A novel jet definition for forward radiation in diffractive Deep Inelastic Scattering

Federico Ceccopieri, L.T.

Phys.Lett. B655,15-25 (2007)

A New Fracture Function Approach to QCD Initial State Radiation.
Federico A. Ceccopieri (Parma U.) , Luca Trentadue (Parma U. & INFN, Parma) . May 2007. 10pp.

Outline

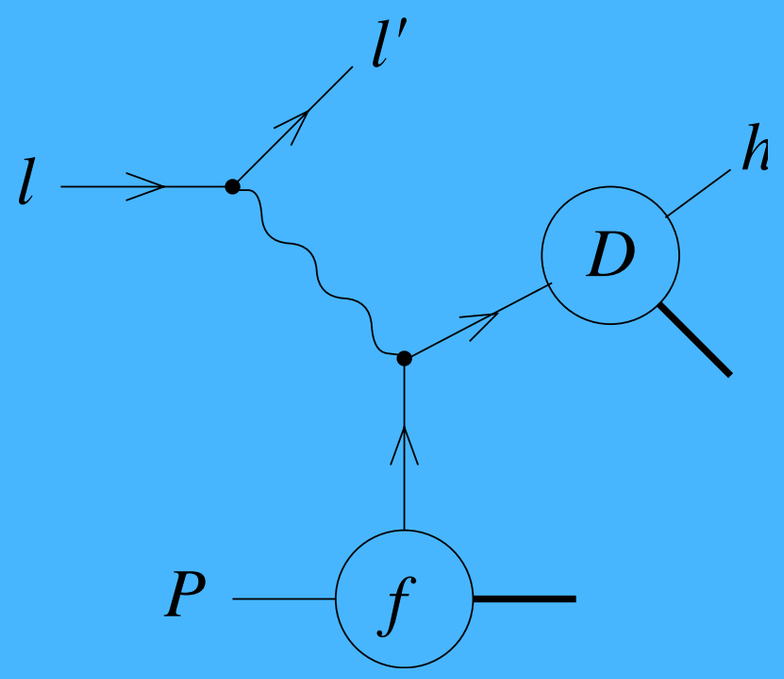
- Fracture Functions

- Semi-inclusive distributions (initial state)
- Applications (Diffraction)

Initial states in hadronic processes:
Multiple hadrons within FF formalism

New Jet-like Fracture Functions

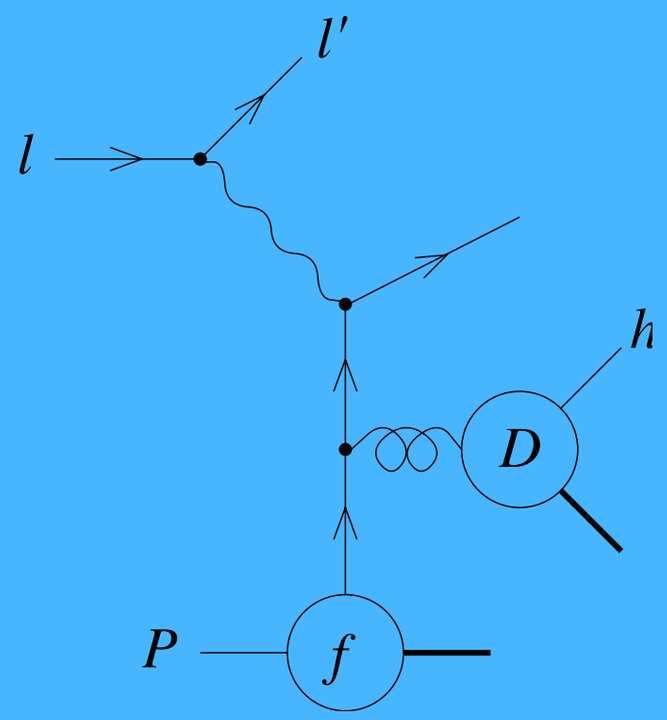
Semi Inclusive Deep Inelastic Scattering
Current Fragmentation

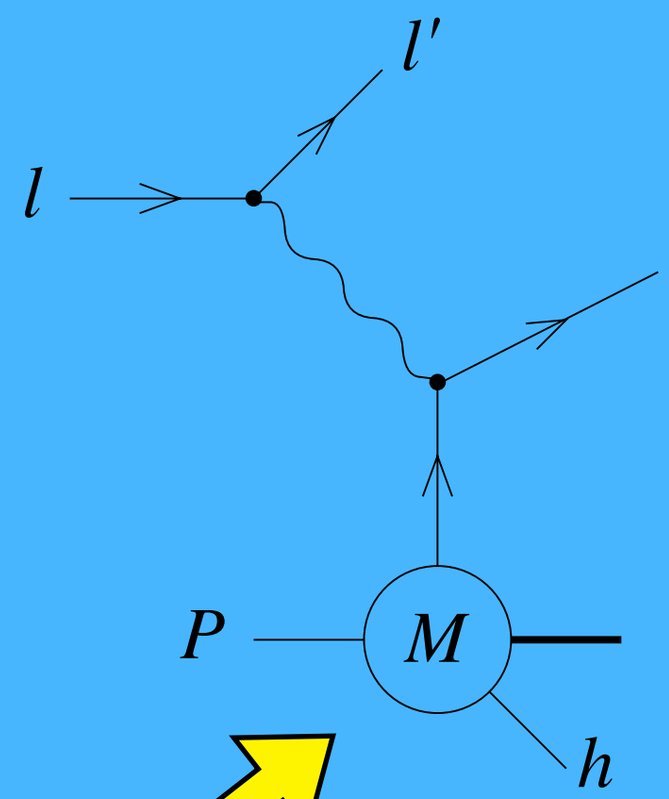


$$\sigma_C = \int \frac{dx'}{x'} \frac{dz'}{z'} F_P^i(x', Q^2)\, \hat{\sigma}_{ij}(x/x', z_h/z', Q^2)\, D_h^j(z', Q^2)$$

Hadrons may also come from elsewhere !

Semi Inclusive Deep Inelastic Scattering
Target Fragmentation





Fracture Function

$$\sigma_T = \int \frac{dx'}{x'} M^i_{Ph'}(x', z_h, Q^2)\, \hat{\sigma}_i(x/x', Q^2)$$

L.Trentadue and G.Veneziano, Phys.Lett. **B323** (1994) 201

Fracture Functions = Fragmentation + structure

Properties:

- Do not depend on the arbitrary choosen scale Q_0^2 i.e.

$$\frac{\partial}{\partial Q_0^2}\, M^j_{p,h}(x,z,Q^2) \;=\; 0$$

- Both $D_l^h(x,Q^2)$ and $F_p^i(x,Q^2)$ satisfy the usual Altarelli Parisi evolution equations and $\sum_h \int_0^1 \; dz \; z \; D_l^h(x,Q^2) = 1$ and $\sum_i \int_0^1 \; dx \; x \; F_p^i(x,Q^2) = 1$ with

$$\sum_i \int_0^1 \; du \; u \; P_i^j(u) \;=\; 0$$

$M^j_{p,h}(x,z,Q^2)$ satisfies the momentum sum rule:

$$\sum_h \int_0^1 \; dz \; z \; M^j_{p,h}(x,z,Q^2) \;=\; (1-x)\; F_p^j(x,Q^2)$$

accounting for the s-channel unitarity constraint.

The combination of the Fracture Function with the target-Fragmentation evolution
gives the evolution equation:

$$\frac{\partial}{\partial \log Q^2} M_{i,h/p}\left(\xi,\zeta,Q^2\right) = \frac{\alpha_s(Q^2)}{2\pi}\int_{\xi/(1-\zeta)}^{1}\frac{du}{u}\,P_j^i(u)\,M_{j,h/N}\left(\frac{\xi}{u},\zeta,Q^2\right)$$
$$+\frac{\alpha_s(Q^2)}{2\pi}\int_{\xi}^{\xi/(\xi+\zeta)}\frac{du}{\xi(1-u)}\,\hat{P}_j^{i,l}(u)\,f_{j/p}\left(\frac{\xi}{u},Q^2\right)\,D_{h/l}\left(\frac{\zeta u}{\xi(1-u)},Q^2\right)$$

Homogeneus (usual Altarelli Parisi type) term + Inhomogeneus term

Several properties:

1) Fracture Functions satisfy unitarity

$$\sum_h \int_0^1 dz\, z\, M^j_{p,h}(x,z;\mu^2) = (1-x)\cdot F^j_p(x,\mu^2).$$

2) Fracture Functions factorize

*M.Grazzini, L.Trentadue and G.Veneziano, Nucl.Phys.B*519(1998)394
*J.Collins, Phys.Rev.D*57(1998)3051

3) Extended M(x,z,t,Q^2)-Fracture Functions satisfy Gribov-Lipatov-Altarelli-Parisi type evolution equations

$$Q^2\frac{\partial}{\partial Q^2}\mathcal{M}^j_{A,A'}(x,z,t,Q^2) = \frac{\alpha_S(Q^2)}{2\pi}\int_{\frac{x}{1-z}}^1 \frac{du}{u}P^j_i(u)\mathcal{M}^i_{A,A'}(x/u,z,t,Q^2)$$

Applications:

Diffraction:

$$e^-(k) + A(P) \to e^-(k') + A(P') + X$$

- According to the Ingelman-Schlein model,

[G.Ingelman and P.Schlein, Phys.Lett. **B152** (1985) 256.]

the diffractive structure function $F_2^{diff}(x, Q^2)$ is

$$F_2^{diff}(x, Q^2) = \sum_a \int d\xi \frac{df_{a/P}^{diff}(\xi, \mu, x_P, t)}{dx_P dt} \hat{\sigma}(\frac{x}{\xi}, Q^2, \mu)$$

But

$$\int_0^\infty d|t \, \frac{df_{a/P}^{diff}(\xi, \mu, x_P, t)}{dx_P dt} = M_{AA}(\xi, \mu, z = 1 - x_P)$$

- Analyzing $F_2^{D(3)}(\beta, \mu, x_P)$, H1 collaboration at HERA observed a possible Q^2 dependence of the diffractive distributions

$$\Downarrow$$

A logarithmic dependence on Q^2 is implicitely contained in fracture functions!

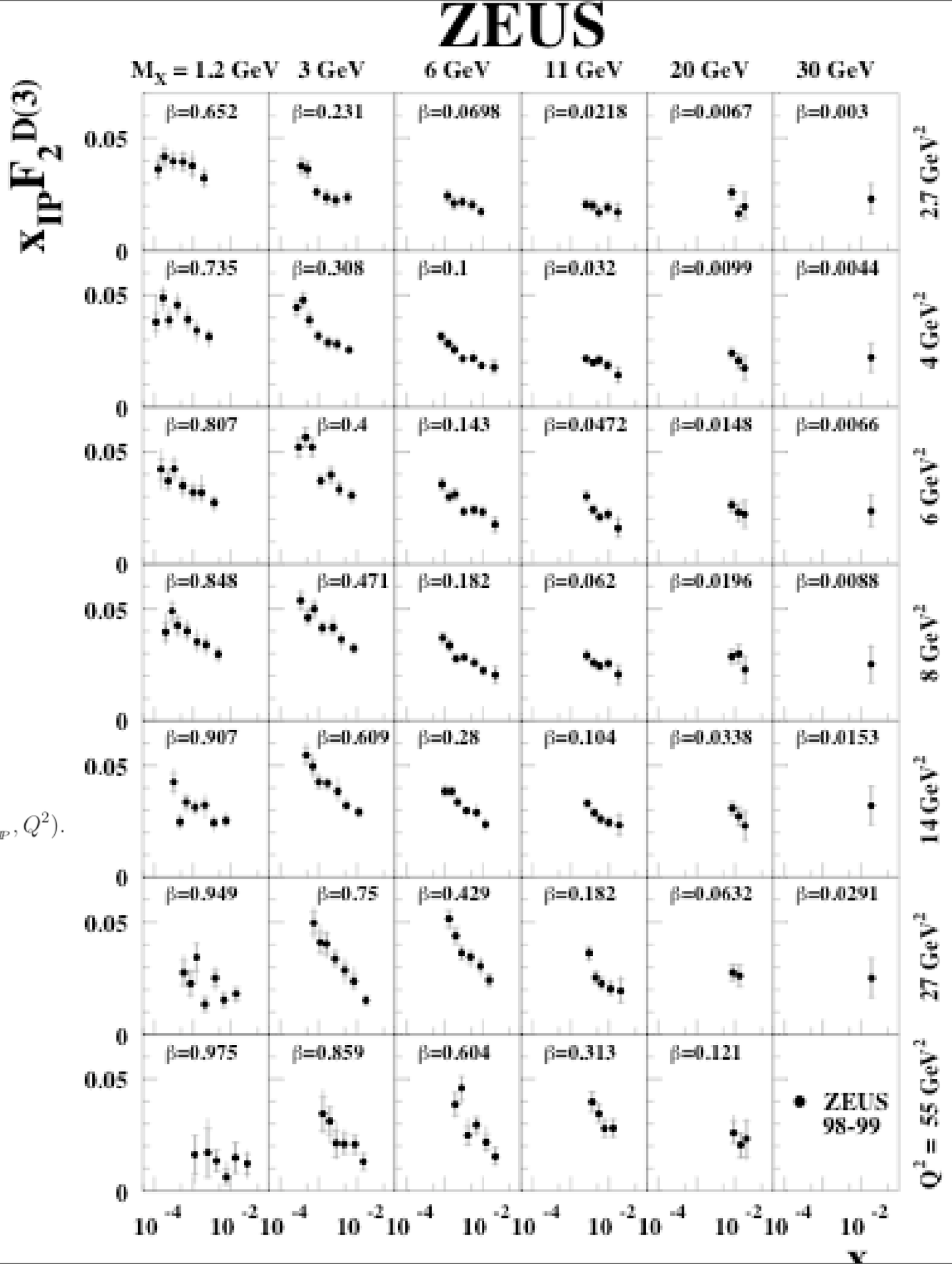
DESY 05-011
hep-ex/0501060
January 2005

Study of deep inelastic inclusive and diffractive scattering with the ZEUS forward plug calorimeter

