
Fits of the small x gluon from J/ψ data



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- I. Introduction; the gluon at small x and low – medium scales
- II. Exclusive J/ψ production in pQCD
- III. Determining the small x gluon from exclusive HERA data
- IV. Conclusions/Outlook

Ref/more details: Martin+Nockles+Ryskin+T, Phys. Lett. B 662 (2008) 252

I. Introduction; the gluon at small x

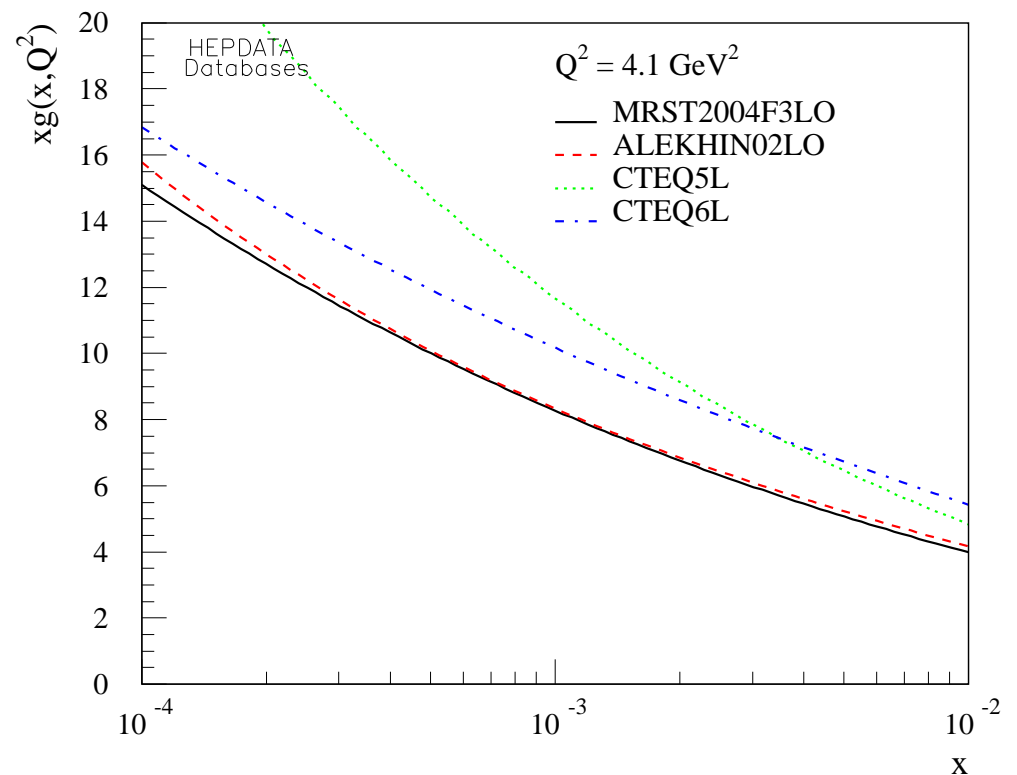
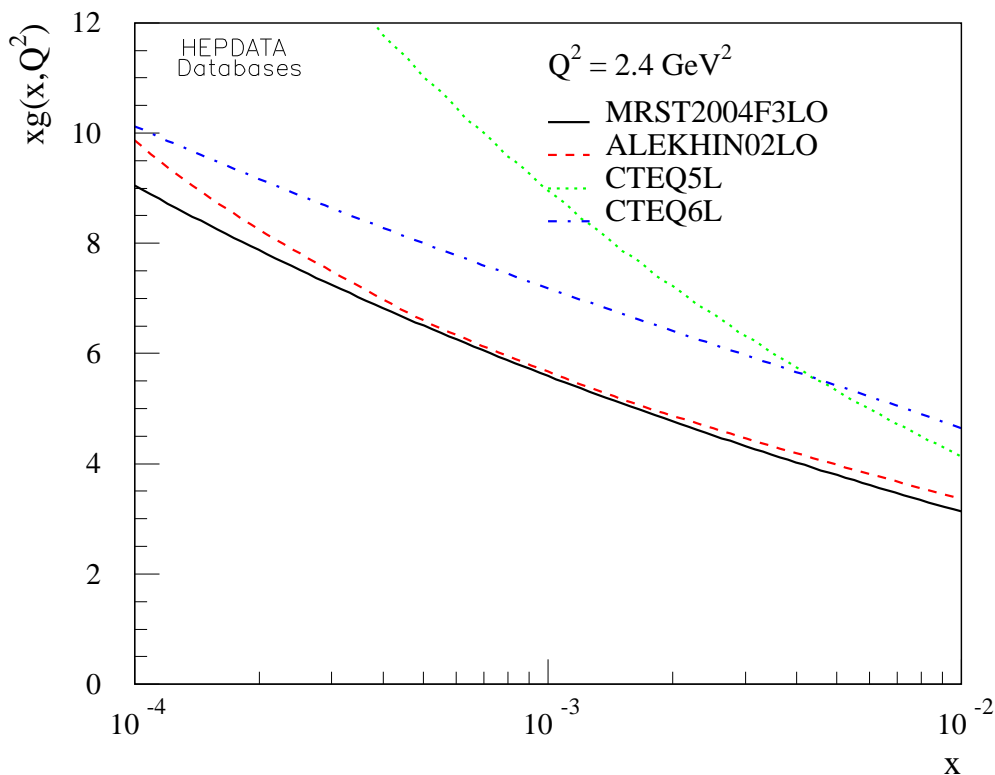
- Global fits constrain the gluon only **poorly** at small x :

→ lack of precise data + F_2 only limited sensitivity to gluon, **but** needed for

