Towards an NNLO prediction of the top-quark pair production cross section

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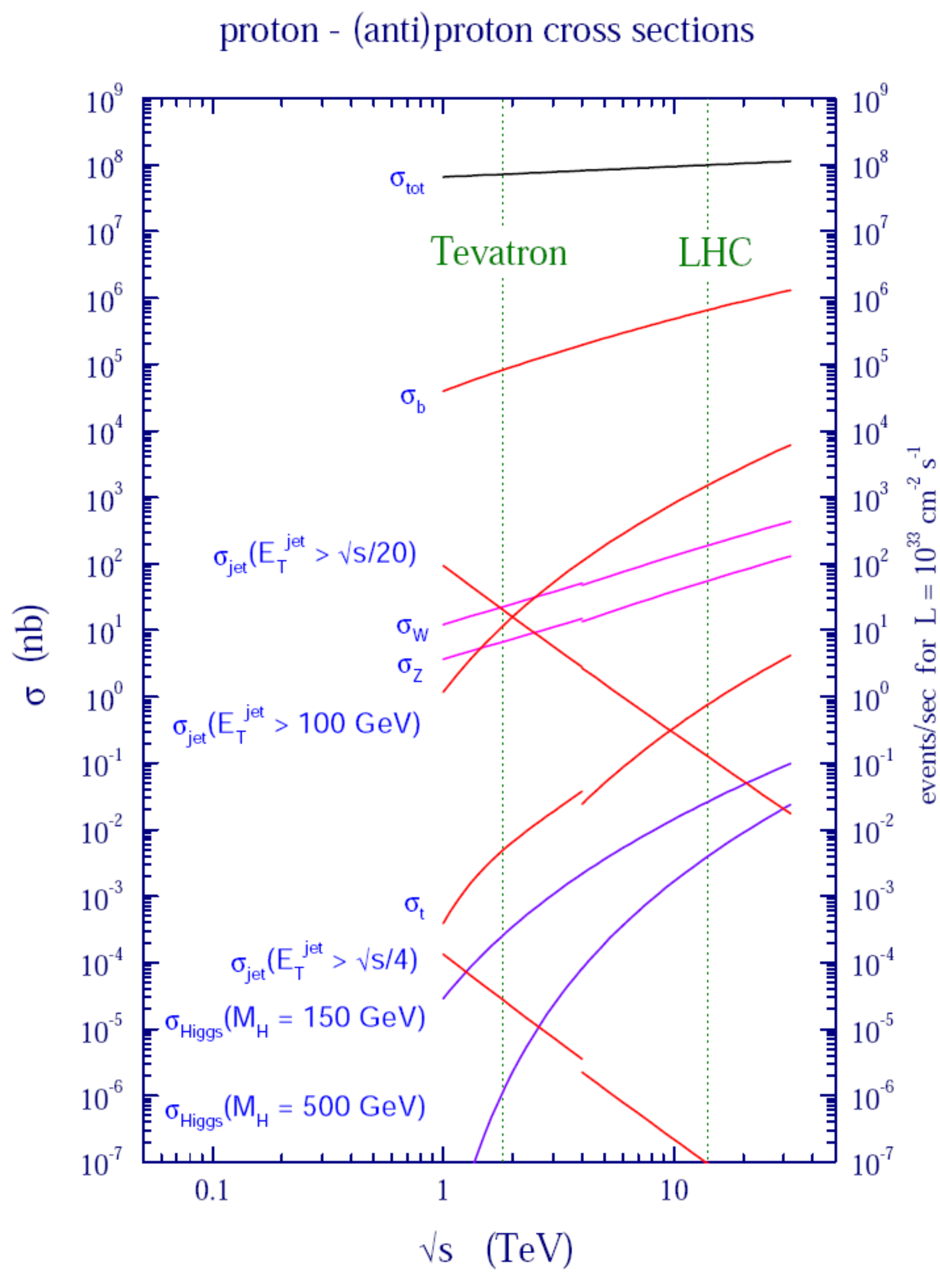
Introduction

- 8 million top quark pairs per year in the low luminosity phase
- Major goals

- top mass measurement with a precision below 1 GeV
- production cross section to better than 10%
- production and decay mechanisms to 1-2%
- spin correlations to 3-5%

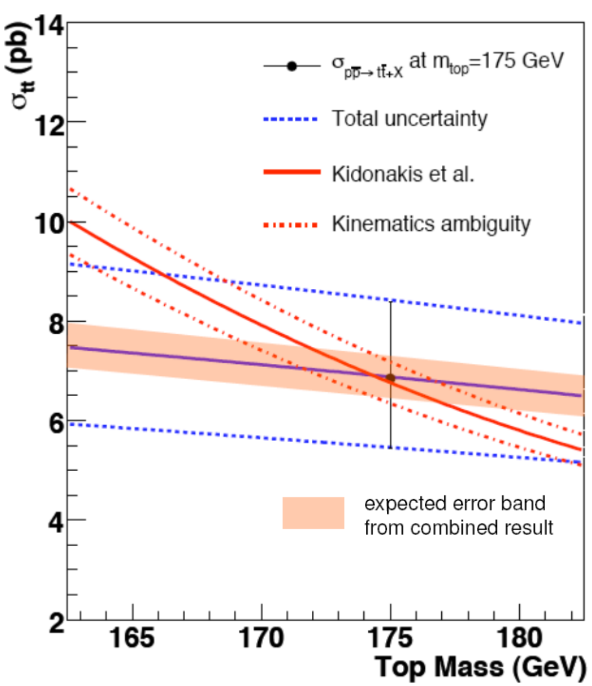
looks like 5% is feasible



Measuring the mass

- The top quark mass has a high impact on electroweak physics !
- The top quark mass is measured in the pair production process exclusively
- The method is based on kinematic reconstruction and fitting
- An interesting alternative is the measurement from the cross section normalization
- At the Tevatron the precision from the cross section shape is about 5 Gev
- At the LHC one expects 2-3 GeV