ZEUS Collaboration

section. The data are also presented in terms of the diffractive structure function, $F_2^{D(3)}(\beta, x_{I\!P}, Q^2)$, of the proton. For fixed β, the Q^2 dependence of $x_{I\!P} F_2^{D(3)}$ changes with $x_{I\!P}$, in violation of Regge factorisation. For fixed $x_{I\!P}$, $x_{I\!P} F_2^{D(3)}$ rises as $\beta \to 0$, the rise accelerating with increasing Q^2. These positive scaling violations suggest substantial contributions of perturbative effects in the diffractive DIS cross section.

$$\frac{1}{2M_X}\frac{d\sigma^{\rm diff}_{\gamma^* p\to XN}(M_X,W,Q^2)}{dM_X}=\frac{4\pi^2\alpha}{Q^2(Q^2+M_X^2)}x_{I\!P}F_2^{D(3)}(\beta,x_{I\!P},Q^2).$$



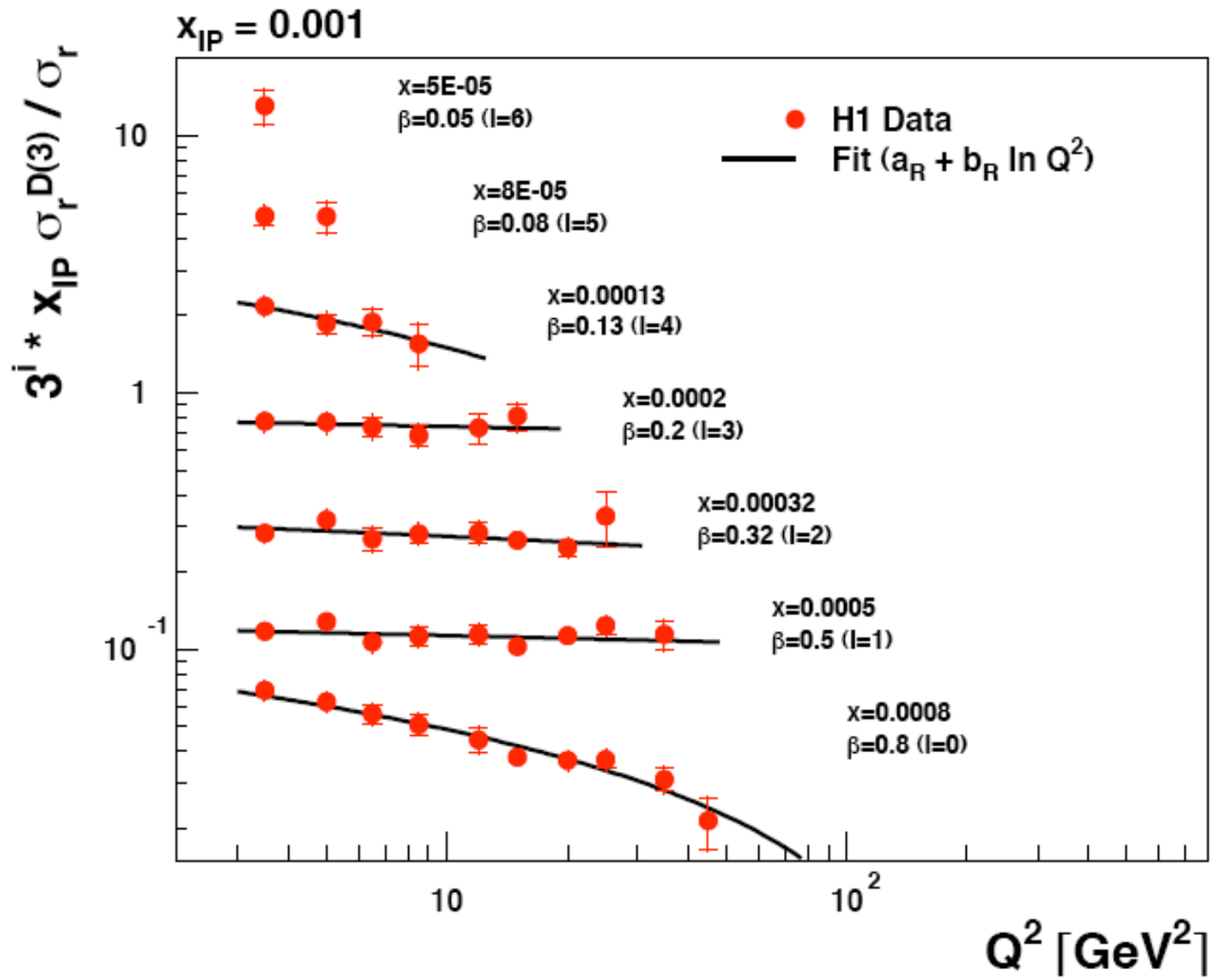


Fig. 14. As Fig.13 with $x_{I\!P} = 0.001$. From Ref.[38]

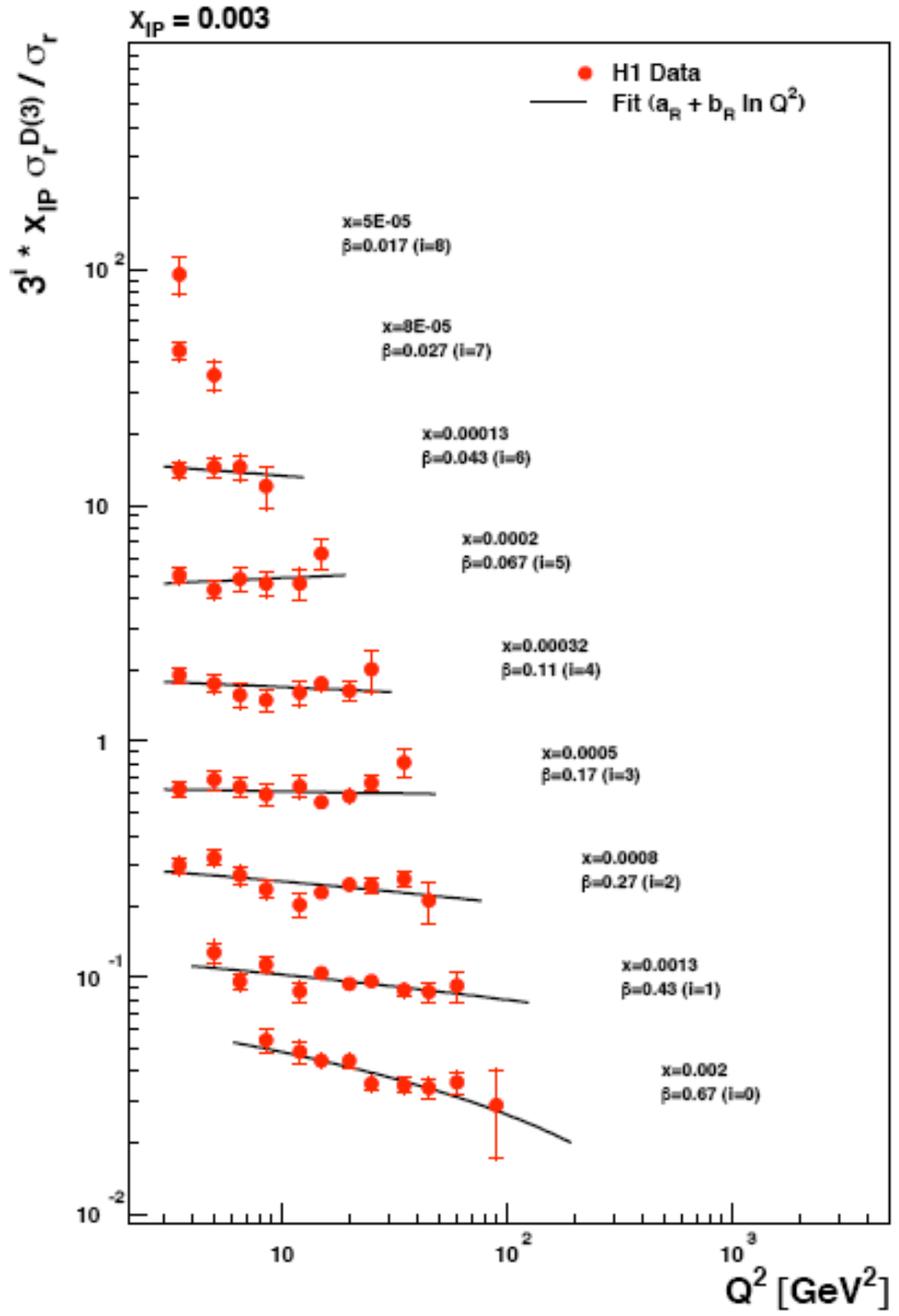
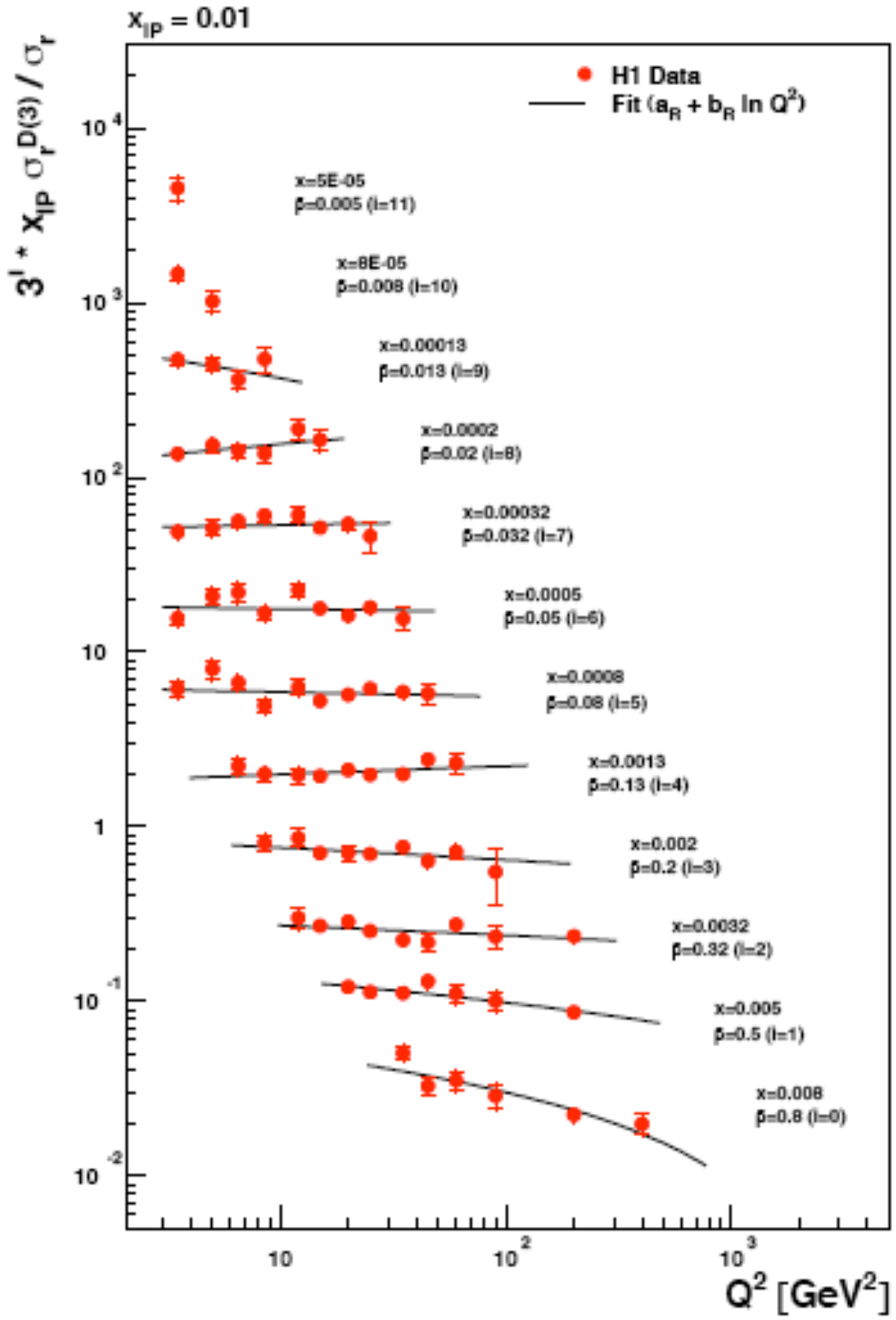


Fig. 15. As Fig.13 with $x_{I\!P} = 0.003$. From Ref.[38]


x_{IP} = 0.01
$3^{l} * x_{IP}\,\sigma_r^{D(3)} / \sigma_r$
H1 Data
Fit ($a_R + b_R \ln Q^2$)
x=5E-05
β=0.005 (l=11)
x=8E-05
β=0.008 (l=10)
x=0.00013
β=0.013 (l=9)
x=0.0002
β=0.02 (l=8)
x=0.00032
β=0.032 (l=7)
x=0.0005
β=0.05 (l=6)
x=0.0008
β=0.08 (l=5)
x=0.0013
β=0.13 (l=4)
x=0.002
β=0.2 (l=3)
x=0.0032
β=0.32 (l=2)
x=0.005
β=0.5 (l=1)
x=0.008
β=0.8 (l=0)
10^4
10^3
10^2
10
1
10^{-1}
10^{-2}
10
10^2
10^3
Q^2 [GeV^2]

Next-to-leading Fracture Functions

Published in **Nucl.Phys.B673:357-384,2003**

Next to leading order evolution of SIDIS processes in the forward region*

A. Daleo[1,†] and R. Sassot[2]

[1] *Laboratorio de Física Teórica*
Departamento de Física, Facultad de Ciencias Exactas
Universidad Nacional de La Plata
C.C. 67 - 1900 La Plata, Argentina
[2] *Departamento de Física, Universidad de Buenos Aires*
Ciudad Universitaria, Pab.1 (1428) Buenos Aires, Argentina

We compute the order α_s^2 quark initiated corrections to semi-inclusive deep inelastic scattering extending the approach developed recently for the gluon contributions. With these corrections we complete the order α_s^2 QCD description of these processes, verifying explicitly the factorization of collinear singularities. We also obtain the corresponding NLO evolution kernels, relevant for the scale dependence of fracture functions. We compare the non-homogeneous evolution effects driven by these kernels with those obtained at leading order accuracy and discuss their phenomenological implications.