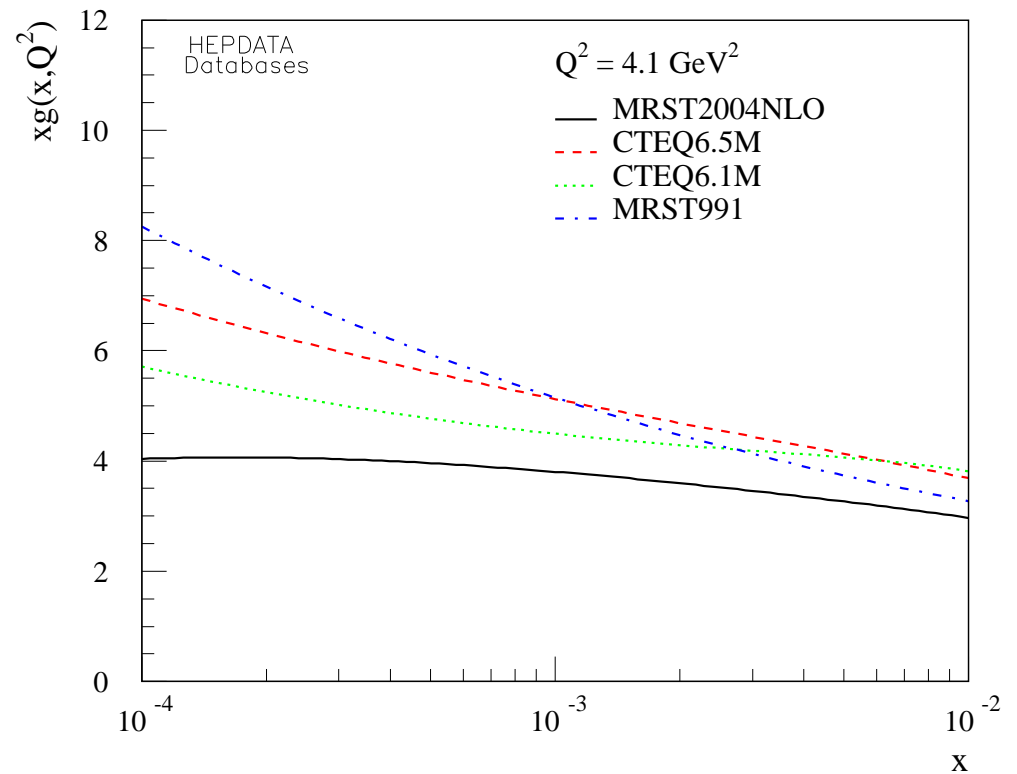
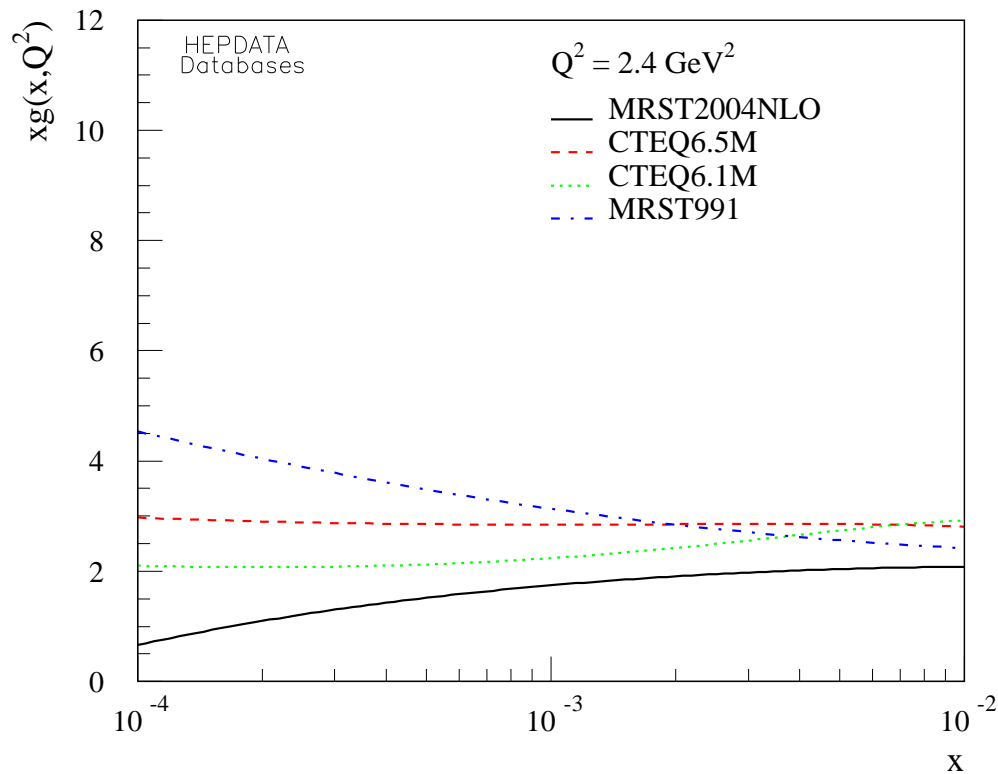
↪ description of semi-hard QCD / high energy scattering

↪ Underlying Events, Multiple Interactions, exclusive Higgs at LHC ...

- Leading Order: (relevant for MC's)

