Review on parton saturation

Krzysztof Golec-Biernat

Institute of Nuclear Physics, Krakow

Rzeszow University

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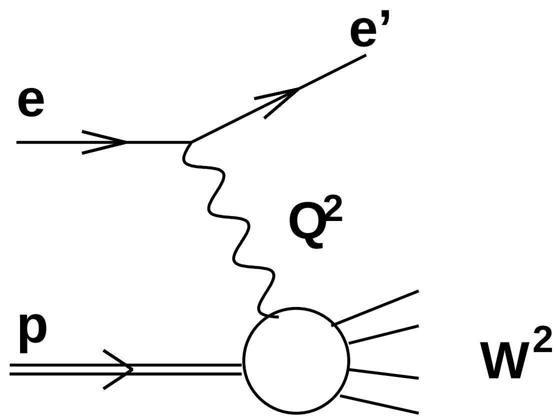
Two limits of DIS



- Bjorken limit – (logarithmic) Bjorken scaling of F_2

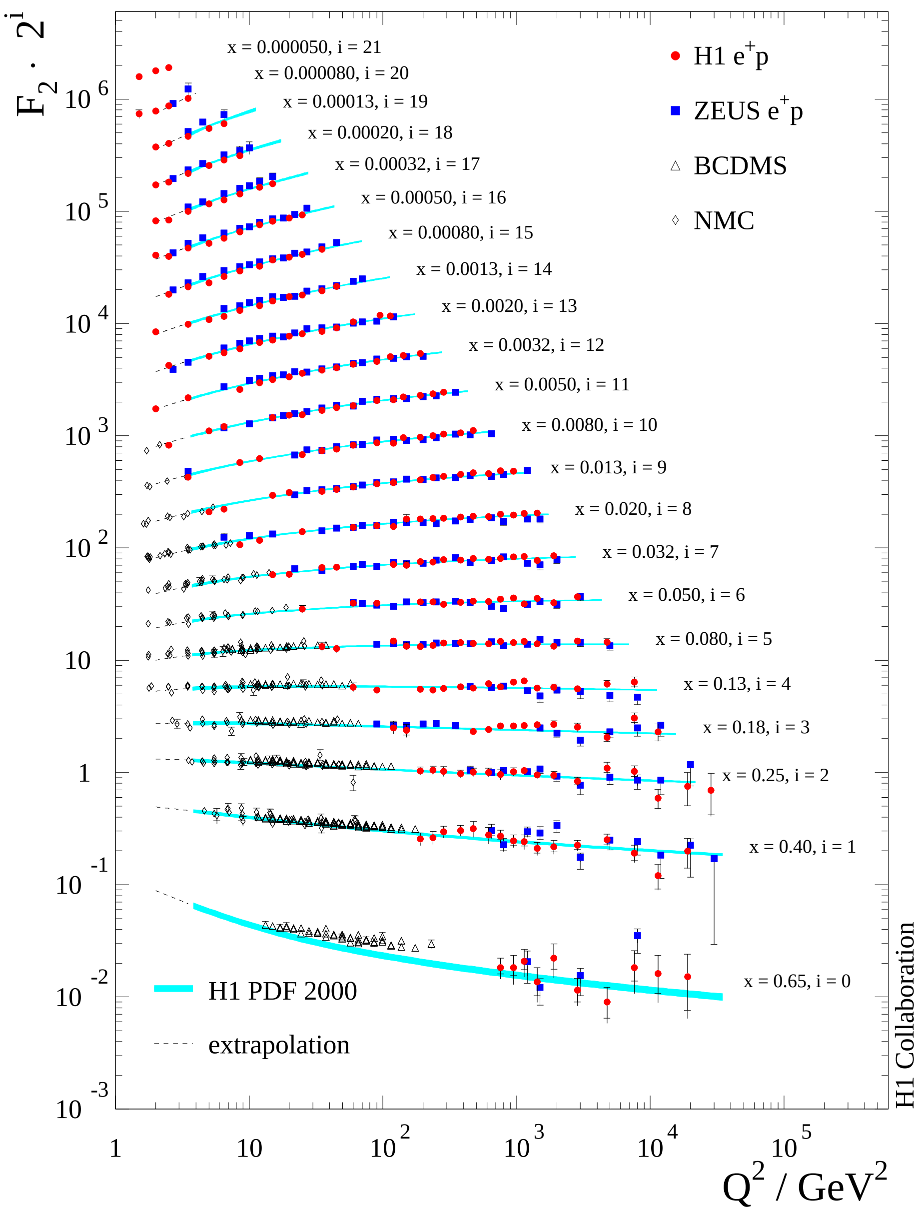
$$Q^2 \sim W^2 \to \infty \qquad x = \frac{Q^2}{Q^2 + W^2} = const$$

- Regge (high energy) limit – Strong rise of F_2

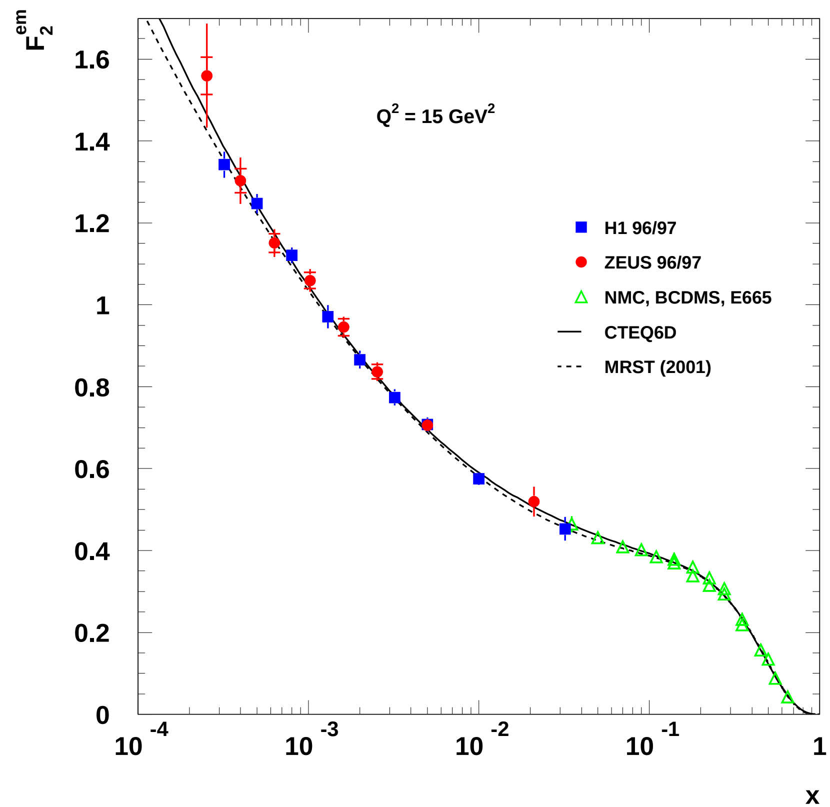
$$Q^2 = const \qquad W^2 \to \infty \qquad x \simeq \frac{Q^2}{W^2} \to 0$$

