3–loop Corrections to Heavy Flavour Wilson Coefficients in Deep–Inelastic Scattering

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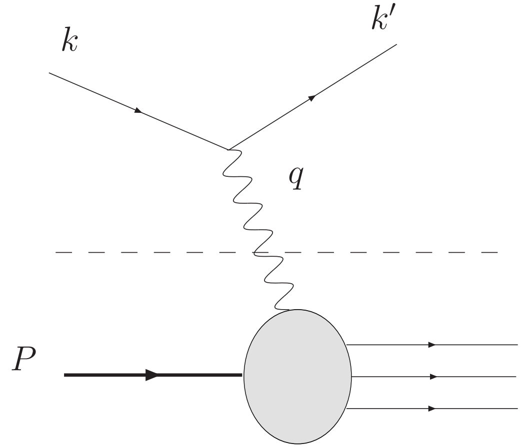
in collaboration with J. Blümlein and S. Klein

- Introduction
- Renormalization of the OMEs to 3 Loops
- O(ϵ) terms at 2 Loops
- Towards fixed moments of $A_{ij}^{(3)}$
- Conclusions

based on:

- I.B., J. Blümlein, S. Klein, and C. Schneider
  `arXiv:0707.4759 [math-ph]`;
  `arXiv:0803.0273 [hep-ph]`.
- I.B., J. Blümlein, and S. Klein,
  Phys. Lett. **B648** (2007) 195;
  Nucl. Phys. **B780** (2007) 40;
  arXiv:0706.2738 [hep-ph];
  Acta Phys. Polon. B **38** (2007) 3543;

Deep–Inelastic Scattering (DIS):



$$Q^2 := -q^2, \quad x := \frac{Q^2}{2pq} \quad \text{Bjorken–x}$$

$$\frac{d\sigma}{dQ^2\, dx} \sim W_{\mu\nu} L^{\mu\nu}$$

Hadronic tensor for heavy quark production via single photon exchange:

$$W_{\mu\nu}^{Q\bar{Q}}(q,P,s) = \frac{1}{4\pi}\int d^4\xi \exp(iq\xi)\langle P,s \mid [J_\mu^{em}(\xi), J_\nu^{em}(0)] \mid P,s\rangle_{Q\bar{Q}}$$

unpol. $\Big\{ \; = \frac{1}{2x}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) F_L^{Q\bar{Q}}(x,Q^2) + \frac{2x}{Q^2}\left(P_\mu P_\nu + \frac{q_\mu P_\nu + q_\nu P_\mu}{2x} - \frac{Q^2}{4x^2} g_{\mu\nu}\right) F_2^{Q\bar{Q}}(x,Q^2)$

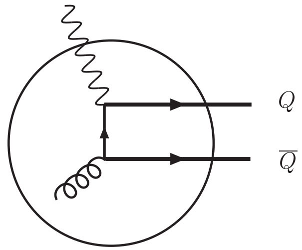
pol. $\Big\{ \; -\frac{M}{2Pq}\varepsilon_{\mu\nu\alpha\beta} q^\alpha \left[s^\beta g_1^{Q\bar{Q}}(x,Q^2) + \left(s^\beta - \frac{sq}{Pq} p^\beta\right) g_2^{Q\bar{Q}}(x,Q^2)\right] .$

Consider extrinsic charm production: charm quark only in

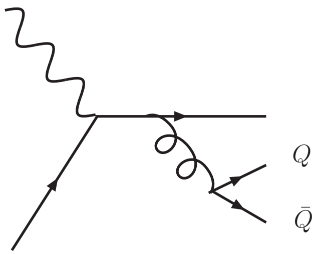
- final state
- virtual corrections

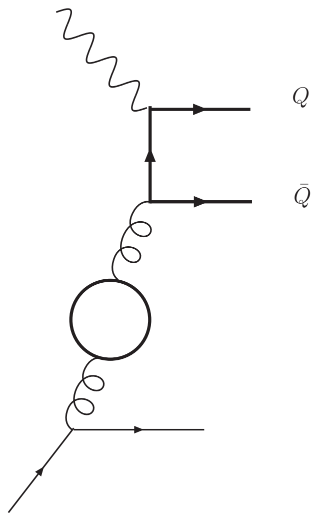
no intital state heavy quark.

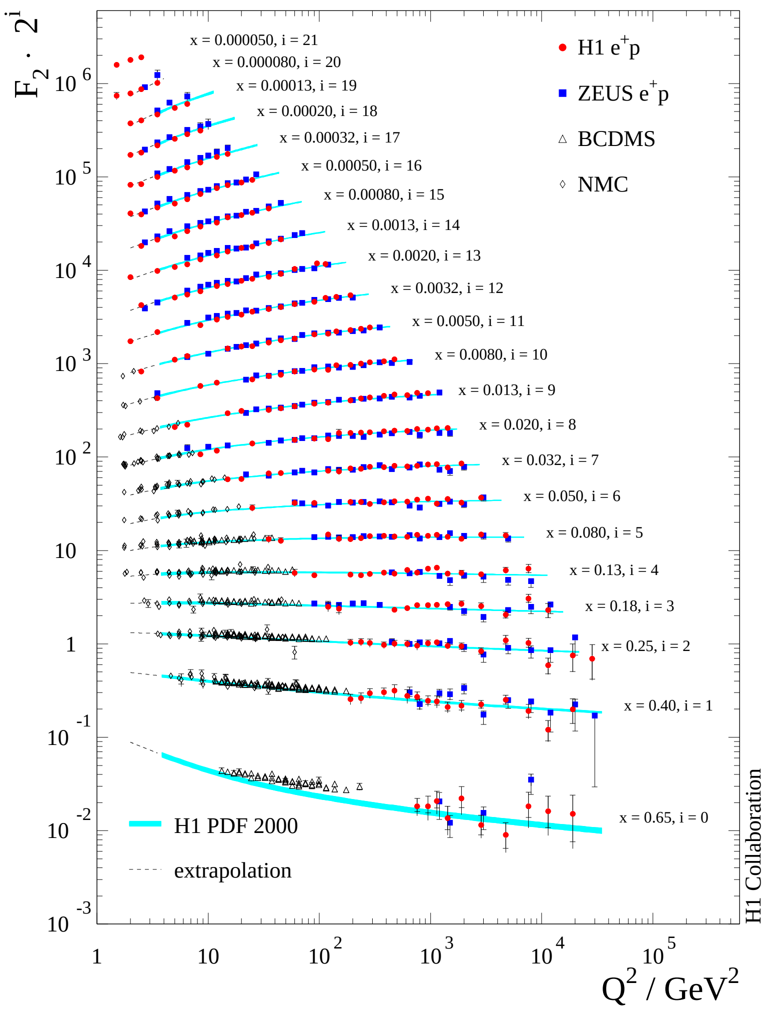
In leading order α_s: photon–gluon fusion process

