Precision Resummed QED$\otimes$QCD Theory for LHC Physics: IR-Improved Scheme for Parton Distributions, Kernels, Reduced Cross Sections with Shower/ME Matching

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Outline

- **Introduction**
- **Review of Exact Amplitude-Based Resummation for QED$\otimes$QCD**
- **IR-Improved DGLAP-CS Theory: Parton Distributions, Kernels, Reduced Cross Sections with Shower/ME Matching**
- **Sample MC data: IR-Improved Kernels in HERWIG6.5**
- **Conclusions**

See B.F.L.W., S. Jadach and B.F.L. Ward, S. Jadach, *et al.*,B.F.L.W. and S. Yost, MPL A **14** (1999) 491, hep-ph/0205062; *ibid.* **12** (1997) 2425; *ibid.***19** (2004) 2113; hep-ph/0503189, 0508140,0509003, 0605054, 0607198, arxiv:0704.0294, 0707.2101, 0707:3424

Motivation

- FOR THE LHC/ILC, THE REQUIREMENTS ARE DEMANDING AND OUR $QED \otimes QCD$ SOFT n(G)-m(γ) MC RESUMMATION RESULTS WILL BE AN IMPORTANT PART OF THE NECESSARY THEORY – (YFS)RESUMMED $\mathcal{O}(\alpha_s^2)L^n, \mathcal{O}(\alpha_s\alpha)L^{n'}, \mathcal{O}(\alpha^2)L^{n''}, n = 0, 1, 2, n' = 0, 1, 2, n'' = 2, 1$, IN THE PRESENCE OF SHOWERS, ON AN EVENT-BY-EVENT BASIS, WITHOUT DOUBLE COUNTING AND WITH EXACT PHASE SPACE.

- HOW RELEVANT ARE QED HIGHER ORDER CORRECTIONS WHEN QCD IS CONTROLLED AT $\sim 1\%$ PRECISION?

- CROSS CHECK OF QCD LITERATURE:
  1. PHASE SPACE – CATANI, CATANI-SEYMOUR, ALL INITIAL PARTONS MASSLESS
  2. RESUMMATION – STERMAN, CATANI ET AL., BERGER ET AL., ...
  3. NO-GO THEOREMS–Di'Lieto et al.,Doria et al.,Catani et al.,Catani
  4. IR QCD EFFECTS IN DGLAP-CS THEORY

- CROSS CHECK OF QED-EW LITERATURE:

  1. ESTIMATES BY SPIESBERGER, STIRLING, ROTH and WEINZIERL, BLUMLEIN and KAWAMURA – FEW PER MILLE EFFECTS FROM QED CORRECTIONS TO STR. FN. EVOLUTION.

  2. WELL-KNOWN POSSIBLE ENHANCEMENT OF QED CORRECTIONS AT THRESHOLD, ESPECIALLY IN RESONANCE PRODUCTION

  3. *See for example, A. Kulesza et al., S. Pozzorini et al., A. Denner et al., in "Proc. RADCOR07", for large (Sudakov log, etc.) EW effects in hadron-hadron scattering – at 1TeV, W's and Z's are almost massless!*

  $\Rightarrow$ HOW TO BEST REALIZE THESE EFFECTS AT THE LHC?

- TREAT QED AND QCD SIMULTANEOUSLY IN THE (YFS) RESUMMATION TO OBTAIN THE ROLE OF THE QED-EW AND TO REALIZE AN APPROACH TO SHOWER/ME MATCHING.

- CURRENT STATE OF AFFAIRS: see N. Adam et al., arxiv.org:0802.3251 – Using MC@NLO and FEWZ, HORACE, PHOTOS, etc., $(4.1 \pm 0.3)\% = (1.51 \pm 0.75)\%(QCD) \oplus 3.79(PDF) \oplus 0.38 \pm 0.26(EW)\%$ accuracy on single Z to leptons at LHC was found, but no exclusive hard gluon/quark radiation phase space available – the latter are truly needed for realistic theoretical results. They are our goal, at $\lesssim 1\%$.

PRELIMINARIES

- WE USE THE GPS CONVENTIONS OF JWW FOR SPINORS; PHOTON-GLUON POLARIZATION VECTORS FOLLOW THEREFROM:

$$(\epsilon^{\mu}_{\sigma}(\beta))^{*} = \frac{\bar{u}_{\sigma}(k)\gamma^{\mu}u_{\sigma}(\beta)}{\sqrt{2}\ \bar{u}_{-\sigma}(k)u_{\sigma}(\beta)}, \quad (\epsilon^{\mu}_{\sigma}(\zeta))^{*} = \frac{\bar{u}_{\sigma}(k)\gamma^{\mu}u_{\sigma}(\zeta)}{\sqrt{2}\ \bar{u}_{-\sigma}(k)u_{\sigma}(\zeta)}, \tag{1}$$

- REPRESENTATIVE PROCESSES

$pp \rightarrow V + m(\gamma) + n(G) + X \rightarrow \bar{\ell}\ell' + m'(\gamma) + n(G) + X$, where $V = W^{\pm}, Z$,and $\ell = e, \mu,\ \ell' = \nu_e, \nu_\mu(e, \mu)$ respectively for $V = W^{+}(Z)$, and $\ell = \nu_e, \nu_\mu,\ \ell' = e, \mu$ respectively for $V = W^{-}$.

QED⊗QCD Resummation

In hep-ph/0210357(ICHEP02), Acta Phys.Polon.B33,1543-1558,2002, Phys.Rev.D52(1995)108;ibid. 66 (2002) 019903(E);PLB342 (1995) 239, we have extended the YFS theory to QCD:

$$d\hat{\sigma}_{\rm exp} = \sum_n d\hat{\sigma}^n$$
$$= e^{\rm SUM_{IR}(QCD)} \sum_{n=0}^{\infty} \int \prod_{j=1}^{n} \frac{d^3 k_j}{k_j} \int \frac{d^4 y}{(2\pi)^4} e^{iy\cdot(P_1+P_2-Q_1-Q_2-\sum k_j)+D_{\rm QCD}}$$
$$* \tilde{\bar{\beta}}_n(k_1,\ldots,k_n) \frac{d^3 P_2}{P_2^{\,0}} \frac{d^3 Q_2}{Q_2^{\,0}} \tag{2}$$

where the new hard gluon residuals $\tilde{\bar{\beta}}_n(k_1,\ldots,k_n)$ defined by

$$\tilde{\bar{\beta}}_n(k_1,\ldots,k_n) = \sum_{\ell=0}^{\infty} \tilde{\bar{\beta}}_n^{(\ell)}(k_1,\ldots,k_n)$$

are free of all infrared divergences to all orders in $\alpha_s(Q)$. $\Rightarrow$

Simultaneous exponentiation of QED and QCD higher order effects,

hep-ph/0404087,

gives

$$\begin{aligned} B^{nls}_{QCD} &\to B^{nls}_{QCD} + B^{nls}_{QED} \equiv B^{nls}_{QCED}, \\ \tilde{B}^{nls}_{QCD} &\to \tilde{B}^{nls}_{QCD} + \tilde{B}^{nls}_{QED} \equiv \tilde{B}^{nls}_{QCED}, \\ \tilde{S}^{nls}_{QCD} &\to \tilde{S}^{nls}_{QCD} + \tilde{S}^{nls}_{QED} \equiv \tilde{S}^{nls}_{QCED} \end{aligned} \tag{3}$$

which leads to

$$d\hat{\sigma}_{\rm exp} = e^{\rm SUM_{IR}(QCED)} \sum_{n,m=0}^{\infty} \int \prod_{j_1=1}^{n} \frac{d^3 k_{j_1}}{k_{j_1}} \prod_{j_2=1}^{m} \frac{d^3 {k'}_{j_2}}{{k'}_{j_2}} \int \frac{d^4 y}{(2\pi)^4} e^{iy\cdot(p_1+q_1-p_2-q_2-\sum k_{j_1}-\sum {k'}_{j_2})+D_{\rm QCED}} \tilde{\bar{\beta}}_{n,m}(k_1,\ldots,k_n;k'_1,\ldots,k'_m) \frac{d^3 p_2}{p_2^0} \frac{d^3 q_2}{q_2^0}, \tag{4}$$

where the new YFS residuals $\tilde{\bar{\beta}}_{n,m}(k_1,\ldots,k_n;k'_1,\ldots,k'_m)$**, with** n **hard gluons and** m **hard photons, represent the successive application of the YFS expansion first for QCD and subsequently for QED.**

The infrared functions are now

$$\mathrm{SUM_{IR}(QCED)} = 2\alpha_s \Re B^{nls}_{QCED} + 2\alpha_s \tilde{B}^{nls}_{QCED}$$
$$D_{\mathrm{QCED}} = \int \frac{dk}{k^0}\left(e^{-iky} - \theta(K_{max} - k^0)\right)\tilde{S}^{nls}_{QCED} \quad (5)$$

where K_{max} is a dummy parameter – here the same for QCD and QED.

Infrared Algebra(QCED):

$x_{avg}(QED) \cong \gamma(QED)/(1+\gamma(QED))$

$x_{avg}(QCD) \cong \gamma(QCD)/(1+\gamma(QCD))$

$\gamma(A) = \frac{2\alpha_A \mathcal{C}_A}{\pi}(L_s - 1)$, $A = QED, QCD$

$\mathcal{C}_A = Q_f^2, C_F$, **respectively, for** $A = QED, QCD$

$\Rightarrow$ **QCD dominant corrections happen an order of magnitude earlier than those for QED.**

$\Rightarrow$ **Leading $\tilde{\tilde{\beta}}^{(0,0)}_{0,0}$-level gives a good estimate of the size of the effects we study:**

see arxiv.org: 0802.0724, and references therein.