PACS numbers: 12.38.Bx, 13.85.Ni
Keywords: Semi-Inclusive DIS; perturbative QCD; Fracture functions

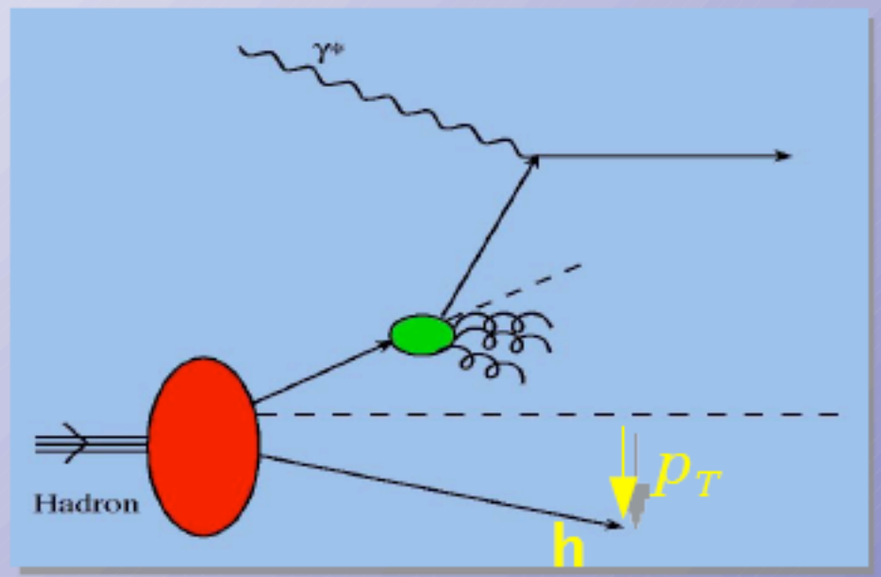
$$\frac{\partial\, M^r_{i,h/P}(\xi,\zeta,M^2)}{\partial \log M^2} = \frac{\alpha_s(M^2)}{2\pi}\int_{\frac{\xi}{1-\zeta}}^1 \frac{du}{u}\left[P^{(0)}_{i\leftarrow j}(u) + \frac{\alpha_s(M^2)}{2\pi}\,P^{(1)}_{i\leftarrow j}(u)\right] M^r_{j,h/P}\left(\frac{\xi}{u},\zeta,M^2\right)$$

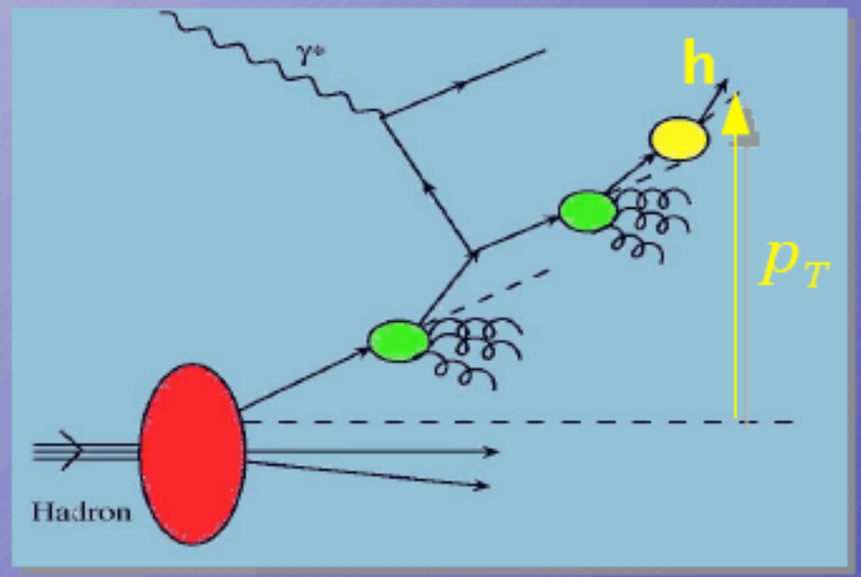
$$+\frac{\alpha_s(M^2)}{2\pi}\,\frac{1}{\xi}\int_{\xi}^{\frac{\xi}{\xi+\zeta}}\frac{du}{u}\int_{\frac{\xi}{\xi}}^{\frac{1-u}{u}}\frac{dv}{v}\left[\tilde{P}^{(0)}_{ki\leftarrow j}(u,v) + \frac{\alpha_s(M^2)}{2\pi}\,P^{(1)}_{ki\leftarrow j}(u,v)\right] f^r_{j/P}\left(\frac{\xi}{u},M^2\right) D^r_{h/k}\left(\frac{\zeta}{\xi\,v},M^2\right),$$

Transverse Momenta

F. Ceccopieri, L.T.,
Phys.Lett.B (2006)

- Fracture functions TMD evolution equation





$$Q^2 \frac{\partial M^j_{p,h}(Q^2, x_B, \boldsymbol{k_T}, z_h, \boldsymbol{p_T})}{\partial Q^2} =$$

$$= \sum_c \frac{\alpha_s(Q^2)}{2\pi} \int \frac{du}{u^3} \int \frac{d^2\boldsymbol{q_T}}{\pi} \delta[(1-u)Q^2 - \boldsymbol{q_T^2}] P^j_i(u) M^i_{p,h}\left(Q^2, \frac{x_B}{u}, \frac{\boldsymbol{k_T} - \boldsymbol{q_T}}{u}, z_h, \boldsymbol{p_T}\right) +$$

$$+ \sum_c \frac{\alpha_s(Q^2)}{2\pi} \int \frac{du}{u} \int \frac{d^2\boldsymbol{q_T}}{\pi} \delta[(1-u)Q^2 - \boldsymbol{q_T^2}] \frac{u}{x_B(1-u)} \hat{P}^{j,l}_i(u) \cdot$$

$$\cdot F^i_P\left(Q^2, \frac{x_B}{u}, \frac{\boldsymbol{k_T} - \boldsymbol{q_T}}{u}\right) D^h_l\left(Q^2, \frac{z_h u}{x_B(1-u)}, \boldsymbol{p_T} - \frac{z_h u}{x_B(1-u)} \boldsymbol{q_T}\right)$$

Increasing phase space

Factorization:

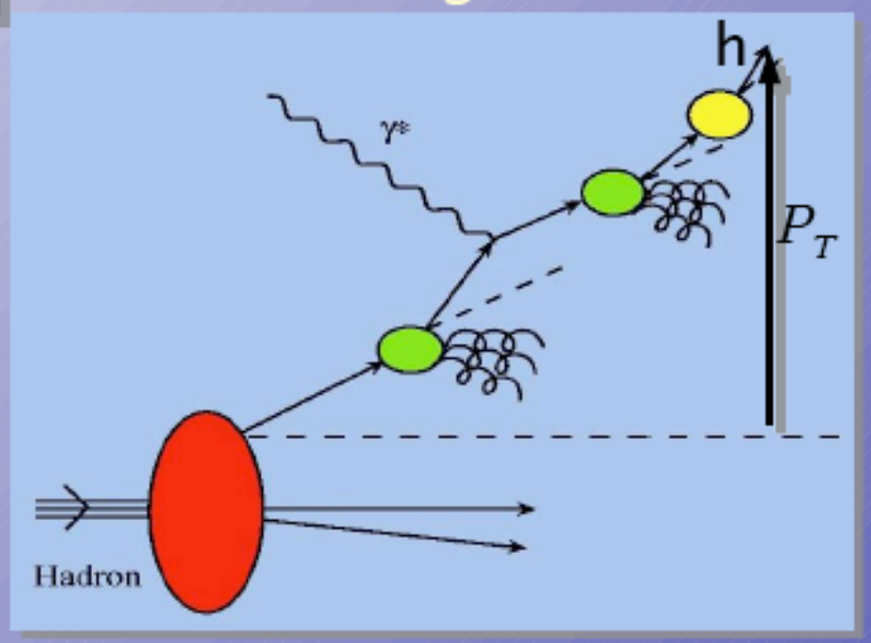
$$d\sigma \propto f(x) \circ D(z)$$

$$d\sigma \propto f(x) \circ D(z) + M(x,z)$$

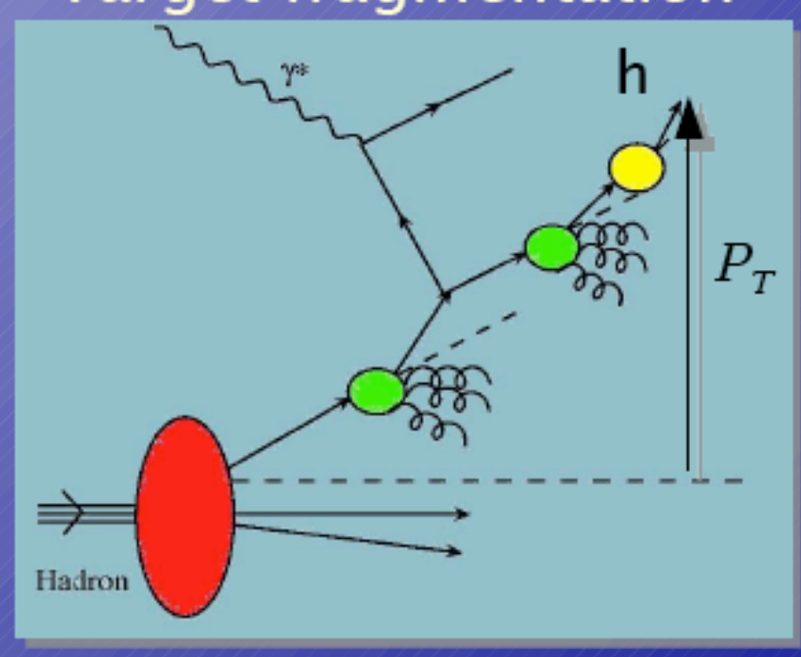
$$\frac{d\sigma}{d\boldsymbol{P}_T} \propto f(x,\boldsymbol{k}_T) \circ D(z,\boldsymbol{p}_T)$$ [9]

$$\frac{d\sigma}{d\boldsymbol{P}_T} \propto f(x,\boldsymbol{k}_T) \circ D(z,\boldsymbol{p}_T) + M(x,\boldsymbol{k}_T,z,\boldsymbol{p}_T)$$

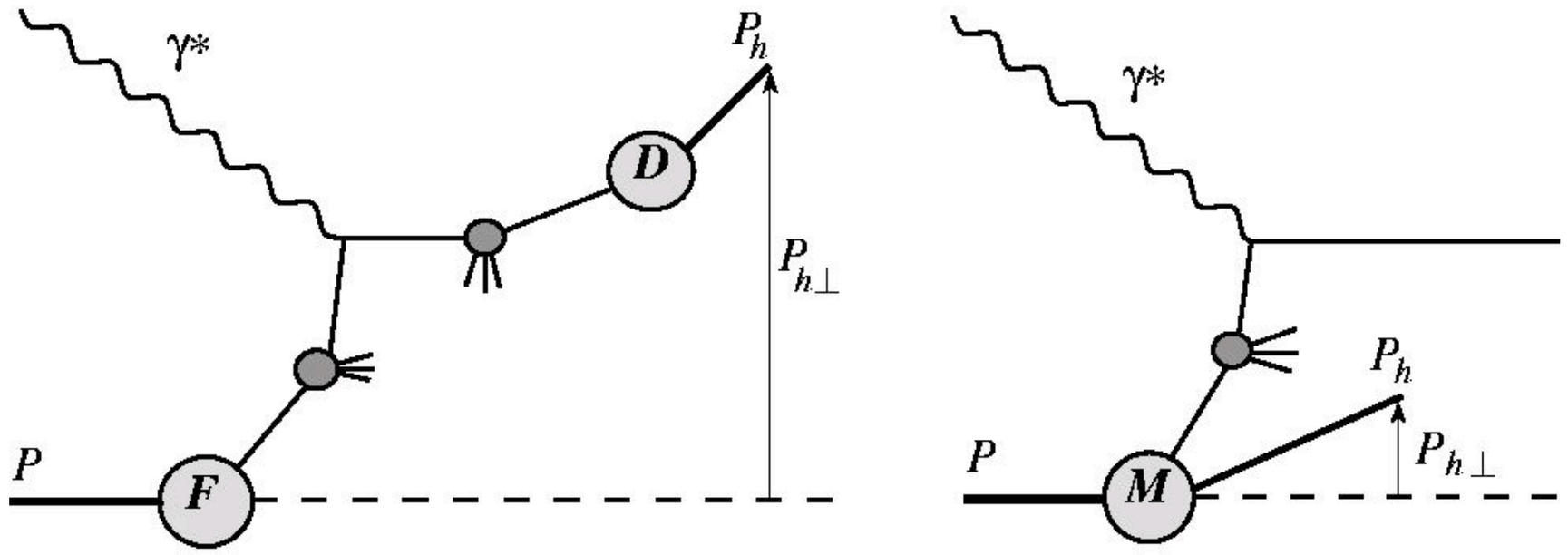
Current fragmentation



Target fragmentation



[9] X. Ji, J. Ma, F. Yuan *Phys. Rev.* D71, 034005 (2005)



$$Q^2\frac{\partial\mathcal{M}^i_{P,h}(x,\boldsymbol{k}_\perp,z,\boldsymbol{p}_\perp,Q^2)}{\partial Q^2}=\frac{\alpha_s(Q^2)}{2\pi}\Bigg\{\int_{\frac{x}{1-z}}^1\frac{du}{u^3}\,P^i_j(u)\int\frac{d^2\boldsymbol{q}_\perp}{\pi}\,\delta(\,(1-u)Q^2-$$