Bjorken scaling and strong rise

Bjorken limit

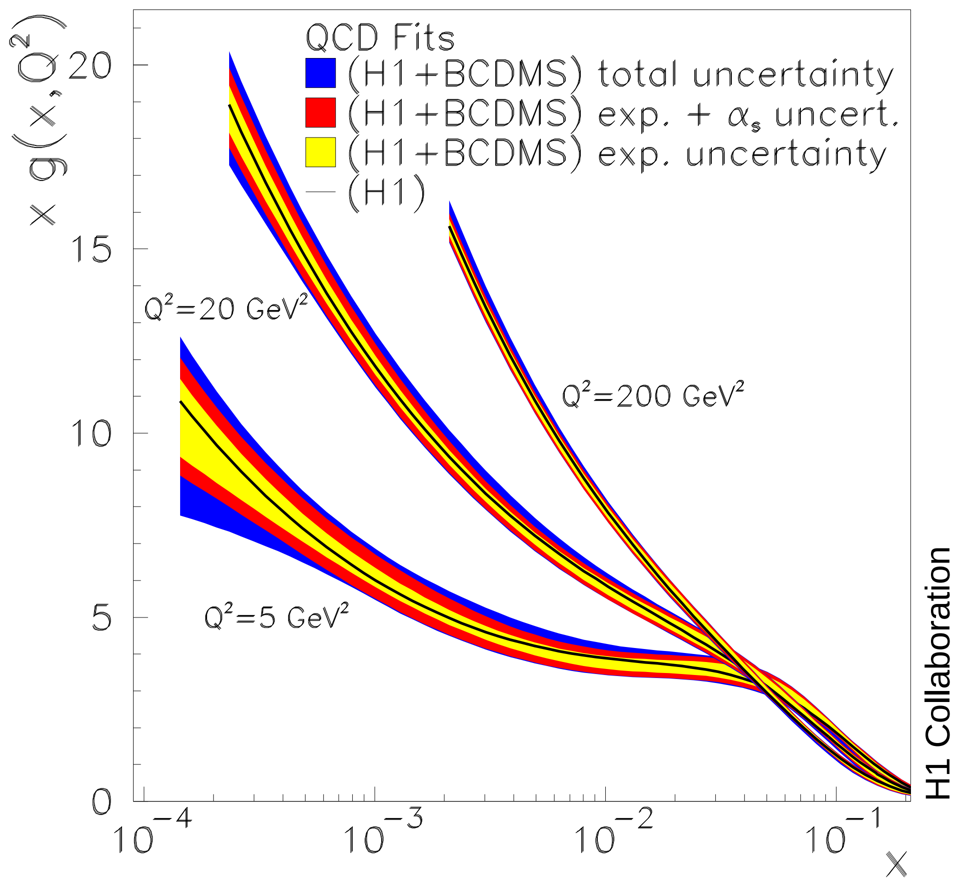


Regge limit

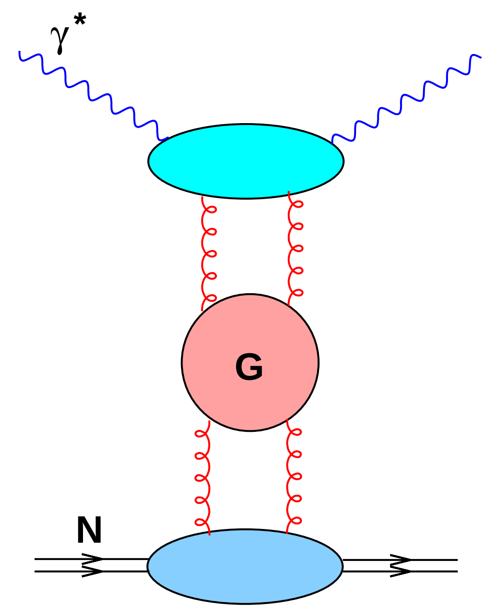


High energy limit of QCD

- Large gluon density for $x \to 0$: $\alpha_s \ln(1/x) \sim 1$



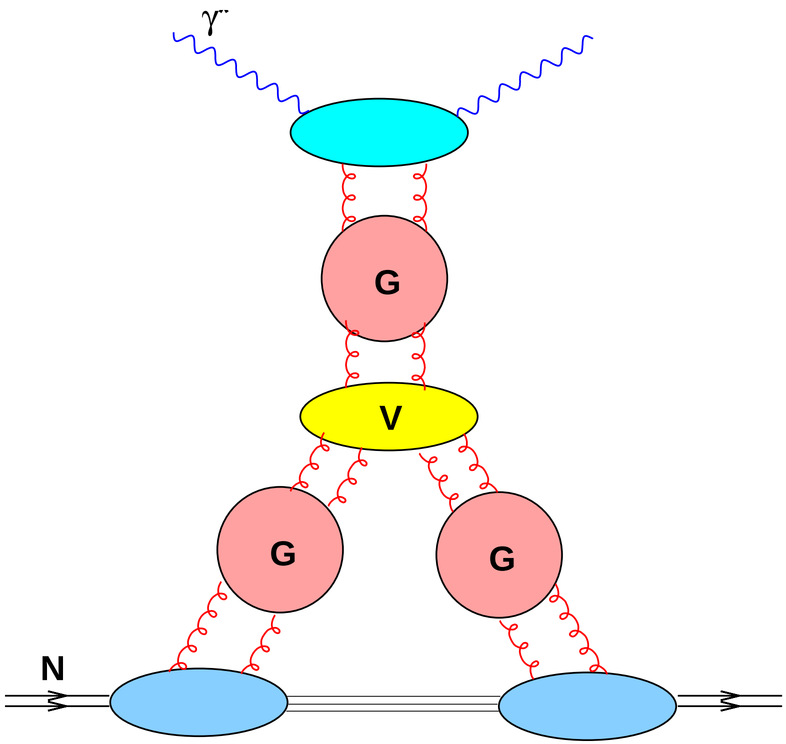
BFKL pomeron



- Power-like dependence: $F_2 \sim x^{-4\overline{\alpha}_s \ln 2}$ - unitarity violation for $x \to 0$

Need of unitarization

- Nonlinear evolution equations of QCD – gluons recombine



Gribov-Levin-Ryski, Mueller-Qiu, Bartels

Venugopalan, McLerran, Balitsky,...

$$\frac{\partial G}{\partial \ln Q^2} = P \otimes G - V \otimes G^2$$

- Power-like rise in x is tamed

- Early studies plagued by DLL approximation: $\alpha_s \ln Q^2 \ln(1/x) \sim 1$
- Significant progress in recent years – high density QCD

High density QCD

Guided by the idea of parton saturation:

- gluons form high density system in which they easily recombine
- nonlinear evolution equations: GLR, BK, JIMWLK
- unitarity restored (Froissart bound: $\sigma_{tot} < C \ln^2 s; \quad |T(b)| < 1$)

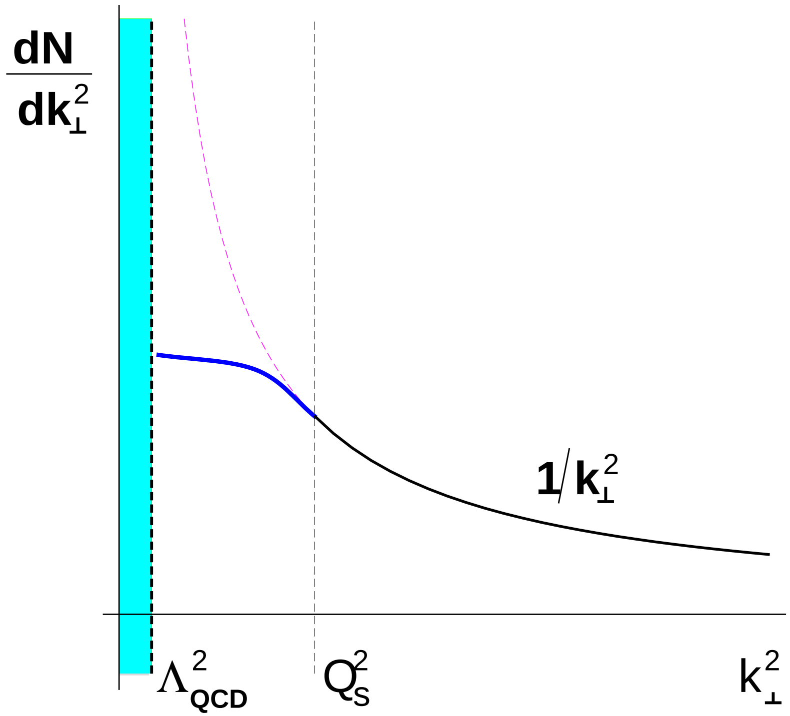
Two basic features:

- saturation scale $Q_s(x)$ - intrinsic scale of a dense gluonic system
- geometric scaling - quantities like $\gamma^* p$ cross section scale

$$\sigma^{\gamma^* p}(x, Q^2) = \frac{F_2}{Q^2} = \sigma^{\gamma^* p}\left(\frac{Q^2}{Q_s^2(x)}\right)$$