Relationship to Sterman-Catani-Trentadue Soft Gluon Resummation

In Phys. Rev. D74 (2006) 074004[MADG], Abayat et al. apply the more familiar resummation for soft gluons to a general $2 \rightarrow n$ parton process [f] at hard scale Q,

$f_1(p_1,r_1) + f_2(p_2,r_2) \rightarrow f_3(p_3,r_3) + f_4(p_4,r_4) + \cdots + f_{n+2}(p_{n+2},r_{n+2})$**,**
where the p_i, r_i label 4-momenta and color indices respectively, with all parton masses set to zero to get

$$\begin{aligned}\mathcal{M}^{[f]}_{\{r_i\}} &= \sum_{L}^{C} \mathcal{M}^{[f]}_{L}(c_L)_{\{r_i\}} \\ &= J^{[f]} \sum_{L}^{C} S_{LI} H^{[f]}_{I}(c_L)_{\{r_i\}},\end{aligned} \tag{6}$$

$J^{[f]}$ **is the jet function**

S_{LI} **is the soft function which describes the exchange of soft gluons between the external lines**

$H_I^{[f]}$ **is the hard coefficient function**

Infrared and collinear poles calculated to 2-loop order.

To make contact with our approach, identify in $\bar{Q}'Q \rightarrow \bar{Q}'''Q'' + m(G)$ **in (2)** $f_1 = Q, \bar{Q}', f_2 = \bar{Q}', f_3 = Q'', f_4 = \bar{Q}''', \{f_5, \cdots, f_{n+2}\} = \{G_1, \cdots, G_m\}$

$\Rightarrow n = m + 2$ **here.**

Observe the following:

- **By its definition in eq.(2.23) of [MADG], the anomalous dimension of the matrix** S_{LI} **does not contain any of the diagonal effects described by our infrared functions** $\Sigma_{IR}(QCD)$ **and** D_{QCD}**.**
- **By its definition in eqs.(2.5) and (2.7) of [MADG], the jet function** $J^{[f]}$ **contains the exponential of the virtual infrared function** $\alpha_s \Re B_{QCD}$**, so that we have to take care that we do not double count when we use (6) in (2) and the equations that lead thereto.**

$\Rightarrow$

We identify $\bar{\rho}^{(m)}$ in our theory as

$$\bar{\rho}^{(m)}(p_1,q_1,p_2,q_2,k_1,\cdots,k_m) = \overline{\sum}_{colors,spin} |\mathcal{M}^{[f]}_{\{r_i\}}|^2$$

$$\equiv \sum_{spins,\{r_i\},\{r'_i\}} \mathfrak{h}^{cs}_{\{r_i\}\{r'_i\}} |\bar{J}^{[f]}|^2 \sum_{L=1}^{C} \sum_{L'=1}^{C} S^{[f]}_{LI} H^{[f]}_{I}(c_L)_{\{r_i\}} \left(S^{[f]}_{L'I'} H^{[f]}_{I'}(c_{L'})_{\{r'_i\}}\right)^\dagger, \tag{7}$$

where here we defined $\bar{J}^{[f]} = e^{-\alpha_s \Re B_{QCD}} J^{[f]}$, and we introduced the color-spin density matrix for the initial state, $\mathfrak{h}^{cs}$.

Here, we recall (see hep-ph/0508140, for example) that in our theory, we have

$$d\hat{\sigma}^n = \frac{e^{2\alpha_s ReB_{QCD}}}{n!} \int \prod_{m=1}^{n} \frac{d^3 k_m}{(k_m^2+\lambda^2)^{1/2}} \delta(p_1+q_1-p_2-q_2-\sum_{i=1}^{n} k_i)$$
$$\bar{\rho}^{(n)}(p_1,q_1,p_2,q_2,k_1,\cdots,k_n)\frac{d^3p_2 d^3q_2}{p_2^0 q_2^0}, \tag{8}$$

for n-gluon emission. $\Rightarrow$

We can repeat thus our usual steps (see hep-ph/0508140) to get our formula (2), without any double counting of effects. This is in progress.

IR-Improved DGLAP-CS Theory: Parton Distributions, Kernels, Reduced Cross Sections with Shower/ME Matching

IR-Improved DGLAP-CS Theory

Exponentiation of QCD higher order effects: Where to apply?

hep-ph/0508140,

consider

$$\frac{dq^{NS}(x,t)}{dt} = \frac{\alpha_s(t)}{2\pi}\int_x^1 \frac{dy}{y} q^{NS}(y,t) P_{qq}(x/y) \qquad (9)$$

where the well-known result for the kernel $P_{qq}(z)$ is, for $z<1$,

$$P_{qq}(z) = C_F \frac{1+z^2}{1-z}, \qquad (10)$$

$t = \ln \mu^2/\mu_0^2$ **for some reference scale μ_0.** $\Rightarrow$

Unintegrable singularity at $z = 1$, usually regularized by

$$\frac{1}{(1-z)} \rightarrow \frac{1}{(1-z)_+} \tag{11}$$

with $\frac{1}{(1-z)_+}$ such that

$$\int_0^1 dz \frac{f(z)}{(1-z)_+} = \int_0^1 dz \frac{f(z)-f(1)}{(1-z)}. \tag{12}$$

$\Rightarrow$

$$\frac{1}{(1-z)_+} = \frac{1}{(1-z)}\theta(1-\epsilon-z) + \ln\epsilon\,\delta(1-z) \tag{13}$$

with the understanding that $\epsilon \downarrow 0$.

Require

$$\int_0^1 dz P_{qq}(z) = 0, \tag{14}$$

$\Rightarrow$ add virtual corrections to get

$$P_{qq}(z) = C_F\left(\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z)\right). \tag{15}$$

Observations

- Smooth, divergent $1/(1-z)$ behavior as $z \rightarrow 1$ replaced with a mathematical artifact: the regime $1-\epsilon < z < 1$ now has no probability at all; at $z = 1$ we have a large negative integrable contribution $\Rightarrow$ a finite (zero) value for the total integral of $P_{qq}(z)$
- LEP1,2 experience: such mathematical artifacts, while correct, impair precision.

Why set $P_{qq}(z)$ to 0 for $1-\epsilon < z < 1$ where it actually has its largest values?

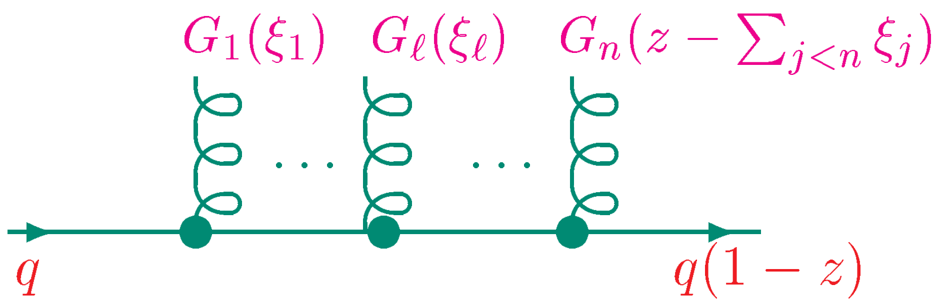
- **USE EXPERIENCE FROM LEP1,2: $\frac{1}{(1-z)_+}$ SHOULD BE EXPONENTIATED –SEE CERN YELLOW-BOOKS, CERN-89-08., YIELDING FROM (2) THE REPLACEMENT**

$$P_{BA} = \frac{1}{2} z(1-z) \overline{\sum_{spins}} \frac{|V_{A\to B+C}|^2}{p_\perp^2}$$

$$\Rightarrow \tag{16}$$

$$P_{BA} = \frac{1}{2} z(1-z) \overline{\sum_{spins}} \frac{|V_{A\to B+C}|^2}{p_\perp^2} z^{\gamma_q} F_{YFS}(\gamma_q) e^{\frac{1}{2}\delta_q}$$

WHERE $A=q$, $B=G$, $C=q$ AND $V_{A\to B+C}$ IS THE LOWEST ORDER AMPLITUDE FOR $q \to G(z) + q(1-z)$.



$$q \to q(1-z) + G_1 + \cdots + G_n$$

$\Rightarrow$

$$P_{qq}(z) = C_F F_{YFS}(\gamma_q) e^{\frac{1}{2}\delta_q} \frac{1+z^2}{1-z}(1-z)^{\gamma_q} \tag{17}$$

where

$$\gamma_q = C_F \frac{\alpha_s}{\pi} t = \frac{4C_F}{\beta_0} \tag{18}$$

$$\delta_q = \frac{\gamma_q}{2} + \frac{\alpha_s C_F}{\pi}(\frac{\pi^2}{3} - \frac{1}{2}) \tag{19}$$

and

$$F_{YFS}(\gamma_q) = \frac{e^{-C_E \gamma_q}}{\Gamma(1+\gamma_q)}. \tag{20}$$

Note: $\int_{k_0} dz/z = C_0 - \ln k_0$

is experimentally distinguishable from

$\int_{k_0} dz/z^{1-\gamma} = C'_0 - k_0^{\gamma}/\gamma$.