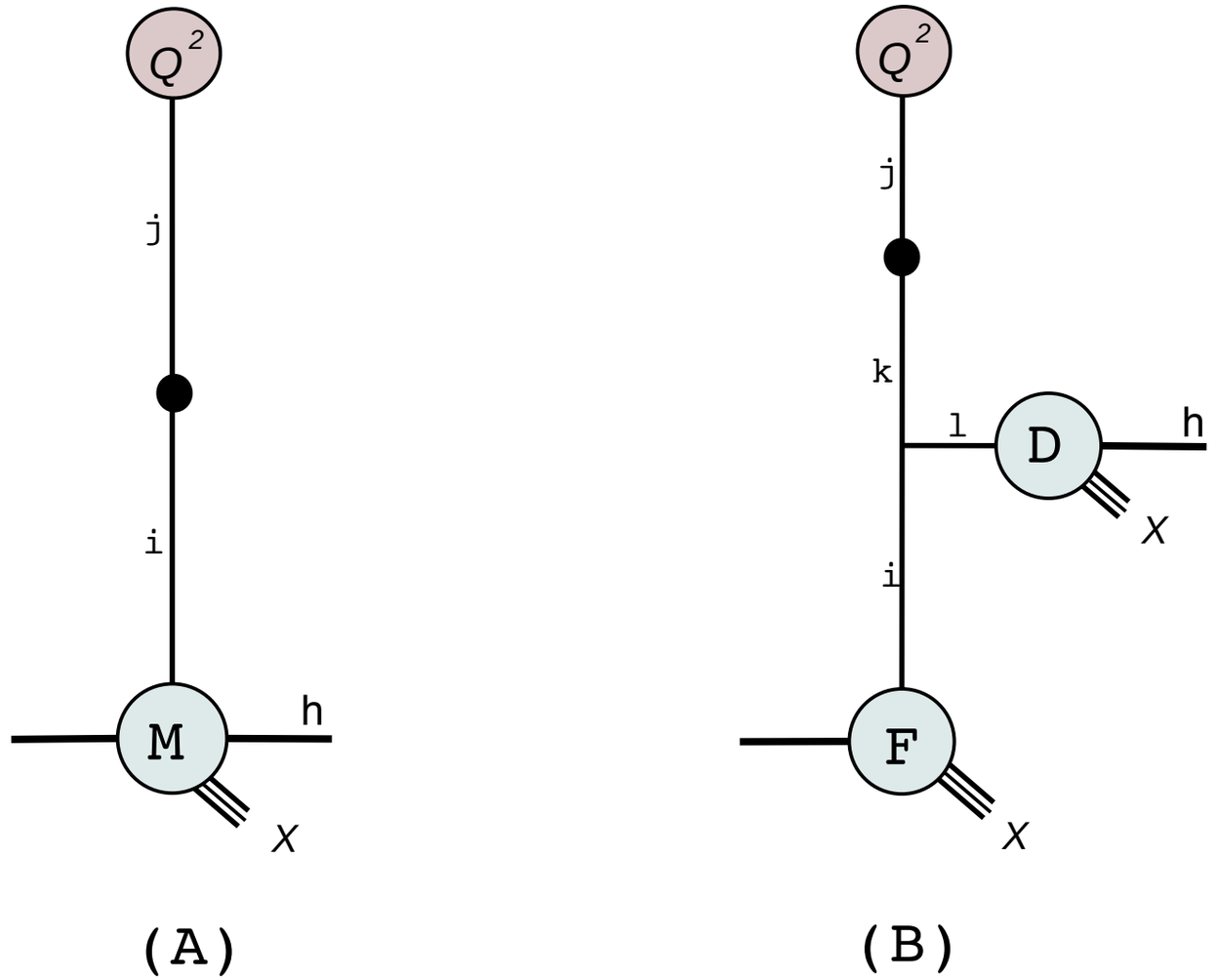
$$\cdot\mathcal{M}^j_{P,h}\Big(Q^2,\frac{x}{u},\frac{\boldsymbol{k}_\perp-\boldsymbol{q}_\perp}{u},z,\boldsymbol{p}_\perp\Big)+\int_x^{\frac{x}{x+z}}\frac{du}{x(1-u)u^2}\hat{P}^{i,l}_j(u)\frac{d^2\boldsymbol{q}_\perp}{\pi}\,\delta(\,(1-u)Q^2-$$

$$\cdot\mathcal{F}^j_P\Big(\frac{x}{u},\frac{\boldsymbol{k}_\perp-\boldsymbol{q}_\perp}{u},Q^2\Big)\,\mathcal{D}^h_l\Big(\frac{zu}{x(1-u)},\boldsymbol{p}_\perp-\frac{zu}{x(1-u)}\,\boldsymbol{q}_\perp,Q^2\Big)\Bigg\}\,.$$

$$\int d^2\boldsymbol{k}_\perp\int d^2\boldsymbol{p}_\perp\mathcal{M}^i_{P,h}(x,\boldsymbol{k}_\perp,z,\boldsymbol{p}_\perp,Q^2)=\mathcal{M}^i_{P,h}(x,z,Q^2)\,,$$

Extension to multiple hadron distributions

$$
\begin{aligned}
M^j_{h/P}(x,z,Y) &= M^j_{A,h/P}(x,z,Y) + M^j_{B,h/P}(x,z,Y)\,, \\
M^j_{A,h/P}(x,z,Y) &= \int_x^{1-z} \frac{dw}{w} E^j_i\Big(\frac{x}{w}, Y-y_0\Big) M^i_{A,h/P}(w,z,y_0)\,, \\
M^j_{B,h/P}(x,z,Y) &= \int_{y_0}^{Y} dy \int_{x+z}^{1} \frac{dw}{w^2} \int_{\frac{x}{w}}^{1-\frac{z}{w}} \frac{du}{u(1-u)}\,\cdot \\
&\quad \cdot E^j_k\Big(\frac{x}{wu}, Y-y\Big) \hat{P}^{kl}_i(u) D^h_l\Big(\frac{z}{w(1-u)}, y\Big) F^i_P(w,y)\,.
\end{aligned}
$$



$$
\begin{aligned}
\frac{\partial}{\partial Y} M^j_{A,h/P}(x,z,Y) &= \int_{\frac{x}{1-z}}^1 \frac{du}{u} P^j_i(u) M^j_{A,h/P}(x/u,z,Y)\,, \\
\frac{\partial}{\partial Y} M^j_{B,h/P}(x,z,Y) &= \int_{\frac{x}{1-z}}^1 \frac{du}{u} P^j_i(u) M^j_{B,h/P}(x/u,z,Y) + \\
&\qquad \int_x^{\frac{x}{x+z}} \frac{du}{u} \frac{u}{x(1-u)} \hat{P}^{jl}_i(u) D^h_l\Big(\frac{zu}{x(1-u)},Y\Big) F^i_P(x/u,Y)\,.
\end{aligned}
$$

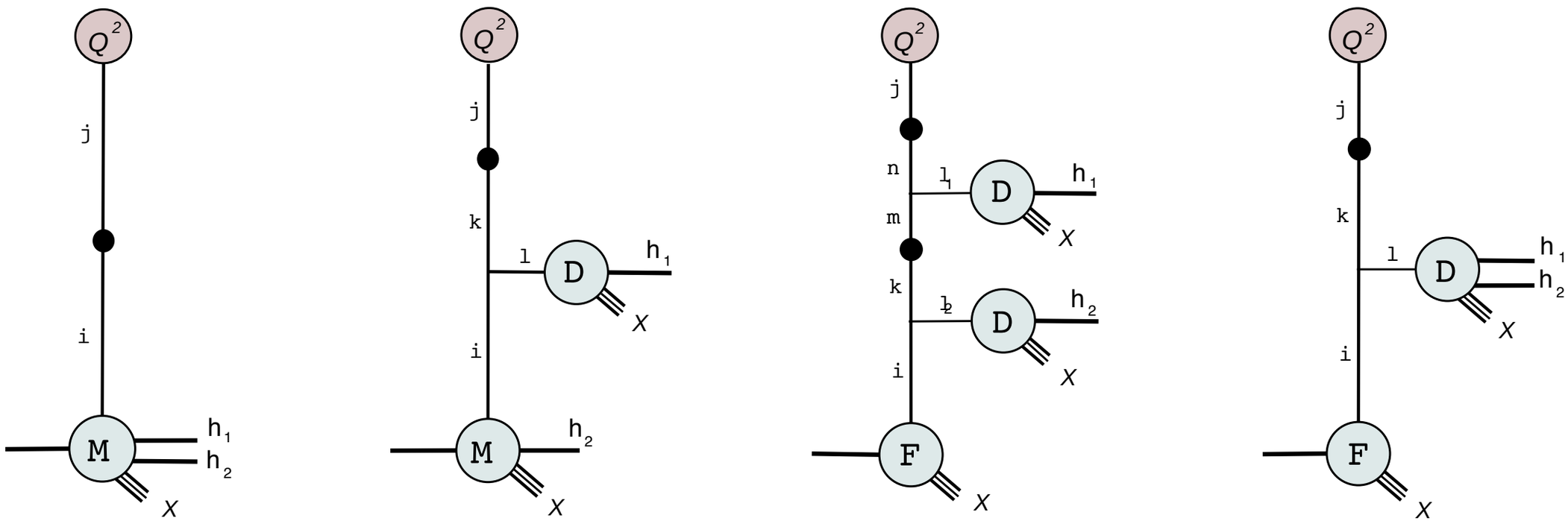
$$
Y = \frac{1}{2\pi\beta_0} \ln\Big[\frac{\alpha_s(\mu_R^2)}{\alpha_s(Q^2)}\Big], \qquad dY = \frac{\alpha_s(Q^2)}{2\pi}\frac{dQ^2}{Q^2}\,.
$$

$$
\begin{aligned}
Q^2\frac{\partial}{\partial Q^2} M^j_{h/P}(x,z,Q^2) &= \frac{\alpha_s(Q^2)}{2\pi} \int_{\frac{x}{1-z}}^1 \frac{du}{u} P^j_i(u) M^i_{h/P}(x/u,z,Q^2) \; + \\
&\qquad \frac{\alpha_s(Q^2)}{2\pi} \int_x^{\frac{x}{x+z}} \frac{du}{u} \frac{u}{x(1-u)} \hat{P}^{jl}_i(u) D^h_l\Big(\frac{zu}{x(1-u)},Q^2\Big) F^i_P(x/u,Q^2)\,.
\end{aligned}
$$

Di-hadron Fracture Functions

$$\sigma_T = \int \frac{du}{u} M^j_{h_1,h_2/P}(u, z_1, z_2, Q^2)\, \hat{\sigma}_j(x/u, Q^2)\,.$$

$$M^j_{h_1,h_2/P}(x, z_1, z_2, Y) = \sum_{X=A,B,C,D} M^j_{X,h_1,h_2/P}(x, z_1, z_2, Y)\,.$$