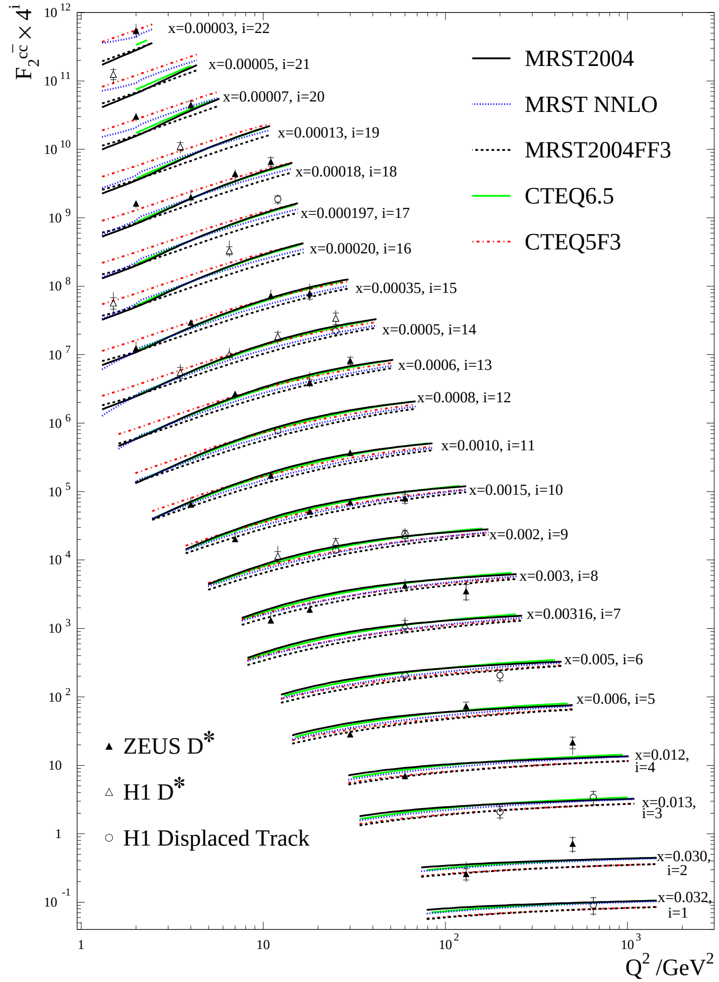


In higher orders of α_s: additional processes, e.g.:









[Thompson, 2007]

High statistics in both cases

$F_2^{c\bar{c}}(x,Q^2) \sim 20 - 40\,\%\ F_2(x,Q^2)$ for small values of x, but different scaling violations

Need for the calculation:

- Heavy flavor (charm) contributions to DIS structure functions are rather large [20–40 % at lower values of x] .
- Increase in accuracy of the perturbative description of DIS structure functions.
- $\Longleftrightarrow$ QCD analysis and determination of $\Lambda_{\rm QCD}$, resp. $\alpha_s(M_Z^2)$, from DIS data
- $\Longleftrightarrow$ Precise determination of the gluon and sea quark distributions
- $\Longleftrightarrow$ Derivation of variable flavor number scheme for heavy quark production to $O(a_s^3)$ ($\rightarrow$ see talk of S. Klein)

Goal: Calculation of the heavy flavor Wilson coefficients to higher orders for $Q^2 \geq 25\,{\rm GeV}^2$ [sufficient in many applications] .

Previous calculations:

Unpolarized DIS :

- LO : [Witten, 1976; Babcock, Sivers, 1978; Shifman, Vainshtein, Zakharov 1978; Leveille, Weiler, 1979]
- NLO : [Laenen, Riemersma, Smith, van Neerven, 1993, 1995]
  asymptotic: [Buza, Matiounine, Smith, Migneron, van Neerven, 1996; I.B., Blümlein, Klein, 2007]

Observation: $F_2^{c\bar{c}}(x, Q^2)$ is very well described by $F_2^{c\bar{c}}(x, Q^2)|_{Q^2 \gg m^2}$ for $Q^2 \gtrsim 10\, m_c^2$.

