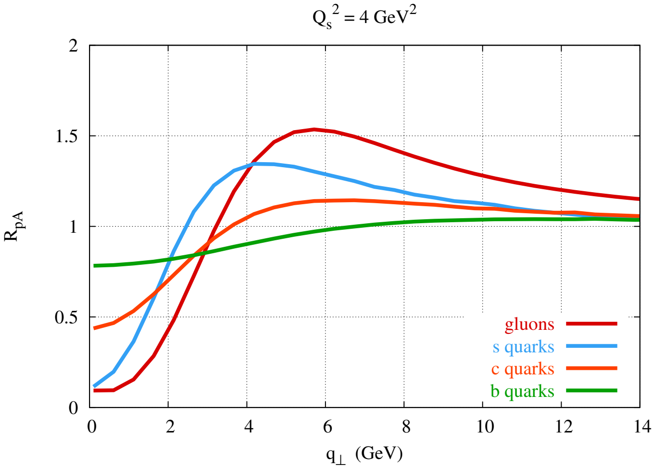
Factorization for c quarks

- Single *c*-quark cross-section :



pA/pp for single particle spectra

- $m = 1.5\ \mathrm{GeV}, Q_s^2 = 4\ \mathrm{GeV}^2$ (MV model)



Summary

- The effects of saturation and multiple scatterings can be divided in two classes:
  - terms that break factorization,
  - terms that could be absorbed into the usual gluon distributions

- In principle, the terms that break factorization may be seen by comparing the cross-sections for various final states produced in the same collisions. However, these effects are only a fairly small part of all the saturation effects

  ▷ it could happen that saturation is there, and is not seen as saturation because of this

- In order to see the terms that do not manifestly break factorization, one has to vary the target, and look for anomalies in the target size dependence