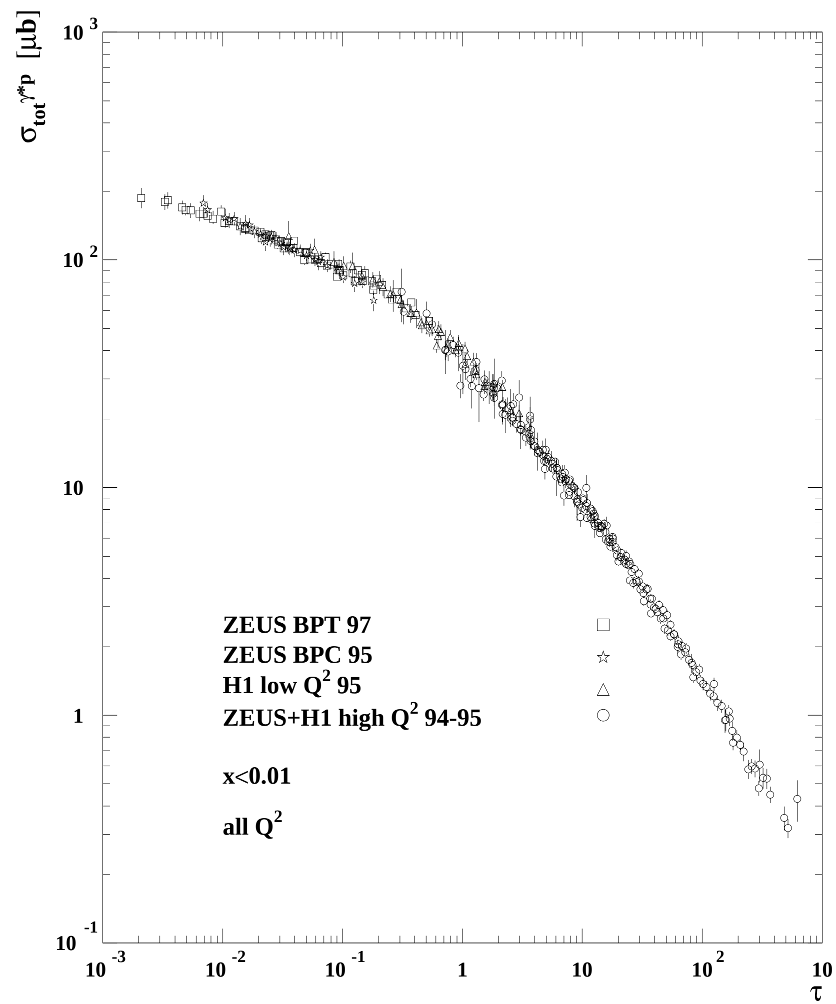
Saturation scale and geometric scaling

Gluon density



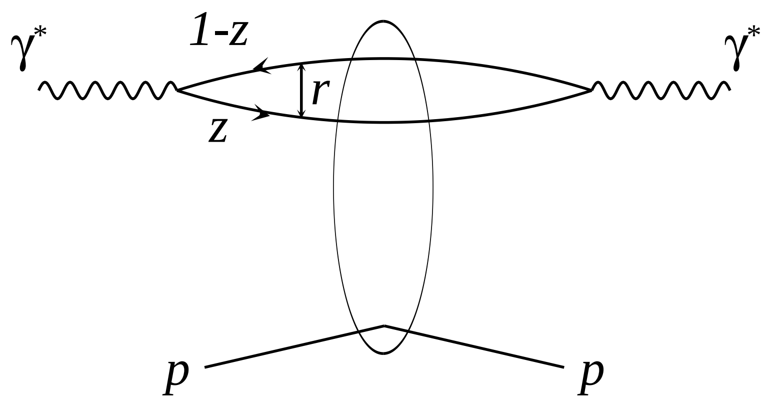
Saturation for $k_\perp < Q_s(x)$

$\gamma^* p$ cross section



Scaling in $\tau = Q^2/Q^2(x)$

Dipole picture of DIS dor $x \to 0$



r - transverse size

z - photon momentum fraction

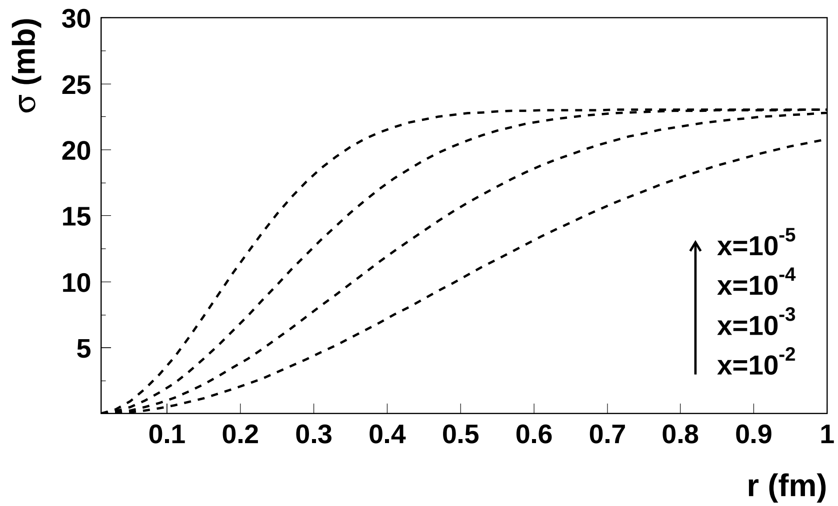
- Color dipole formation + dipole interaction

$$\sigma^{\gamma^* p} = \frac{F_2}{Q^2} = \int d^2r \int_0^1 dz \, |\Psi^\gamma(r,z,Q^2)|^2 \, \hat{\sigma}(r,x)$$

- Dipole cross section $\hat{\sigma}$ – unitarized interaction

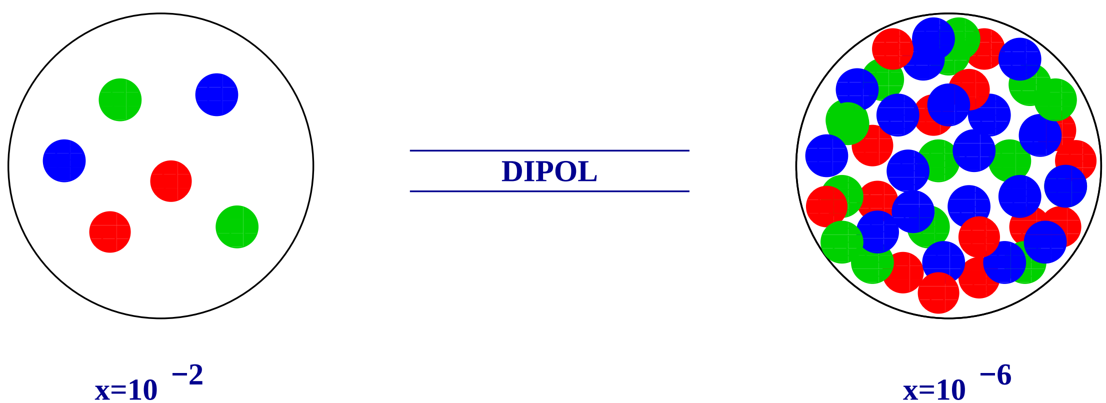
Dipole cross section

G-B, Wüsthoff, 1999



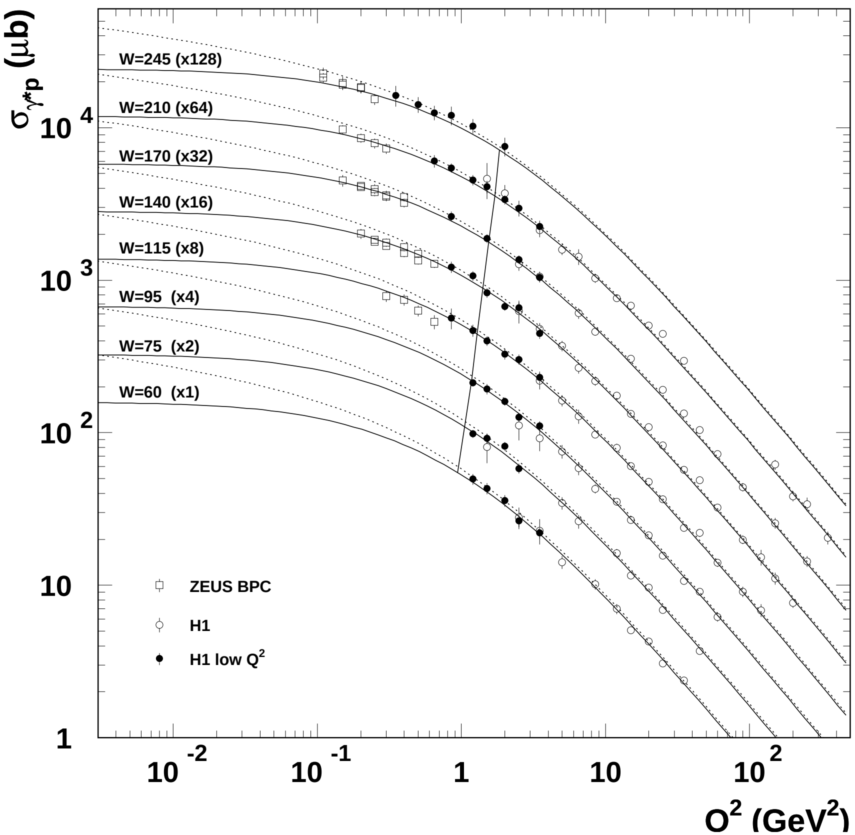
- $\hat{\sigma}(r,x) = \sigma_0 \, \{1 - \mathrm{e}^{-r^2 Q_s^2(x)/4}\}$
- Saturation scale: $Q_s^2 = Q_0^2 \; x^{-\lambda}$
- Geometric scaling: $\hat{\sigma}(r Q_s(x))$