$$
\begin{aligned}
M^j_{A,h_1,h_2/P}(x,z_1,z_2,Y) &= \int_x^{1-z_1-z_2} \frac{dw}{w} E^j_i\Big(\frac{x}{w}, Y-y_0\Big) M^i_{A,h_1,h_2/P}(w,z_1,z_2,y_0)\,,\\
M^j_{B,h_1,h_2/P}(x,z_1,z_2,Y) &= \int_{y_0}^{Y} dy \int_{x+z_1}^{1-z_2} \frac{dw}{w^2} \int_{x/w}^{1-z_1/w} \frac{du}{u(1-u)} E^j_k\Big(\frac{x}{wu}, Y-y\Big) \hat{P}^{kl}_i(u)\cdot\\
&\quad M^i_{A,h_2/P}(w,z_2,y) D^{h_1}_l\Big(\frac{z_1}{w(1-u)}, y\Big) + (h_1,z_1)\leftrightarrow(h_2,z_2)\,,\\
M^j_{C,h_1,h_2/P}(x,z_1,z_2,Y) &= \int_{y_0}^{Y} dy_2 \int_{y_2}^{Y} dy_1 \int_{x+z_1+z_2}^{1} \frac{dw}{w^2} \int_{x+z_1}^{w-z_2} \frac{dx_2}{x_2^2} \int_{x/x_2}^{1-z_1/x_2} \frac{du_1}{u_1(1-u_1)} \hat{P}^{nl_1}_m(u_1)\cdot\\
&\quad \int_{x_2/w}^{1-z_2/w} \frac{du_2}{u_2(1-u_2)} \hat{P}^{kl_2}_i(u_2) E^m_k\Big(\frac{x_2}{wu_2}, y_1-y_2\Big) E^j_n\Big(\frac{x}{u_1x_2}, Y-y_1\Big)\cdot\\
&\quad D^{h_2}_{l_2}\Big(\frac{z_2}{w(1-u_2)}, y_2\Big) D^{h_1}_{l_1}\Big(\frac{z_1}{x_2(1-u_1)}, y_1\Big) F^i_P(w,y_1) + (h_1,z_1)\leftrightarrow(h_2,z_2)\,,\\
M^j_{D,h_1,h_2/P}(x,z_1,z_2,Y) &= \int_{y_0}^{Y} dy \int_{x+z_1+z_2}^{1} \frac{dw}{w} \int_{x/w}^{1-(z_1+z_2)/w} \frac{1}{w^2(1-u)^2} \frac{du}{u} E^j_k\Big(\frac{x}{wu}, Y-y\Big)\cdot\\
&\quad \hat{P}^{kl}_i(u) F^i_P(w,y) D^{h_1,h_2}_l\Big(\frac{z_1}{w(1-u)}, \frac{z_2}{w(1-u)}, y\Big)\,.
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial}{\partial Y} M^j_{A,h_1,h_2/P} &= \int_{\frac{x}{1-z_1-z_2}}^1 \frac{du}{u} P^j_i(u) M^i_{A,h_1,h_2/P}(x/u,z_1,z_2,Y) + \,, \\
\frac{\partial}{\partial Y} M^j_{B,h_1,h_2/P} &= \int_{\frac{x}{1-z_2}}^{\frac{x}{x+z_1}} \frac{du}{u} \frac{u}{x(1-u)} \hat{P}^{jl}_i(u) M^i_{A,h_2/P}(x/u,z_2,Y) D^{h_1}_l\left(\frac{z_1 u}{x(1-u)},Y\right) + \\
&\quad + (h_1,z_1) \leftrightarrow (h_2,z_2) + \int_{\frac{x}{1-z_1-z_2}}^1 \frac{du}{u} P^j_i(u) M^i_{B,h_1,h_2/P}(x/u,z_1,z_2,Y)\,, \\
\frac{\partial}{\partial Y} M^j_{C,h_1,h_2/P} &= \int_{\frac{x}{1-z_2}}^{\frac{x}{x+z_1}} \frac{du}{u} \frac{u}{x(1-u)} \hat{P}^{jl_1}_m(u) M^m_{B,h_2/P}(x/u,z_2,Y) D^{h_1}_{l_1}\left(\frac{z_1 u}{x(1-u)},Y\right) \\
&\quad + (h_1,z_1) \leftrightarrow (h_2,z_2) + \int_{\frac{x}{1-z_1-z_2}}^1 \frac{du}{u} P^j_i(u) M^i_{C,h_1,h_2/P}(x/u,z_1,z_2,Y)\,, \\
\frac{\partial}{\partial Y} M^j_{D,h_1,h_2/P} &= \int_x^{\frac{x}{x+z_1+z_2}} \frac{du}{u} \frac{u^2}{x^2(1-u)^2} \hat{P}^{jl}_i(u) F^i_P(x/u,Y) D^{h_1,h_2}_l\left(\frac{z_1 u}{x(1-u)},\frac{z_2 u}{x(1-u)},Y\right) \\
&\quad + \int_{\frac{x}{1-z_1-z_2}}^1 \frac{du}{u} P^j_i(u) M^i_{D,h_1,h_2/P}(x/u,z_1,z_2,Y).
\end{aligned}
$$

$$Q^2\frac{\partial}{\partial Q^2}M^j_{h_1,h_2/P}(x,z_1,z_2,Q^2)=\frac{\alpha_s(Q^2)}{2\pi}\int_{\frac{x}{1-z_1-z_2}}^1\frac{du}{u}P^j_i(u)M^i_{h_1,h_2/P}(x/u,z_1,z_2,Q^2)+$$

$$+\frac{\alpha_s(Q^2)}{2\pi}\int_{\frac{x}{1-z_2}}^{\frac{x}{x+z_1}}\frac{du}{u}\frac{u}{x(1-u)}\hat{P}^{jl}_i(u)M^i_{h_2/P}(x/u,z_2,Q^2)D^{h_1}_l\left(\frac{z_1u}{x(1-u)},Q^2\right)+(h_1,z_1)\leftrightarrow(h_2,z_2)+$$

$$+\frac{\alpha_s(Q^2)}{2\pi}\int_x^{\frac{x}{x+z_1+z_2}}\frac{du}{u}\frac{u^2}{x^2(1-u)^2}\hat{P}^{jl}_i(u)F^i_P(x/u,Q^2)D^{h_1,h_2}_l\left(\frac{z_1u}{x(1-u)},\frac{z_2u}{x(1-u)},Q^2\right).$$

$$\sum_{h_2}\int dz_2\,z_2\,M^i_{h_1,h_2/P}(x,z_1,z_2,Q^2)=(1-x-z_1)M^i_{h_1/P}(x,z_1,Q^2)\,,$$

$$\sum_{h_1,h_2}\int dz_1\,z_1\int dz_2\,z_2\,M^i_{2,h_1h_2/P}(x,z_1,z_2,Q^2)=(1-x)F^i_P(x,Q^2)\,.$$

n-hadrons

$$Q^2\frac{\partial M^j_{n/P}}{\partial Q^2}(x,z_1,\ldots,z_n,Q^2)=\frac{\alpha_s(Q^2)}{2\pi}\int_{\frac{x}{1-\sum_{k=1}^n z_k}}^1\frac{du}{u}P_i^j(u)M^i_{n/P}(x/u,z_1,..,z_n,Q^2)+\frac{\alpha_s(Q^2)}{2\pi}\sum_{q=1}^{n-1}\mathcal{P}_n\{h,z\}$$

$$\cdot\int_{\frac{x}{1-\sum_{k=q+1}^n z_k}}^{\frac{x}{x+\sum_{k=1}^q z_k}}\frac{du}{u}\Big(\frac{u}{x(1-u)}\Big)^q\hat{P}_i^{jl}(u)M^i_{n-q}(x/u,z_{q+1},..,z_n,Q^2)D_l^q\Big(\frac{z_1u}{x(1-u)},..,\frac{z_qu}{x(1-u)},Q^2\Big)+$$

$$+\frac{\alpha_s(Q^2)}{2\pi}\int_x^{\frac{x}{x+\sum_k^n z_k}}\frac{du}{u}\Big(\frac{u}{x(1-u)}\Big)^n\hat{P}_i^{jl}(u)F^i_P(x/u,Q^2)D_l^n\Big(\frac{z_1u}{x(1-u)},..,\frac{z_nu}{x(1-u)},Q^2\Big).$$

Jet-like Fracture Functions

for

inclusive initial state radiation

A jet algorithm for Deep Inelastic Scattering and Hadron-Hadron collisions

S. Catani, Y. Dokshitzer, B. Webber, *Phys. Lett.* **B285**,291 (1992)

Differences with respect to e+e- annihilation:

Proton remnants have to be separated from high p_t jets produced both in Current and Target fragmentation

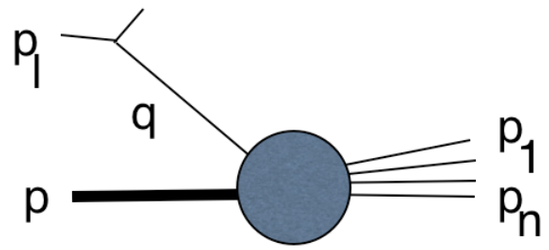
Initial state collinear singularities have to be factorized out in a universal process independent way

a hard scale has to be introduced with respect to e+e- annihilation

Two step procedure:

1) pre-clustering of jets in beam-jets and final state macro- jets
2) resolution of final state jets

$$\ell(p_\ell) + h(p) \to \ell'(p'_\ell) + h_1(p_1) + \cdots h_n(p_n) \ .$$



$$q = p_\ell - p'_\ell \ , \qquad Q^2 = -q^2$$

$$x = \frac{Q^2}{2pq} \ , \qquad y = \frac{pq}{pp_\ell} \ .$$

A) Pre-clustering of hadrons.

a_1) Define a hard scattering scale E_t ($Q^2 \geq E_t^2 \gg \Lambda^2$, Λ being the QCD scale)

a_2) For every final state hadron h_k compute

$$y_{kp} = \frac{2(1-\cos\theta_{kp})}{E_t^2} E_k^2 \ , \tag{7}$$

as well as for every pair h_k, h_l,

$$y_{kl} = \frac{2(1-\cos\theta_{kl})}{E_t^2} \min(E_k^2, E_l^2) \ . \tag{8}$$

a_3) Take the smallest value among $\{y_{kl}, y_{kp}\}$. If y_{ij} is the smallest value and $y_{ij} < 1$, combine (p_i, p_j) into a single pre-cluster ('pseudoparticle') $p_{ij} = p_i + p_j$. If $y_{ip} < 1$ is the smallest value, include p_i in the 'beam jet'.

a_4) Repeat this procedure from step a_2) for all particles (pseudoparticles) not included in the beam jet, until all objects (particles and/or pseudoparticles) have $y_{kl}, y_{kp} > 1$. One ends up with the beam jet and objects which we call final-state macro-jets.

B) Resolving jet structure.

b_1) Define a resolution parameter $y_{\text{cut}} = Q_0^2/E_t^2 < 1$.

b_2) For any hadron in a final-state macro-jet consider y_{ij} and define final-state jets as for e^+e^- annihilation in Sec. 1.1.

In experimental as well as theoretical analyses one can use any hard scattering scale $E_t^2 \leq Q^2$. For fixed values of y_{cut} and Q^2, one can increase the number of hadrons included in the final-state jets (note that we do not count the beam jet as a final-state jet) by decreasing the value of E_t.

$$F_2(x,Q^2) = \sum_{n=1}^{\infty} F_2^{(n)}(x,Q^2)\,.$$

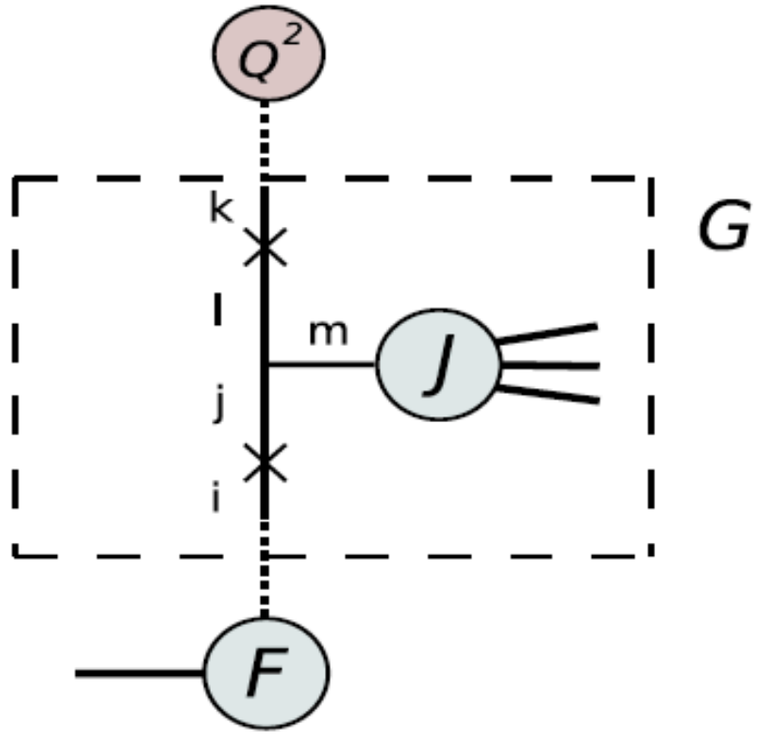
$$F_2^{(n)}(x,Q^2;E_t^2,y_{\rm cut}) = \sum_{i=q,\bar{q}} \int_x^1 \frac{dz}{z} F_P^i(x/z,\mu_F^2) R_{2,i}^{(n)}\left(z,\alpha_s,\frac{Q^2}{E_t^2},y_{\rm cut}\right),$$

$$\Lambda^2 \ll E_t^2 \leq Q^2.$$

parton distributions (collinear singularities)

jet coefficient

$$G_i^k(u, Q_i^2, Q_j^2) \equiv \Delta_i^j(Q_i^2, Q_j^2)\, \hat{P}_j^{lm}(u)\, J_m(Q_j^2, Q_0^2)\, \Delta_l^k(Q_l^2, Q_k^2)$$



$$J(Q^2, k^2) = \int_0^1 dz\, d(z, Q^2, k^2)\,.$$

jet function

$$\int^{Q^2} dk^2\, d(z, Q^2, k^2) \equiv D(z, Q^2)\,.$$

parton fragmentation function

$$\Delta_i^j(Q_i^2, Q_j^2) \equiv \exp\Big[-\sum_k \int_{Q_i^2}^{Q_j^2} \frac{dt}{t} \int_{\frac{Q_i^2}{Q_j^2}}^{1-\frac{Q_i^2}{Q_j^2}} dz \frac{\alpha_s(t)}{2\pi} \hat{P}_i^{jk}(z) \Big]\,,$$

The beam-jet is excluded from the picture

it is mainly of soft and kinematical nature

it results from the fragmentation of the spectator partons of the hadron remnants plus semi-hard radiation coming from the evolution of the active parton

we define new semi-inclusive jet-like distributions

$$\mathcal{M}^i_{\triangleleft}(x, Q^2, z, t)$$

the probability of finding a parton i with fractional momentum x of the incoming hadron while a cluster {h} of hadrons is detected in R

$$\mathcal{R}: \quad t_i = -(P - h_i)^2 < t, \qquad t_0 \leq t \leq Q^2 \ .$$

$$\frac{1}{\sigma_{tot}}\frac{d\sigma}{dxdQ^2dzdt} \propto x \sum_{i=q,\bar{q}} e_i^2 \, \mathcal{M}^i_{\triangleleft}(x, Q^2, z, t)\,.$$