- **NORMALIZATION CONDITION (14) $\Rightarrow$:**

$$P_{qq}(z) = C_F F_{YFS}(\gamma_q) e^{\frac{1}{2}\delta_q} \left[\frac{1+z^2}{1-z}(1-z)^{\gamma_q} - f_q(\gamma_q)\delta(1-z) \right] \tag{21}$$

where

$$f_q(\gamma_q) = \frac{2}{\gamma_q} - \frac{2}{\gamma_q+1} + \frac{1}{\gamma_q+2}. \tag{22}$$

- **THIS IS OUR IR-IMPROVED P_{qq} DGLAP-CS KERNEL.**

$\Rightarrow$ **STANDARD DGLAP-CS THEORY:**

for $z < 1$, we have

$$P_{Gq}(z) = P_{qq}(1-z) = C_F F_{YFS}(\gamma_q) e^{\frac{1}{2}\delta_q} \frac{1+(1-z)^2}{z} z^{\gamma_q}. \tag{23}$$

$\Rightarrow$ **TEST OF NEW THEORY – QUARK MOMENTUM SUM RULE:**

$$\int_0^1 dz z \left(P_{Gq}(z) + P_{qq}(z)\right) = 0. \tag{24}$$

$\Rightarrow$ **CHECK VANISHING OF**

$$I = \int_0^1 dz z \left(\frac{1+(1-z)^2}{z} z^{\gamma_q} + \frac{1+z^2}{1-z}(1-z)^{\gamma_q} - f_q(\gamma_q)\delta(1-z) \right). \quad (25)$$

NOTE,

$$\frac{z}{1-z} = \frac{z-1+1}{1-z} = -1 + \frac{1}{1-z}. \quad (26)$$

$\Rightarrow$

$$\begin{aligned} I &= \int_0^1 dz \{(1+(1-z)^2)z^{\gamma_q} - (1+z^2)(1-z)^{\gamma_q} \\ &\quad + \frac{1+z^2}{1-z}(1-z)^{\gamma_q} - f_q(\gamma_q)\delta(1-z)\} \\ &= 0 \end{aligned}$$

QUARK MOMENTUM SUM RULE IS SATISFIED.

- For $P_{qG}(z),\ P_{GG}(z)$, **we get, with the replacement** $C_F \rightarrow C_G$ **in the IR algebra, that the usual results**

$$P_{GG}(z) = 2C_G(\frac{1-z}{z} + \frac{z}{1-z} + z(1-z))$$
$$P_{qG}(z) = \frac{1}{2}(z^2 + (1-z)^2) \tag{27}$$

become

$$P_{GG}(z) = 2C_G F_{YFS}(\gamma_G) e^{\frac{1}{2}\delta_G}\{\frac{1-z}{z} z^{\gamma_G} + \frac{z}{1-z}(1-z)^{\gamma_G}$$
$$+ \frac{1}{2}(z^{1+\gamma_G}(1-z) + z(1-z)^{1+\gamma_G}) - f_G(\gamma_G)\delta(1-z)\}, \tag{28}$$
$$P_{qG}(z) = F_{YFS}(\gamma_G) e^{\frac{1}{2}\delta_G}\frac{1}{2}\{z^2(1-z)^{\gamma_G} + (1-z)^2 z^{\gamma_G}\}, \tag{29}$$

where

$$\gamma_G = C_G \frac{\alpha_s}{\pi} t = \frac{4C_G}{\beta_0} \tag{30}$$

$$\delta_G = \frac{\gamma_G}{2} + \frac{\alpha_s C_G}{\pi}\left(\frac{\pi^2}{3} - \frac{1}{2}\right), \tag{31}$$

$$f_G(\gamma_G) = \frac{n_f}{C_G} \frac{1}{(1+\gamma_G)(2+\gamma_G)(3+\gamma_G)} + \frac{2}{\gamma_G(1+\gamma_G)(2+\gamma_G)} \tag{32}$$

$$+ \frac{1}{(1+\gamma_G)(2+\gamma_G)} + \frac{1}{2(3+\gamma_G)(4+\gamma_G)} \tag{33}$$

$$+ \frac{1}{(2+\gamma_G)(3+\gamma_G)(4+\gamma_G)}. \tag{34}$$

THE GLUON MOMENTUM SUM RULE HAS BEEN USED.

- **THIS DEFINES THE NEW IR-IMPROVED DGLAP-CS THEORY.**

IR-IMPROVED DGLAP-CS KERNELS

$$P_{qq}(z) = C_F F_{YFS}(\gamma_q) e^{\frac{1}{2}\delta_q} \left[\frac{1+z^2}{1-z}(1-z)^{\gamma_q} - f_q(\gamma_q)\delta(1-z)\right], \tag{35}$$

$$P_{Gq}(z) = C_F F_{YFS}(\gamma_q) e^{\frac{1}{2}\delta_q} \frac{1+(1-z)^2}{z} z^{\gamma_q}, \tag{36}$$

$$\begin{aligned} P_{GG}(z) = {} & 2C_G F_{YFS}(\gamma_G) e^{\frac{1}{2}\delta_G} \{\frac{1-z}{z} z^{\gamma_G} + \frac{z}{1-z}(1-z)^{\gamma_G} \\ & + \frac{1}{2}(z^{1+\gamma_G}(1-z) + z(1-z)^{1+\gamma_G}) - f_G(\gamma_G)\delta(1-z)\}, \end{aligned} \tag{37}$$

$$P_{qG}(z) = F_{YFS}(\gamma_G) e^{\frac{1}{2}\delta_G} \frac{1}{2}\{z^2(1-z)^{\gamma_G} + (1-z)^2 z^{\gamma_G}\}. \tag{38}$$

Higher Order DGLAP-CS Kernels

Connection with the exact $\mathcal{O}(\alpha_s^2),\ \mathcal{O}(\alpha_s^3)$ kernel results of Curci, Furmanski and Petronzio, Floratos et al., Moch et al., etc., is immediate:

For example, non-singlet case, using standard notation,

$$P_{ns}^{+} = P_{qq}^{v} + P_{q\bar{q}}^{v} \equiv \sum_{n=0}^{\infty} (\frac{\alpha_s}{4\pi})^{n+1} P_{ns}^{(n)+} \tag{39}$$

where at order $\mathcal{O}(\alpha_s)$ we have

$$P_{ns}^{(0)+}(z) = 2C_F\{\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z)\} \tag{40}$$

$\Rightarrow P_{ns}^{(0)+}(z)$ agrees with the unexponentiated result for P_{qq} except for an overall factor of 2. Floratos et al., etc., have exact result for $P_{ns}^{(1)+}(z)$, and Moch et al. have

exact results for $P^{(2)+}_{ns}(z)$. Applying (2) to $q \to q + X$, $\bar{q} \to q + X'$, we get

$$P^{+,exp}_{ns}(z) = (\frac{\alpha_s}{4\pi})2P^{exp}_{qq}(z) + F_{YFS}(\gamma_q)e^{\frac{1}{2}\delta_q}\Big[(\frac{\alpha_s}{4\pi})^2\{(1-z)^{\gamma_q}\bar{P}^{(1)+}_{ns}(z) + \bar{B}_2\delta(1-z)\} + (\frac{\alpha_s}{4\pi})^3\{(1-z)^{\gamma_q}\bar{P}^{(2)+}_{ns}(z) + \bar{B}_3\delta(1-z)\}\Big] \tag{41}$$

where $P^{exp}_{qq}(z)$ is given above and the resummed residuals $\bar{P}^{(i)+}_{ns}$, $i = 1, 2$ are related to the exact results for $P^{(i)+}_{ns}$, $i = 1, 2$, as follows:

$$\bar{P}^{(i)+}_{ns}(z) = P^{(i)+}_{ns}(z) - B_{1+i}\delta(1-z) + \Delta^{(i)+}_{ns}(z) \tag{42}$$

where

$$\begin{aligned}\Delta^{(1)+}_{ns}(z) &= -4C_F\pi\delta_1\{\frac{1+z^2}{1-z} - f_q\delta(1-z)\}\\ \Delta^{(2)+}_{ns}(z) &= -4C_F(\pi\delta_1)^2\{\frac{1+z^2}{1-z} - f_q\delta(1-z)\}\\ &\quad - 2\pi\delta_1\bar{P}^{(1)+}_{ns}(z)\end{aligned} \tag{43}$$

and

$$\bar{B}_2 = B_2 + 4C_F\pi\delta_1 f_q$$
$$\bar{B}_3 = B_3 + 4C_F(\pi\delta_1)^2 f_q - 2\pi\delta_1\bar{B}_2. \tag{44}$$

The constants $B_i,\ i=2,3$ are given by

$$B_2 = 4C_GC_F(\frac{17}{24} + \frac{11}{3}\zeta_2 - 3\zeta_3) - 4C_Fn_f(\frac{1}{12} + \frac{2}{3}\zeta_2) + 4C_F^2(\frac{3}{8} - 3\zeta_2 + 6\zeta_3)$$
$$B_3 = 16C_GC_Fn_f(\frac{5}{4} - \frac{167}{54}\zeta_2 + \frac{1}{20}\zeta_2^2 + \frac{25}{18}\zeta_3)$$
$$+16C_GC_F^2(\frac{151}{64} + \zeta_2\zeta_3 - \frac{205}{24}\zeta_2 - \frac{247}{60}\zeta_2^2 + \frac{211}{12}\zeta_3 + \frac{15}{2}\zeta_5)$$
$$+16C_G^2C_F(-\frac{1657}{576} + \frac{281}{27}\zeta_2 - \frac{1}{8}\zeta_2^2 - \frac{97}{9}\zeta_3 + \frac{5}{2}\zeta_5)$$
$$+16C_Fn_F^2(-\frac{17}{144} + \frac{5}{27}\zeta_2 - \frac{1}{9}\zeta_3)$$
$$+16C_F^2n_F(-\frac{23}{16} + \frac{5}{12}\zeta_2 + \frac{29}{30}\zeta_2^2 - \frac{17}{6}\zeta_3)$$
$$+16C_F^3(\frac{29}{32} - 2\zeta_2\zeta_3 + \frac{9}{8}\zeta_2 + \frac{18}{5}\zeta_2^2 + \frac{17}{4}\zeta_3 - 15\zeta_5). \tag{45}$$

Contact with Wilson Expansion

N-th moment of the invariants $T_{i,\ell},\ i = L, 2, 3,\ \ell = q, G,$ of the forward Compton amplitude in DIS:(Gorishni et al.)

$$\mathcal{P}_N \equiv \left[\frac{q^{\{\mu_1} \cdots q^{\mu_N\}}}{N!} \frac{\partial^N}{\partial p^{\mu_1} \cdots \partial p^{\mu_N}} \right] |_{p=0}, \tag{46}$$

$x_{Bj} = Q^2/(2qp)$ in the standard DIS notation – Projects the coefficient of $1/(2x_{Bj})^N$. Terms which we resum here $\Leftrightarrow$ Formally γ_q-dependent anomalous dimensions associated with the respective coefficient, not in Wilson's expansion by usual definition.:

LARGE λ NOT ALL ON TIP OF LIGHTCONE.