Polarized DIS :

- LO : [Watson, 1982; Glück, Reya, Vogelsang, 1991; Vogelsang, 1991]
- NLO : asymptotic: [Buza, Matiounine, Smith, van Neerven, 1997]

Mellin–Space Expressions: [Alekhin, Blümlein, 2003].

massless RGE & light–cone expansion in Bjorken–limit $\{Q^2,\ \nu\} \ \rightarrow \ \infty,\ x$ fixed:

$$\lim_{\xi^2\rightarrow 0}\Big[J(\xi),J(0)\Big] \propto \sum_{i,N,\tau} c^N_{i,\tau}(\xi^2,\mu^2)\xi_{\mu_1}...\xi_{\mu_N} O^{\mu_1...\mu_m}_{i,\tau}(0,\mu^2)$$

Operators: flavor non-singlet , pure–singlet and singlet; consider leading twist 2.

RGE for collinear singularities

$\Longrightarrow$ mass factorization of the structure functions into Wilson coefficients and parton densities:

$$F_i(x,Q^2) = \sum_j \underbrace{C_i^j\left(x,\frac{Q^2}{\mu^2}\right)}_{\text{perturbative}} \otimes \underbrace{f_j(x,\mu^2)}_{\text{non-perturbative}}$$

Light–flavour Wilson coefficients: process dependent, to $O(a_s^3)$: [Moch, Vermaseren, Vogt, 2005]

$$C^{\rm fl}_{(2,L);i}\left(\frac{Q^2}{\mu^2}\right) = \delta_{i,q} + \sum_{l=1}^{\infty} a_s^l C^{{\rm fl},(l)}_{(2,L),i}, \qquad i=q,g$$

Heavy quark contributions given by heavy quark Wilson coefficient, $H^{\rm S,NS}_{(2,L),i}\left(\frac{Q^2}{\mu^2},\frac{m^2}{Q^2}\right)$.

In the limit $Q^2 \gg m_Q^2$:

massive RGE, derivative $m^2\partial/\partial m^2$ acts on Wilson coefficients only:
all terms but power corrections calculable through partonic operator matrix elements, $\langle i|A_l|j\rangle$, which are process independent objects!

$$H_{(2,L),i}^{\rm S,NS}\left(\frac{Q^2}{\mu^2},\frac{m^2}{\mu^2}\right) = \underbrace{A_{k,i}^{\rm S,NS}\left(\frac{m^2}{\mu^2}\right)}_{\text{massive OMEs}} \otimes \underbrace{C_{(2,L),k}^{\rm S,NS}\left(\frac{Q^2}{\mu^2}\right)}_{\text{light–parton–Wilson coefficients}}.$$

holds for polarized and unpolarized case. OMEs obey expansion

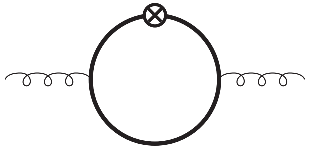
$$A_{k,i}^{\rm S,NS}\left(\frac{m^2}{\mu^2}\right) = \langle i|O_k^{\rm S,NS}|i\rangle = \delta_{k,i} + \sum_{l=1}^{\infty} a_s^l A_{k,i}^{{\rm S,NS},(l)}\left(\frac{m^2}{\mu^2}\right), \qquad i = q, g$$

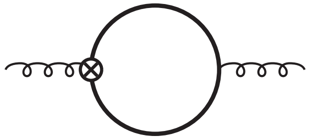
[Buza, Matiounine, Migneron, Smith, van Neerven, 1996; Buza, Matiounine, Smith, van Neerven, 1997.]

$$H_{2,g}^{S}\left(\frac{Q^2}{m^2},\frac{m^2}{\mu^2}\right) = a_s\left[A_{Qg}^{(1)}\left(\frac{m^2}{\mu^2}\right) + \widehat{C}_{2,g}^{(1)}\left(\frac{Q^2}{\mu^2}\right)\right] + a_s^2\left[A_{Qg}^{(2)}\left(\frac{m^2}{\mu^2}\right) + A_{Qg}^{(1)}\left(\frac{m^2}{\mu^2}\right) \otimes C_{2,q}^{(1)}\left(\frac{Q^2}{\mu^2}\right) + \widehat{C}_{2,g}^{(2)}\left(\frac{Q^2}{\mu^2}\right)\right]$$

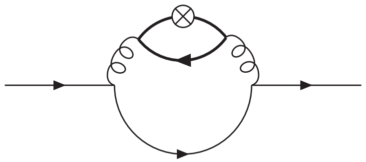
Examples:

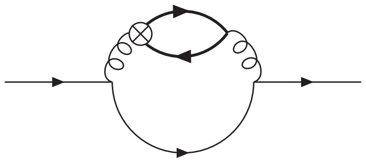
One–loop:



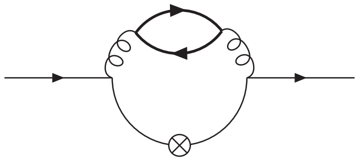


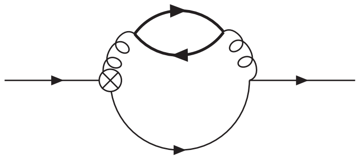
Pure–Singlet:

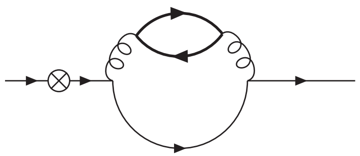




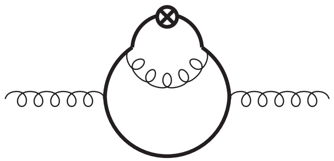
Non–Singlet:

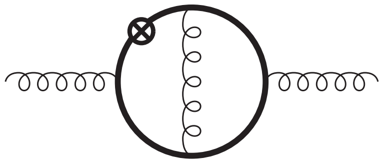


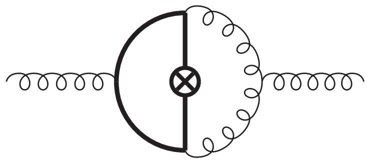




Singlet:







Unrenormalized massive operator matrix elements:

$$\hat{\hat{A}}_{ij} = \delta_{ij} + \sum_{k=0}^{\infty} \hat{a}_s^k \hat{\hat{A}}_{ij}^{(k)}$$

need for:

- mass renormalization: on–mass–shell scheme
- charge renormalization
- Renormalization of ultraviolet singularities
- Factorization of collinear singularities

→ use $\overline{\mathrm{MS}}$ scheme and decoupling formalism [Ovrut, Schnitzer 1981; Bernreuther, Wetzel 1982].