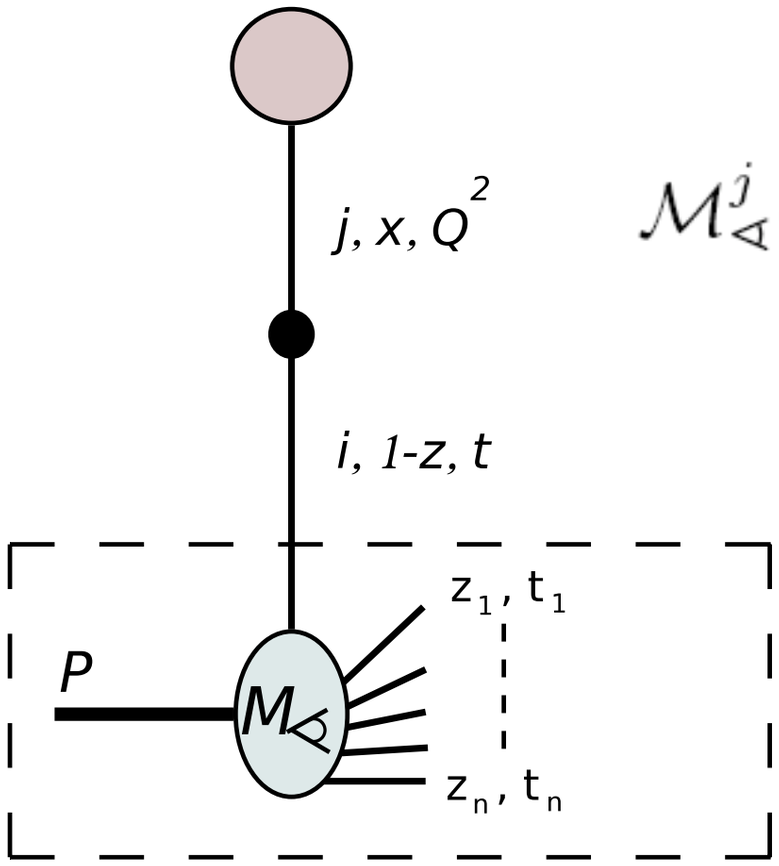
$$\mathcal{R}: \quad t_i = -(P - h_i)^2 < t, \quad t_0 \le t \le Q^2\,. \qquad z = \sum_i z_i, \quad h_i \in \mathcal{R}\,.$$

$$\Sigma^{(n)}_{excl} \equiv \frac{1}{n!}\frac{d^{2n+2}\sigma^{(n)}}{dxdQ^2\prod_{m=1}^{n} dz_m dt_m}$$

n-particle exclusive cross-section from the experiment

$$\frac{1}{\sigma_{tot}}\frac{d\sigma}{dxdQ^2dzdt} \equiv \frac{1}{\sigma_{tot}}\sum_{k=1}^{\infty}\left\{\prod_{m=1}^{k}\int_{t_0}^{t} dt_m \int_0^1 dz_m\right\}\Sigma^{(k)}_{excl}\,\delta\Big(z - \sum_{k=1}^{n} z_k\Big)$$

t_0 corresponds to beam pipe acceptance loss



$$\mathcal{M}^j_{\sphericalangle}(x,Q^2,z,t) = \int_x^{1-z} \frac{dw}{w} \mathcal{M}^i_{\sphericalangle}(w,t,z,t)\; E^j_i(x/w,t,Q^2)\,.$$

$$P^j_k(u) = P^{j\,(0)}_k(u) + \frac{\alpha_s}{2\pi} P^{j\,(1)}_k(u) + \ldots$$

$$Q^2 \frac{\partial}{\partial Q^2} \mathcal{M}^j_{\sphericalangle}(x,Q^2,z,t) = \frac{\alpha_s(Q^2)}{2\pi} \int_{\frac{x}{1-z}}^1 \frac{du}{u} P^j_k(u) \mathcal{M}^k_{\sphericalangle}(x/u,Q^2,z,t)\,.$$

$$F_2^{(n+1)}(x,Q^2;E_t^2,y_{\rm cut}) = \sum_{i=q,\bar{q}} \int_0^1 dz \int_{\frac{x}{1-z}}^1 \frac{du}{u} \mathcal{M}^i_{\sphericalangle}(x/u,Q^2,z,y_{\rm cut}E_t^2)\; R^{(n)}_{2,i}\left(u,\alpha_s,\frac{Q^2}{E_t^2},y_{\rm cut}\right)$$

$$t \simeq y_{\rm cut} E_t^2$$

$\mathcal{M}^j_{\sphericalangle}$ is more inclusive than extended Fracture Functions

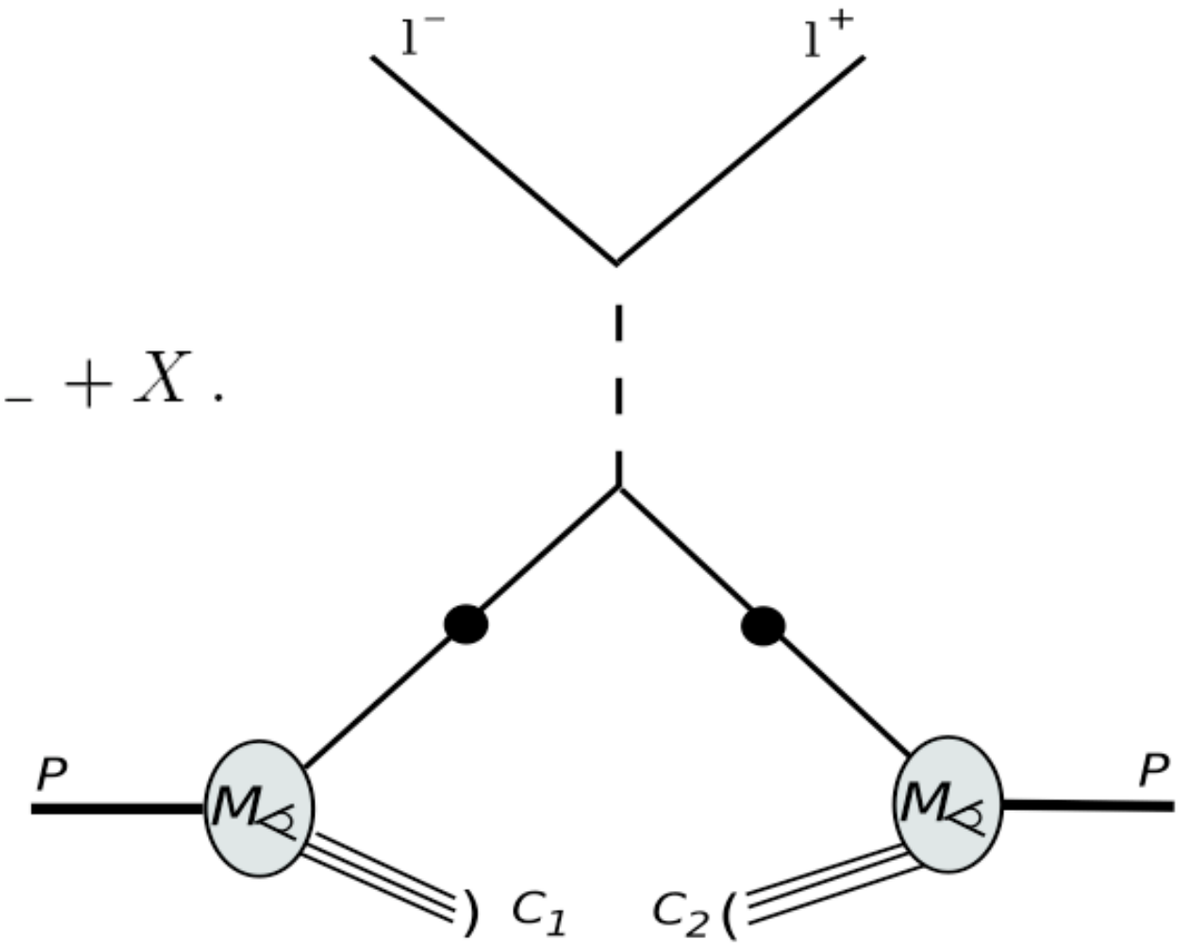
we do not expect any factorization breaking effect in DIS

semi-inclusive

Drell-Yan

and factorization

$$p + p \to C_1 + C_2 + \gamma^* \hookrightarrow {}_{l^+l^-} + X\,.$$



$$\frac{d\sigma^{DY}}{dt_1 dt_2 dQ^2 dz_1 dz_2} = \sum_{i,j=q,\bar{q}} \int\int dx_1 dx_2 \Big(\mathcal{M}^i_{\triangleleft}(x_1, Q^2, z_1, t_1) \mathcal{M}^j_{\triangleleft}(x_2, Q^2, z_2, t_2) + i \leftrightarrow j \Big)\ \delta(s - x_1 x_2 Q^2)\,,$$

can be used to measure the strenght of the factorization breaking in semi-inclusive hadron hadron collisions

factorization properties of the cross sections can be connected with the observed forward radiation pattern (see also F. Ceccopieri talk at this wkshop)

Conclusions

Alternative QCD based approach to initial state radiation including target remnants