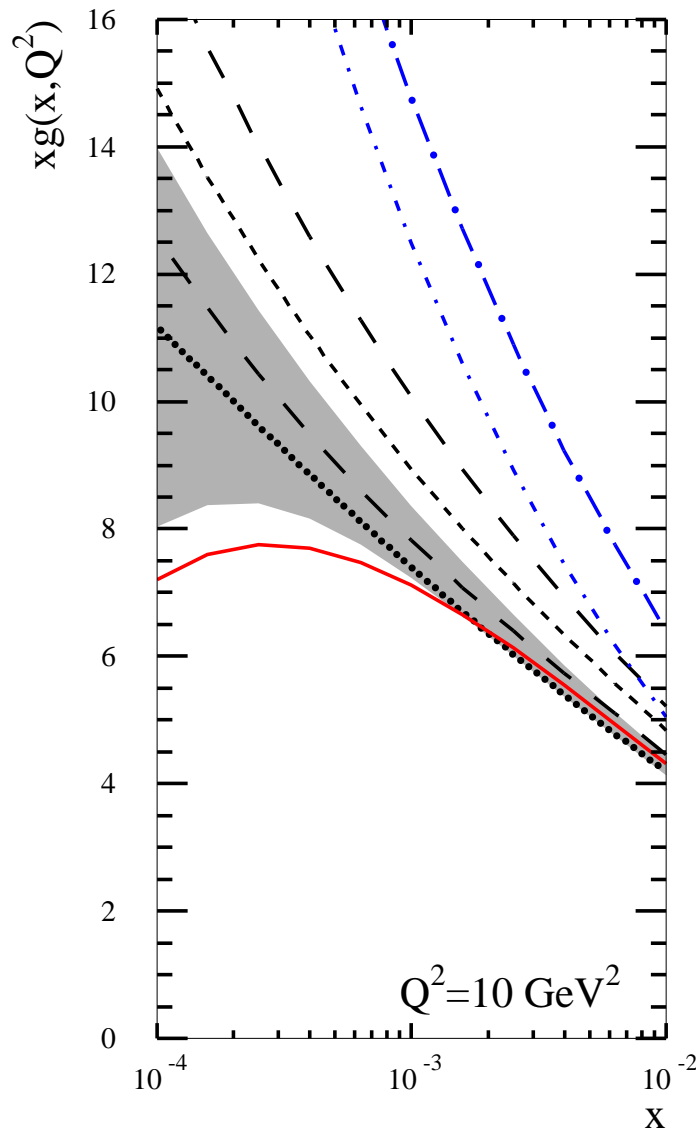
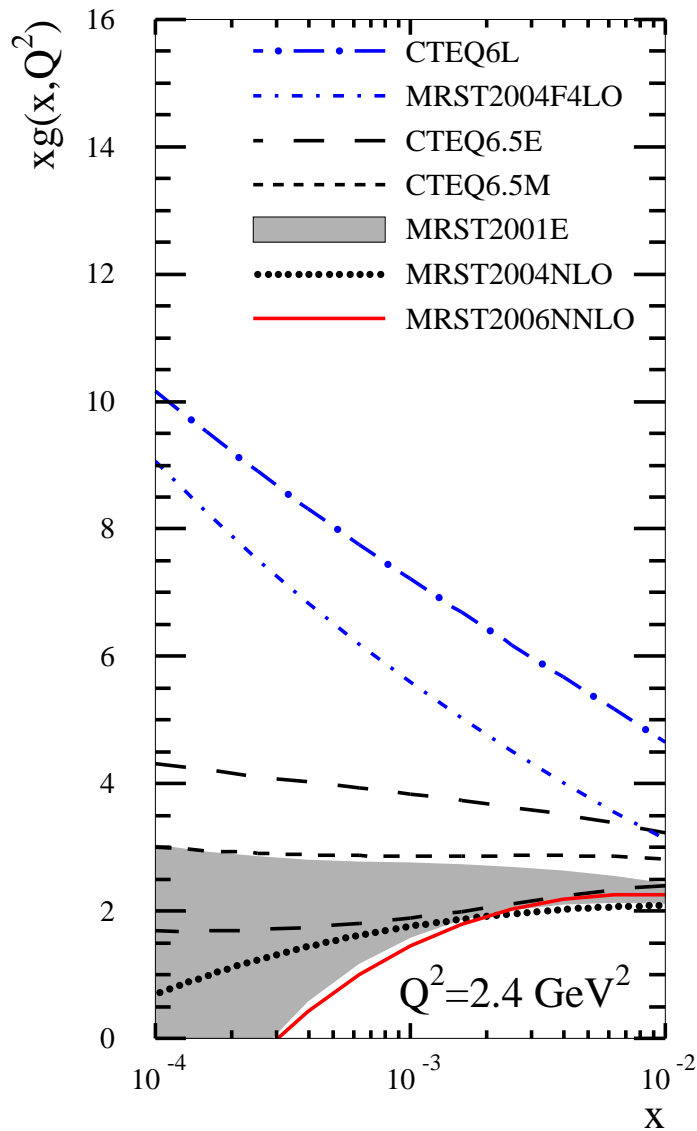


• Next-to-Leading-Order, again at 2.4, 4.1 GeV²:



- Big differences between variants; also different shapes
- MRST99 the last ‘physical’ gluon at small x , used as default in [MartinRyskinTeubner](#) predictions for diffractive production of ρ , ϕ , J/ψ , Υ .

- Big differences between different fits at small x even at 10 GeV^2 :



- Differences 'ok' within predicted errors,
- ... which are huge for small x .
- Large effects LO \rightarrow NLO \rightarrow NNLO.
- Unphysical NNLO gluon.

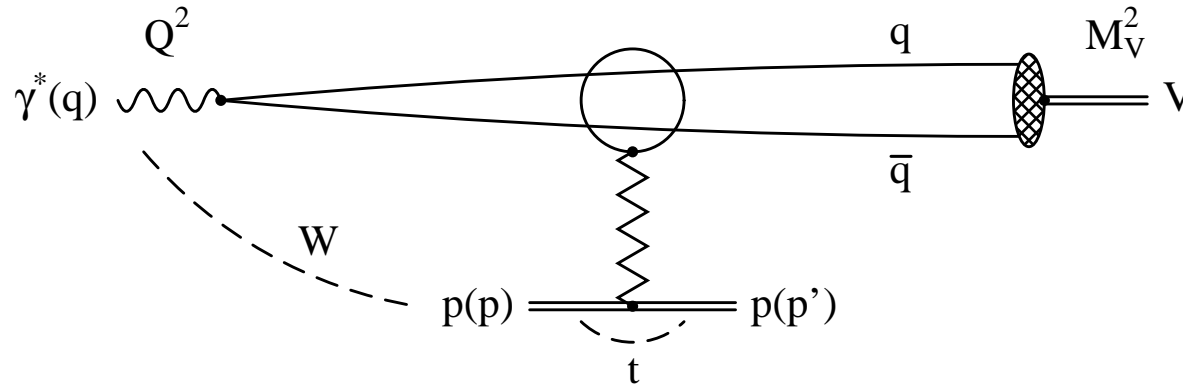
- This is the regime where we have a lot of diffractive data from HERA, in a wide range of the parameter space (W, Q^2, M^2, t) , e.g. for diffractive Vector Meson production:

$$\sigma(\gamma^* p \xrightarrow{IP \sim 2g} V p) \sim [xg(x, scale)]^2$$

→ *potential to directly constrain the gluon!*

- Light mesons: Large uncertainty from modelling of wave function and ‘IR pollution’ (only partly cured for electroproduction).
- Heavy mesons: can use non-relativistic approximation for J/ψ .
[Hoodbhoy: rel. corrections incl. $c\bar{c}g$ component $\mathcal{O}(v^2)$ and 4% only.]
- Υ even better suited, unfortunately data much worse (see below).
- Our strategy: Use available J/ψ data to extract gluon in this regime.
(No global fit, no inclusive data.)

II. Exclusive J/ψ production in pQCD



$$x = \frac{Q^2 + M_V^2}{Q^2 + W^2}$$

$$W^2 = (q + p)^2$$

$$t = (p - p')^2$$

- Factorization of amplitude: $\mathcal{A}(\gamma^* p \rightarrow V p) = \psi_{q\bar{q}}^\gamma \otimes \mathcal{A}_{q\bar{q}+p} \otimes \psi_{q\bar{q}}^V$
 \rightarrow convol. with non-rel. VM wave function $\psi_{q\bar{q}}^V$ (alternatively: open $q\bar{q}$ + PHD)
- ‘Colourless’ strong interaction in LO QCD: $\mathcal{A}_{q\bar{q}+p} \sim$ two gluon exchange:

$$\frac{d\sigma}{dt}(\gamma^* p \rightarrow V p) \Big|_{t=0} = \frac{\Gamma_{ee}^V M_V^3 \pi^3 \alpha_s (\bar{Q}^2)^2}{48\alpha \bar{Q}^8} [x g(x, \bar{Q}^2)]^2 \left(1 + \frac{Q^2}{M_V^2}\right)$$

with the *effective scale* $\bar{Q}^2 = (Q^2 + M_V^2) / 4$ and the experimental Γ_{ee}^V .