Since the light-cone expansion is used, external legs obtain self-energy insertions due to heavy quarks.

Operator renormalization:

ultraviolet divergences, renormalized by Z–factors.

Generic formula in terms of anomalous dimensions $\gamma_{ij,k}$. Has to be adapted e.g. for the various cases – three-loop non-singlet, pure-singlet, etc. $(i, j, m, n \in \{q, g\})$:

$$
\begin{aligned}
Z_{ij}(N, a_s, \varepsilon) \quad = \quad & \delta_{i,j} + a_s S_\varepsilon \frac{\gamma_{ij,0}}{\varepsilon} + a_s^2 S_\varepsilon^2 \left\{ \frac{1}{\varepsilon^2} \left[\frac{1}{2} \gamma_{im,0} \gamma_{mj,0} + \beta_0 \gamma_{ij,0} \right] + \frac{1}{2\varepsilon} \gamma_{ij,1} \right\} \\
& + a_s^3 S_\varepsilon^3 \left\{ \frac{1}{\varepsilon^3} \left[\frac{1}{6} \gamma_{in,0} \gamma_{nm,0} \gamma_{mj,0} + \beta_0 \gamma_{im,0} \gamma_{mj,0} + \frac{4}{3} \beta_0^2 \gamma_{ij,0} \right] \right. \\
& + \frac{1}{\varepsilon^2} \left[\frac{1}{6} \left(\gamma_{im,1} \gamma_{mj,0} + 2 \gamma_{im,0} \gamma_{mj,1} \right) + \frac{2}{3} \left(\beta_0 \gamma_{ij,1} + \beta_1 \gamma_{ij,0} \right) \right] \\
& \left. + \frac{\gamma_{ij,2}}{3\varepsilon} \right\}
\end{aligned}
$$

$$
Z_{qq}^{PS} = Z_{qq} - Z_{NS} .
$$

The anomalous dimensions $\gamma_{ij,k}(N)$ are related to the splitting functions by

$$
\gamma_{ij,k}(N) = - \int_0^1 dz z^{N-1} P_{ij}^{(k)}(z) \ .
$$

Mass factorization: Collinear singularities are factored into Γ_{NS}, $\Gamma_{ij,S}$ and $\Gamma_{qq,PS}$.

For massless quarks: $\Gamma_{\rm NS} = Z^{-1}_{\rm NS}$, $\Gamma_{ij,\rm S} = Z^{-1}_{ij,\rm S}$, $\Gamma_{qq,\rm PS} = Z^{-1}_{qq,\rm PS}$

$$\begin{aligned}
\Gamma_{\rm NS}(N,a_s,\varepsilon) &= 1 - a_s S_\varepsilon \frac{\gamma_{\rm NS,0}}{\varepsilon} + a_s^2 S_\varepsilon^2 \left[\frac{1}{\varepsilon^2}\left(\frac{1}{2}\gamma_{NS,0}^2 - \beta_0\gamma_{\rm NS,0}\right) - \frac{1}{2\varepsilon}\gamma_{\rm NS,1}\right] \\
\Gamma_{ij,\rm S}(N,a_s,\varepsilon) &= \delta_{ij} - a_s S_\varepsilon \frac{\gamma_{ij,0}}{\varepsilon} + a_s^2 S_\varepsilon^2 \left[\frac{1}{\varepsilon^2}\left(\frac{1}{2}\gamma_{ik,0}\gamma_{kj,0} - \beta_0\gamma_{ij,0}\right) - \frac{1}{2\varepsilon}\gamma_{ij,1}\right] \\
\Gamma_{qq,\rm PS}(N,a_s,\varepsilon) &= -a_s^2 S_\varepsilon^2 \left[\frac{1}{2\varepsilon^2}\gamma_{qg,0}\gamma_{gq,0} + \frac{1}{2\varepsilon}\gamma_{qq,\rm PS,1}\right] .
\end{aligned}$$

Here: in each diagram at least one quark line is massive

Γ–matrices apply to massless lines of the diagrams only $\leftrightarrow$ at most 2–loop sub-graphs

$\leftrightarrow$ mass factorization is different in various sub–classes of contributing Feynman diagrams

$\rightarrow$ singularities contained in $\Gamma_{\rm NS}, \Gamma_{ij,\rm S}$, and $\Gamma_{qq,\rm PS}$ are absorbed into the bare parton densities, which become scale–dependent

The renormalized operator matrix elements are obtained removing the ultraviolet singularities and collinear singularities of the operator matrix elements,

$$A_{ij} \quad = \quad Z^{-1}_{ik}\hat{A}_{kl}\Gamma^{-1}_{lj} = \delta_{ij} + a_s A^{(1)}_{ij} + a_s^2 A^{(2)}_{ij} + a_s^3 A^{(3)}_{ij} \; .$$

For example:

$$A_{Qg} \quad = \quad Z^{-1}_{qq}\hat{A}^{PS}_{Qq}\Gamma^{-1}_{qg} + Z^{-1}_{qq}\hat{A}_{Qg}\Gamma^{-1}_{gg} + Z^{-1}_{qg}\hat{A}_{gq,Q}\Gamma^{-1}_{qg} + Z^{-1}_{qg}\hat{A}_{gg,Q}\Gamma^{-1}_{gg} \; .$$

$\rightarrow$ mixing with $\hat{A}^{PS}_{Qq}$ and $\hat{A}_{gg,Q}$ at $O(a_s^3)$.