QCD evolution known

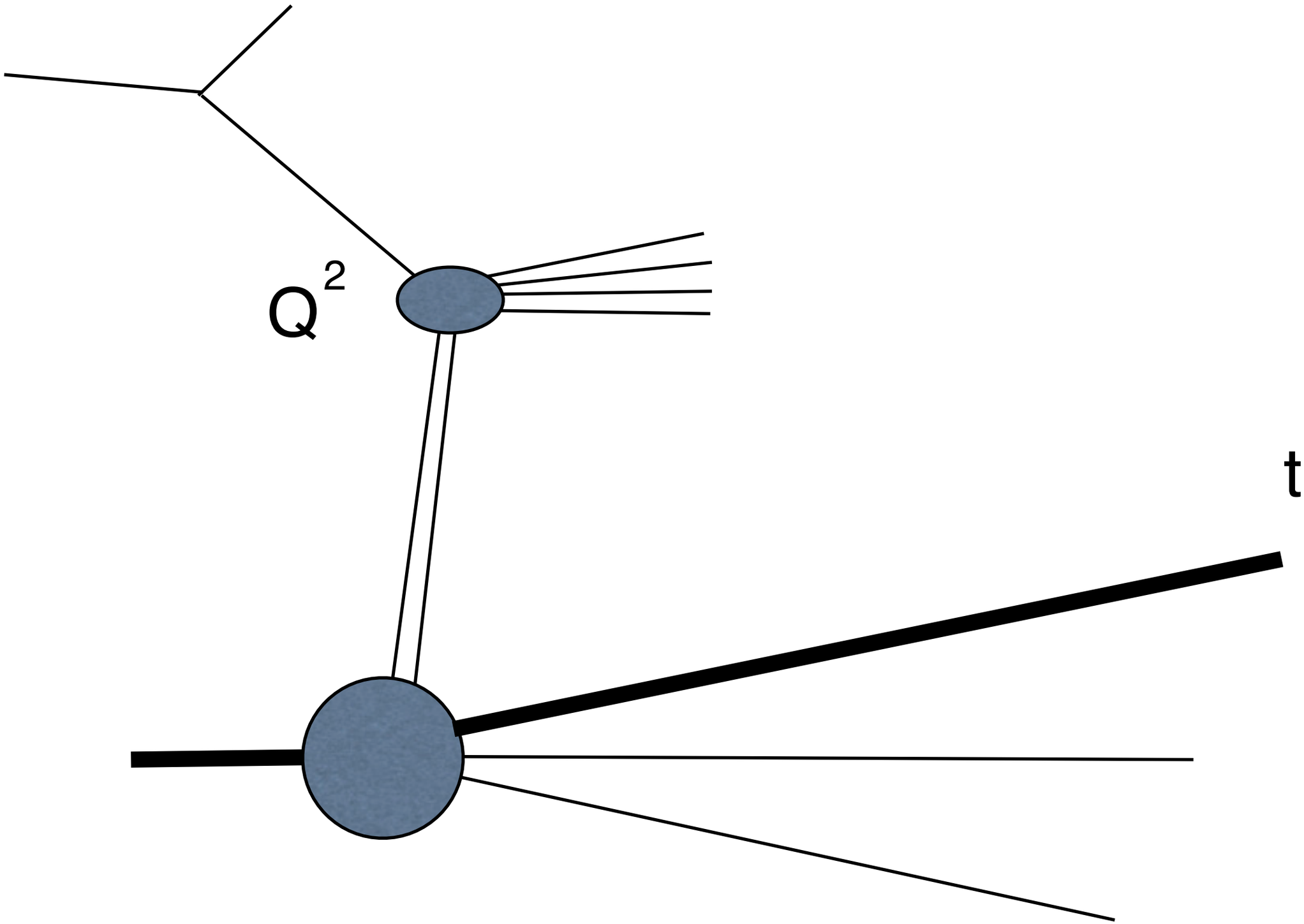
NLO and NNLO improvement + coherence

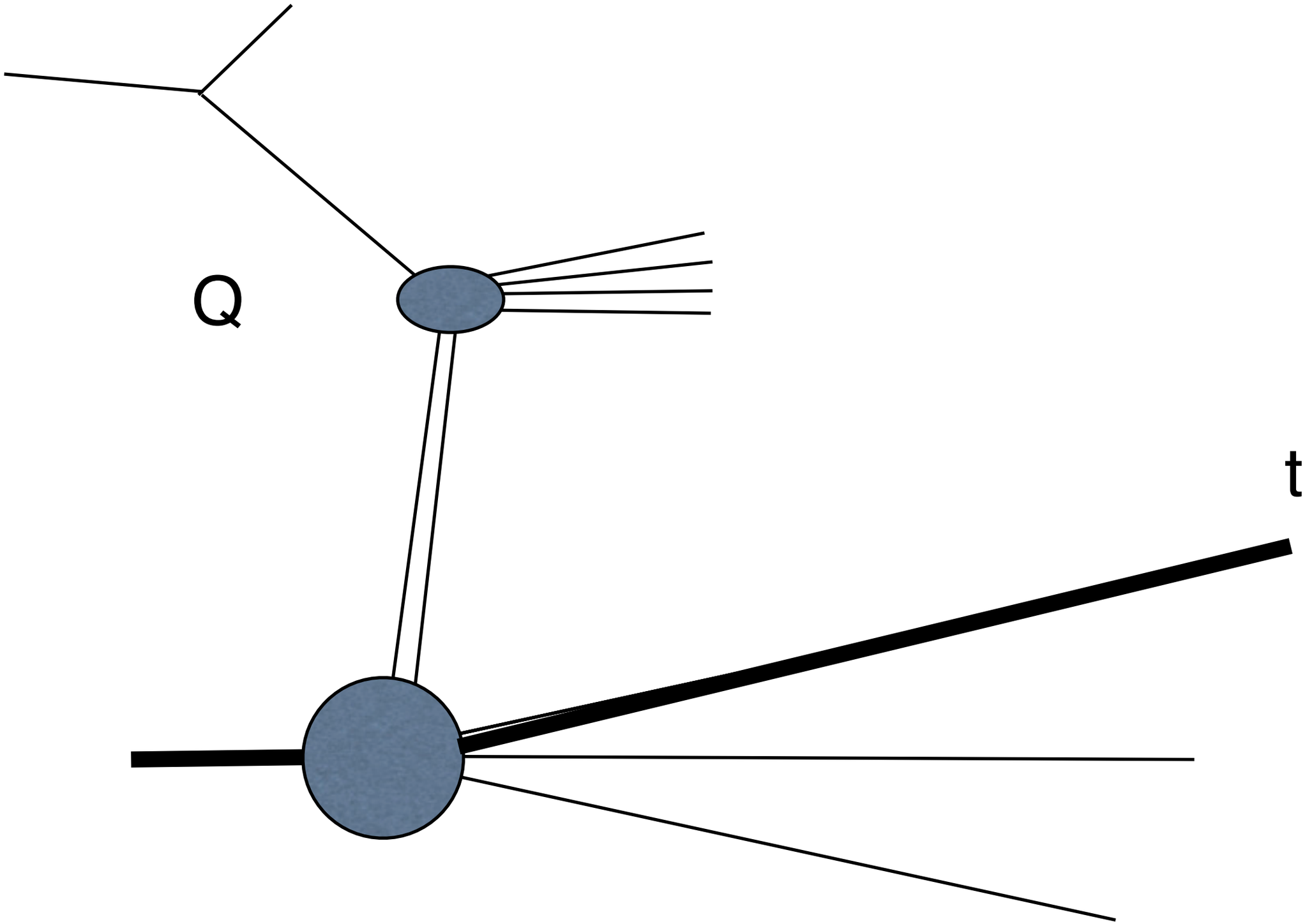
New jet-like Fracture Functions to describe beam (target) jets and their evolution

Useful framework to investigate factorization

A potentially useful tool for minimum bias and underlying event

Additional slides


Q 2
t


Q
t

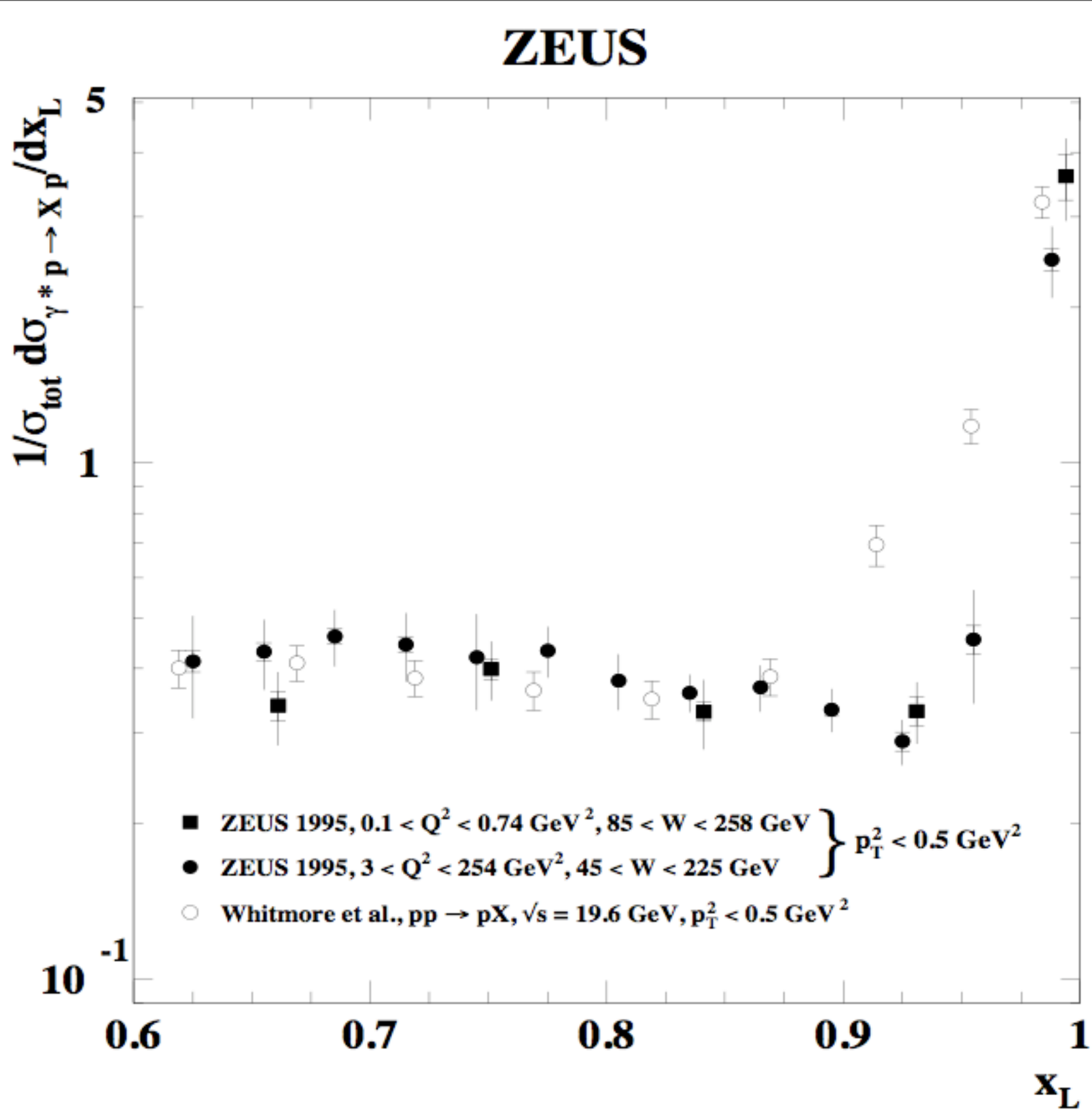


Figure 6: *The normalised cross-section* $(1/\sigma_{\rm tot}) \cdot d\sigma_{\gamma^* p \to Xp}/dx_L$ *for the BPC and DIS data compared to the pp data [54] in the region* $p_T^2 < 0.5$ GeV^2. *The inner bars indicate the statistical uncertainties and the outer bars are the statistical and systematic uncertainties summed in quadrature.*

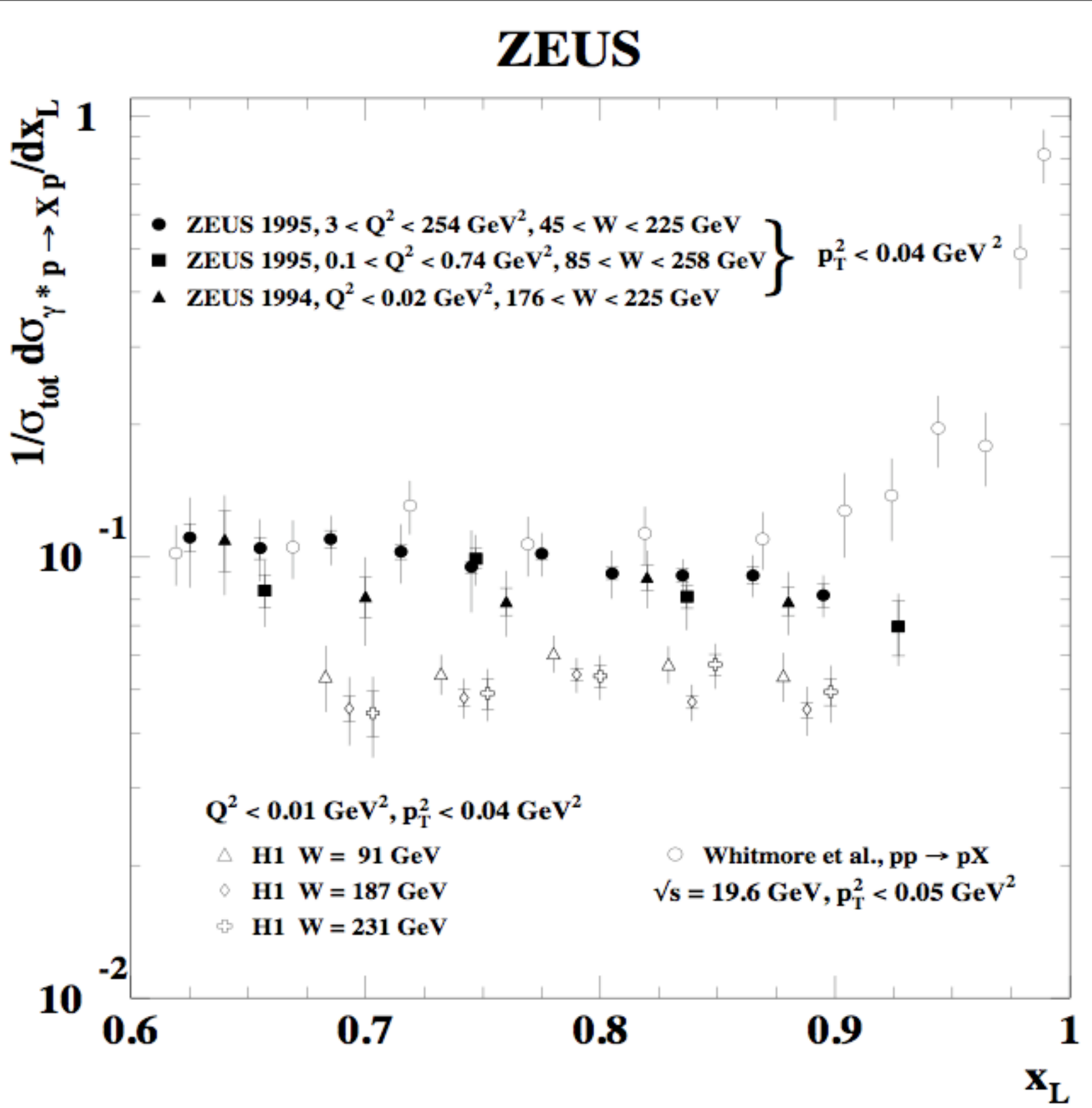
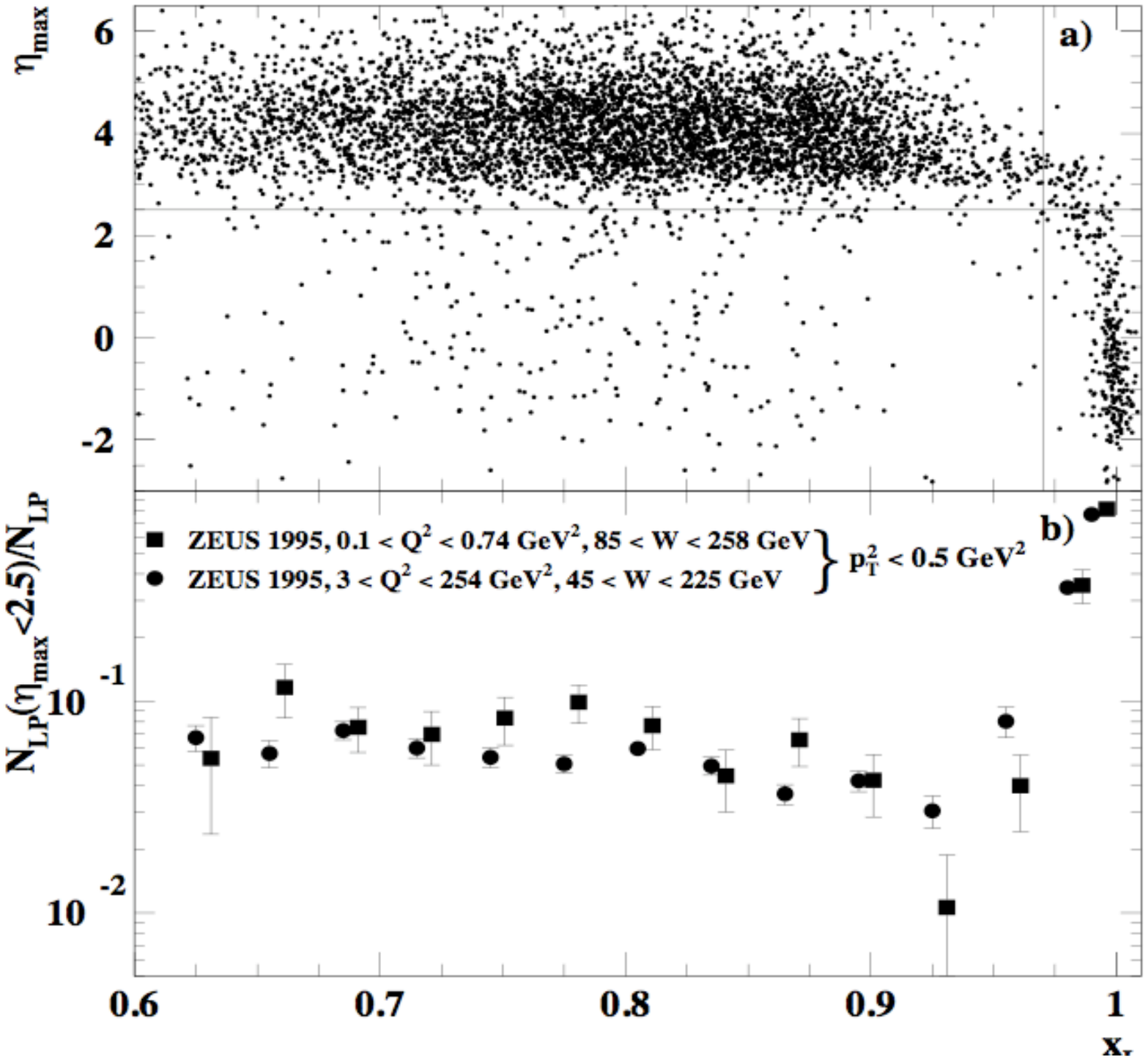


