Event shapes at NLLA+NNLO

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27.05.2008

HERA and the LHC

Outline

- Matching of NLLA and NNLO calculations.
- Determination of α_S with the new NNLO and NLLA+NNLO results.

Event shape observables

- Popular observables for testing QCD $\leftrightarrow$ IR & collinear safe,
- parametrize geometrical properties of energy-momentum flow of an event,
- canonical example: Thrust, $T = \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_i |\vec{p}_i|}$

$T = 1$ $\qquad T = \frac{2}{3}$ $\qquad T = \frac{1}{2}$

- phenomenologically very important:
  - understand QCD dynamics, determination of the QCD coupling constant,
- measured very precisely at LEP:
  - error in the determination of α_S mainly from theoretical uncertainty

$$\alpha_s\,(M_Z) \;=\; 0.1202 \pm 0.0003(\text{stat}) \pm 0.0009(\text{sys}) \pm 0.0013(\text{had}) \pm 0.0047(\text{scale})$$ [LEPQCDWG]

Event shape observables

Theoretical calculations:

- State-of-the-art one year ago:
  - NLO calculations, [Ellis, Ross, Terrano; Kunszt, Nason; Giele, Glover; Catani, Seymour.]
  - NLL resummation, [Catani, Trentadue, Turnock, Webber; Banfi, Salam, Zanderighi.]

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  - NLL resummation, [Catani, Trentadue, Turnock, Webber; Banfi, Salam, Zanderighi.]
- Very important progress in the last year
  - NNLO calculations, [Gehrmann, Gehrmann-De-Ridder, Glover, Heinrich]
  - Matching of NLLA + NNLO of the LEP standard set of event shape observables. [Gehrmann, G.L., Stenzel]
  - Beyond NLL resummation in SCET and matching with NNLO for Thrust, [Schwartz; Becher, Schwartz]

Fixed Order Calculations

- For an observable y the differential cross section is typically given by $\left(\bar{\alpha}_s = \frac{\alpha_s}{2\pi}\ , x_\mu = \frac{\mu}{Q}\right)$:

$$\frac{1}{\sigma_{\text{had}}}\frac{\mathrm{d}\sigma}{\mathrm{d}y}(y,Q,\mu) = \underbrace{\bar{\alpha}_s(\mu)\frac{\mathrm{d}\bar{A}}{\mathrm{d}y}(y)}_{LO} + \underbrace{\bar{\alpha}_s^2(\mu)\frac{\mathrm{d}\bar{B}}{\mathrm{d}y}(y,x_\mu)}_{NLO} + \underbrace{\bar{\alpha}_s^3(\mu)\frac{\mathrm{d}\bar{C}}{\mathrm{d}y}(y,x_\mu)}_{NNLO} + \mathcal{O}(\bar{\alpha}_s^4)\,.$$

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- NNLO calculations: [Gehrmann, Gehrmann-De-Ridder, Glover, Heinrich]
  - careful subtraction of real and virtual divergencies,
  - subtraction obtained by antenna method,
  - implemented in the EERAD3 integration programme.

LO	$\gamma^* \longrightarrow q\bar{q}g$	tree level
NLO	$\gamma^* \longrightarrow q\bar{q}g$	one loop
	$\gamma^* \longrightarrow q\bar{q}gg$	tree level
	$\gamma^* \longrightarrow q\bar{q}q\bar{q}$	tree level
NNLO	$\gamma^* \longrightarrow q\bar{q}g$	two loop
	$\gamma^* \longrightarrow q\bar{q}gg$	one loop
	$\gamma^* \longrightarrow q\bar{q}q\bar{q}$	one loop
	$\gamma^* \longrightarrow q\bar{q}q\bar{q}g$	tree level
	$\gamma^* \longrightarrow q\bar{q}ggg$	tree level

- Coefficient functions $\frac{d\bar{A}}{dy}$, $\frac{d\bar{B}}{dy}$, $\frac{d\bar{C}}{dy}$ are functions of $L \equiv \ln\frac{1}{y}$,
  - describes the enhancement due to soft and collinear emissions.