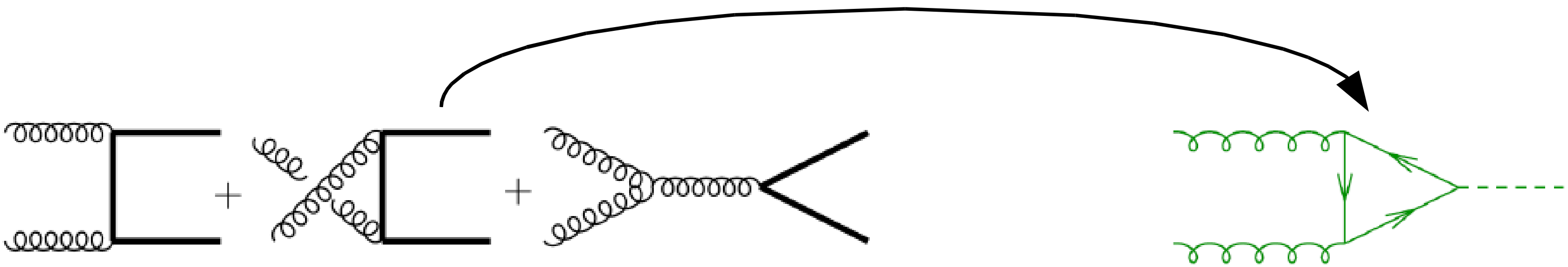
Source of uncertainty	Hadronic top δm_{top} (GeV/c^2)	Kinematic fit δm_{top} (GeV/c^2)	High p_T top sample δm_{top} (GeV/c^2)
Light jet energy scale (1 %)	0.2	0.2	
b-jet energy scale (1 %)	0.7	0.7	
b-quark fragmentation	0.1	0.1	0.3
ISR	0.1	0.1	0.1
FSR	1.	0.5	0.1
Combinatorial background	0.1	0.1	
Mass rescaling			0.9
UE estimate ($\pm$ 10 %)			1.3
Total	1.3	0.9	1.6
Statistical error	0.05	0.1	0.2



Calibration

- A more important use is luminosity determination for processes induced by the gluon flux

STANDARD CANDLE !

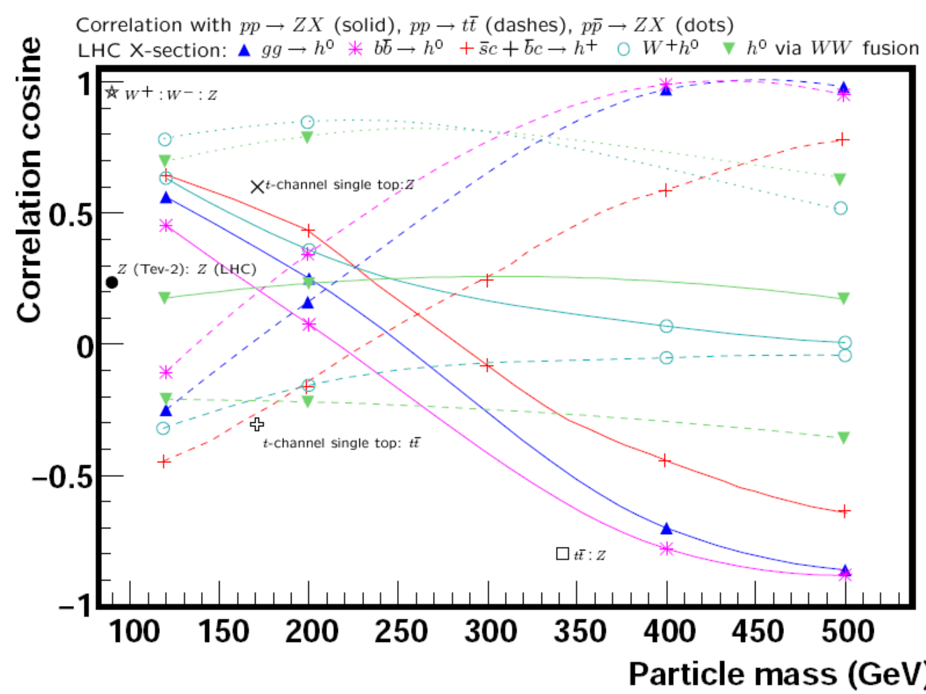


90 % !

overcomes part of the uncertainty in the gluon PDF !

- Conclusion from recent CTEQ analysis: 3 – 5 % precision required on the theory and experimental sides !!!

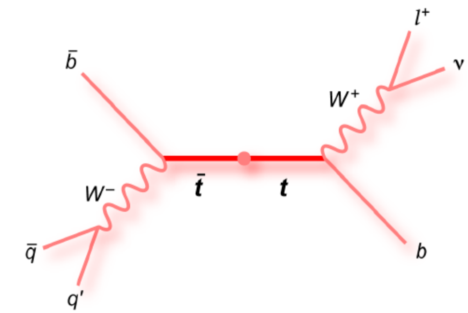
Nadolsky et al '08
see also talk by M. Mangano