- Unitarity condition: $\hat{\sigma}$ never exceeds σ_0 when $x \to 0$



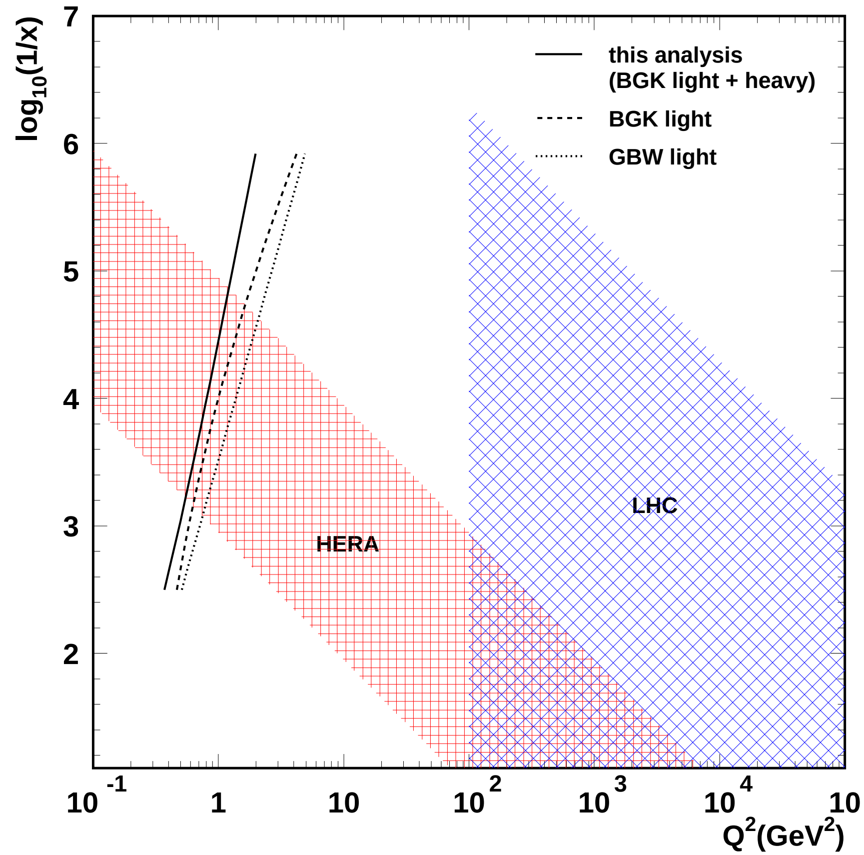
Saturation model and HERA data

Transition to low Q^2



$$\ln(Q_s^2(x)/Q^2) \;\rightarrow\; Q_s^2(x)/Q^2$$

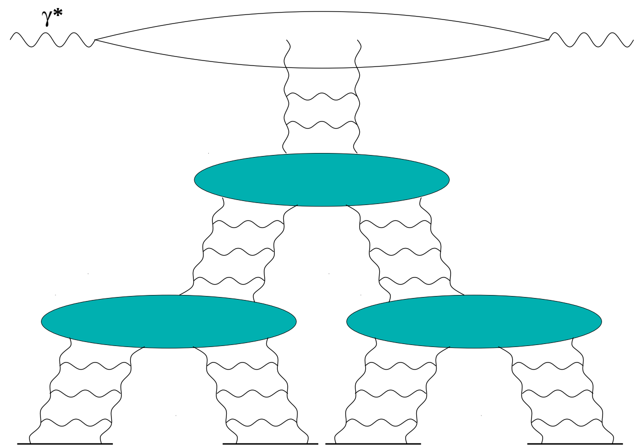
Saturation line



GB Sapeta '06

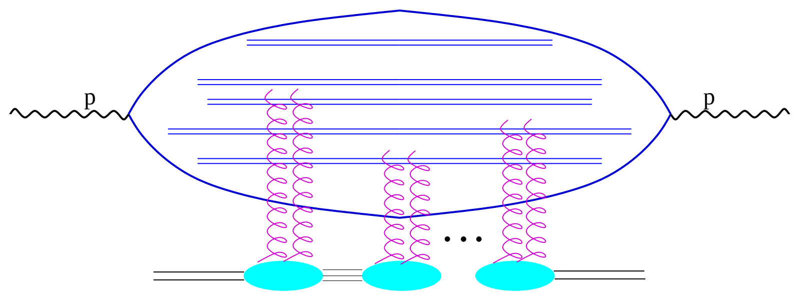
QCD justification

In momentum space



QCD justification

In coordinate space



- Dipole cross section ($Y = \ln(1/x)$):

$$\hat{\sigma}(\vec{r} =, Y) = 2 \int d^2\vec{b}\, N_{xy}(Y) \qquad \vec{r} = \vec{x} - \vec{y} \qquad \vec{b} = \frac{1}{2}(\vec{x} + \vec{y})$$

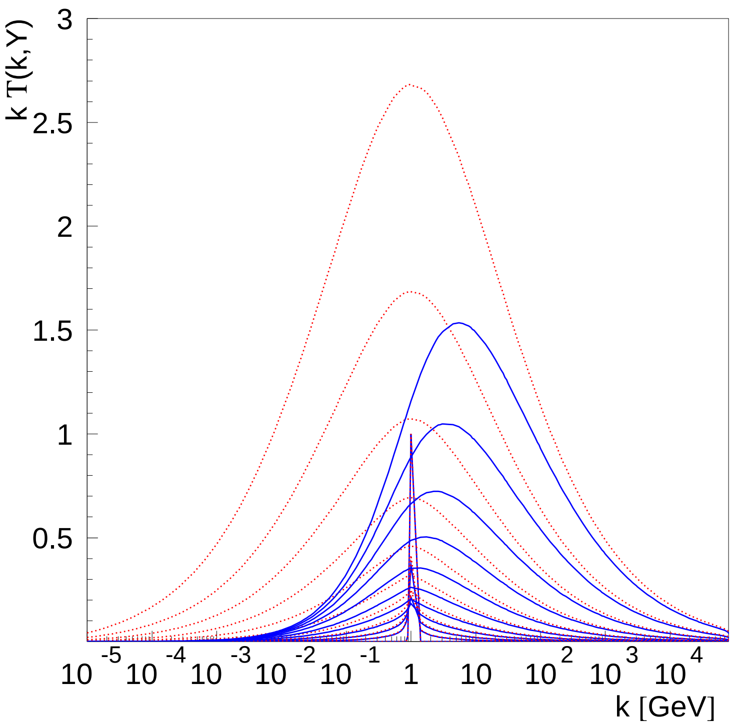
- Balitsky-Kovchegov equation for dipole scattering amplitude $N_{xy}(Y)$

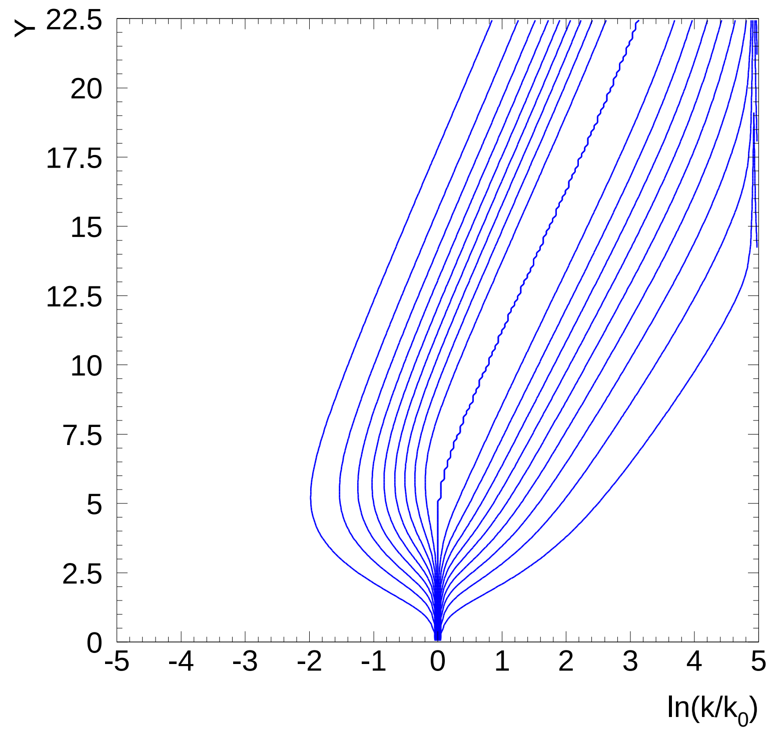
$$\frac{\partial N_{xy}(Y)}{\partial Y} = \overline{\alpha}_s \int d^2\vec{z}\, \frac{(\vec{x} - \vec{y})^2}{(\vec{x} - \vec{z})^2 (\vec{z} - \vec{y})^2} \left\{ \underbrace{N_{xz} + N_{yz} - N_{xy}}_{BFKL} - N_{xz} N_{yz} \right\}$$