Corrections beyond the leading $\ln 1/x$ limit:

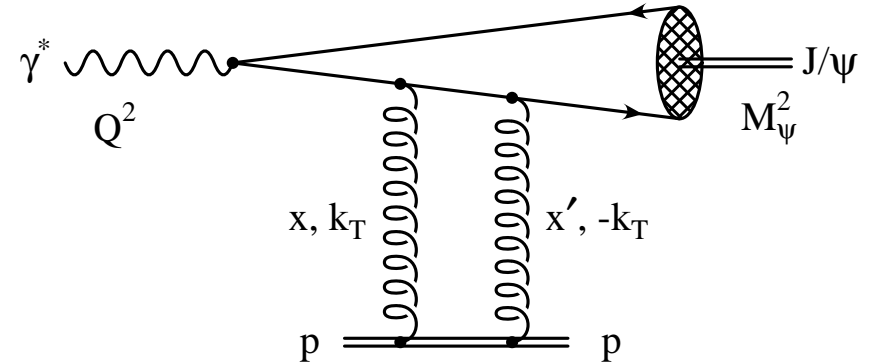
- $x \neq x' \rightsquigarrow$ **skewing**, $\mathcal{A} \sim$ **general**. PDF.

→ Correction factor à la Shuvaev:

$$R_g = \frac{H_g(x, x' \ll x)}{H_g(x, x)} = \frac{2^{2\lambda+3} \Gamma(\lambda + \frac{5}{2})}{\sqrt{\pi} \Gamma(\lambda + 4)},$$

with $\lambda(Q^2)$ the *effective power* of the gluon

$$xg \sim x^{-\lambda}.$$



- Contributions from the **real part** of the amplitude through

$$\text{Re } \mathcal{A} = \tan(\pi\lambda/2) \text{Im } \mathcal{A}.$$

(Crossing symmetry + power behaviour $\text{Im } \mathcal{A} \sim s^\lambda$.)

NLO corrections beyond the leading $\ln Q^2$ limit:

k_T factorization using unintegrated gluon $f(x, k_T^2)$

- In the LLA formula the k_T of the gluons is neglected ($k_T^2 \ll \bar{Q}^2$):

$$\mathcal{A}^{\text{LLA}} \sim \frac{\alpha_s(\bar{Q}^2)}{\bar{Q}^4} \int^{\bar{Q}^2} \frac{dk_T^2}{k_T^2} f(x, k_T^2) = \frac{\alpha_s(\bar{Q}^2)}{\bar{Q}^4} xg(x, \bar{Q}^2)$$

- Effect numerically important \rightsquigarrow large corrections in DGLAP when going to (N)NLO
- Use k_T factorized amplitude:

$$\mathcal{A}^{\text{'NLO'}} \sim \frac{\alpha_s(Q_0^2) xg(x, Q_0^2)}{\bar{Q}^4} + \frac{\alpha_s(\bar{Q}^2)}{\bar{Q}^2} \int_{Q_0^2}^{(W^2 - M^2)/4} \frac{dk_T^2}{\bar{Q}^2 + k_T^2} \frac{\partial xg(x, k_T^2)}{\partial k_T^2}$$

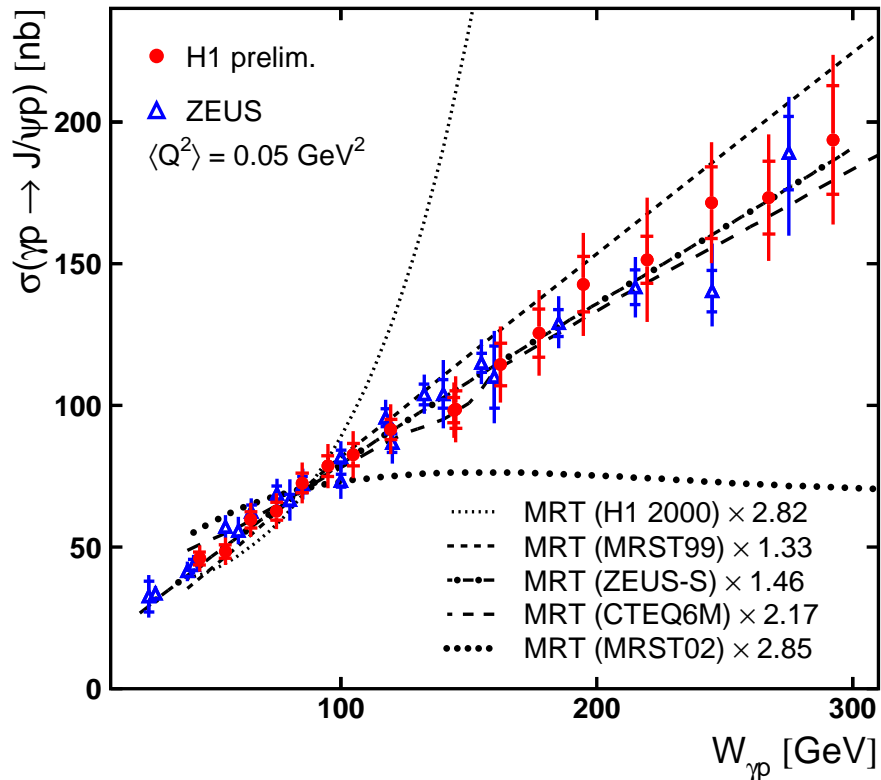
with small sensitivity on infrared transition parameter $Q_0^2 \sim 2 \text{ GeV}^2$ in case of J/ψ .

- Small effect when including Sudakov suppression: $f(x, k_T^2) = \left. \frac{\partial [xg(x, q_0^2) T(q_0^2, \mu^2)]}{\partial \ln q_0^2} \right|_{q_0^2 = k_T^2}$

with *Sudakov* factor $T = \exp\left[\frac{-C_A \alpha_s(\mu^2)}{4\pi} \ln^2 \frac{\mu^2}{q_0^2}\right]$ resums virtual corrections \sim probability for no gluon emission in the interval $q_0^2 \dots \mu^2 \sim \bar{Q}^2$.