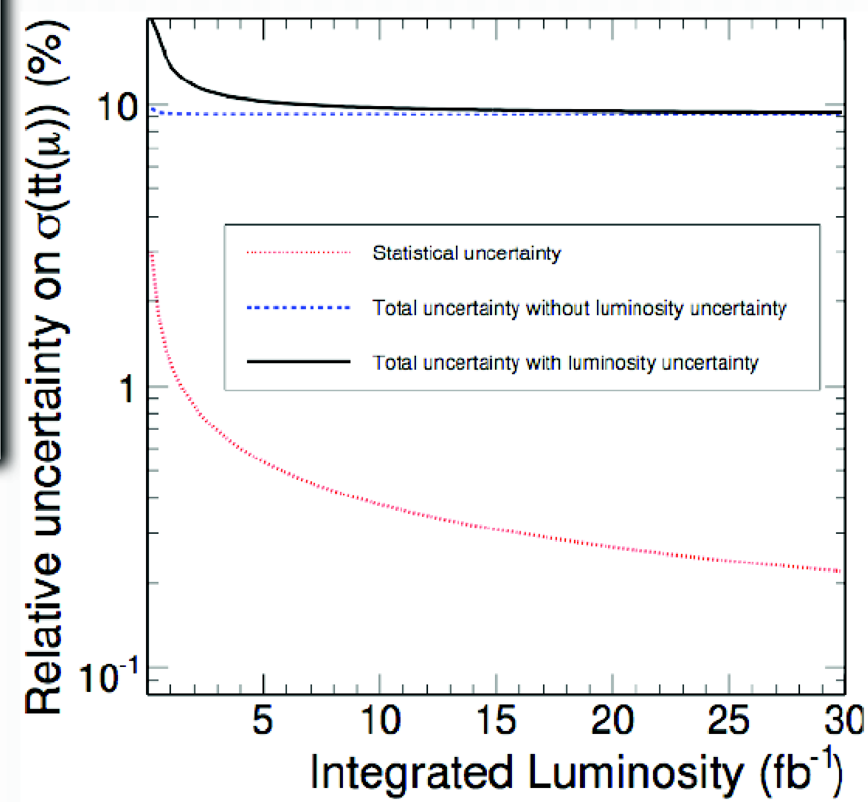


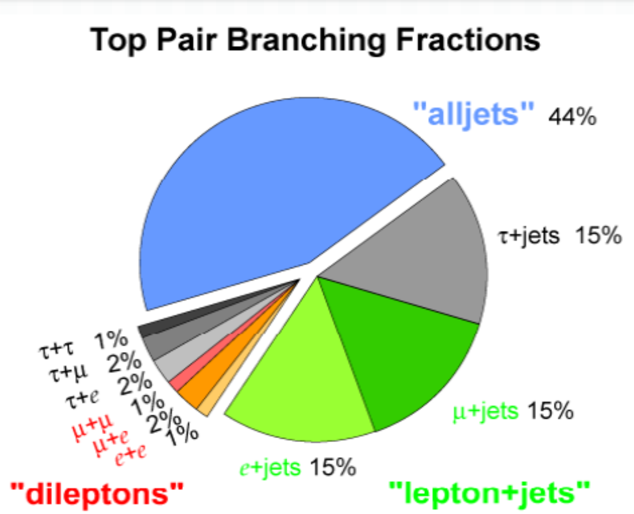
Uncertainties at the LHC

	$\Delta\hat{\sigma}_{t\bar{t}(\mu)}/\hat{\sigma}_{t\bar{t}(\mu)}$		
	$1\,\text{fb}^{-1}$	$5\,\text{fb}^{-1}$	$10\,\text{fb}^{-1}$
Simulation samples (ϵ_{sim})		0.6%	
Simulation samples (F_{sim})		0.2%	
Pile-Up (30% On-Off)		3.2%	
Underlying Event		0.8%	
Jet Energy Scale (light quarks) (2%)		1.6%	
Jet Energy Scale (heavy quarks) (2%)		1.6%	
Radiation (Λ_{QCD}, Q_0^2)		2.6%	
Fragmentation (Lund b, σ_q)		1.0%	
b-tagging (5%)		7.0%	
Parton Density Functions		3.4%	
Background level		0.9%	
Integrated luminosity	10%	5%	3%
Statistical Uncertainty	1.2%	0.6%	0.4%
Total Systematic Uncertainty	13.6%	10.5%	9.7%
Total Uncertainty	13.7%	10.5%	9.7%

CMS PTDR

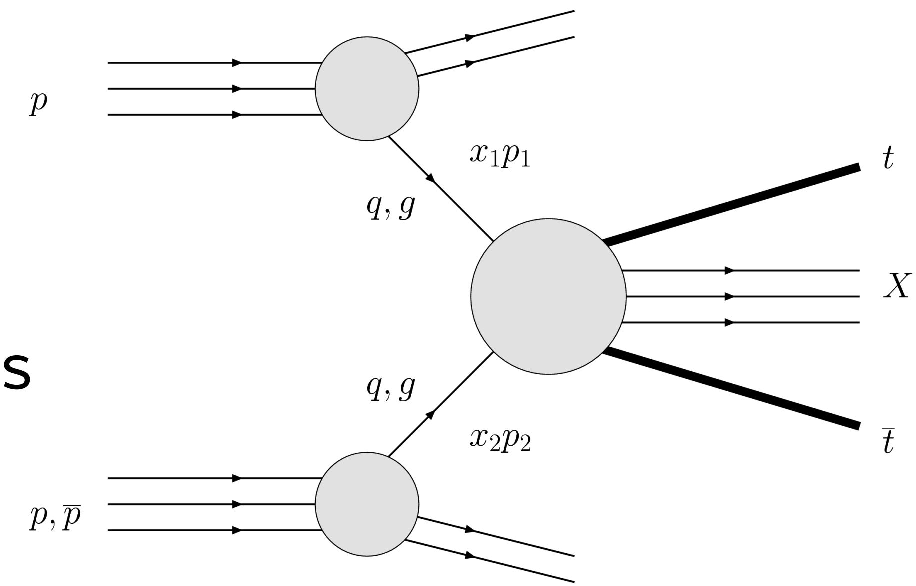






Theoretical framework

factorization theorem up to power corrections



renormalization scale

parton distribution functions

$x_1 x_2 s$

$$\sigma_{h_1 h_2}(s, m_t) = \sum_{ij} \iint_0^1 dx_1 dx_2 \, \phi_{i/h_1}(x_1, \mu_F) \, \phi_{j/h_2}(x_2, \mu_F) \, \hat{\sigma}_{ij}(\hat{s}, m_t, \alpha_S(\mu_R), \mu_R, \mu_F)$$