Saturation scale from BK equation

- Fourier transform: $T(k_\perp, Y) = \int d^2r \, e^{ik_\perp \cdot r} \, N(r,Y)/r^2$
- BK equation in spherical and uniform case *(GB, Motyka, Staśto (2003))*

$$\partial_Y T = \underbrace{\chi\left(-\partial_{\ln k_\perp}\right) T}_{BFKL} - T^2$$





Infrared suppression

Scaling: $T = T(k_\perp / Q_s(Y))$

Traveling wave

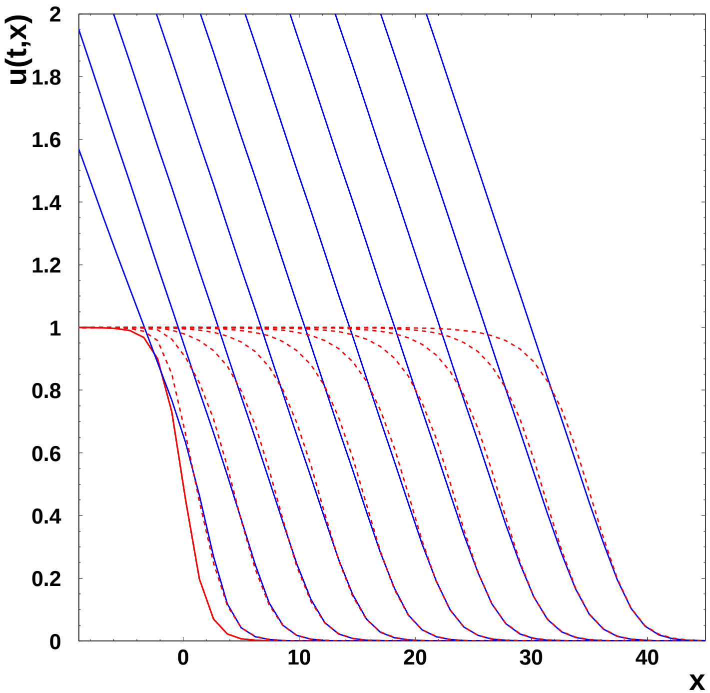
- Expand BFKL kernel: *(Munier, Peschanski (2003-04))*

$$\chi(\gamma) = \chi_c + \chi'_c(\gamma - \gamma_c) + \frac{1}{2}\chi''_c(\gamma - \gamma_c)^2$$

and change variable: $T(k_\perp, Y) \to u(x,t)$.

- Fisher-Kolmogorov-Petrov-Piscounov (FKPP) equation

$$\partial_t u(x,t) \; = \; \partial_{xx} u \; + \; u\,(1-u)$$



- Traveling wave: $u = u(x - v_\infty t)$
- Geometric scaling: $T = T\left(\frac{k_\perp}{Q_s(Y)}\right)$
- Saturation scale from the tail $u \ll 1$

$$\ln Q_s^2(Y) = \frac{\bar{\alpha}_s\,\chi_c}{\gamma_c} Y - \frac{3}{2\gamma_c}\ln Y - \frac{A(\gamma_c)}{\sqrt{Y}}$$

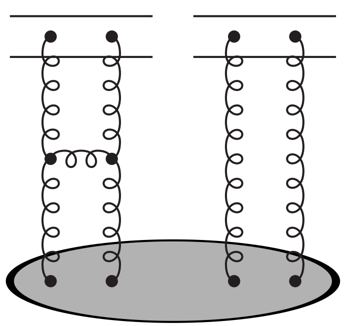
Saturation and fluctuation

In BK equation T is averaged over partonic configurations:

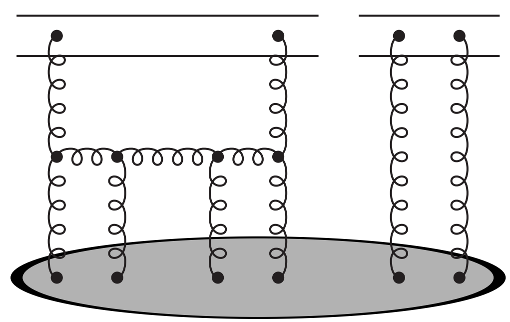
$$\langle T^2 \rangle \simeq \langle T \rangle \langle T \rangle \qquad \text{(mean field approx.)}$$

BK equation is an approximation to Balitsky – JIMWALK hierarchy

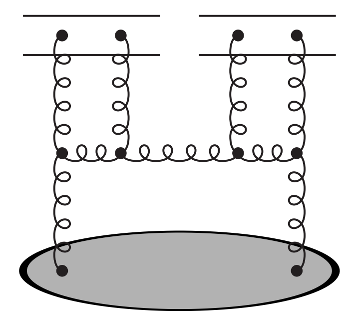
$$\langle T \rangle \rightarrow \langle T^2 \rangle \rightarrow \langle T^3 \rangle \rightarrow \langle T^4 \rangle \rightarrow \cdots$$



evolution T^2



splitting $T^2 \rightarrow T^3$



merging $T^2 \rightarrow T$

Merging is not present in the Balitsky's hierarchy – no pomeron loops

Stochastic FKPP equation

- HE scattering as a stochastic process with death/birth processes.
- this process is in universality class of stochastic sFKPP equation *(Munier '04)*:

$$\partial_t u = \partial_{xx} u + u\,(1-u) + \alpha_s \sqrt{u(1-u)}\,\eta$$

η is white noise

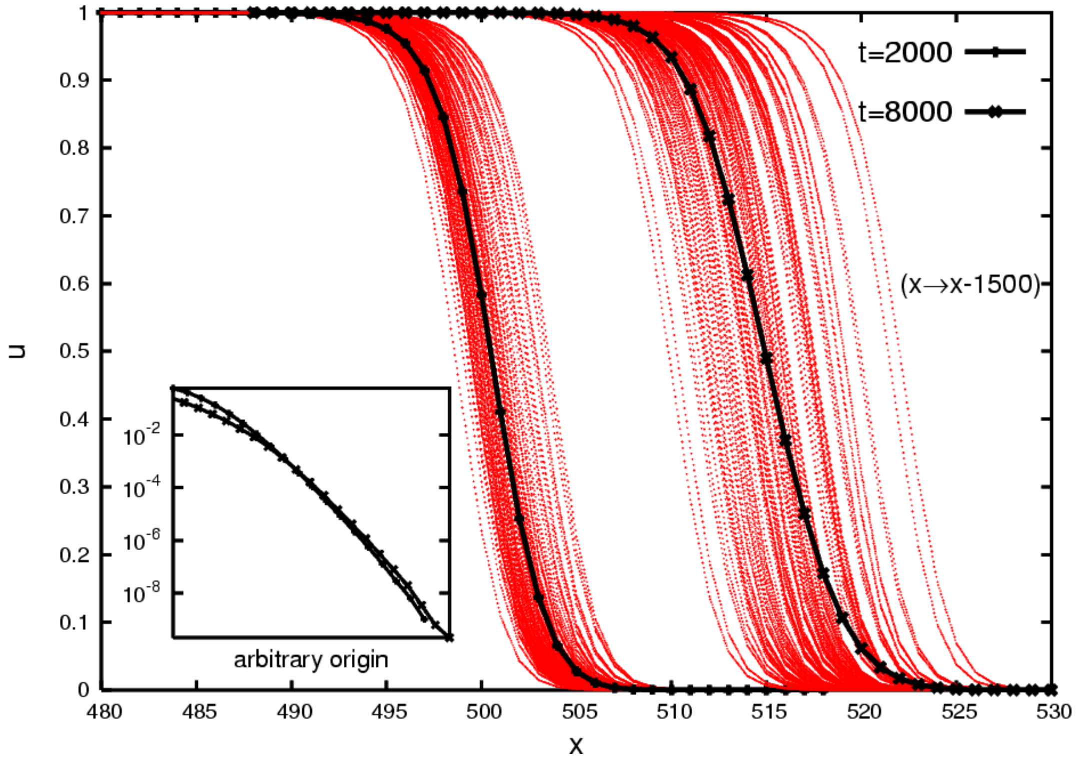
$$\langle \eta(x,t) \rangle = 0 \qquad \langle \eta(x,t)\,\eta(x',t') \rangle = \delta(x-x')\,\delta(t-t')$$

- saturation scale $Q_s(Y)$ is a random variable with dispersion $\sigma^2 \sim Y$

$$\langle \ln Q_s^2(Y) \rangle = \left(\frac{\bar{\alpha}_s \chi_c}{\gamma_c} - \frac{\bar{\alpha}_s \pi^2 \gamma_c \chi_c''}{2 \ln^2(1/\alpha_s^2)} \right) Y \equiv \ln \overline{Q_s^2}(Y)$$