COMMENTS

$(*)$ IRI-DGLAP-CS RESUMS IR SINGULAR ISR;BY FACTORIZATION THIS IS NOT CONTAINED IN ANY RESUMMATION OF HARD SHORT-DISTANCE COEFFICIENT FN CORRECTIONS AS IN THE STERMAN, CATANI-TRENTADUE, COLLINS ET AL. FORMULAS

$(**)$ WE DO NOT CHANGE THE PREDICTED HADRON CROSS SECTION:

$$\begin{aligned}\sigma &= \sum_{i,j}\int dx_1 dx_2 F_i(x_1)F_j(x_2)\hat{\sigma}(x_1x_2s) \\ &= \sum_{i,j}\int dx_1 dx_2 F'_i(x_1)F'_j(x_2)\hat{\sigma}'(x_1x_2s)\end{aligned} \tag{47}$$

ORDER BY ORDER IN PERTURBATION THEORY.

$\{P^{exp}\}$ factorize $\hat{\sigma}_{\text{unfactorized}} \Rightarrow \hat{\sigma}'$ – NEW SCHEME

$\{P\}$ factorize $\hat{\sigma}_{\text{unfactorized}} \Rightarrow \hat{\sigma}$

$(***)$ QUARK NUMBER CONSERVATION AND CANCELLATION OF IR SINGULARITIES IN XSECTS: Guaranteed by fundamental quantum field theoretic principles: Global Gauge Invariance, Unitarity – Everybody may use these principles.

Effects on Parton Distributions

Moments of kernels ⇔ Logarithmic exponents for evolution

$$\frac{dM_n^{NS}(t)}{dt} = \frac{\alpha_s(t)}{2\pi} A_n^{NS} M_n^{NS}(t) \tag{48}$$

where

$$M_n^{NS}(t) = \int_0^1 dz z^{n-1} q^{NS}(z,t) \tag{49}$$

and the quantity A_n^{NS} is given by

$$\begin{aligned} A_n^{NS} &= \int_0^1 dz z^{n-1} P_{qq}(z), \\ &= C_F F_{YFS}(\gamma_q) e^{\frac{1}{2}\delta_q} [B(n,\gamma_q) + B(n+2,\gamma_q) - f_q(\gamma_q)] \end{aligned} \tag{50}$$

where $B(x,y)$ is the beta function given by

$$B(x,y) = \Gamma(x)\Gamma(y)/\Gamma(x+y)$$

.

Compare the usual result

$$A_n^{NS^o} \equiv C_F \left[-\frac{1}{2} + \frac{1}{n(n+1)} - 2\sum_{j=2}^{n} \frac{1}{j} \right]. \tag{51}$$

- ASYMPTOTIC BEHAVIOR: IR-improved goes to a multiple of $-f_q$, consistent with $\lim_{n\to\infty} z^{n-1} = 0$ for $0 \le z < 1$; usual result diverges as $-2C_F \ln n$.
- Different for finite n as well: for $n = 2$ we get, for example, for $\alpha_s \cong .118$,

$$A_2^{NS} = \begin{cases} C_F(-1.33) & , \text{ un-IR-improved} \\ C_F(-0.966) & , \text{ IR-improved} \end{cases} \tag{52}$$

- **For completeness we note**

$$\begin{aligned} M_n^{NS}(t) &= M_n^{NS}(t_0) e^{\int_{t_0}^{t} dt' \frac{\alpha_s(t')}{2\pi} A_n^{NS}(t')} \\ &= M_n^{NS}(t_0) e^{\bar{a}_n [Ei(\frac{1}{2}\delta_1 \alpha_s(t_0)) - Ei(\frac{1}{2}\delta_1 \alpha_s(t))]} \\ &\underset{t,t_0 \text{ large with } t>>t_0}{\Longrightarrow} M_n^{NS}(t_0) \left(\frac{\alpha_s(t_0)}{\alpha_s(t)}\right)^{\bar{a'}_n} \end{aligned} \quad (53)$$

where $Ei(x) = \int_{-\infty}^{x} dr e^r/r$ **is the exponential integral function,**

$$\begin{aligned} \bar{a}_n &= \frac{2C_F}{\beta_0} F_{YFS}(\gamma_q) e^{\frac{\gamma_q}{4}} [B(n,\gamma_q) + B(n+2,\gamma_q) - f_q(\gamma_q)] \\ \bar{a'}_n &= \bar{a}_n \left(1 + \frac{\delta_1}{2} \frac{(\alpha_s(t_0) - \alpha_s(t))}{\ln(\alpha_s(t_0)/\alpha_s(t))}\right) \end{aligned} \quad (54)$$

with

$\delta_1 = \frac{C_F}{\pi}\left(\frac{\pi^2}{3} - \frac{1}{2}\right)$.

Compare with un-IR-improved result where last line in eq.(53) holds exactly with $\bar{a'}_n = 2A_n^{NS^o}/\beta_0$**.**

- For $n = 2$, taking $Q_0 = 2$GeV and evolving to $Q = 100$GeV, with $\Lambda_{QCD} \cong .2GeV$ and $n_f = 5$ for illustration, (53,54) $\Rightarrow$ a shift of evolved NS moment by $\sim\ 5\%$, of some interest in view of the expected HERA precision (see for example, T. Carli et al., Proc. HERA-LHC Wkshp, 2005).

ANOTHER EXAMPLE: THRESHOLD CORRECTIONS

We have applied the new simultaneous QED$\otimes$QCD exponentiation calculus to the single Z production with leptonic decay at the LHC (and at FNAL) to focus on the ISR alone, for definiteness. See also the work of Baur *et al.*, Dittmaier and Kramer, Zykunov for exact $\mathcal{O}(\alpha)$ results and Hamberg *et al.*, van Neerven and Matsuura and Anastasiou *et al.* for exact $\mathcal{O}(\alpha_s^2)$ results.

For the basic formula

$$d\sigma_{exp}(pp \to V+X \to \bar{\ell}\ell'+X') = \sum_{i,j}\int dx_i dx_j F_i(x_i)F_j(x_j)d\hat{\sigma}_{exp}(x_ix_js), \tag{55}$$

we use the result in (4) here with semi-analytical methods and structure functions from Martin *et al.*. A MC realization will appear, but see below.

SHOWER/ME MATCHING

- **Note the following: In (55) WE DO NOT ATTEMPT *HERE* TO REPLACE HERWIG and/or PYTHIA – WE INTEND *HERE* TO COMBINE OUR EXACT YFS CALCULUS, $d\hat{\sigma}_{exp}(x_i x_j s)$, WITH HERWIG and/or PYTHIA BY USING THEM/IT TO GENERATE A PARTON SHOWER STARTING FROM (x_1, x_2) AT FACTORIZATION SCALE μ AFTER THIS POINT IS PROVIDED BY $\{F_i\}$: THERE ARE TWO APPROACHES TO THE MATCHING UNDER STUDY, ONE BASED ON p_T-MATCHING AND ONE BASED ON SHOWER-SUBTRACTED RESIDUALS $\{\hat{\bar{\bar{\beta}}}_{n,m}(k_1, \ldots, k_n; k'_1, \ldots, k'_m)\}$, WHEREIN THE SHOWER FORMULA AND THE $QED \otimes QCD$ EXPONENTIATION FORMULA CAN BE EXPANDED IN PRODUCT AND REQUIRED TO MATCH THE GIVEN EXACT RESULT TO THE SPECIFIED ORDER – SEE hep-ph/0509003.**
- **THIS COMBINATION OF THEORETICAL CONSTRUCTS CAN BE SYSTEMATICALLY IMPROVED WITH EXACT RESULTS ORDER-BY-ORDER IN α_s, α, WITH EXACT PHASE SPACE.**
- **THE RECENT ALTERNATIVE PARTON EVOLUTION ALGORITHM BY JADACH and SKRZYPEK,Acta. Phys. Pol.B35, 745 (2004), CAN ALSO BE USED.**
- **LACK OF COLOR COHERENCE $\Rightarrow$ ISAJET NOT CONSIDERED HERE.**

With this said, we compute , with and without QED, the ratio

$r_{exp} = \sigma_{exp}/\sigma_{Born}$

to get the results (We stress that we *do not* use the narrow resonance approximation here.)

$$r_{exp} = \begin{cases} 1.1901 & , \textbf{QCED} \equiv \textbf{QCD+QED, LHC} \\ 1.1872 & , \textbf{QCD, LHC} \\ 1.1911 & , \textbf{QCED} \equiv \textbf{QCD+QED, Tevatron} \\ 1.1879 & , \textbf{QCD, Tevatron} \end{cases} \quad (56)$$

$\Rightarrow$

$*$QED IS AT .3% AT BOTH LHC and FNAL.

$*$THIS IS STABLE UNDER SCALE VARIATIONS.

$*$WE AGREE WITH BAUR ET AL., HAMBERG ET AL., van NEERVEN and ZIJLSTRA–NOTE THAT AS WE HAVE AN EXPONENTITED FORMULA, IT MAKES SENSE TO COMPARE WITH THE LATTER.