factorization scale

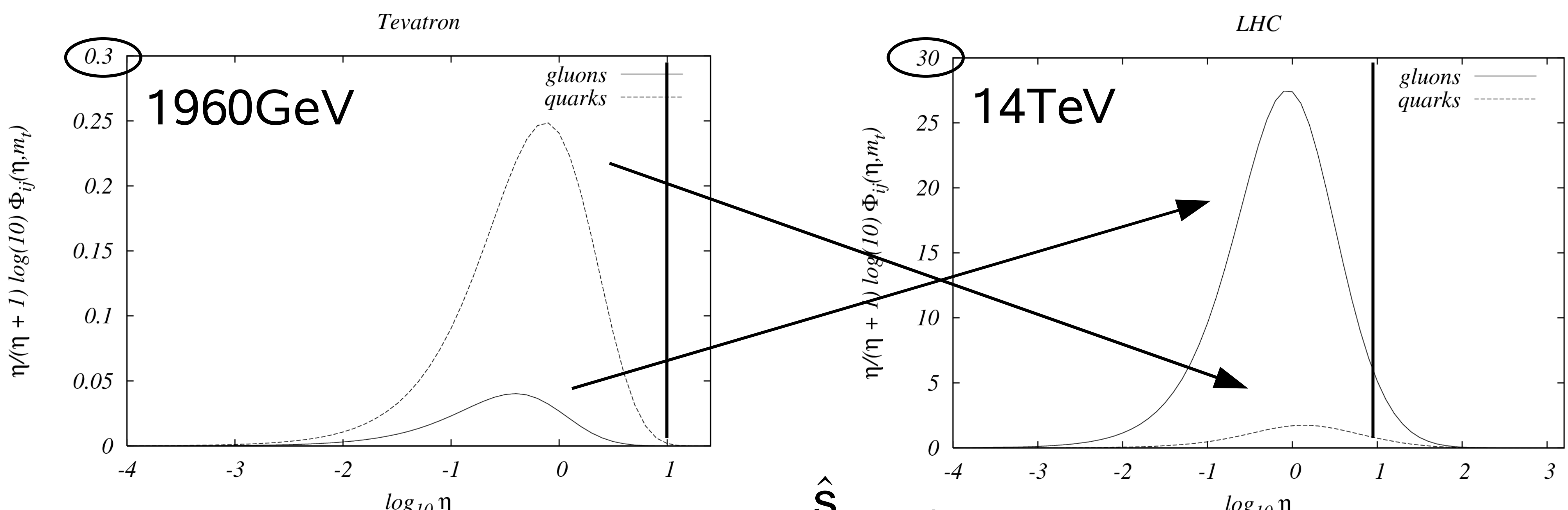
usually it is assumed that $\mu_F = \mu_R = \mu$

Fluxes

$$\Phi_{ij}^{h_1 h_2}(\tau,\mu)=\tau\iint_0^1 dx_1 dx_2\,\phi_{i/h_1}(x_1,\mu)\,\phi_{j/h_2}(x_2,\mu)\,\delta(x_1 x_2-\tau)$$

$$\sigma_{h_1 h_2}(s,m_t)=\frac{\alpha_S^2(\mu)}{m_t^2}\sum_{ij}\int_{\rho_H}^1\frac{d\tau}{\tau}\Phi_{ij}^{h_1 h_2}(\tau,\mu)\,f_{ij}\left(\frac{\rho_H}{\tau},\frac{\mu}{m_t}\right),\quad \rho_H=\frac{4m_t^2}{s}$$