O(ϵ) terms at 2 Loops

For example: Renormalized gluonic massive operator matrix elements up to O(a_s^2):

Unrenormalized $\hat{A}_{Qg}$ to O(a_s^2):

$$\hat{A}_{Qg}^{(1)} = S_\varepsilon \left(\frac{m^2}{\mu^2}\right)^{\varepsilon/2} \left\{-\frac{1}{\varepsilon}\hat{P}_{qg}^{(0)} + a_{Qg}^{(1)} + \varepsilon\, \bar{a}_{Qg}^{(1)} + \varepsilon^2\, \bar{\bar{a}}_{Qg}^{(1)}\right\}$$

Renormalized A_{Qg} to O(a_s^2):

$$A_{Qg}^{(1)} = -\frac{1}{2}\widehat{P}_{qg}^{(0)}\ln\left(\frac{m^2}{\mu^2}\right)$$

$$A_{Qg}^{(2)} = \frac{1}{8}\left\{\widehat{P}_{qg}^{(0)} \otimes \left[P_{qq}^{(0)} - P_{gg}^{(0)} + 2\beta_0\right]\right\}\ln^2\left(\frac{m^2}{\mu^2}\right) - \frac{1}{2}\widehat{P}_{qg}^{(1)}\ln\left(\frac{m^2}{\mu^2}\right)$$
$$+\overline{a}_{Qg}^{(1)}\left[P_{qq}^{(0)} - P_{gg}^{(0)} + 2\beta_0\right] + a_{Qg}^{(2)} \quad ; \qquad a_{Qg}^{(1)} \equiv 0$$

$\Rightarrow$ For the renormalization of $A_{ij}^{(3)}$: $\overline{a}_{kl}^{(2)}$ are needed.

Unpolarized case: all $\overline{a}_{kl}^{(2)}$ are calculated; polarized case: partially

- $\rightsquigarrow$ Calculation in Mellin-space for space–like q^2, $Q^2 = -q^2$: $\quad 0 \le x \le 1$
- $\rightsquigarrow$ use of generalized hypergeometric functions for general analytic results
- $\rightsquigarrow$ use of Mellin-Barnes integrals for numerical checks (MB, [Czakon, 2006]) and some analytic results
- $\rightsquigarrow$ Summation of lots of new infinite one-parameter sums into harmonic sums. E.g.:

$$N \sum_{i,j=1}^{\infty} \frac{S_1(i) S_1(i+j+N)}{i(i+j)(j+N)} = 4S_{2,1,1} - 2S_{3,1} + S_1\left(-3S_{2,1} + \frac{4S_3}{3}\right) - \frac{S_4}{2} - S_2^2 + S_1^2 S_2 + \frac{S_1^4}{6} + 6S_1\zeta_3 + \zeta_2\left(2S_1^2 + S_2\right) .$$

use of integral techniques and the Mathematica package SIGMA [C. Schneider, 2007], [I.B., Blümlein, Klein, Schneider, 2007; 2008]

- $\rightsquigarrow$ Partial checks for fixed values of N using SUMMER, [Vermaseren, 1999].
- $\rightsquigarrow$ Algebraic and structural simplification of the harmonic sums [Blümlein, 2003, 2007].

Unpolarized case, Singlet, O(ε)

$$
\begin{aligned}
\overline{a}_{Qg}^{(2)} = {} & T_F C_F \Bigg\{ \frac{2}{3}\frac{(N^2+N+2)(3N^2+3N+2)}{N^2(N+1)^2(N+2)}\zeta_3 + \frac{P_1}{N^3(N+1)^3(N+2)} S_2 + \frac{N^4-5N^3-32N^2-18N-4}{N^2(N+1)^2(N+2)} S_1^2 \\
& + \frac{N^2+N+2}{N(N+1)(N+2)}\Big(16S_{2,1,1} - 8S_{3,1} - 8S_{2,1}S_1 + 3S_4 - \frac{4}{3} S_3S_1 - \frac{1}{2} S_2^2 - S_2S_1^2 - \frac{1}{6} S_1^4 + 2\zeta_2S_2 - 2\zeta_2S_1^2 - \frac{8}{3}\zeta_3S_1\Big) \\
& - 8\frac{N^2-3N-2}{N^2(N+1)(N+2)}S_{2,1} + \frac{2}{3}\frac{3N+2}{N^2(N+2)}S_1^3 + \frac{2}{3}\frac{3N^4+48N^3+43N^2-22N-8}{N^2(N+1)^2(N+2)}S_3 + 2\frac{3N+2}{N^2(N+2)}S_2S_1 + 4\frac{S_1}{N^2}\zeta_2 \\
& + \frac{N^5+N^4-8N^3-5N^2-3N-2}{N^3(N+1)^3}\zeta_2 - 2\frac{2N^5-2N^4-11N^3-19N^2-44N-12}{N^2(N+1)^3(N+2)}S_1 + \frac{P_2}{N^5(N+1)^5(N+2)}\Bigg\} \\
& + T_F C_A \Bigg\{\frac{N^2+N+2}{N(N+1)(N+2)}\Big(16S_{-2,1,1} - 4S_{2,1,1} - 8S_{-3,1} - 8S_{-2,2} - 4S_{3,1} - \frac{2}{3}\beta''' + 9S_4 - 16S_{-2,1}S_1 \\
& + \frac{40}{3}S_1S_3 + 4\beta''S_1 - 8\beta'S_2 + \frac{1}{2}S_2^2 - 8\beta'S_1^2 + 5S_1^2S_2 + \frac{1}{6}S_1^4 - \frac{10}{3}S_1\zeta_3 - 2S_2\zeta_2 - 2S_1^2\zeta_2 - 4\beta'\zeta_2 - \frac{17}{5}\zeta_2^2\Big) \\
& + \frac{4(N^2-N-4)}{(N+1)^2(N+2)^2}\Big(-4S_{-2,1} + \beta'' - 4\beta'S_1\Big) - \frac{2}{3}\frac{N^3+8N^2+11N+2}{N(N+1)^2(N+2)^2}S_1^3 + 8\frac{N^4+2N^3+7N^2+22N+20}{(N+1)^3(N+2)^3}\beta' \\
& + 2\frac{3N^3-12N^2-27N-2}{N(N+1)^2(N+2)^2}S_2S_1 - \frac{16}{3}\frac{N^5+10N^4+9N^3+3N^2+7N+6}{(N-1)N^2(N+1)^2(N+2)^2}S_3 - 8\frac{N^2+N-1}{(N+1)^2(N+2)^2}\zeta_2S_1 \\
& - \frac{2}{3}\frac{9N^5-10N^4-11N^3+68N^2+24N+16}{(N-1)N^2(N+1)^2(N+2)^2}\zeta_3 - \frac{P_3}{(N-1)N^3(N+1)^3(N+2)^3}S_2 - \frac{2P_4}{(N-1)N^3(N+1)^3(N+2)^2}\zeta_2 \\
& - \frac{P_5}{N(N+1)^3(N+2)^3}S_1^2 + \frac{2P_6}{N(N+1)^4(N+2)^4}S_1 - \frac{2P_7}{(N-1)N^5(N+1)^5(N+2)^5}\Bigg\}.
\end{aligned}
$$