$*$QED EFFECT SIMILAR IN SIZE TO STR. FN. RESULTS.

$*$DGLAP-CS SYNTHESIZATION HAS NOT COMPROMISED THE NORMALIZATION.

QUARK MASSES and RESUMMATION in PRECISION QCD THEORY

- Di'Lieto et al.(NPB183(1981)223), Doria et al.(*ibid.*168(1980)93), Catani et al.(*ibid.*264(1986)588;Catani(ZPC37(1988)357): IN ISR, BLOCH-NORDSIECK CANCELLATION FAILS AT $\mathcal{O}(\alpha_s^2)$ for $m_q \neq 0$.
- FOR $q + q' \to q'' + q''' + V + X$, THEY GET

$$\text{flux}\ \frac{d\sigma}{d^3Q} = \frac{-g^4\bar{H}}{(d-4)32\pi^2}\left(\frac{1-\beta}{\beta}\right)\left(\frac{1}{\beta}\ln\left(\frac{1+\beta}{1-\beta}\right)-2\right) \tag{57}$$

- HERE, $\bar{H}$ IS THE HARD PROCESS DRESSED AS

$$F_1 = C_2(G)H_{ab}^{\alpha\beta}(T_i)^{\beta\alpha}(T_i)_b a \tag{58}$$

FOR

$$f_{ijk}f_{ijl} = C_2(G)\delta_{kl}$$
$$(T_iT_i)_{ab} = C_2(F)I_{ab}.$$

THEY EVALUATE THE GRAPHS IN FIG.1 USING MUELLER'S THM.

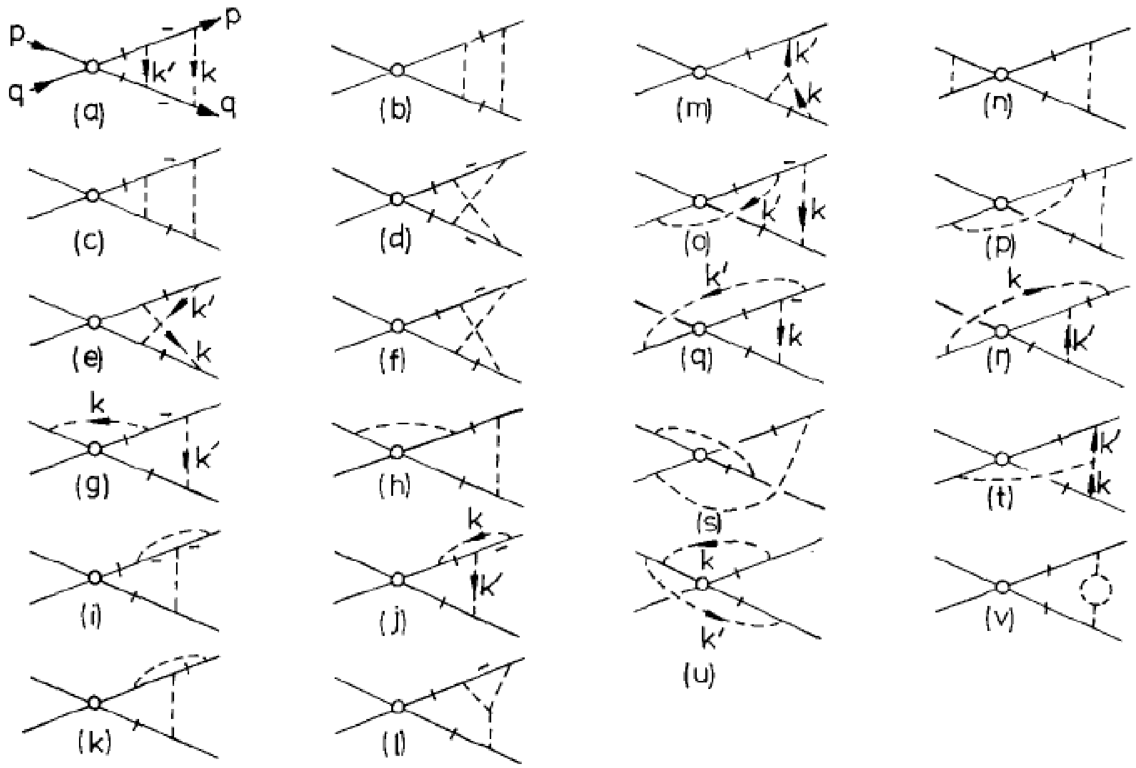


Figure 1. Graphs evaluated in Ref. [2] (see the first paper therein especially) in arriving at the result in (3) using Mueller's theorem for the respective cross section. The usual Landau-Bjorken-Cutkosky (LBC) [10] rules obtain so that a slash puts the line on-shell and a dash changes the iε-prescription; and, graphs that have cancelled or whose contributions are implied by those in the figure are not shown explicitly.

- SINCE BN VIOLATION VANISHES FOR $m_q \to 0$, MUST SET $m_q = 0$ IN ISR for $\mathcal{O}(\alpha_s^n),\ n \geq 2$: NOTE, $m_b \cong 5$GeV.

- SOURCE OF BN-VIOLATION: LOOK AT CONTRIBUTION OF DIAGRAMS (q-o) IN FIG.1:

$$A_{q-o} = \frac{1}{\beta^2}\int \frac{d^3k d^3k' 2k_z}{(k_z + k'_z + i\epsilon)(\beta^2 k_z^2 - \mathbf{k}^2)(\beta^2 k_z^2 - \mathbf{k}'^2 + i\epsilon)(k_z^2 + \epsilon^2)} \tag{59}$$

UV-REGULATED RESULT: USE THE REGULATOR $e^{-\mathbf{k}^2/\Lambda^2}$,

$\Rightarrow$

$$A_{q-o}|_{UV-reg} = \frac{4\pi^{n+1}(\Lambda^2)^{n-3}}{\beta^2}\left\{\frac{1}{(n-3)^2} + \frac{1}{2(n-3)}\ln\left(\frac{1+\beta}{1-\beta}\right)\right\}. \tag{60}$$

$\Rightarrow$

$$F_{nbn} = \frac{(1-\beta)(\ln\left(\frac{1+\beta}{1-\beta}\right) - 2\beta)}{\ln\left(\frac{1+\beta}{1-\beta}\right)} \tag{61}$$

IS FRACTION OF SINGLE-POLE TERM UN-CANCELLED.

- LANDAU-BJORKEN-CUTKOSKY ANALYSIS: INTEGRATE OVER ${k'}_z$ IN (59)

  $\Rightarrow$ TWO POLES BELOW REAL AXIS

  ${k'}_z = -k_z - i\epsilon\,,\, {k'}_z = -\sqrt{\beta^2 k_z^2 - {k'}_\perp{}^2 + i\epsilon}$

  WHERE ENERGY OF k'-gluon $= -\beta k_z$ BY LBC RULES.

  ONLY THE LATTER POLE GIVES ON-SHELL k' RADIATION:

  $\mathfrak{R} = \{0 \le {k'}_\perp{}^2 \le \beta^2 k_z^2\}$ IS ON-SHELL k'-gluon REGIME.

- WE GET THERE

$$A_{q-o}|_{\mathfrak{R}} = \Re\frac{1}{\beta^2}\int d^3k \int_0^{\beta^2 k_z^2} \pi d({k'}_\perp{}^2)\frac{-2\pi i}{-(-2)\sqrt{\beta^2 k_z^2 - {k'}_\perp{}^2}}$$
$$\frac{1}{k_z - \sqrt{\beta^2 k_z^2 - {k'}_\perp{}^2} + i\epsilon}\,\frac{1}{\beta^2 k_z^2 - \mathbf{k}^2}\,\frac{2k_z}{k_z^2 + \epsilon^2}. \qquad (62)$$

WE NEED THE REAL PART:

$$
\begin{aligned}
A_{q-o}|_{\Re} = \Re\frac{-\pi i}{\beta^2}\int d^3k \int_0^{\beta^2 k_z^2} \pi d({k'_\perp}^2) \frac{1}{\sqrt{\beta^2 k_z^2 - {k'_\perp}^2}} \\
\frac{k_z + i\epsilon + \sqrt{\beta^2 k_z^2 - {k'_\perp}^2}}{(k_z + i\epsilon)^2 - (\beta^2 k_z^2 - {k'_\perp}^2)} \frac{1}{\beta^2 k_z^2 - \mathbf{k}^2} \frac{2k_z}{k_z^2 + \epsilon^2} \\
= \Re\frac{-\pi i}{\beta^2}\int d^3k \int_0^{\beta^2 k_z^2} \pi d({k'_\perp}^2) \frac{1}{\sqrt{\beta^2 k_z^2 - {k'_\perp}^2}} \\
\frac{k_z + i\epsilon}{(k_z + i\epsilon)^2 - (\beta^2 k_z^2 - {k'_\perp}^2)} \frac{1}{\beta^2 k_z^2 - \mathbf{k}^2} \frac{2k_z}{k_z^2 + \epsilon^2} \\
= \Re\frac{-\pi i}{\beta^2}\int d^3k \int_0^{\beta^2 k_z^2} \pi d({k'_\perp}^2) \frac{1}{\sqrt{\beta^2 k_z^2 - {k'_\perp}^2}} \\
\frac{1}{2}\left(\frac{1}{k_z + i\epsilon - \sqrt{\beta^2 k_z^2 - {k'_\perp}^2}} + \frac{1}{k_z + i\epsilon - \sqrt{\beta^2 k_z^2 - {k'_\perp}^2}}\right) \\
\frac{1}{\beta^2 k_z^2 - \mathbf{k}^2} \frac{2k_z}{k_z^2 + \epsilon^2} \\
= \Re\frac{-i\pi^2}{\beta^2}\int d^3k \left(-\ln(k_z + i\epsilon - \beta|k_z|) + \ln(k_z + i\epsilon + \beta|k_z|)\right) \\
\frac{1}{\beta^2 k_z^2 - \mathbf{k}^2} \frac{2k_z}{k_z^2 + \epsilon^2},
\end{aligned}
\tag{63}
$$

WHERE ON-SHELL REGIME ACTUALLY HAS $k'_0 = -\beta k_z < 0$, REAL RADIATIVE CONTRIBUTION, BY THE STANDARD LBC METHODS, HAS $k_z > 0$.