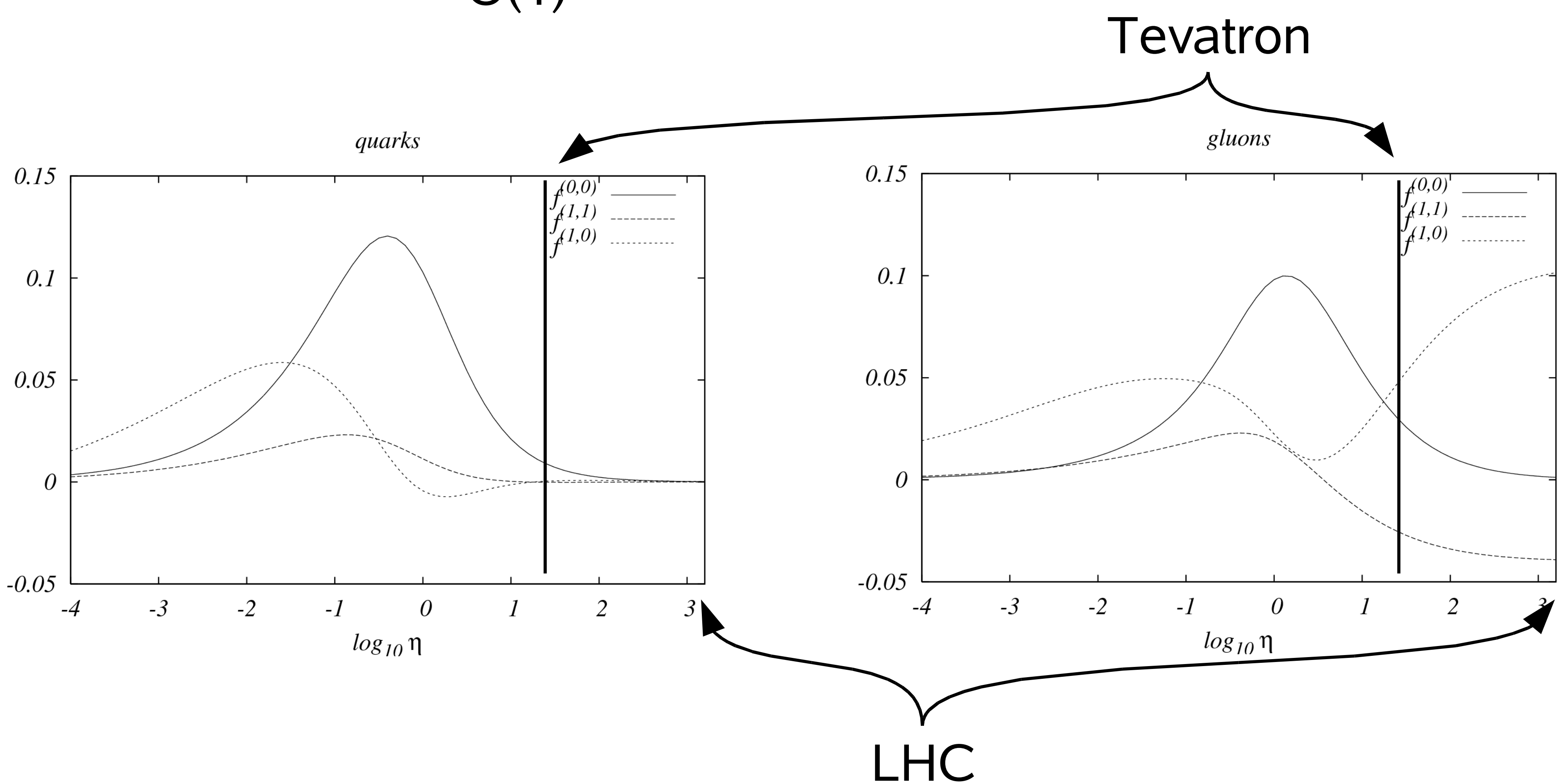
scaling functions



$$\eta=\frac{\hat{s}}{4m_t^2}-1$$

Scaling functions

$$f_{ij}\left(\rho, \frac{\mu^2}{m_t^2}\right) = \sum_{k=0}^{\infty} \underbrace{(4\pi\alpha_S(\mu))}_{O(1)}{}^{k} \sum_{l=0}^{k} f_{ij}^{(k,l)}(\rho) \log^l\left(\frac{\mu^2}{m_t^2}\right), \quad \rho = \frac{4 m_t^2}{\hat{s}}$$



Threshold behaviour

$$f_{q\bar{q}}^{(1,0)} \to \frac{1}{72\pi}\left(-\frac{\mathrm{pi}^2}{6} + \beta\left(\frac{16}{3}\log^2(8\beta^2) - \frac{82}{3}\log(8\beta^2)\right) + O(\beta)\right)$$

velocity

$$\beta = \sqrt{1 - \frac{4m_t^2}{\hat{s}}}$$

color octet: repulsion

Coulomb singularity $\sim \beta\left(\frac{\alpha_s}{\beta}\right)^l$

soft gluon radiation

singlet dominates: attraction

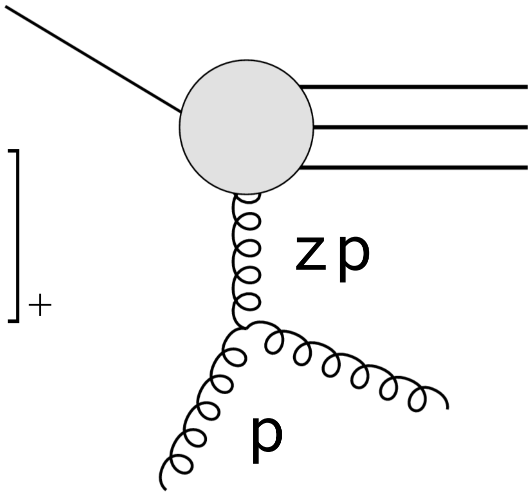
$$f_{gg}^{(1,0)} \to \frac{7}{1536\pi}\left(\frac{11\,\mathrm{pi}^2}{42} + \beta\left(12\log^2(8\beta^2) - \frac{366}{7}\log(8\beta^2)\right) + O(\beta)\right)$$

Coulomb singularity is color suppressed, but needs to be resummed for higher orders

$$\hat{\sigma}_{\mathrm{Coulomb}}(\rho) = \hat{\sigma}_{(8)}(\rho)\frac{\pi\alpha_s/(6\beta)}{\exp(\pi\alpha_s/(6\beta)) - 1} + \hat{\sigma}_{(1)}(\rho)\frac{4\pi\alpha_s/(3\beta)}{\exp(-4\pi\alpha_s/(3\beta)) - 1}$$

Threshold behaviour

- Soft gluon radiation important because of the vanishing phase space at threshold



- At l-loops the logarithms go up to $\left[\frac{\log^{2l-1}(1-z)}{1-z}\right]_+$

- Currently known up to NNLL Moch, Uwer '08

$$\hat{\sigma}^{(1)}_{q\bar{q}\to t\bar{t}} = \hat{\sigma}^{(0)}_{q\bar{q}\to t\bar{t}}\left\{42.667\ln^2\beta - 20.610\ln\beta + 13.910 - 3.2899\frac{1}{\beta}\right\},$$

$$\hat{\sigma}^{(2)}_{q\bar{q}\to t\bar{t}} = \hat{\sigma}^{(0)}_{q\bar{q}\to t\bar{t}}\left\{910.22\ln^4\beta - 1315.5\ln^3\beta + \left(565.80 - 140.37\frac{1}{\beta}\right)\ln^2\beta + \left(862.42 + 32.106\frac{1}{\beta}\right)\ln\beta - 28.862\frac{1}{\beta^2} + 89.431\frac{1}{\beta} + C^{(2)}_{q\bar{q}}\right\}$$

$$\hat{\sigma}^{(1)}_{gg\to t\bar{t}} = \hat{\sigma}^{(0)}_{gg\to t\bar{t}}\left\{96\ln^2\beta - 9.5165\ln\beta + 35.322 + 5.1698\frac{1}{\beta}\right\},$$

$$\hat{\sigma}^{(2)}_{gg\to t\bar{t}} = \hat{\sigma}^{(0)}_{gg\to t\bar{t}}\left\{4608\ln^4\beta - 1894.9\ln^3\beta + \left(-3.4811 + 496.30\frac{1}{\beta}\right)\ln^2\beta + \left(3144.4 + 321.17\frac{1}{\beta}\right)\ln\beta + 45.354\frac{1}{\beta^2} - 140.53\frac{1}{\beta} + C^{(2)}_{gg}\right\}$$

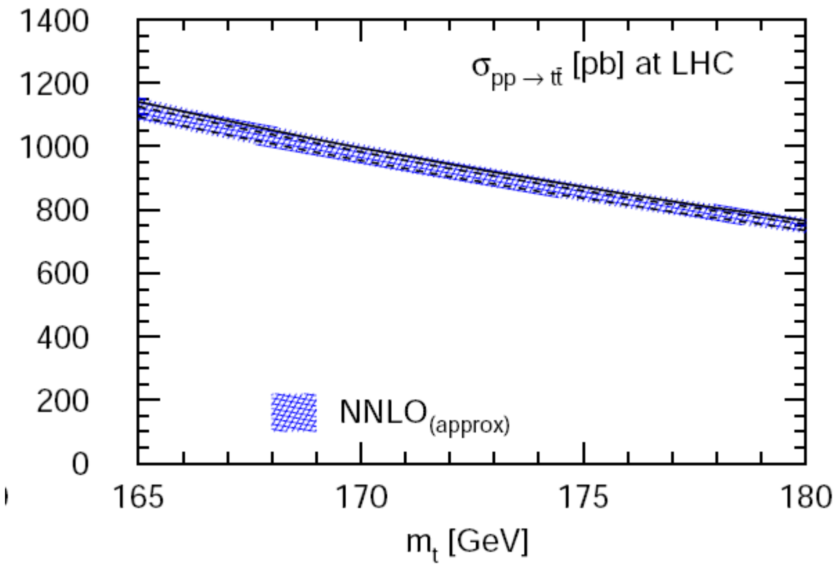
Theory uncertainties

- At the LHC the scale dependence gives an error of about 12% at NLO
- Input parameter sensitivity at NLO