Contributing OMEs:

Singlet	A_{Qg}	$A_{qg,Q}$	$A_{gg,Q}$	$A_{gq,Q}$	mixing
Pure–Singlet		$A_{Qq}^{\rm PS}$	$A_{qq,Q}^{\rm PS}$		mixing
Non–Singlet	$A_{qq,Q}^{\rm NS,+}$	$A_{qq,Q}^{\rm NS,-}$	$A_{qq,Q}^{\rm NS,v}$		

All 2–loop $O(\varepsilon)$–terms in the unpolarized case are known:

$$\overline{a}_{Qg}^{(2)},\quad \overline{a}_{Qq}^{(2),\mathbf{PS}},\quad \overline{a}_{gg,Q}^{(2)},\quad \overline{a}_{gq,Q}^{(2)},\quad \overline{a}_{qq,Q}^{(2),\mathbf{NS}}.$$

Unpolarized anomalous dimensions are known up to $O(a_s^3)$ [Moch, Vermaseren, Vogt, 2004.]

$\Longrightarrow$ All terms needed for the renormalization of unpolarized 3–Loop heavy OMEs are present.

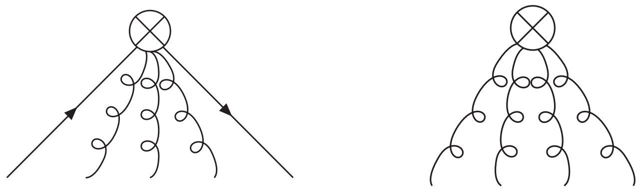
$\Longrightarrow$ Calculation will provide first independent checks on $\gamma_{qg}^{(3)}$, $\gamma_{qq}^{(3),\rm PS}$ and on respective color projections of $\gamma_{qq}^{(3),\rm NS\pm,v}$, $\gamma_{gg}^{(3)}$ and $\gamma_{gq}^{(3)}$.

Calculation proceeds in the same way in the polarized case. Known so far :

$$\Delta\overline{a}_{Qg}^{(2)},\quad \Delta\overline{a}_{Qq}^{(2),\mathbf{PS}},\quad \Delta\overline{a}_{qq,Q}^{(2),\mathbf{NS}} = \overline{a}_{qq,Q}^{(2),\mathbf{NS}}\ .$$

3–loop OMEs are generated with QGRAF [Nogueira 1993]

New operator insertions emerge:



For example: terms with 2 quark loops (at least one heavy): # of diagrams for $A_{Qg}^{(3)}$

- 489 diagrams with two quark loops
- 1478 diagrams with one quark loop

$\Longrightarrow$ number of diagrams can be reduced by using symmetry arguments.

First step: Calculation of fixed moments of $A_{ij}^{(3)}(N)$, $N = 2, 4, 6, ...$

- three–loop "self-energy" type diagrams with an operator insertion
- Extension: additional scale compared to massive propagators: Mellin variable N
- Genuine tensor integrals due to

$$\Delta^{\mu_1}...\Delta^{\mu_n} \langle p|O_{\mu_1...\mu_n}|p\rangle = \Delta^{\mu_1}...\Delta^{\mu_n} \langle p|S\,\bar{\Psi}\gamma_{\mu_1}D_{\mu_2}...D_{\mu_n}\Psi|p\rangle = A(N)\cdot(\Delta p)^N$$

$$D_\mu = \partial_\mu - igt_a A_\mu^a \quad , \qquad \Delta^2 = 0.$$

- Construction of a projector to obtain the desired moment in N [undo Δ-contraction] $\surd$
- Color factors are calculated with [Ritbergen, Schellekens, Vermaseren 1998]
- Translation to suitable input for MATAD [Steinhauser, 2001] $\surd$

