Factorization With Incoming Wilson Lines

S. Mert Aybat

ETH, Zurich

in collaboration with G. Sterman

HERA and the LHC, 2008

Outline

Prologue

- perturbative cross sections involving Wilson Lines play an important role in the analysis of high-energy scattering in QCD.
- definitions of various parton distribution functions
- their analysis is central to the proof of factorization theorems.

We will discuss : a new proof of the cancellation of infrared divergences in cross sections with a light-like Wilson line in the initial state.

Why Care ?

- most recent extensive argument for factorization in the full perturbation theory assumes color-singlet particles in the initial state.
  (Collins, Soper and Sterman 1985, 1988)
- all perturbative calculations of QCD hard-scattering functions use infrared-regulated perturbation theory with colored incoming lines.
- cross section predictions using pQCD $\Rightarrow$ factorization for incoming partons as well as color-singlet hadrons

QCD Factorization

- Consider the following process (as an example)

$$h(p) + h'(p') \to ll'(Q^\mu) + X\,.$$

The cross section may be written as

$$\frac{d\sigma_{hh'\to Q^2}(s,Q^2)}{dQ^2} = \sum_{i,j=f,\bar{f},g} \int_0^1 d\zeta d\zeta'\, \phi_{i/h}(\zeta,\mu^2)\, H_{ij}\left(\frac{Q^2}{\zeta\zeta' s}, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right) \times \phi_{j/h'}(\zeta',\mu^2)\,.$$

- $\phi_{i/h}$, $\phi_{j/h'}$: parton distribution functions $\Rightarrow$ experiment.

  H_{ij} : hard scattering function $\Rightarrow$ pQCD calculations beyond the leading order.

How Do We Calculate H_{ij}?

- Start with the following assumption : factorization holds for partonic cross sections.

$$\frac{d\sigma_{aa' \to Q^2}(s, Q^2)}{dQ^2} = \sum_{i,j=f,\bar{f},g} \int_0^1 d\zeta d\zeta' \, \phi_{i/a}(\zeta, \mu^2) \, H_{ij}\left(\frac{Q^2}{\zeta\zeta' s}, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right) \times \phi_{j/a'}(\zeta', \mu^2)\,,$$

  with the <u>same</u> hard scattering function.
- To calculate the hard scattering function to any order in perturbation theory,
  - construct the partonic cross section to that order (colored initial states),
  - factorize initial state collinear dynamics into the “parton in parton” distribution functions, $\phi_{i/a}$,
  - remainders give the perturbative expansion for the IR safe H_{ij}.

Soft Gluons and Wilson Lines

Where exactly in the proof of factorization do we need the cancellation of soft divergences with incoming Wilson lines?

main steps

1. identify leading regions (Sterman 1978)
2. cancellation of final state interactions using time ordered perturbation theory
3. factorization of collinear gluons from the hard part
4. soft gluon factorization and cancellation

Eikonal Cross Sections

Consider an eikonal annihilation cross section for an initial state parton and a Wilson line.

$$\sigma_{eik}[S] = \sum_{N} S(N)\, \mathrm{Tr_c}\left[\langle p|\bar{\mathrm{T}}\left[(\phi_f(0)\Phi_\beta(0,-\infty))^\dagger\right]|N\rangle\langle N|\mathrm{T}\left(\phi_f(0)\Phi_\beta(0,-\infty)\right)|p\rangle\right] ,$$

- $S(N)$ assigns a weight to each final state $|N\rangle$, IR safe.
- ϕ_f : field of the incoming parton of flavor f with momentum p
- $$\Phi_\beta^{(f)}(b,a) = \mathrm{P}\exp\left[-i\,g\int_a^b d\lambda\,\beta\cdot A^{(f)}(\lambda\beta)\right]$$

  with $\beta^2 = 0$ (we will choose $\beta^\mu = \delta^{\mu+}$) .

Eikonal Cross Sections

Expand the Wilson lines in terms of their fields to get

$$\begin{aligned}\sigma_{eik}[S] \quad = \quad & \sum_N S(N) \sum_m (-ig)^m \sum_{j=1}^m \int_{-\infty}^{0} d\lambda_m \ldots \int_{-\infty}^{\lambda_{j+2}} d\lambda_{j+1} \\ & \times \mathrm{Tr_c}\Big\{ \langle p|\, \bar{\mathrm{P}} \left[\beta \cdot A(\beta\lambda_{j+1}) \ldots \beta \cdot A(\beta\lambda_m)\right] \phi_f^\dagger(0)|N\rangle \\ & \times \int_{-\infty}^{0} d\lambda_j \ldots \int_{-\infty}^{\lambda_2} d\lambda_1 \langle N|\phi_f(0)\, \mathrm{P}\left[\beta \cdot A(\beta\lambda_j) \ldots \beta \cdot A(\beta\lambda_1)\right] |p\rangle \Big\},\end{aligned}$$

Eikonal Cross Sections

Expand the Wilson lines in terms of their fields to get

$$\sigma_{eik}[S] = \sum_N S(N) \sum_m (-ig)^m \sum_{j=1}^m \int_{-\infty}^0 d\lambda_m \ldots \int_{-\infty}^{\lambda_{j+2}} d\lambda_{j+1} \times \mathrm{Tr_c}\left\{ \langle p| \,\bar{\mathrm{P}} \left[\beta\cdot A(\beta\lambda_{j+1})\ldots\beta\cdot A(\beta\lambda_m)\right] \phi_f^\dagger(0)|N\rangle \times \int_{-\infty}^0 d\lambda_j \ldots \int_{-\infty}^{\lambda_2} d\lambda_1 \langle N|\phi_f(0)\,\mathrm{P}\left[\beta\cdot A(\beta\lambda_j)\ldots\beta\cdot A(\beta\lambda_1)\right]|p\rangle \right\},$$