with $m_t = 172.6 \pm 1.4\,\text{GeV}$

$$\frac{5\,\Delta m_t}{m_t} \simeq 4\%$$ (CDF & D0)

by varying the PDFs in CTEQ $\simeq 5\%$



- Excellent prospects at NNLO

3% scale, 2% PDF Moch, Uwer '08

- Conservative estimate with NLL resummation

$$\sigma_{t\bar{t}}^{\rm NLO+NLL}({\rm LHC}, m_t = 171~{\rm GeV}, {\rm CTEQ6.5}) = 908~^{+82(9.0\%)}_{-85(9.3\%)}~({\rm scales})~^{+30(3.3\%)}_{-29(3.2\%)}~({\rm PDFs})~~{\rm pb}$$

$$\sigma_{t\bar{t}}^{\rm NLO+NLL}({\rm LHC}, m_t = 171~{\rm GeV}, {\rm MSTW2006nnlo}) = 961~^{+89(9.2\%)}_{-91(9.4\%)}~({\rm scales})~^{+11(1.1\%)}_{-12(1.2\%)}~({\rm PDFs})$$

Cacciari, Frixione, Mangano, Nason & Ridolfi '08

Historical perspective

- NLO corrections

  Nason, Dawson, Ellis '88

- implemented in MCFM

  Campbell, Ellis

- 1-loop squared

  Korner, Merebashvili, Rogal '06

- NLO tT + jet

  Dittmaier, Uwer, Weinzierl '07

- LL resummation

  Laenen, Smith, van Neerven '92

- NLL resummation

  Bonciani, Catani, Mangano, Nason '98

- NLL resummation + ...

  Kidonakis, Vogt '03

- NNLL resummation

  Moch, Uwer '08

- High energy asymptotics of 2-loop amplitudes

  quark annihilation MC, Mitov, Moch '07

  gluon fusion MC, Mitov, Moch '07

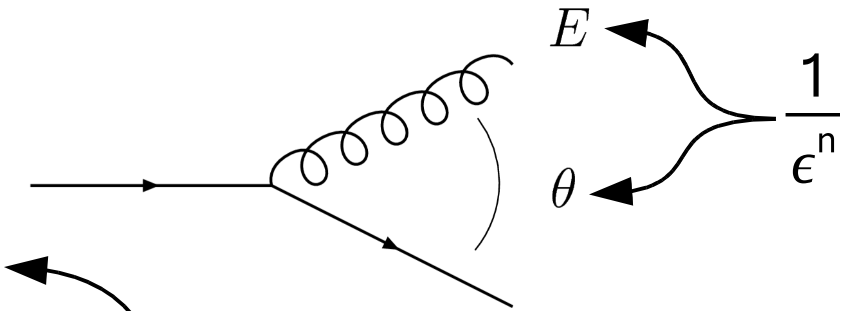
- Full mass dependence of 2-loop amplitudes

  quark annihilation MC '08

  gluon fusion Baernreuther, MC (in preparation)

Contributions to the cross section

difficult 2-loop amplitudes
1-loop amplitude squared not easy
trivial phase space



similar to tT + jet, but
difficulties at the phase space
boundaries

ongoing collaboration with C. Papadopoulos, Athens
(OPP algorithm for NLO computations)

$$d\hat{\sigma}_{t\bar{t}}^{NNLO} = d\hat{\sigma}_{2}^{VV} + d\hat{\sigma}_{2+1}^{VR} + d\hat{\sigma}_{2+2}^{RR}$$

trivial amplitude
difficult phase space

$$d\hat{\sigma}_n = d\Phi_n \left|M_n\right|^2$$

DREG singularities

$$M^l = \frac{1}{\epsilon^{2l}} + \ldots$$

ongoing collaboration with
G. Heinrich, Durham
(Sector Decomposition)

phase space

amplitude

Other ideas? Sure... (ongoing collaboration with A. Mitov, Stony Brook)

Virtual corrections for quark annihilation... numerics

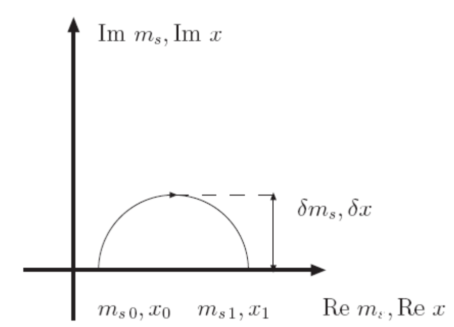
The new invention — based on some earlier ideas by Czyz, Caffo, Remiddi '02

- Determine the coefficients of the mass expansions using differential equations in m_s obtaining the power corrections

$$m_s \frac{d}{dm_s} M_i(m_s, x, \epsilon) = \sum_j C_{ij}(m_s, x, \epsilon) M_j(m_s, x, \epsilon)$$

- Evaluate the expansions for $m_s \ll 1$ to obtain the desired numerical precision of the boundaries

- Evolve the functions from the boundary point with differential equations first in m_s and then in x (**ZVODE**)



The hardest part — invented for Bhabha scattering by MC, Gluza, Riemann '06

- Compute the high energy asymptotics of the master integrals using Mellin-Barnes representations in order to obtain the leading behaviour of the amplitude