$\Rightarrow$ automated chain to evaluate the OMEs $\surd$

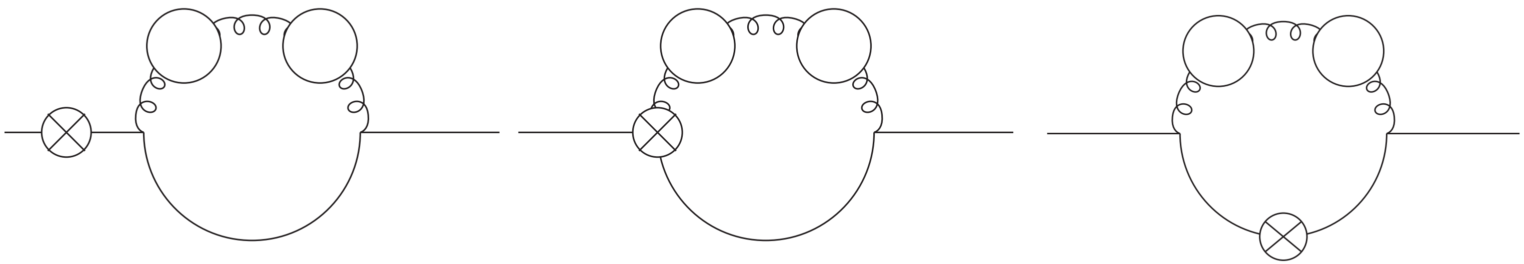
Tests performed:

Various 2–loop calculations for $N = 2, 4, 6, ...$ were repeated

$\rightarrow$ agreement with our previous calculation;

check against an analytic result of $A_{gg,Q}^{(2)}$ ($\rightarrow$ see talk of S. Klein) currently investigated: terms $\propto$ $T_F^2 C_F$, $T_F^2 C_A$–terms

First example: Non-singlet terms : $O(T_F^2 C_F)$ $A_{qq,Q}^{(3),\mathrm{NS}}(N)$:



with one or two heavy quark loops

First results: 2 massive quarks

$$\hat{\hat{A}}_{qq,QQ}^{(3),\mathsf{NS}} = \left(\frac{m^2}{\mu^2}\right)^{3\varepsilon/2} \left(-\frac{8\beta_{0,Q}^2 \gamma_{qq}^{(0),\mathsf{NS}}}{3\varepsilon^3} - \frac{4\beta_{0,Q}\gamma_{qq,Q}^{(1),\mathsf{NS}}}{3\varepsilon^2} + \frac{\gamma_{qq,QQ}^{(2),\mathsf{NS}} - 12\beta_{0,Q} a_{qq,Q}^{(2),\mathsf{NS}}}{3\varepsilon} + a_{qq,QQ}^{(3),\mathsf{NS}} \right) .$$

$$A_{qq,QQ}^{(3),\mathsf{NS}} = \frac{1}{6}\ln^3\left(\frac{m^2}{\mu^2}\right)\beta_{0,Q}^2\gamma_{qq}^{(0),\mathsf{NS}} + \frac{1}{2}\ln^2\left(\frac{m^2}{\mu^2}\right)\beta_{0,Q}\gamma_{qq,Q}^{(1),\mathsf{NS}} + \frac{1}{2}\ln\left(\frac{m^2}{\mu^2}\right)\gamma_{qq,QQ}^{(2),\mathsf{NS}} + a_{qq,QQ}^{(3),\mathsf{NS}} + 4\beta_{0,Q}\overline{a}_{qq,Q}^{(2),\mathsf{NS}}$$

$$\gamma_{qq}^{(0),\mathsf{NS}} = 4C_F\left[2S_1 - \frac{3N^2+3N+2}{2N(N+1)}\right]$$

$$\gamma_{qq,Q}^{(1),\mathsf{NS}} = 4C_FT_F\left\{\frac{8}{3}S_2 - \frac{40}{9}S_1 + \frac{3N^4+6N^3+47N^2+20N-12}{9N^2(N+1)^2}\right\}$$

$$\gamma_{qq,QQ}^{(2),\mathsf{NS}} = C_FT_F^2\left(\frac{128}{9}S_3 - \frac{640}{27}S_2 - \frac{128}{27}S_1 + 8\frac{51N^6+153N^5+57N^4+35N^3+96N^2+16N-24}{27N^3(N+1)^3}\right)$$

$$\hat{\hat{A}}_{qq,QQ}^{(3),\mathsf{NS}}(2)$$
$$= \frac{1}{\varepsilon^3}\left(-\frac{2048}{81}\right) + \frac{1}{\varepsilon^2}\left(-\frac{4096}{243}\right) + \frac{1}{\varepsilon}\left(-\frac{20224}{729} - \frac{256}{27}\zeta_2\right) - \frac{28736}{2187} - \frac{2048}{81}\zeta_3 - \frac{512}{81}\zeta_2 \ ,$$

$$\hat{\hat{A}}_{qq,QQ}^{(3),\mathsf{NS}}(4)$$
$$= \frac{1}{\varepsilon^3}\left(-\frac{20096}{405}\right) + \frac{1}{\varepsilon^2}\left(-\frac{212336}{6075}\right) + \frac{1}{\varepsilon}\left(-\frac{5071846}{91125} - \frac{2512}{135}\zeta_2\right) - \frac{151928299}{5467500} - \frac{20096}{405}\zeta_3 - \frac{26542}{2025}\zeta_2 \ ,$$

$$\hat{\hat{A}}_{qq,QQ}^{(3),\mathsf{NS}}(6)$$
$$= \frac{1}{\varepsilon^3}\left(-\frac{181504}{2835}\right) + \frac{1}{\varepsilon^2}\left(-\frac{13699808}{297675}\right) + \frac{1}{\varepsilon}\left(-\frac{2263340116}{31255875} - \frac{22688}{945}\zeta_2\right)$$
$$- \frac{26884517771}{729303750} - \frac{181504}{2835}\zeta_3 - \frac{1712476}{99225}\zeta_2 \ ,$$