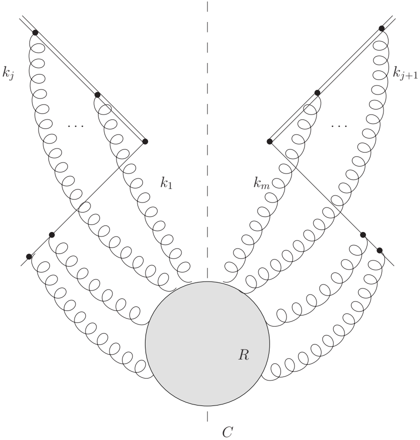
in terms of momentum space integrals over the Fourier transforms of the fields,

$$\sigma_{eik}[S] = \sum_N S(N) \sum_m (-ig)^m \prod_{i=1}^m \int d^4 q_i \sum_{j=1}^m \left\{ E_j^m(q_1,\ldots,q_m) \times \mathrm{Tr_c}\left\{ \langle p|\bar{\mathrm{P}}\left[\beta\cdot A(q_{j+1})\ldots\beta\cdot A(q_m)\right] \phi_f^\dagger(0)|N\rangle \times \langle N|\phi_f(0)\mathrm{P}\left[\beta\cdot A(q_j)\ldots\beta\cdot A(q_1)\right]|p\rangle\right\}\right\},$$

E_j^m : propagators on the Wilson lines in eikonal approximation

R: remainder function $\Rightarrow$ everything except for E

Infrared Cancellation Using Eikonal Identity



The Eikonal Factor

The eikonal factor E_j^m is given by

$$E_j^m(q_1,\ldots,q_m) = \left\{\frac{1}{q^-_{m-j+1}+i\epsilon}\,\frac{1}{q^-_{m-j+1}+q^-_{m-j+2}+i\epsilon}\cdots\frac{1}{q^-_{m-j+1}+\cdots+q^-_m+i\epsilon}\right\}$$
$$\times(-1)^{m-j}\left\{\frac{1}{q^-_{m-j}+i\epsilon}\,\frac{1}{q^-_{m-j-1}+q^-_{m-j}+i\epsilon}\cdots\frac{1}{q^-_1+\cdots+q^-_{m-j}+i\epsilon}\right\},$$

which satisfies the following important eikonal identity

$$\sum_{j=0}^{m} E_j^m(q_1,\ldots,q_m) = 0\,.$$

(Libby and Sterman 1979)

How Do We Make Use of the Eikonal Identity ?

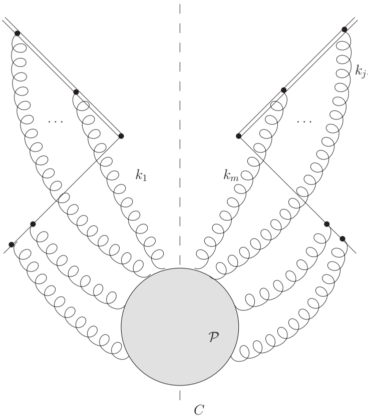
- rewrite the eikonal cross section as a sum over cut diagrams
- write each cut diagram as product of an eikonal factor E and a remainder function R
- use the property$\Rightarrow$ E_j^m for an uncut diagram is the same for all its cuts
- (Collin Soper and Sterman 1988) : the remainder function, when summed over its final states, is independent of the way soft gluons attach the eikonal lines $\Rightarrow$ after studying R in LCOPT

the eikonal cross section can be written as

$$\sigma_{eik} = \sum_m \left\{ \sum_j E_j^m \otimes \sum_R \sum_{\text{cuts of R}} R \right\}$$

Infrared Cancellation Using Eikonal Identity

$$\sum_{j=1}^{m}$$



$$= 0$$

Problem With Infrared Cancellation

- E_j^m has singularities for fixed values of q_T while $q^- \rightarrow 0$
- eikonal identity $\Rightarrow$ sum over all possible connections $\Rightarrow$ gluons appear in the final state
- final states typically have a cutoff on the amount of energy they can have $\Rightarrow$ Q
- onshell gluons with $q^- \rightarrow 0$ would have plus momenta $q^+ \sim q_T^2/q^- >> Q$ therefore they can not appear in the final state
- eikonal identity can not apply for such gluons and moreover they generate singularities in the eikonal cross section

Problem With Infrared Cancellation

- singularities correspond to radiation of energetic lines in the direction of the eikonal line
- classical physical evolution $\Rightarrow$ the particle is emitted far in the past, travels parallel to the eikonal, arrives at hard scattering at the same time
- complex in general when the gluon couples to another incoming line arriving from another direction.

Solution

Any such leading momentum configuration

- is either factorized into the incoming jet
- or cancels in the inclusive cross section

Summary and Outlook

- problem : cancellation of initial state soft gluon singularities
- motivation : partonic scattering cross sections
- shown : initial state IR singularities either cancel or factorize as a part of an enhanced jet
- closes a well recognized gap in the standard proof of factorization for inclusive processes
- extensions of classic factorization theorems (transverse-momentum and spin dependent distributions)
- assumptions still to be relaxed (for future work) : gluon attachments to the eikonals are <u>all soft</u> ! $\Rightarrow$ study of nested collinear subtractions