H1 and ZEUS data compared to MRT predictions; gluon fit strategy

Plot thanks to Philipp Fleischmann (H1)



- MRT predictions for VM production based on pQCD able to describe large body of data; however:

→ Huge uncertainty due to input gluon pdf!

→ And: Data much more precise now.

↪ **This study:** use data to *determine* gluon at small x and low-medium scales via χ^2_{\min} fit.

- Use of theory described above and a **simple gluon ansatz** with three free parameters:

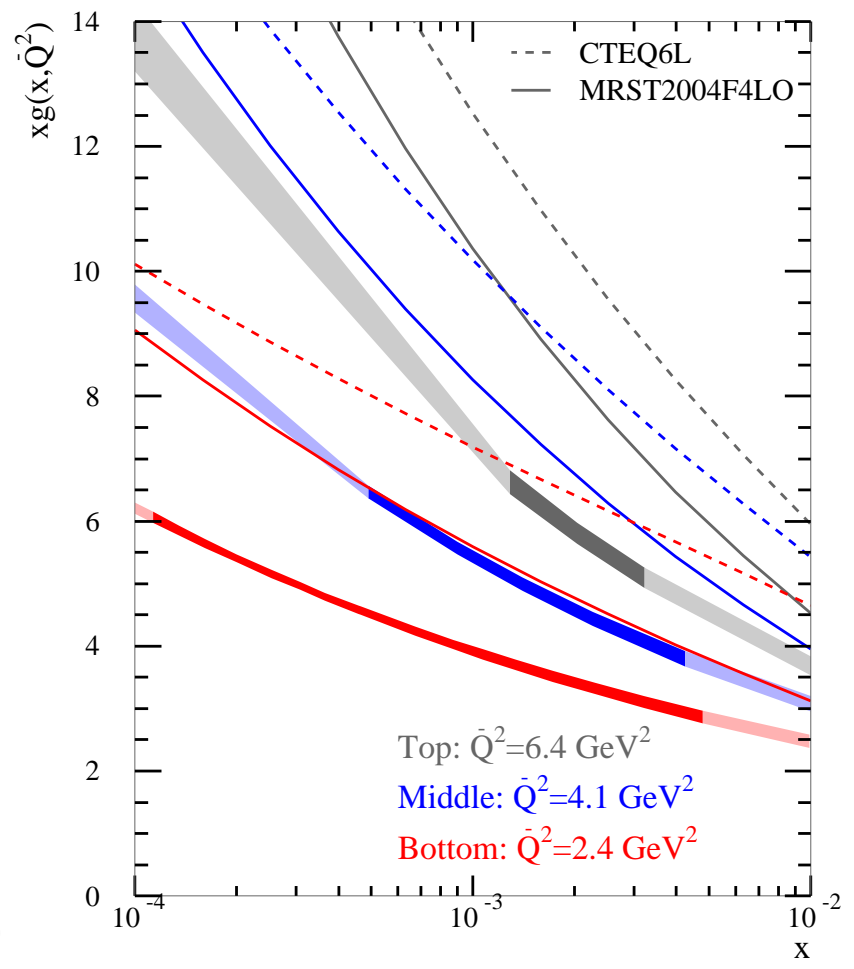
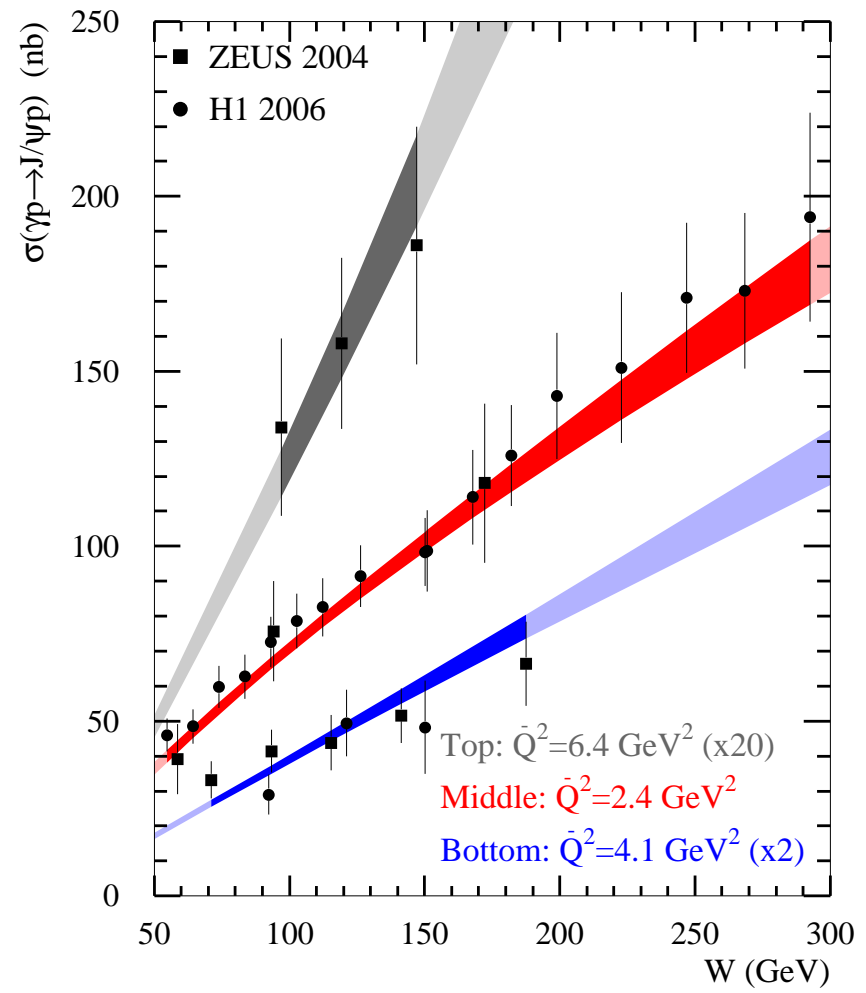
$$xg(x, \mu^2) = N \cdot x^{-\lambda}, \quad \text{LO: } \lambda = a + b \ln \frac{\mu^2}{0.45 \text{ GeV}^2}, \quad \text{NLO: } \lambda = a + b \ln \ln \frac{\mu^2}{0.09 \text{ GeV}^2}$$

(form used successfully by Martin+Ryskin+Watt, EPJC37,285), 'QCD anal. of diffr. DIS data'