Virtual corrections for quark annihilation... numerics

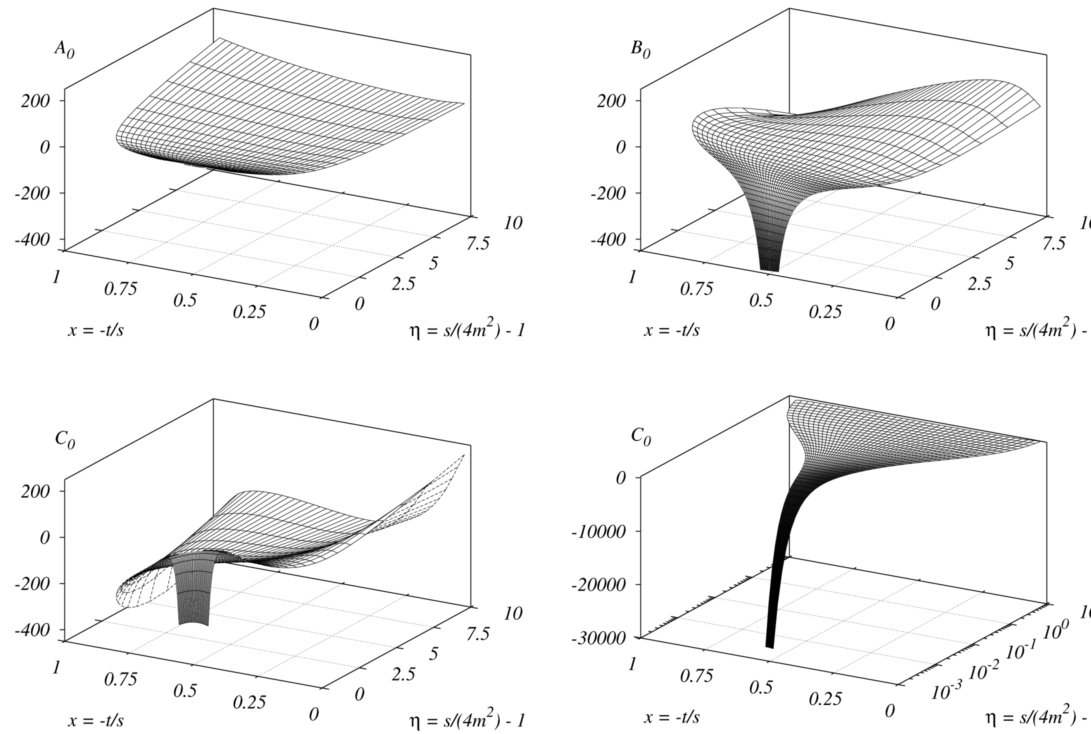
- Computational complexity

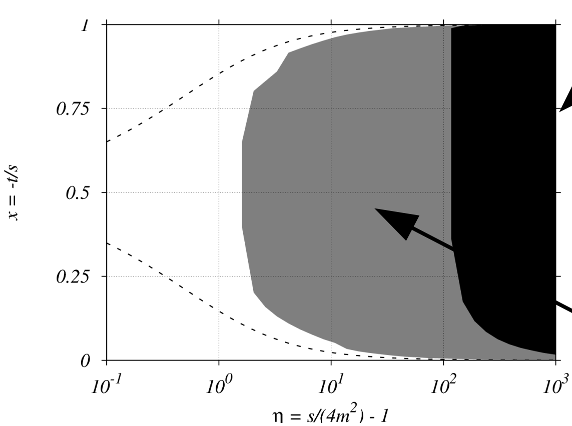
190 diagrams
2812 integrals
145 master integrals

- Even worse for gluon fusion

726 diagrams
8676 integrals
422 master integrals

- Convergence regions for a small mass expansion





1 % accuracy region of the leading asymptotics (MC, Mitov, Moch '07)

1 ‰ accuracy region of the expansion

$m^2 = .2\ s$, $t = -0.45\ s$

	ϵ^{-4}	ϵ^{-3}	ϵ^{-2}	ϵ^{-1}	ϵ^{0}
A	0.22625	1.391733154	-2.298174307	-4.145752449	17.37136599
B	-0.4525	-1.323646320	8.507455541	6.035611156	-35.12861106
C	0.22625	-0.06808683395	-18.00716652	6.302454931	3.524044913
D_l		-0.22625	0.2605057339	-0.7250180282	-1.935417247
D_h			0.5623350684	0.1045606449	-1.704747998
E_l		0.22625	-0.3323207300	7.904121951	2.848697837
E_h			-0.5623350684	4.528240788	12.73232424
F_l					-1.984228442
F_{lh}					-2.442562819
F_h					-0.07924540546

Virtual corrections for quark annihilation... numerics

- Numerical stability requires higher precision
- Global error determined by contour variation
- Due to relatively slow evaluation one needs interpolation based on a grid of values for Monte-Carlo generation (grid available on arXiv)

	leading color			full color		
number of masters	36			145		
number of functions	155			595		
precision	quadruple		double	quadruple		double
evolution in m_s						
requested local error	10^{-20}	10^{-12}	10^{-12}	10^{-20}	10^{-12}	10^{-12}
contour deformation δm_s	0.1	0.1	0.1	0.1	0.1	0.1
number of steps taken	2319	618	534	2932	777	1302
Jacobian evaluation time [ms]	3.4	3.4	0.2	37	37	4.9
evolution in x						
requested local error	10^{-18}	10^{-10}	10^{-10}	10^{-18}	10^{-10}	10^{-10}
contour deformation δx	0.1	0.1	0.1	0.1	0.1	0.1
number of steps taken	545	139	139	739	174	432
Jacobian evaluation time [ms]	8.3	8.3	0.4	150	150	17
total evaluation time [s]	49	13	< 1	957	243	26