$$\hat{\hat{A}}_{qq,QQ}^{(3),\mathsf{NS}}(8)$$
$$= \frac{1}{\varepsilon^3}\left(-\frac{632512}{8505}\right) + \frac{1}{\varepsilon^2}\left(-\frac{144967772}{2679075}\right) + \frac{1}{\varepsilon}\left(-\frac{285344205403}{3375634500} - \frac{79064}{2835}\zeta_2\right)$$
$$- \frac{740566685766263}{17013197880000} - \frac{632512}{8505}\zeta_3 - \frac{36241943}{1786050}\zeta_2 \ .$$

$$A_{qq,Q Q}^{(3),\mathsf{NS}}(2)$$
$$= T_F^2 C_F \Big[\frac{128}{81} \ln^3\Big(\frac{m^2}{\mu^2}\Big) + \frac{512}{81} \ln^2\Big(\frac{m^2}{\mu^2}\Big) - \frac{896}{243} \ln\Big(\frac{m^2}{\mu^2}\Big) + \frac{25024}{2187} - \frac{1792}{81}\zeta_3 \Big] \ ,$$

$$A_{qq,Q Q}^{(3),\mathsf{NS}}(4)$$
$$= T_F^2 C_F \Big[\frac{1256}{405} \ln^3\Big(\frac{m^2}{\mu^2}\Big) + \frac{26542}{2025} \ln^2\Big(\frac{m^2}{\mu^2}\Big) - \frac{384277}{60750} \ln\Big(\frac{m^2}{\mu^2}\Big) + \frac{243265699}{10935000} - \frac{17584}{405}\zeta_3 \Big] \ ,$$

$$A_{qq,Q Q}^{(3),\mathsf{NS}}(6)$$
$$= T_F^2 C_F \Big[\frac{11344}{2835} \ln^3\Big(\frac{m^2}{\mu^2}\Big) + \frac{1712476}{99225} \ln^2\Big(\frac{m^2}{\mu^2}\Big) - \frac{80347571}{10418625} \ln\Big(\frac{m^2}{\mu^2}\Big) + \frac{125199267113}{4375822500} - \frac{22688}{405}\zeta_3 \Big] \ ,$$

$$A_{qq,Q Q}^{(3),\mathsf{NS}}(8)$$
$$= T_F^2 C_F \Big[\frac{39532}{8505} \ln^3\Big(\frac{m^2}{\mu^2}\Big) + \frac{36241943}{1786050} \ln^2\Big(\frac{m^2}{\mu^2}\Big) - \frac{38920977797}{4500846000} \ln\Big(\frac{m^2}{\mu^2}\Big) + \frac{1128638049575063}{34026395760000} - \frac{79064}{1215}\zeta_3 \Big] \ .$$

All ζ_2 terms vanish after renormalization.

1 heavy - 1 massless quark

$$A_{qq,qQ}^{(3),\mathsf{NS}}(2)$$
$$= n_f T_F^2 C_F \Big[-\frac{128}{81} \ln^3\Big(\frac{m^2}{\mu^2}\Big) - \frac{4864}{243} \ln\Big(\frac{m^2}{\mu^2}\Big) - \frac{46336}{2187} + \frac{1024}{81}\zeta_3 \Big] ,$$

$$A_{qq,qQ}^{(3),\mathsf{NS}}(4)$$
$$= n_f T_F^2 C_F \Big[-\frac{1256}{405} \ln^3\Big(\frac{m^2}{\mu^2}\Big) - \frac{2331077}{60750} \ln\Big(\frac{m^2}{\mu^2}\Big) - \frac{229618301}{5467500} + \frac{10048}{405}\zeta_3 \Big] ,$$

$$A_{qq,qQ}^{(3),\mathsf{NS}}(6)$$
$$= n_f T_F^2 C_F \Big[-\frac{11344}{2835} \ln^3\Big(\frac{m^2}{\mu^2}\Big) - \frac{170378657}{3472875} \ln\Big(\frac{m^2}{\mu^2}\Big) - \frac{39653493629}{729303750} + \frac{90752}{2835}\zeta_3 \Big] ,$$

$$A_{qq,qQ}^{(3),\mathsf{NS}}(8)$$
$$= n_f T_F^2 C_F \Big[-\frac{39532}{8505} \ln^3\Big(\frac{m^2}{\mu^2}\Big) - \frac{255097766597}{4500846000} \ln\Big(\frac{m^2}{\mu^2}\Big) - \frac{1076783602872937}{17013197880000} + \frac{316256}{8505}\zeta_3 \Big] .$$

Conclusion

- The heavy flavour contributions to $F_2(x, Q^2)$ are rather large in the small-x region.
- To $O(a_s^3)$, heavy–flavour corrections require to perform a 3–loop QCD analysis.
- We newly calculated first contributions to these corrections given by the $O(\varepsilon)$ terms of the two–loop OMEs in the unpolarized and polarized case for general values of the Mellin variable. These terms contribute to $H_{ij}^{(3)}$, $\Delta H_{ij}^{(3)}$ respectively, through renormalization.
- The calculation
  - The calculation is performed in Mellin space $\rightarrow$ simplification
  - We used Mellin–Barnes integrals for numerical and generalized hypergeometric functions for analytic results.
  - Integral techniques and the program package SIGMA have been used for summation. The results are given in the form of nested harmonic sums.
- We are establishing a program chain which allows to calculate $a_{ij}^{(3)}$ for fixed Mellin moments. $\rightarrow$ first 3–loop result given by $A_{qq,Q}^{(3),\mathsf{NS}}$.