III. Determining the small x gluon from diffractive HERA data

LO combined fit of H1 and ZEUS J/ψ data:

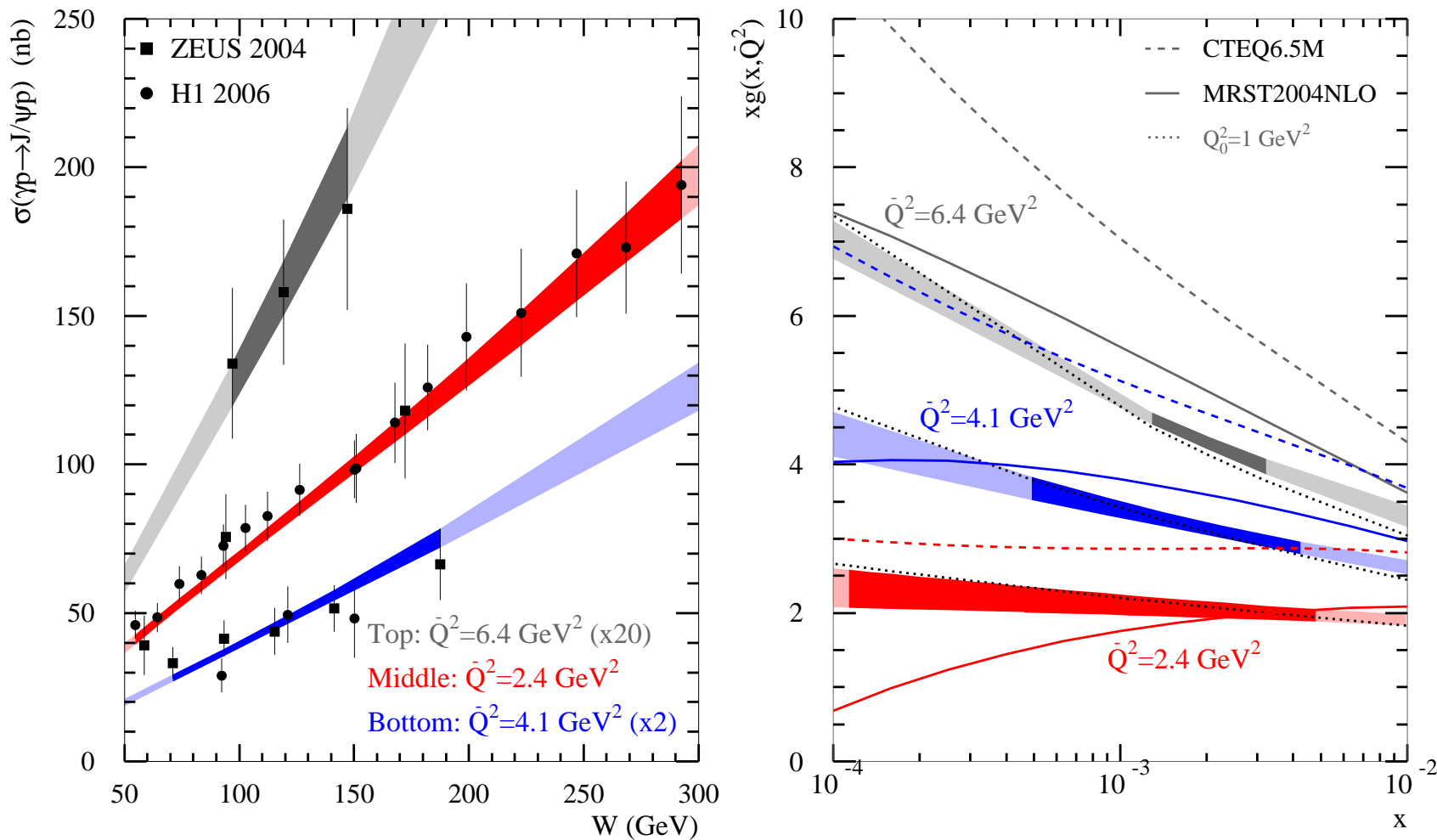
(error bands including correlations)



$$\chi_{\min}^2 / (d.o.f. = 48) = 0.9$$

$$N = 0.99 \pm 0.09, \quad a = 0.051 \pm 0.012, \quad b = 0.088 \pm 0.005$$

NLO combined fit of H1 and ZEUS J/ψ data:



- Excellent overall fit to available J/ψ data: $\chi_{\min}^2 / (d.o.f. = 48) = 0.8$
 $N = 1.55 \pm 0.18$, $a = -0.50 \pm 0.06$, $b = 0.46 \pm 0.03$.
- Tendency for slightly too steep higher- Q^2 cross sections, but statistically less significant.

NLO combined fit of H1 and ZEUS J/ψ data:

- Quality of data sufficient to give strong constraint on gluon.
- Considerable **less 'evolution'** compared to MRST gluon, fair agreement with CTEQ; generally less steep at larger scales, but no unphysical 'turn-over' at low scales.

- k_T factorization using *unintegrated* gluon:

$$A \sim \int_{Q_0^2}^{(W^2 - M^2)/4} dk_T^2 \frac{f(x, k_T^2)}{\bar{Q}^2 + k_T^2}$$

↪ significant large-scale contributions due to **anomalous dimension rising** with k_T^2 , damped only by $\ln \ln$.

- In standard DGLAP fits based on collinear factorization we have strong k_T ordering