- FOR INTEGRATION OVER $k_z > \sqrt{\epsilon}$, RHS OF THE LAST EQUATION HAS NO REAL PART AS $\epsilon \to 0$.

  THE REAL EMISSION PART OF (63) ARISES FROM $0 \leq k_z \leq \sqrt{\epsilon}$.

  BRANCH CUTS FOR THE LOGS:JOIN THEM BETWEEN $k_{z1} = -i\epsilon/(1-\beta)$ and $k_{z2} = -i\epsilon/(1+\beta)$;

  AND WE CLOSE THE CONTOUR BELOW THE REAL AXIS AS SHOWN IN Fig. 2

  $\Rightarrow$

$$\oint_C dk_z\left(-\ln(k_z + i\epsilon - \beta k_z) + \ln(k_z + i\epsilon + \beta k_z)\right)\frac{1}{\beta^2 k_z^2 - \mathbf{k}^2}\frac{2k_z}{k_z^2 + \bar{\epsilon}^2} = 0, \tag{64}$$

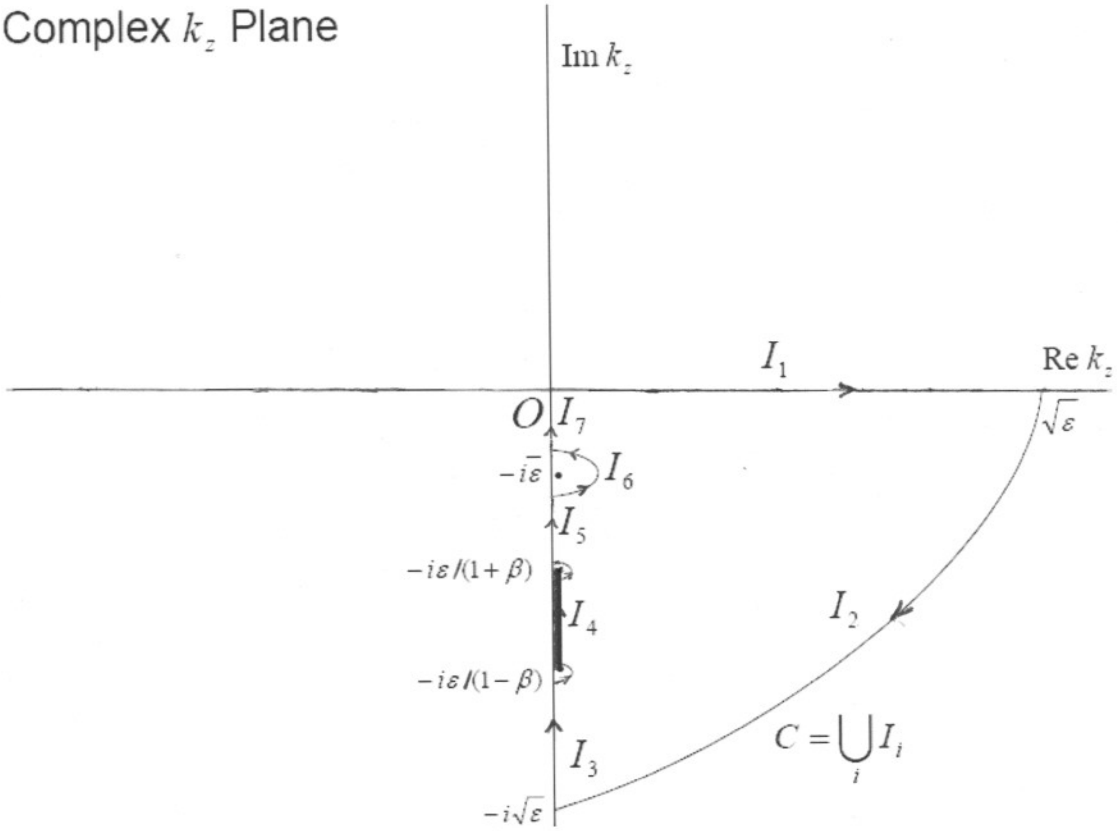


Figure 2. The contour C used in the complex k_z–plane to evaluate the real emission part of the contribution of diagrams (q-o) in Fig. 1 to the RHS of (3). See the text for further discussion.

WHERE WE USE THE INTRINSIC FREEDOM IN THE FEYNMAN $i\epsilon$-PRESCRIPTION TO TAKE EACH SUCH INFINITESIMAL PARAMETER INDEPENDENTLY TO 0 FROM ABOVE AND THE CURVE C IS GIVEN IN FIG. 2. WE TAKE HERE $k_\perp > \sqrt{\epsilon},\ \bar{\epsilon} = \epsilon^{\frac{3}{2}}$

- BY CAUCHY'S THEOREM,

$$
\begin{aligned}
I_1 &= \int_0^{\sqrt{\epsilon}} dk_z \left(-\ln(k_z + i\epsilon - \beta^2|k_z|) + \ln(k_z + i\epsilon + \beta|k_z|\right) \\
&\qquad \frac{1}{\beta^2 k_z^2 - \mathbf{k}^2} \frac{2k_z}{k_z^2 + \bar{\epsilon}^2} \\
&= -\sum_{i=2}^{7} I_i .
\end{aligned} \tag{65}
$$

- TREAT EACH INTEGRAL IN TURN:

  FOR I_2, USE THE CHANGE OF VARIABLE $k_z = \sqrt{\epsilon}e^{i\theta}$, FOR$0 \geq \theta \geq -\frac{\pi}{2}$.

  THEN, WE GET

$$
\begin{aligned}
I_2 &= \int_0^{-\frac{\pi}{2}} id\theta k_z \left(-\ln(k_z + i\epsilon - \beta k_z) + \ln(k_z + i\epsilon + \beta k_z)\right) \\
&\qquad \frac{1}{\beta^2 k_z^2 - \mathbf{k}^2} \frac{2k_z}{k_z^2 + \vec{\epsilon}^2} \\
&= 2\int_0^{-\frac{\pi}{2}} id\theta \left(-\ln(1-\beta) + \ln(1+\beta)\right) \frac{1}{-{\mathbf{k}_\perp}^2} \\
&= -i\pi \ln\left(\frac{1+\beta}{1-\beta}\right) \frac{1}{(-{\mathbf{k}_\perp}^2)}.
\end{aligned}
\tag{66}
$$

- For I_3, USE THE CHANGE OF VARIABLE $k_z = -iy$: IT IS PURE REAL SO THAT IT WILL NOT CONTRIBUTE THE THE IMAGINARY PART OF I_1 VIA (64).

- FOR I_4 WE SEE FROM PASSING AROUND THE LOWER BRANCH POINT IN FIG. 2 THAT THE RESPECTIVE IMAGINARY CONTRIBUTION IS

$$\begin{aligned} i\Im I_4 &= \int_{\frac{-i\epsilon}{1-\beta}}^{\frac{-i\epsilon}{1+\beta}} dk_z \, (-\pi i) \, \frac{1}{-\mathbf{k}_\perp{}^2} \frac{2k_z}{k_z^2 + \bar{\epsilon}^2} \\ &= 2\pi i \ln\left(\frac{1+\beta}{1-\beta}\right) \frac{1}{(-\mathbf{k}_\perp{}^2)}. \end{aligned} \tag{67}$$

- FOR I_5, WE SEE BY THE CHANGE OF VARIABLE $k_z = -iy$ THAT IT TOO IS PURE REAL AND DOES NOT CONTRIBUTE TO THE IMAGINARY PART OF I_1 VIA (64).

- For I_6, WE GET THE RESULT

$$I_6 = \pi i Res(-i\bar{\epsilon}) = 0 \tag{68}$$

SINCE $\bar{\epsilon}/\epsilon \to 0$ WHEN $\epsilon \to 0$.

- FOR I_7 THE CHANGE OF VARIABLE $k_z = -iy$ SHOWS THAT IT TOO IS PURE REAL AND DOES NOT CONTRIBUTE TO THE IMAGINARY PART OF I_1 VIA (64).

- NET RESULT:

$$\begin{aligned} i\Im I_1 &= -\{2\pi i - \pi i\}\frac{-1}{\mathbf{k}_\perp{}^2}\ln\left(\frac{1+\beta}{1-\beta}\right) \\ &= \frac{\pi i}{\mathbf{k}_\perp{}^2}\ln\left(\frac{1+\beta}{1-\beta}\right). \end{aligned} \tag{69}$$

$\Rightarrow$

$$A_{q-o}|_{\mathfrak{R},\text{real rad.}} = \frac{2\pi^3}{\beta^2}\left(\frac{1}{2}\ln\left(\frac{1+\beta}{1-\beta}\right)\right)\int\frac{d^2k_\perp}{\mathbf{k}_\perp{}^2}, \tag{70}$$

- INTEGRAL OVER $\mathbf{k}_\perp$ in (70):

$$
\begin{aligned}
\mathcal{I}_{\text{UV reg.}} &= \int \frac{d^2 k_\perp e^{-{\mathbf{k}_\perp}^2/\Lambda^2}}{{\mathbf{k}_\perp}^2} \\
&= \int \frac{d^3 k \delta(k_z) e^{-\mathbf{k}^2/\Lambda^2}}{\mathbf{k}^2} \\
&= \int \frac{d^n k \delta(k_z) e^{-\mathbf{k}^2/\Lambda^2}}{\mathbf{k}^2} \\
&= \int_0^\infty d\rho \int d^n k \delta(k_z) e^{-\mathbf{k}^2/\Lambda^2 - \rho \mathbf{k}^2} \\
&= \frac{2\pi^{\frac{(n-1)}{2}}}{n-3} (\Lambda^2)^{\frac{n-3}{2}}.
\end{aligned}
\tag{71}
$$

- $\Rightarrow$

$$
A_{q-o}\Big|_{\mathfrak{R},\text{real rad., UV reg.}} = \frac{4\pi^4 (\Lambda^2)^{\frac{n-3}{2}}}{\beta^2} \left(\frac{1}{2(n-3)} \ln\left(\frac{1+\beta}{1-\beta}\right)\right), \tag{72}
$$

$\Rightarrow$ REAL EMISSION IN A_{q-o} SATURATES SINGLE IR POLE.