Diffusive scaling

(Munier '05)

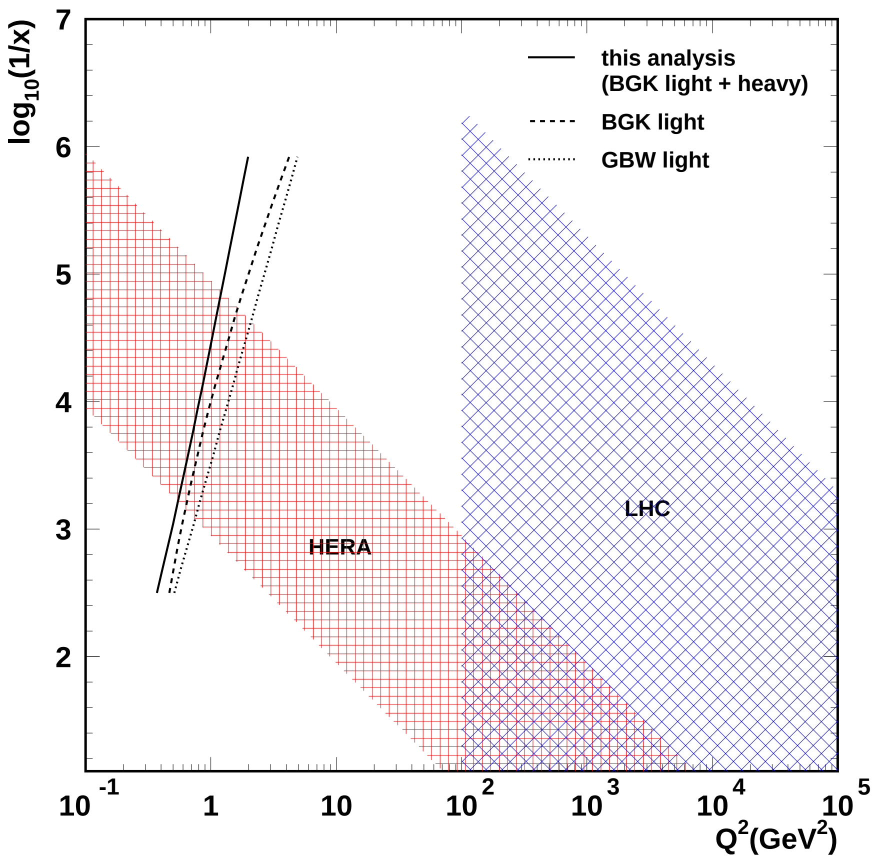


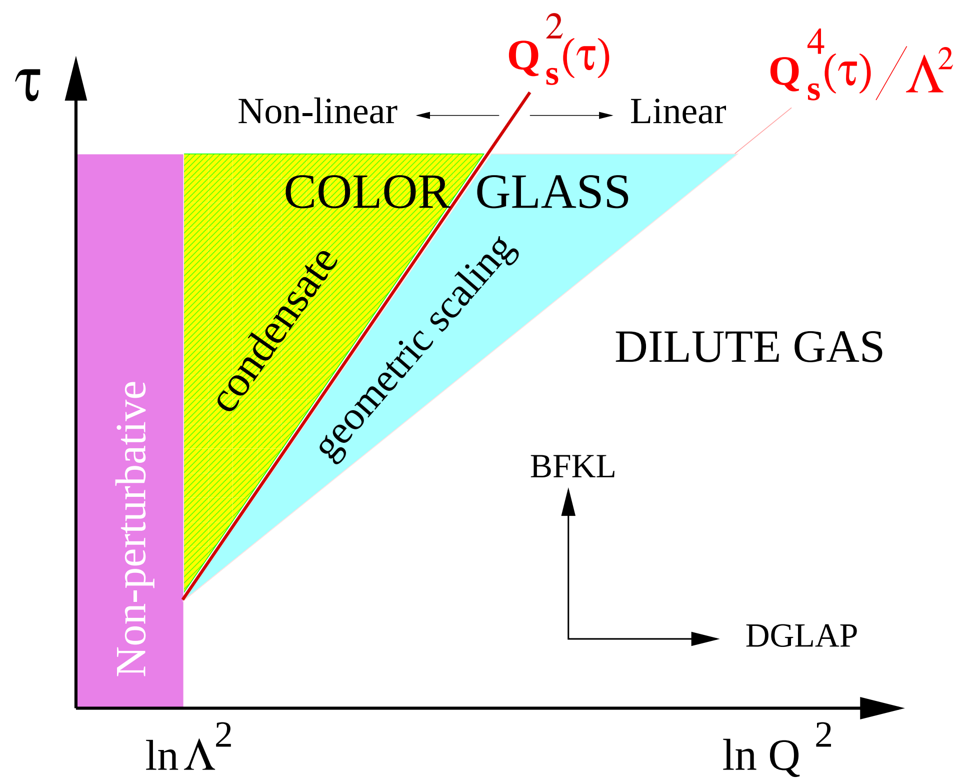
- Average amplitude – diffusive scaling when $\bar{\alpha}_s Y \gg \ln^2(1/\alpha_s^2) \gg 1$

$$\langle T(k_\perp, Y)\rangle = T\left(\frac{\ln(k_\perp^2/\overline{Q_s^2}(Y))}{D\sqrt{Y}}\right)$$

"Phase diagram" of QCD

(Brunet, Derida, Enberg, GB, Iancu, Marquet, Mueller, Munier, Peschanski, Soyez, Shoshi, Triantafyllopoulos, Xiao)





Color glass condensate - CGC

- In high energy collisions of heavy ions gluon density is enhanced by large number of nucleons A.

- Collision of two dense gluonic systems (CGC).

$$\mathrm{CGC} \quad \rightarrow \quad \mathrm{quark-gluon\ \ plasma} \quad \rightarrow \quad \mathrm{hadrons}$$

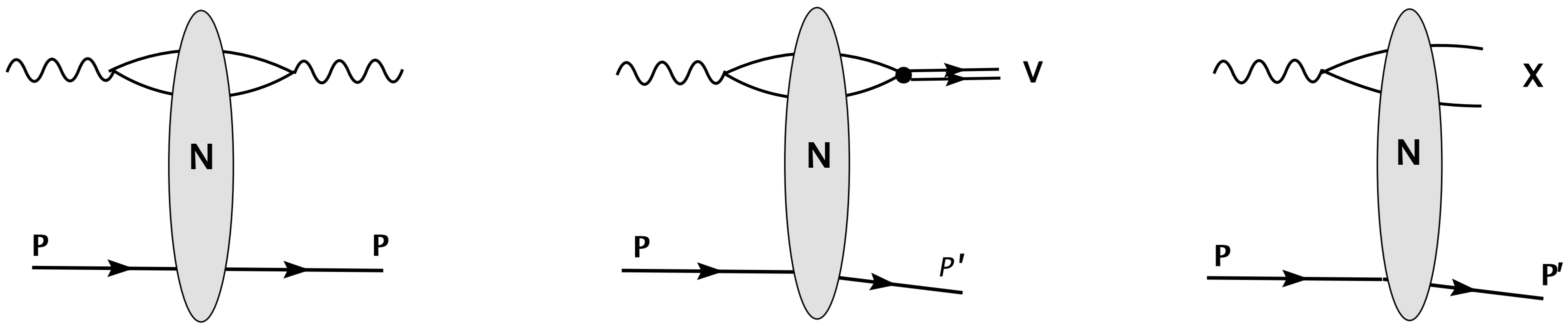
- CGC – effective QCD description of high density systems

Experimental applications

- inclusive diffraction in ep
- exclusive diffractive processes in ep and pp *(G. Watt talk)*
- seminclusive process in pp (e.g. Drell-Yan) *(J. Bartels talk, much to be done)*
- twist analysis in DIS *(L. Motyka talk)*
- heavy ion collisions *(F. Gelis talk, much to be done)*