↪ no such large scale contributions from the rising anomalous dimension, but

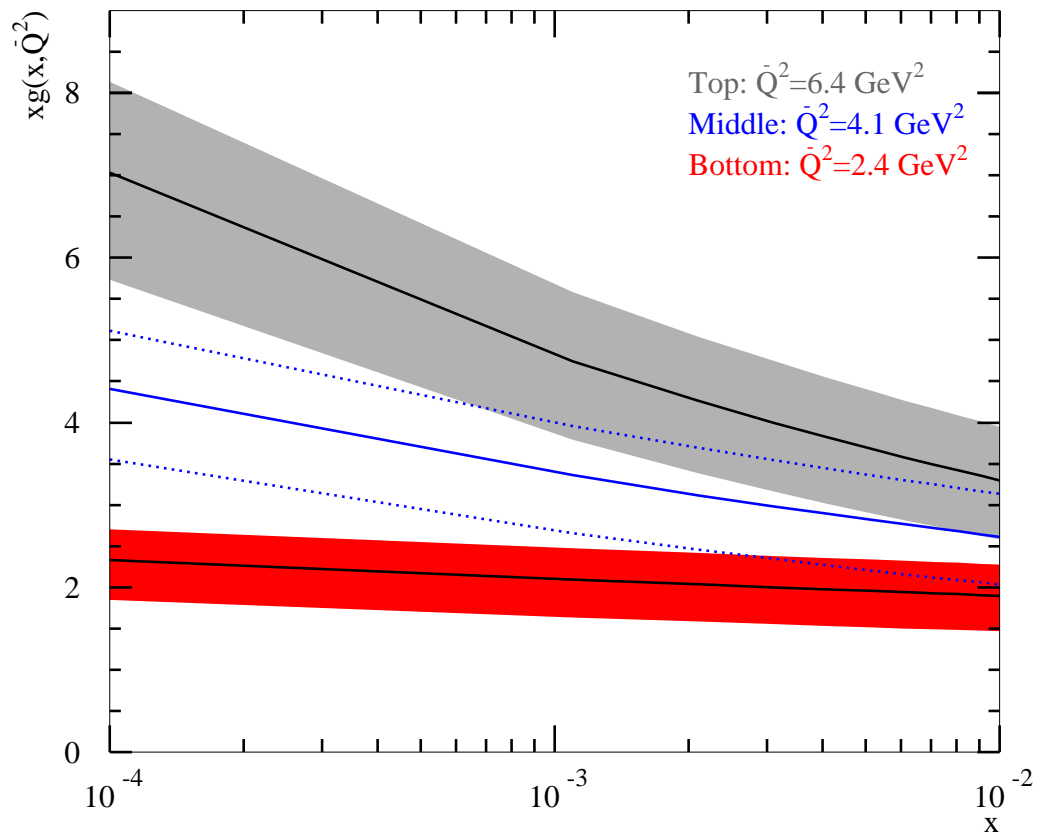
↪ corrections captured order by order through coefficient function.

- q contributions absent, i.e. included in fitted gluon; possibly not negligible in collinear factorization; large skewing corrections.

- Our NLO gluon should be ideal for calculations based on k_T factorization.

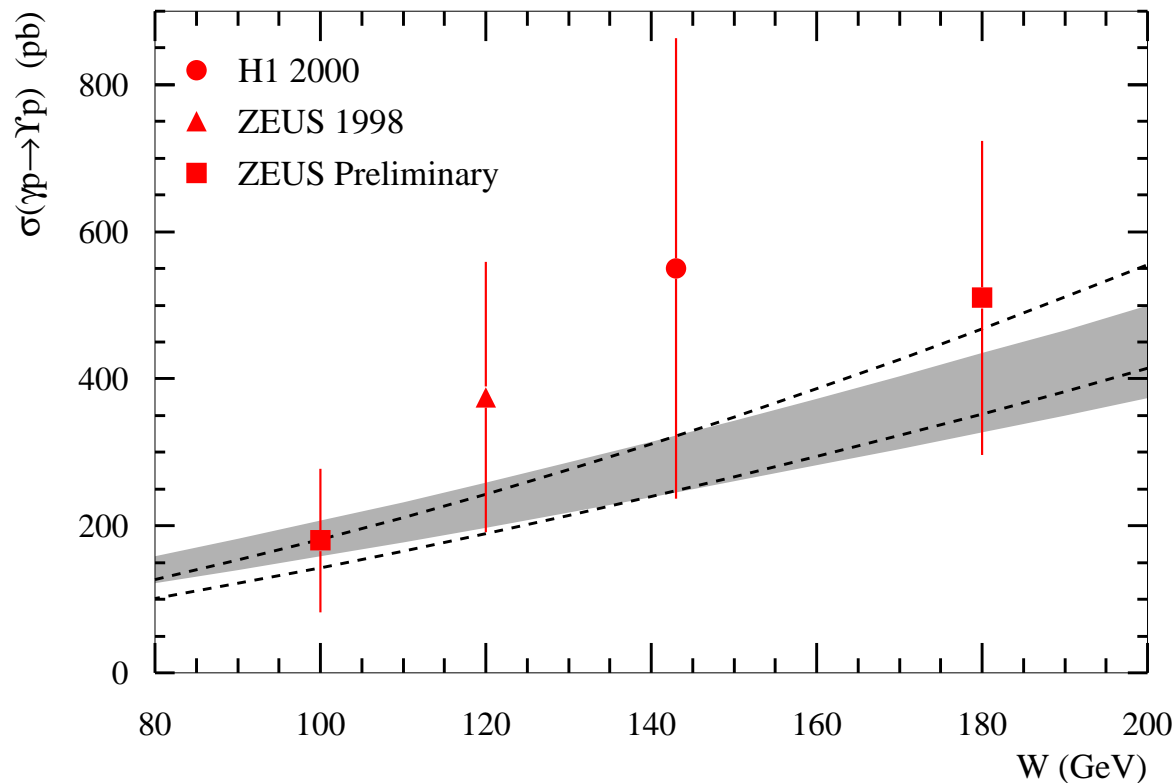
NLO Gluons; scale uncertainty

- Estimate of theoretical uncertainty via scale variation: $\mu^2/2 \dots 2\mu^2$ in both α_s and the Sudakov factor results in an uncertainty of about $\pm 20\%$.
- Sizeable but conservative as cancellations expected and should be seen in comparison with large differences between global fit gluons.



Predictions for elastic Υ production

- Use of NLO gluon to predict Υ photoproduction in the same framework; applicability of pQCD and non-rel. approximation even better justified
- change of mass, electric charge, Γ_{ee} , but no other adjustments
- Comparison with HERA data: good description



IV. Conclusions/Outlook

- Current PDF fits do not constrain the gluon well at small x and low–medium scales.
 - Diffractive Vector Meson production is very sensitive to the gluon in this regime.
 - We have used the pQCD approach to determine the LO gluon pdf from exclusive J/ψ data from H1 and ZEUS.
Our gluon fit is in fair agreement with MRST fits (undershooting CTEQ) at $x \sim 10^{-2}$ and shows a slightly flatter small x behaviour.
 - Within k_T factorization a fit of a ‘NLO’ gluon turns out to have less evolution compared to DGLAP NLO global fits, as expected from the different framework.
 - Further studies ongoing to quantify the correlation, including role of q contributions.
 - Gluons applicable for predictions at LO, well suited for use of exclusive predictions within k_T factorization.
-

Other NLO corrections beyond the leading $\ln Q^2$ limit:

- Relativistic corrections $\mathcal{O}(v^2/c^2)$ from the wave function:

If treated properly including higher Fock component $c\bar{c}g$ states of the J/ψ , and with use of experimentally measured Γ_{ee} , these corrections were shown to be small, $\mathcal{O}(4\%)$ [Hoodbhoy]

- q contributions suppressed for small x production of J/ψ :

$$\text{rough estimate: } \mathcal{A}_{\text{sea}} \sim \mathcal{A}_{\text{gluon}} \frac{1}{6} \frac{R_q}{R_g} \frac{\alpha_s}{4\pi}, \quad R_q/R_g \sim 2 - 3$$

- Genuine 'hard' NLO corrections to the $\gamma^*gg \rightarrow VM$ impact factor:

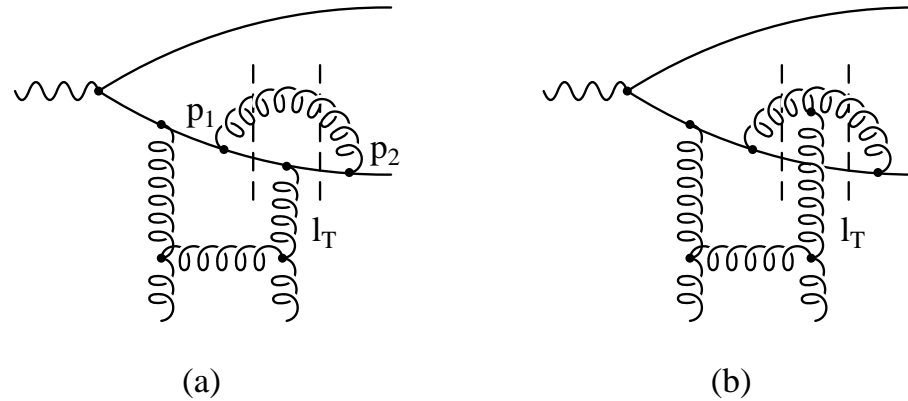
Still not available \rightsquigarrow additional K factor \rightsquigarrow normalization uncertainty

However: large logarithmic contributions captured with k_T factorization scheme and scale choice; quantitative study based on work of Ivanov et al. under way.

Some details of the original MRT approach:

K factor:

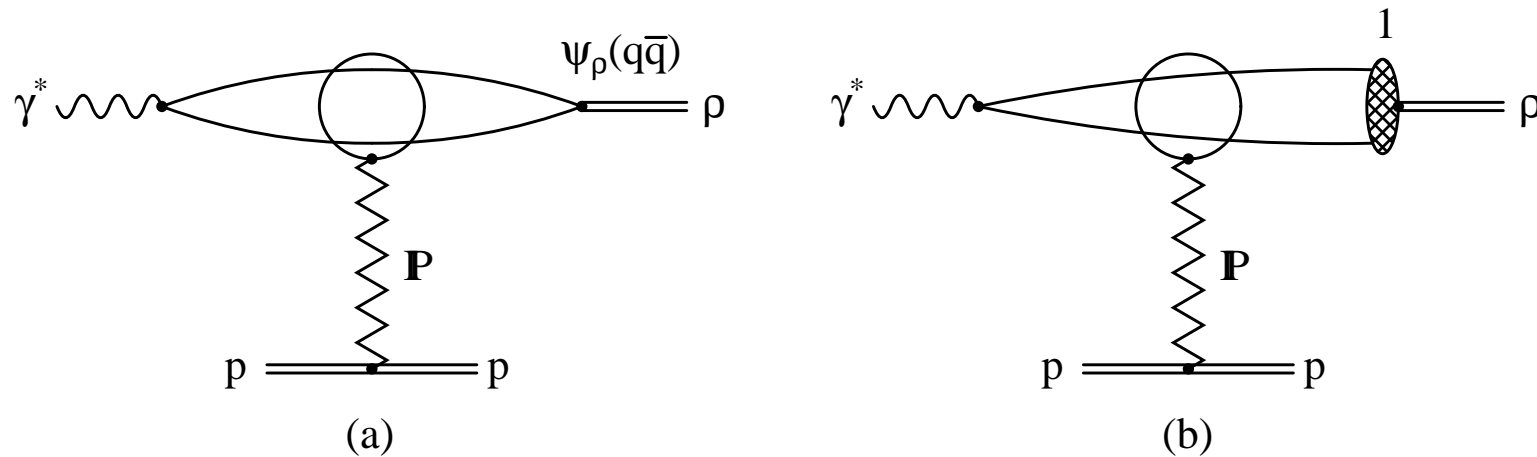
Important missing ingredient for a full NLO prediction: One loop corrections to the $[(q\bar{q})(2g)]$ vertex



- Typically lead to a significant enhancement in the normalization of QCD processes \rightarrow K factor (may also be fitted from data)
- Up to now no full calculation within k_T factorization
- MRT estimate the K factor from π^2 enhanced terms, analogous to the well known corrections in Drell-Yan $\rightsquigarrow \sigma = \sigma^0 \exp[\pi^2 C_F \alpha_s(\cdot)/\pi]$.*
- First results for diagrammatic calculation for \mathcal{A}^L by D.Yu. Ivanov et al.

* Exp. of the double logarithmic Sudakov form factor $\sim \ln^2(-M^2)$, $\ln(-M^2) = \ln M^2 + i\pi$

Alternative to use of VM wave function: **Parton Hadron Duality:**



Assumption (case of ρ): $\gamma^* \rightarrow q\bar{q}$ cross section in the region $M_{q\bar{q}} \sim M_\rho$ saturated by ρ (up to $\sim 10\%$ for ω) when integrated over a *suitable* (universal?!) mass interval ΔM :

$$\sigma(\gamma^* p \rightarrow \rho p) \simeq 0.9 \sum_{q=u,d} \int_{M_{min}^2}^{M_{max}^2} \frac{d\sigma(\gamma^* p \rightarrow (q\bar{q})p)}{dM^2} dM^2$$

+ Projection of $q\bar{q}$ state on the correct VM Quantum Numbers $J^P = 1^-$.
 (\rightsquigarrow Suppression of IR divergencies for contr. from transverse photon!)

Structure of the calculation à la Martin+Ryskin+Teubner:

Contributions to $\sigma(\gamma_{L,T}^* p \rightarrow V p)|_{t=0}$ from **Re, Im** for **L, T**, numerically
(‘straightforward’, no iterative procedure for effective scales):

$$\text{PHD: } \int dM^2 \left[\text{Projection: } \int dk_T^2 \left(\text{Skewed } \mathcal{A}'s \text{ w. } K \text{ fact.}; \text{ Re: } \int dl_T^2 \right) \right]^2$$