- THUS, WE WRITE

$$\text{flux}\,\frac{d\sigma}{d^3Q} = \frac{-g^4\bar{H}}{64\pi^6} F_{nbn} A_{q-o}\big|_{\mathfrak{R},\text{real rad., IR pole part}}, \tag{73}$$

WHERE FROM (72) WE HAVE

$$A_{q-o}\big|_{\mathfrak{R},\text{real rad., IR pole part}} = \frac{4\pi^4}{\beta^2}\left(\frac{1}{2(n-3)}\ln\left(\frac{1+\beta}{1-\beta}\right)\right). \tag{74}$$

- APPLY QCD RESUMMATION TO REAL EMISSION IN $A_{q-o}|_{\mathfrak{R}}$: APPLY IT TO THE FRACTION F_{nbn}; REMAINING $1-F_{nbn}$ CANCELLED BY VIRTUAL CORRECTIONS

- USING

$$d\hat{\sigma}_{\rm exp} = e^{{\rm SUM_{IR}(QCD)}} \sum_{n=0}^{\infty} \int \prod_{j=1}^{n} \frac{d^3 k_j}{k_j} \int \frac{d^4 y}{(2\pi)^4} e^{iy\cdot(p_1+q_1-p_2-q_2-p_X-\sum k_j)}$$
$$* \, e^{D_{\rm QCD}} \tilde{\bar{\beta}}_n(k_1,\ldots,k_n) \frac{d^3 p_2}{p_2^{\,0}} \frac{d^3 q_2}{q_2^{\,0}} \frac{d^3 p_X}{p_X^{\,0}} \tag{75}$$

WE GET

$$F_{nbn} A_{q-o}\big|_{\mathfrak{R}\text{,real rad., resummed}} = F_{nbn} \Re \frac{-i\pi^2}{\beta^2} \int d^2 k_\perp \int_0^{\sqrt{\epsilon}} dk_z F_{YFS}(\bar{\gamma}_q) e^{\bar{\delta}_q/2}$$
$$(\beta k_z)^{\bar{\gamma}_q} \left(-\ln(k_z + i\epsilon - \beta k_z) + \ln(k_z + i\epsilon + \beta k_z)\right)$$
$$\frac{1}{\beta^2 k_z^2 - \mathbf{k}^2} \frac{2k_z}{k_z^2 + \epsilon^2}, \tag{76}$$

WHERE WE HAVE DEFINED

$$\bar{\gamma}_q = 2C_F \frac{\alpha_s(Q^2)}{\pi}(\ln(s/m^2) - 1) \quad (77)$$

$$\bar{\delta}_q = \frac{\bar{\gamma}_q}{2} + \frac{2\alpha_s C_F}{\pi}(\frac{\pi^2}{3} - \frac{1}{2}). \quad (78)$$

- USING THE SUBSTITUTION $k_z = \sqrt{\epsilon}\bar{k}_z$, WE HAVE

$$F_{nbn} A_{q-o}|_{\Re,\text{real rad., resummed}} = F_{nbn} \Re \frac{-i\pi^2 \epsilon^{\frac{\bar{\gamma}_q}{2}}}{\beta^2} \int d^2 k_\perp \int_0^1 d\bar{k}_z F_{YFS}(\bar{\gamma}_q) e^{\bar{\delta}_q/2}$$
$$(\beta\bar{k}_z)^{\gamma_q} \left(-\ln(\bar{k}_z + i\sqrt{\epsilon} - \beta\bar{k}_z) + \ln(\bar{k}_z + i\sqrt{\epsilon} + \beta\bar{k}_z)\right)$$
$$\frac{1}{-(1-\beta^2)\epsilon\bar{k}_z^2 - \mathbf{k}_\perp{}^2} \frac{2\bar{k}_z}{\bar{k}_z^2 + \epsilon}. \quad (79)$$

THE RHS OF THIS LAST EQUATION VANISHES AS $\epsilon \longrightarrow 0$, REMOVING THE VIOLATION OF BLOCH-NORDSIECK CANCELLATION IN (57).

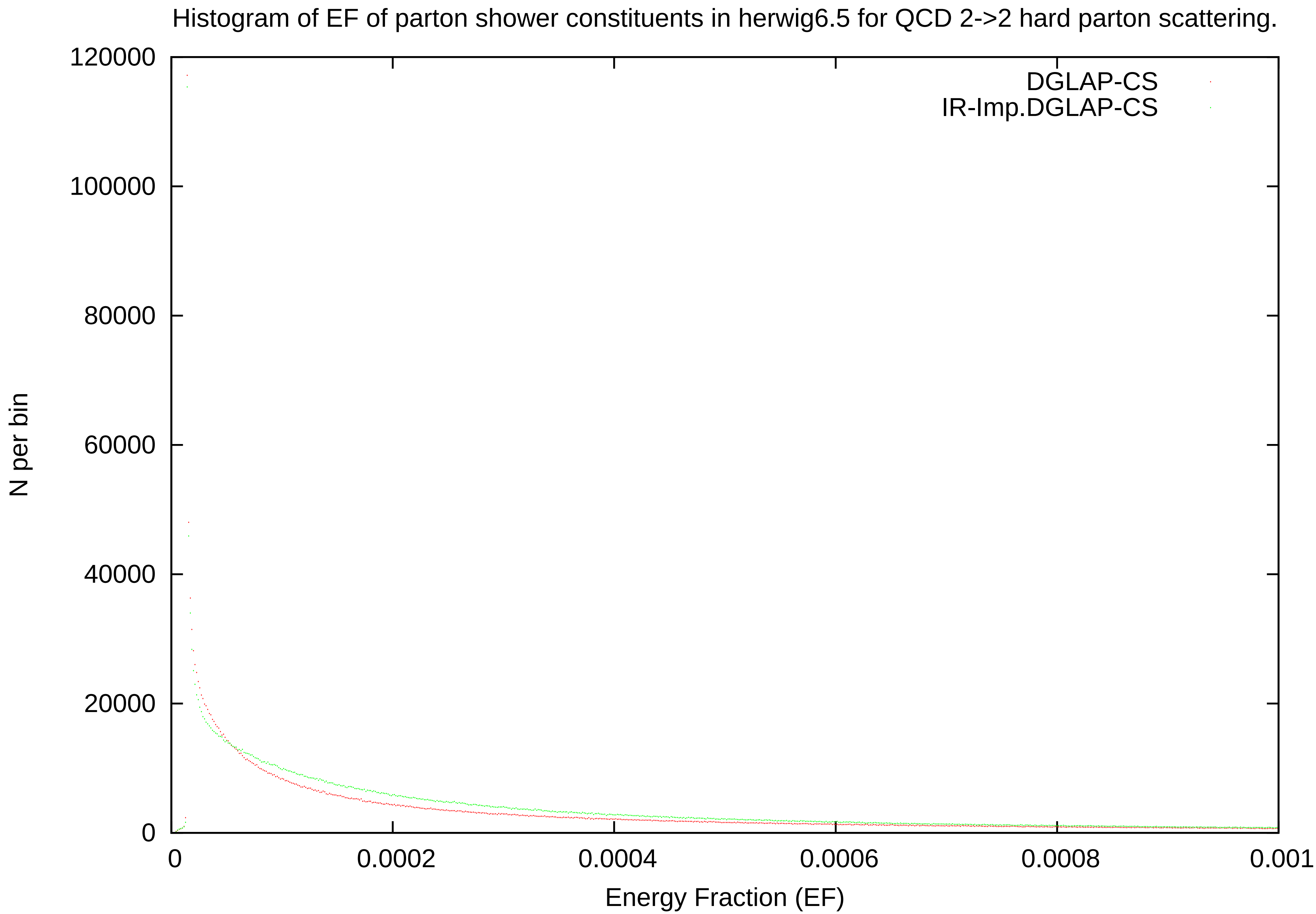
CONCLUSION:

- RESUMMATION CURES LACK OF BN CANCELLATION IN MASSIVE QCD

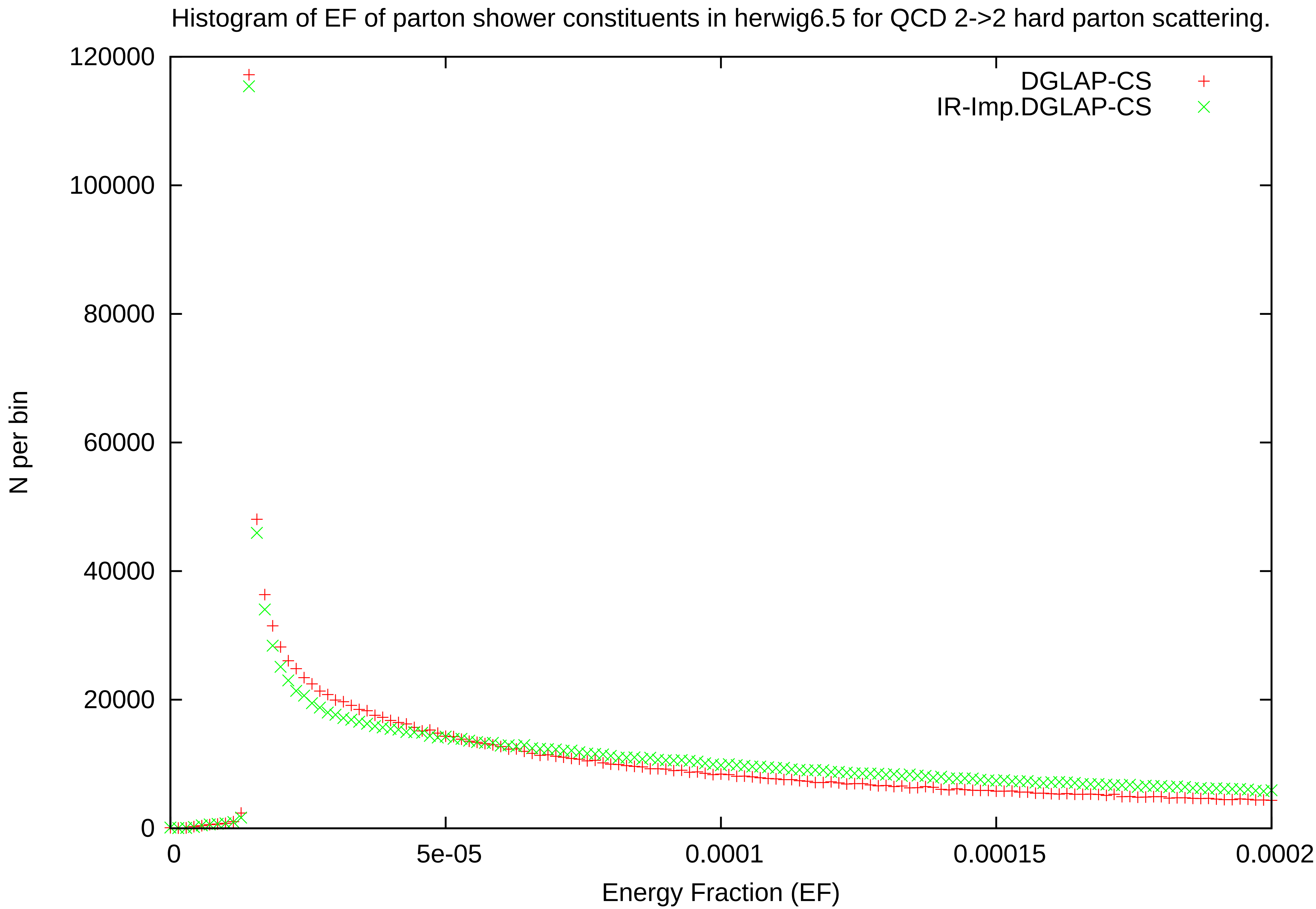
Sample MC data: IR-Improved Kernels in HERWIG6.5

We have preliminary results on IR-Improved Showers in HERWIG6.5: we compare the z-distributions and the p_T of the IR-Improved and usual DGLAP-CS showers in the Figs. 3, 4, 5.

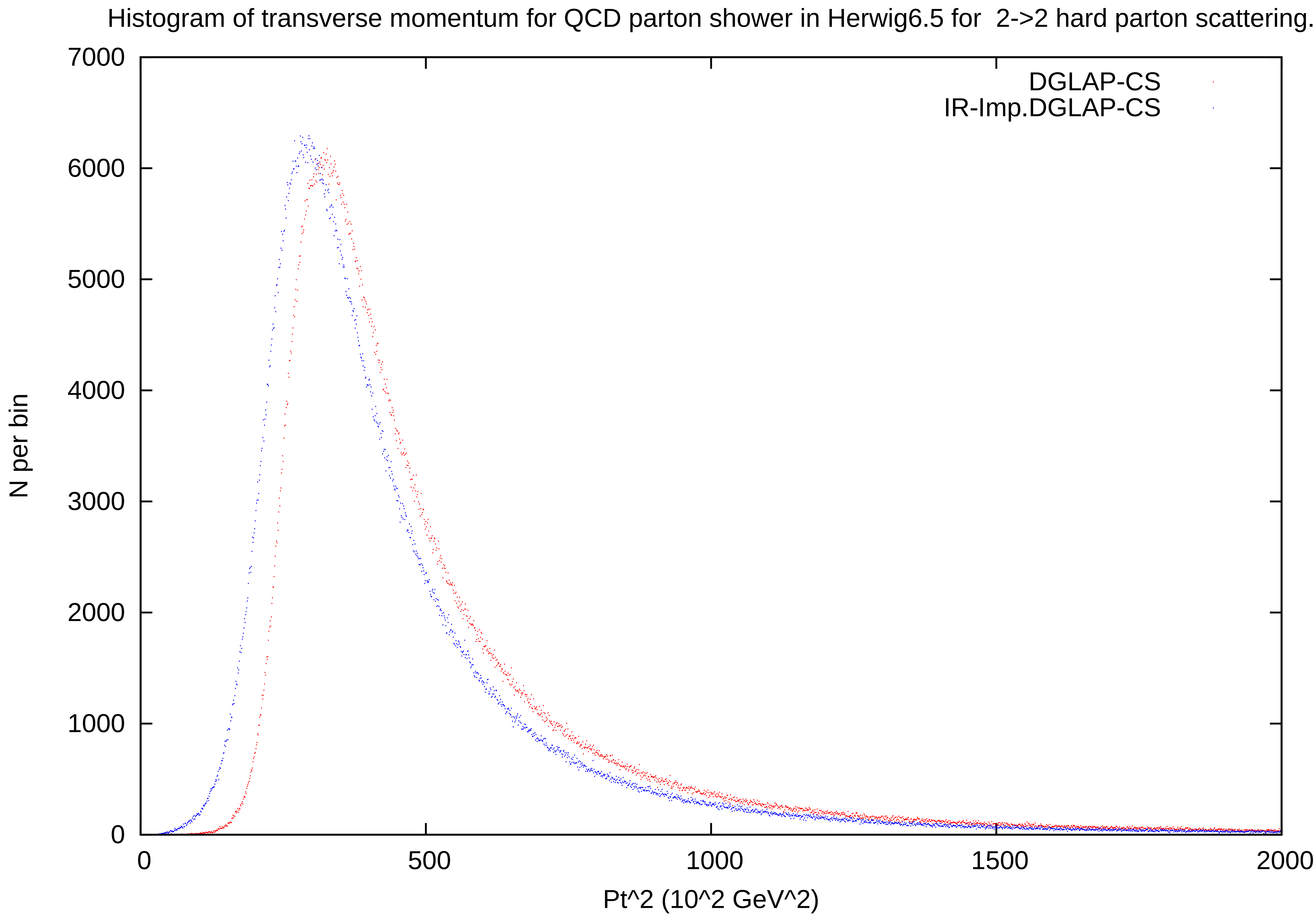
SIMILAR RESULTS FOR PYTHIA and MC@NLO IN PROGRESS.



Histogram of EF of parton shower constituents in herwig6.5 for QCD 2->2 hard parton scattering.
DGLAP-CS
IR-Imp.DGLAP-CS
N per bin
Energy Fraction (EF)
120000
100000
80000
60000
40000
20000
0
0
0.0002
0.0004
0.0006
0.0008
0.001



Histogram of EF of parton shower constituents in herwig6.5 for QCD 2->2 hard parton scattering.
DGLAP-CS
IR-Imp.DGLAP-CS
N per bin
Energy Fraction (EF)
0
20000
40000
60000
80000
100000
120000
0
5e-05
0.0001
0.00015
0.0002


Histogram of transverse momentum for QCD parton shower in Herwig6.5 for 2->2 hard parton scattering.
DGLAP-CS
IR-Imp.DGLAP-CS
N per bin
Pt^2 (10^2 GeV^2)
0
1000
2000
3000
4000
5000
6000
7000
0
500
1000
1500
2000

Conclusions

YFS-TYPE METHODS (EEX AND CEEX) EXTEND TO NON-ABELIAN GAUGE THEORY AND ALLOWS SIMULTANEOUS RESMN OF QED AND QCD WITH PROPER SHOWER/ME MATCHING BUILT-IN.

FOR QED$\otimes$QCD

- FULL MC EVENT GENERATOR REALIZATION OPEN.
- SEMI-ANALYTICAL RESULTS FOR QED (AND QCD) THRESHOLD EFFECTS AGREE WITH LITERATURE ON Z PRODUCTION; CHECK WITH W-PROD. IMMINENT
- AS QED ALONE IS AT THE .3% LEVEL, IT IS NEEDED FOR 1% LHC THEORY PREDICTIONS.

- A FIRM BASIS FOR THE COMPLETE $\mathcal{O}(\alpha_s^2, \alpha\alpha_s, \alpha^2)$ MC RESULTS NEEDED FOR THE PRECISION FNAL/LHC/RHIC/ILC PHYSICS HAS BEEN DEMONSTRATED AND ALL THE LATTER IS IN PROGRESS, WITH M. Kalmykov, S. Majhi, S. Yost and S. Joseph.– SEE JHEP0702(2007)040,arxiv:0707.3654,0708.0803, NEW RESULTS FOR HO F-Int's,etc. –no time to discuss here

- FIRST MC DATA ON NEW IR-IMPROVED SHOWERS IN HERWIG6.5: SPECTRA ARE SOFTER AS EXPECTED.