Diffractive processes in DIS

Universal description in leading order in $\log(s)$

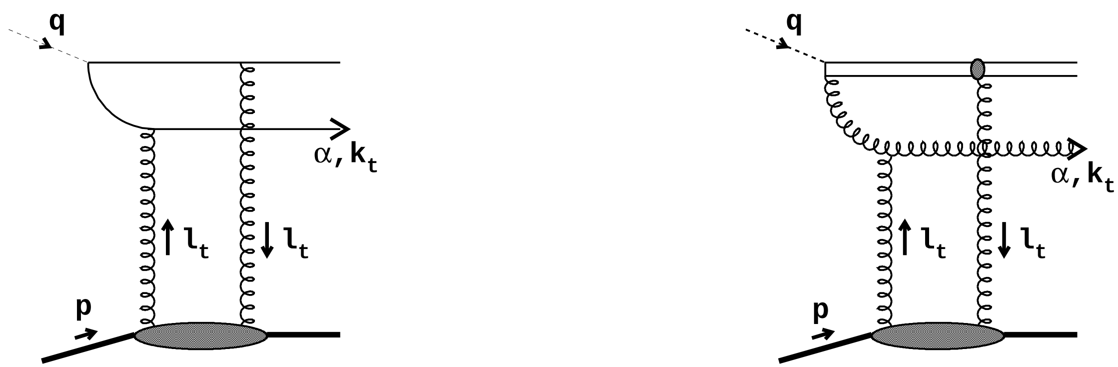


- Scattering amplitude: $\mathcal{A}(\gamma^* + p \to A + p) = \int d^2r dz\, \Psi_A^*\, N_{q\bar{q}}\, \Psi_\gamma$
- Dipole scattering amplitude $N_{q\bar{q}}$ found from inclusive DIS.
- In GBW model: $N_{q\bar{q}}(r,b,x) \;=\; \theta(b<b_0)\left(1-\mathrm{e}^{-r^2 Q_s^2(x)/4}\right)$
- Improvement: *(Bartels, GB, Kowalski, Teaney, Motyka, Watt)*

$$N(r,b,x) = 1 - \exp\left\{-\frac{\pi^2}{2N_c} r^2 \alpha_s(\mu^2)\, xg(x,\mu^2) T(b)\right\}$$

Inclusive DIS diffraction and saturation

- Successful description of DDIS with two component diffractive state

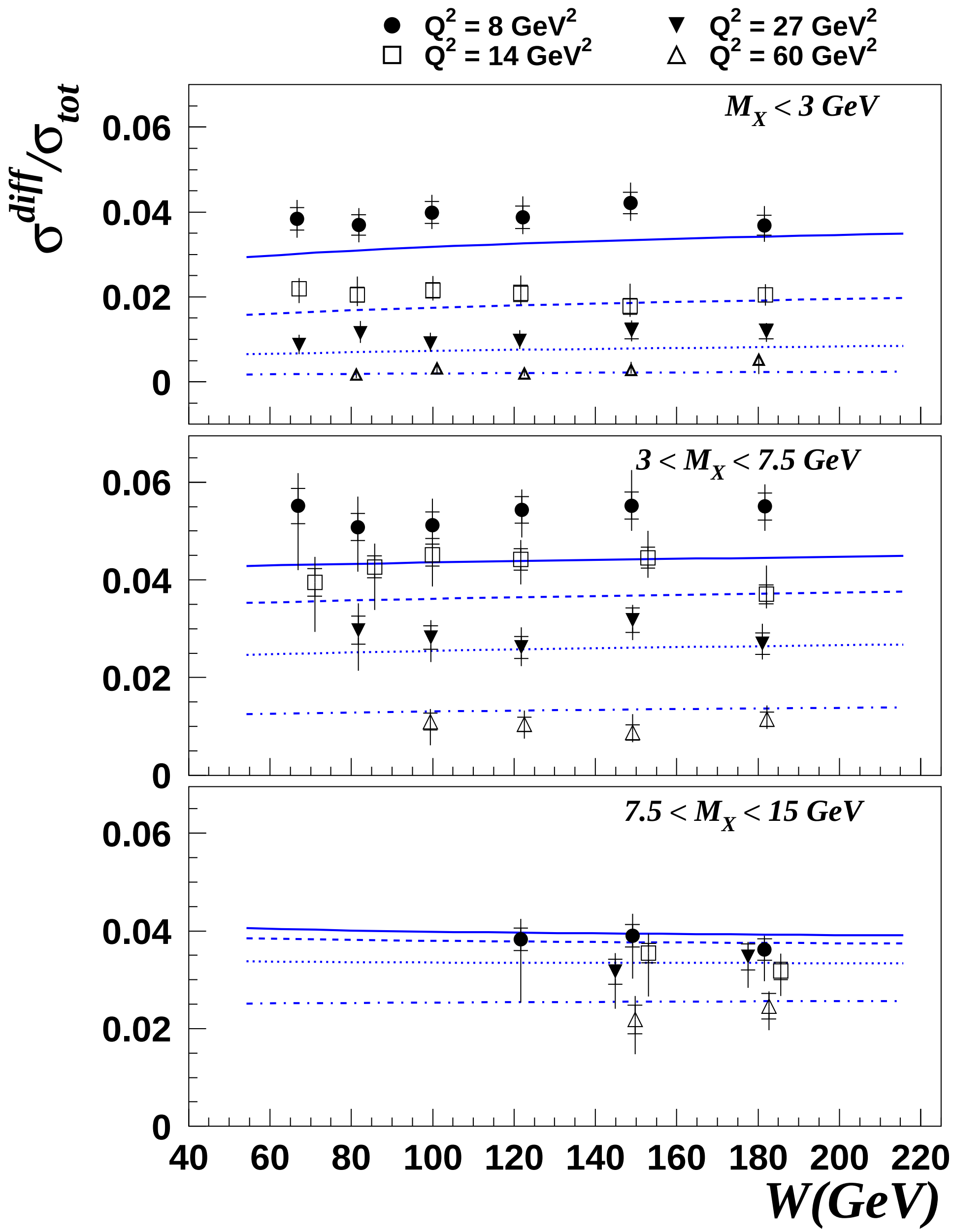


- DDIS sensitive to semihard: $r \sim 1/Q_s$ and soft dipoles $r \gg 1/Q_s$
- The only approach which explains constant ratio with energy

$$\frac{\sigma_{diff}}{\sigma_{tot}} \sim \frac{1}{\log(Q^2/Q_s^2(x))}$$

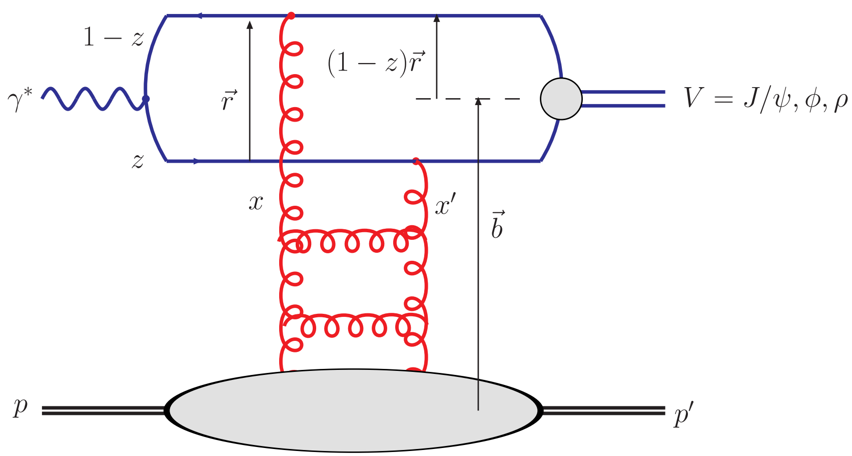
- Unitarity bound for the dipole cross section is crucial for this

Constant ratio

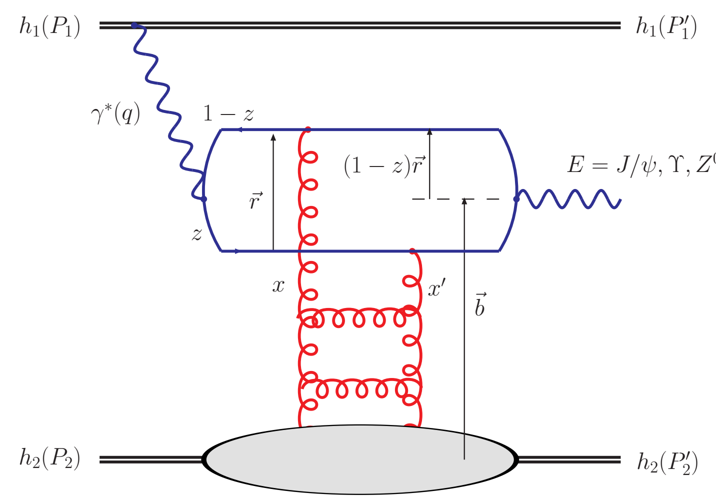


Diffractive processes in pp

(G. Watt talk)