Virtual corrections for quark annihilation... numerics

- Problems at the singular points of the differential equations

Jacobian singularity	branching	allowed	interpretation
$m_s = 0$	yes		collinear singularity
$m_s = 1/4$	yes		s-channel threshold
$m_s = -1/4$			
$x = 0$	yes		t-channel threshold
$x = 1$	yes		u-channel threshold
$x = 1/2$		yes	perpendicular scattering
$m_s = x\,(1-x)$			forward/backward scattering
$m_s = x$			
$m_s = 1-x$			
$m_s = -x$			
$m_s = x-1$			
$m_s = 1/2\,x\,(1-x)$		yes	
$m_s = 1/2\,x$		yes	
$m_s = 1/2\,(1-x)$		yes	
$m_s = 1/2\,(1-x^2)$			
$m_s = -1/2\,(1-x)^2$			

finite part of the bosonic + heavy lepton contribution

$m_s = 0.2, \qquad d = 1/10\,(x - 0.45)$

- Taylor expansions around arbitrary points as a possible solution

$$469.555 - 14.5383\,d + 17.4298\,d^2 - 0.448744\,d^3 - 0.0600637\,d^4 + 0.0325809\,d^5 - 0.00888086\,d^6 + 0.00212687\,d^7 - 0.000528458\,d^8 + 0.000118944\,d^9 - 0.0000286279\,d^{10} + 6.39261\times 10^{-6}\,d^{11} - 1.51017\times 10^{-6}\,d^{12} + 3.37839\times 10^{-7}\,d^{13} - 7.88119\times 10^{-8}\,d^{14} + 1.76849\times 10^{-8}\,d^{15} - 4.09387\times 10^{-9}\,d^{16} + 9.10803\times 10^{-10}\,d^{17} - 2.2892\times 10^{-10}\,d^{18} + 9.93296\times 10^{-12}\,d^{19} - 8.09343\times 10^{-11}\,d^{20}$$

- Extremely efficient implement with sparse matrix multiplication and multiple precision (e.g. 128 digits needs 10 seconds for 21 terms)

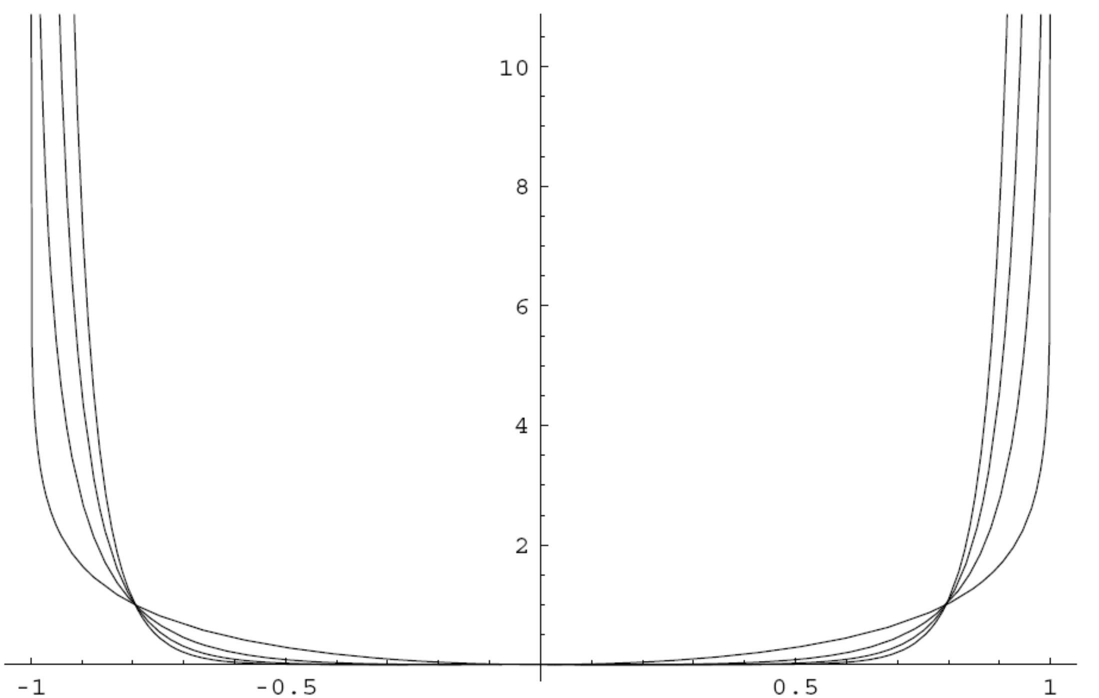
Virtual corrections for quark annihilation... numerics

- Decent precision needed at the edges of the phase space for total cross section contribution in dimensional regularization

$$d\Omega \sim \int_{-1}^{+1} d\cos\theta (1-\cos^2\theta)^{-\epsilon}$$

$$\approx \int_{-1}^{+1} d\cos\theta \left(1-\epsilon\log(1-\cos^2\theta)+\frac{\epsilon^2}{2}\log^2(1-\cos^2\theta)+\dots\right)$$

$$x=\frac{1}{2}(1-\beta\cos\theta)$$



Virtual corrections for quark annihilation... numerics

- Contribution to the total cross section at $m_s = 0.2$, obtained with 2 Taylor expansions around $x = 0.45$ and $x = 0.55$ (unrenormalized)

$$\int_{-1}^{+1} d\cos\theta\,(1-\cos^2\theta)^{-\epsilon}\, 2\,\Re\,\langle M^{(0)}|M^{(2)}\rangle \approx$$

with 17 terms

$$\frac{53.0963}{\epsilon^4}-\frac{665.629}{\epsilon^3}+\frac{2524.60}{\epsilon^2}-\frac{21.6349}{\epsilon}+16206.6$$

with 18 terms

$$\frac{53.0963}{\epsilon^4}-\frac{665.630}{\epsilon^3}+\frac{2524.60}{\epsilon^2}-\frac{21.6359}{\epsilon}+16206.6$$

with 19 terms

$$\frac{53.0963}{\epsilon^4}-\frac{665.630}{\epsilon^3}+\frac{2524.60}{\epsilon^2}-\frac{21.6353}{\epsilon}+16206.6$$

Conclusions

- The total cross section will be measured to better than 10% at the LHC and can be used for callibration and alternative mass measure
- The error from scale variation at NLO is about 12%
- Soft gluon resummations not sufficient and do not fit into MC
- Known at NNLO are

  - PDFs

  - square of the one-loop matrix element

  - tT + jet cross section

  - leading behaviour at high energy of the 2-loop virtual corrections

  - exact virtual corrections in the quark annihilation channel
- Next: remaining virtuals and real radiation
- Lots of work before a satisfactory Monte-Carlo implementation