Figure 7: *The normalised cross-section* $(1/\sigma_{\rm tot}) \cdot d\sigma_{\gamma^* p \to Xp}/dx_L$ *for the photoproduction, BPC and DIS data compared to the pp data [54] in the region* $p_T^2 < 0.04$ GeV^2. *The inner bars indicate the statistical uncertainties and the outer bars are the statistical and systematic uncertainties summed in quadrature. The H1 results [7] are also shown.*



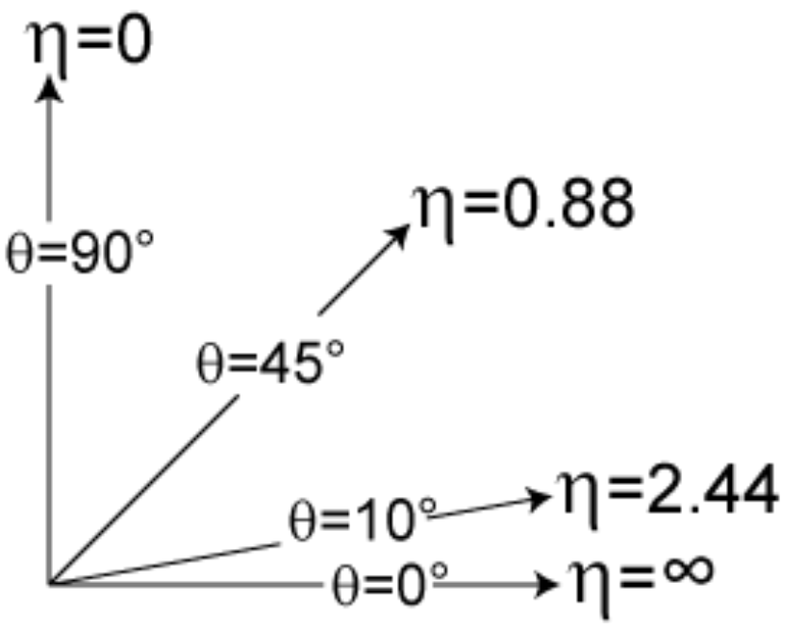


Figure 5a) shows the distribution of the DIS events in the ($\eta_{\max}$, x_L) plane, where $\eta_{\max}$ is the pseudorapidity of the most-forward energy deposit of at least 400 MeV in the CAL. The accumulation of events at $x_L \approx 1$, which mostly have $\eta_{\max} < 2.5$, is due to diffractive events [52], $e^+p \to e^+Xp$, in which the final-state proton remains intact and carries approximately the same momentum as the incoming proton. Events with

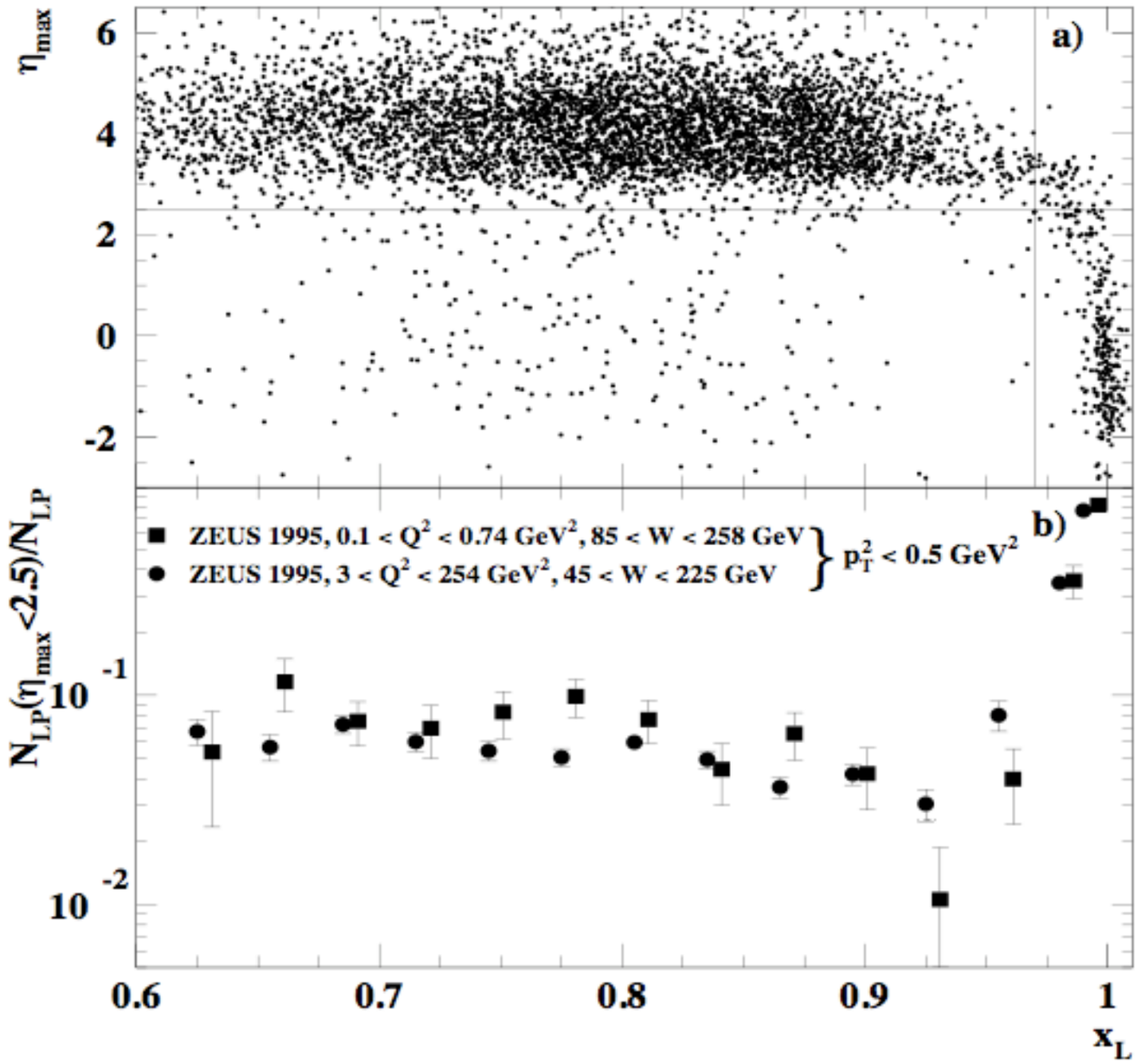


Figure 5: *a) Distribution of the DIS events in the ($\eta_{\max}$, x_L) plane. The horizontal and vertical lines indicate $\eta_{\max} = 2.5$ and $x_L = 0.97$, respectively. b) Fraction of events with $\eta_{\max} < 2.5$ for both the BPC and the DIS samples. The BPC data are slightly shifted in x_L for clarity of presentation. The bars indicate the statistical uncertainties; systematic uncertainties mostly cancel in the ratio.*

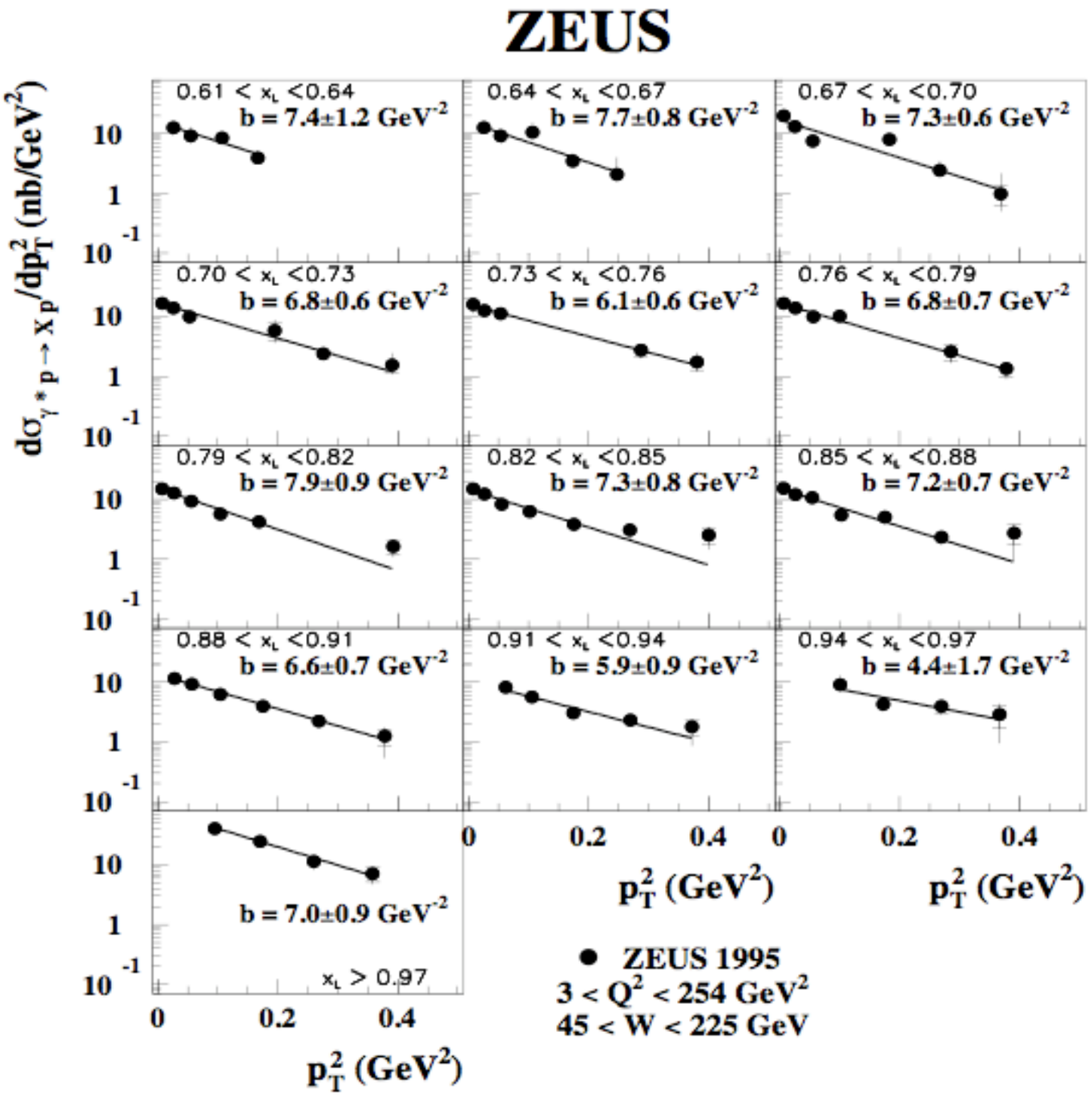
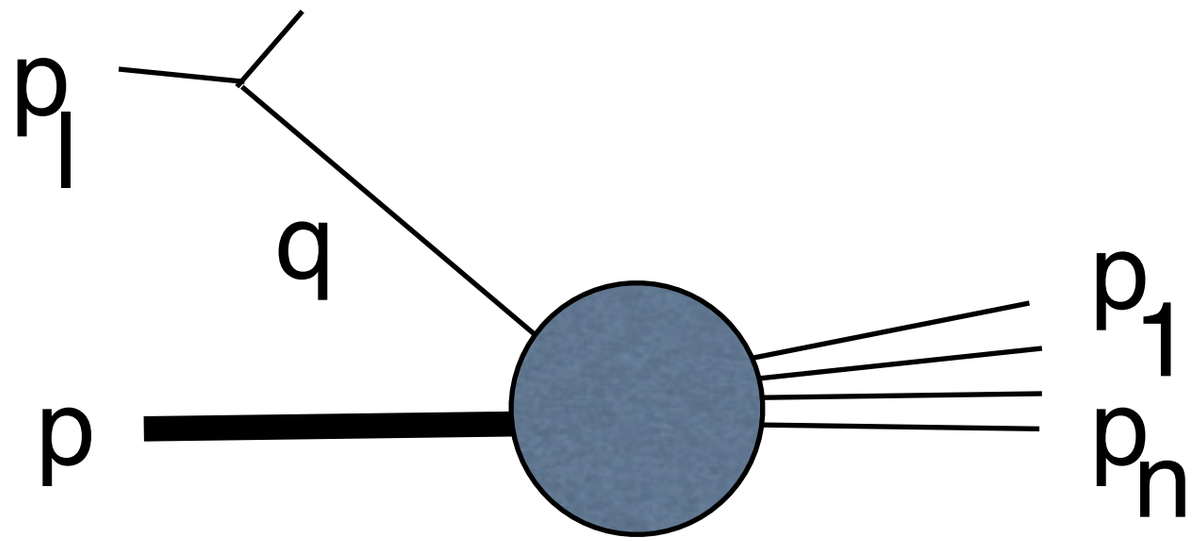


Figure 10: *The differential cross-section $d\sigma_{\gamma^* p \to X p}/dp_T^2$ for several x_L bins for the DIS sample. The lines represent the results of fits to the functional form $d\sigma_{\gamma^* p \to X p}/dp_T^2 \propto e^{-bp_T^2}$. The fitted values of b and their statistical uncertainties are also given. The inner bars indicate the size of the statistical uncertainties, the outer bars show the statistical and systematic uncertainties summed in quadrature.*


p_l
q
p
p_1
p_n