ep scattering

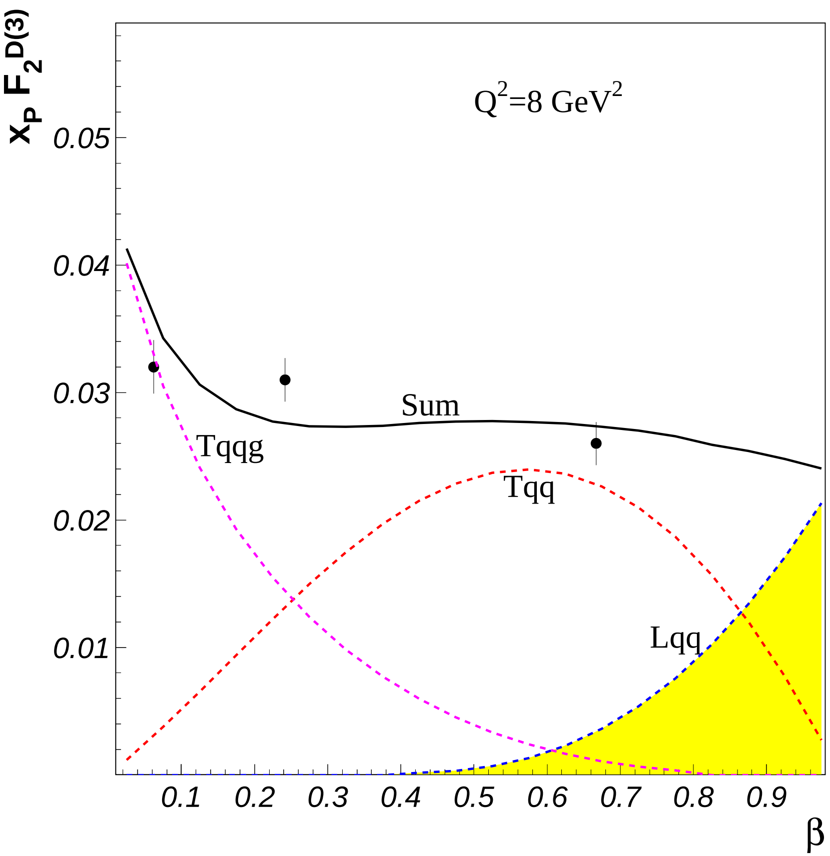


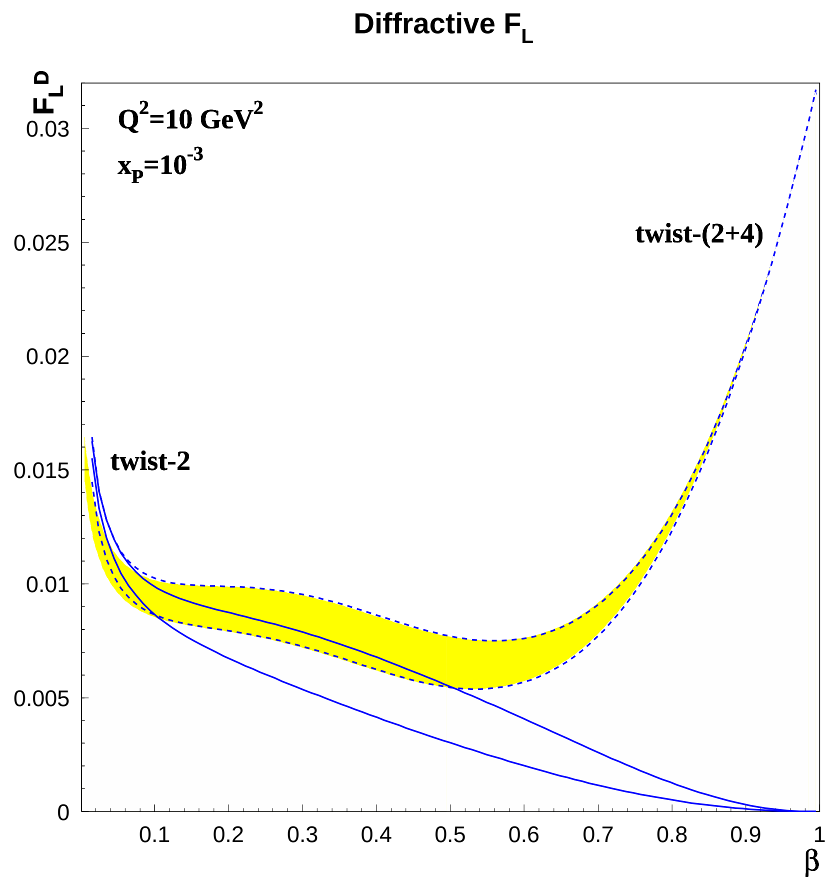
pp scattering

- Determined from ep processes dipole scattering amplitude $N(r, b, x)$ used to predict cross sections for pp processes

Diffractive F_L

(GB A. Luszczak)





Twist expansion of $F_{T,L}$ at small x

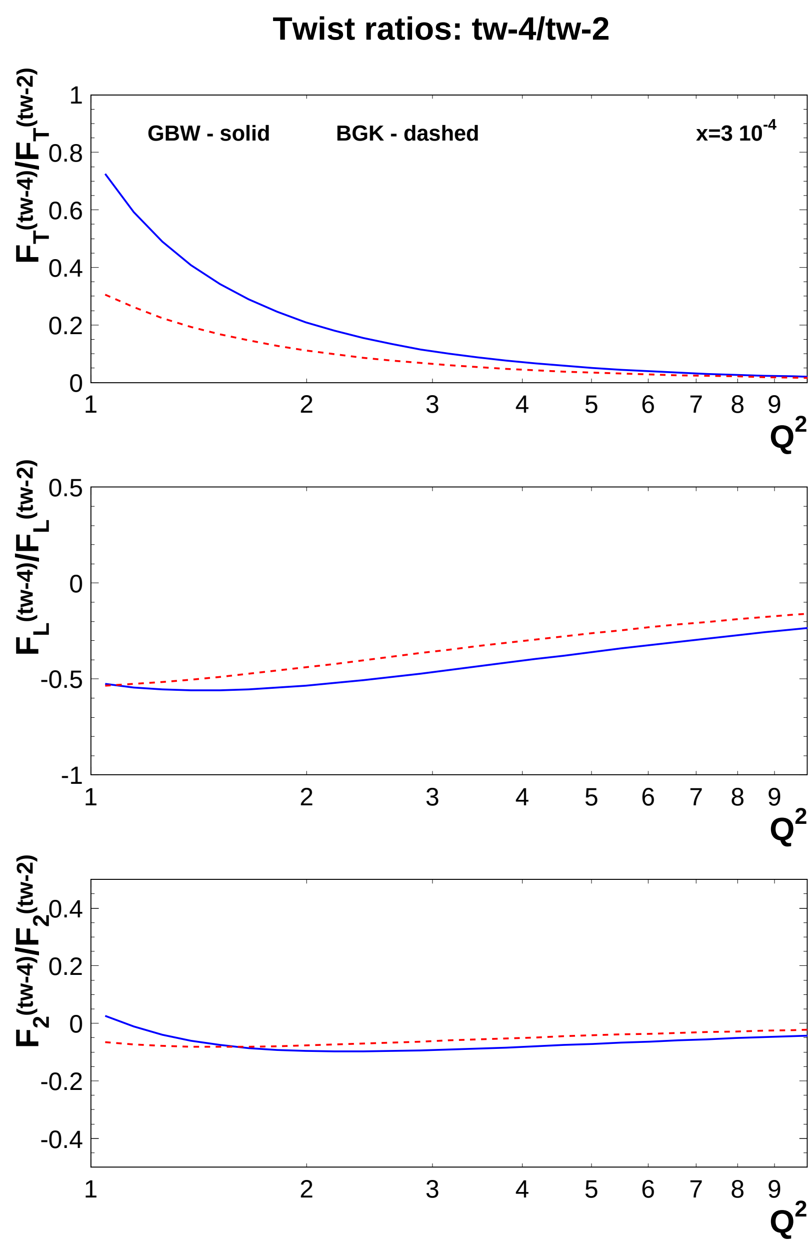
(L. Motyka talk)

- Twist expansion for $F_2 = F_T + F_L$ with a generic form of the dipole cross section:

$$F_T = F_T^{(2)}(\ln Q^2) + F_T^{(4)}\left(\frac{1}{Q^2}\right) + \cdots$$
$$F_L = F_L^{(2)}(\ln Q^2) + F_L^{(4)}\left(\frac{1}{Q^2}\right) + \cdots$$

- Large and negative correction from $F_L^{(4)}$ (up to 50%)
- In $F_T + F_L$ contributions from $twist - 4$ almost cancel.

Twist ratios



Summary

- Saturation is used as a code name for unitary description of semihard processes.
- Parton saturation is an intuitive picture of gluon interactions which lead to unitary cross sections.
- Phenomenological models of parton saturation are partially confirmed by perturbative QCD analyses.
- The crucial role in parton saturation is played by the saturation scale which reduce the dependence of results on the soft region.
- We are still far from rigorous understanding of the high density regime of QCD.
- The LHC and future projects should provide enough motivation for dedicated effort in this direction.