Fixed Order Calculations

- Consider cumulative cross section $R(y,Q,\mu) \equiv \frac{1}{\sigma_{\rm had}} \int_0^y \frac{d\sigma(x,Q,\mu)}{dx} dx,$

$$R(y,Q,\mu) = 1 + \mathcal{A}(y)\,\bar{\alpha}_s(\mu) + \mathcal{B}(y,x_\mu)\,\bar{\alpha}_s^2(\mu) + \mathcal{C}(y,x_\mu)\,\bar{\alpha}_s^3(\mu).$$

$\bar{\alpha}_s\mathcal{A}(y)$	$\alpha_s L$	$\alpha_s L^2$				
$\bar{\alpha}_s^2\mathcal{B}(y,x_\mu)$	$\alpha_s^2 L$	$\alpha_s^2 L^2$	$\alpha_s^2 L^3$	$\alpha_s^2 L^4$		
$\bar{\alpha}_s^3\mathcal{C}(y,x_\mu)$	$\alpha_s^3 L$	$\alpha_s^3 L^2$	$\alpha_s^3 L^3$	$\alpha_s^3 L^4$	$\alpha_s^3 L^5$	$\alpha_s^3 L^6$

Contribution becomes smaller ↓

- If L is NOT large, contributions become smaller line-by-line.
- In phase space region where $y \to 0$, $L \to \infty$:

$\longrightarrow$ coefficient functions becomes large spoiling the convergence of the series expansion.

- Main contribution comes from highest power of the logarithms.

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$\bar\alpha_s^3\mathcal{C}(y,x_\mu)$	$\alpha_s^3 L$	$\alpha_s^3 L^2$	[illegible] L^3	$\alpha_s^3 L^4$	$\alpha_s^3 L^5$	$\alpha_s^3 L^6$

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Need RESUMMATION!

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Resummed Calculations

- Idea: resum the highest powers of the logarithms to all orders in perturbation theory

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- Leading logarithms
- Next-to-Leading logarithms
- From trivial exponentiation

Resummed Calculations

- For suitable observables, resummation of logarithms leads to exponentiation

$$\Sigma(y) = e^{L\, g_1(\alpha_s L) + g_2(\alpha_s L) + \ldots}$$

with

$$L\, g_1(\alpha_s L) = G_{12} L^2 \alpha_s + G_{23} L^2 \alpha_s^2 + G_{34} L^4 \alpha_s^3 + \ldots \quad \text{(LL)}$$

$$g_2(\alpha_s L) = G_{11} L \alpha_s + G_{22} L^2 \alpha_s^2 + G_{33} L^3 \alpha_s^3 + \ldots \quad \text{(NLL)}$$

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- Integrated cross section at NLLA to be matched with NNLO:

$$R(y) = \left(C_1 \alpha_s + C_2 \alpha_s^2 + C_3 \alpha_s^3\right) \times e^{L\, g_1(\alpha_s L) + g_2(\alpha_s L) + \bar{\alpha}_s^2 G_{21} L + \bar{\alpha}_s^3 G_{32} L^2 + \bar{\alpha}_s^3 G_{31} L} + D(y)$$

$$= \underbrace{C(\alpha_s)\, \Sigma(y)}_{\text{logarithmic part}} + \underbrace{D(y)}_{\text{remainder function: } \to 0 \text{ as } y \to 0}$$

C_1, C_2, C_3, G_{21}, G_{32}, G_{31}, $D(y)$: to be determined by matching with fixed order.

Matching

- Different matching schemes
  - R-matching scheme:
    - Two predictions for $R(y)$ are matched and double-counting terms are subtracted.
    - Unknown matching coefficients $C_1, C_2, C_3, G_{21}, G_{32}, G_{31}$ numerically determined from fixed order result.

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  - Log(R)-matching scheme:
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Log-R matching scheme

- To NLLA + NNLO the integrated cross section in the Log-R matching scheme is given by

$$
\begin{aligned}
\ln\left(R\left(y,\alpha_S\right)\right) &= L\, g_1\left(\alpha_s L\right) + g_2\left(\alpha_s L\right) \\
&+ \bar{\alpha}_S\left(\mathcal{A}\left(y\right) - G_{11}L - G_{12}L^2\right) + \\
&+ \bar{\alpha}_S^2\left(\mathcal{B}\left(y\right) - \frac{1}{2}\mathcal{A}^2\left(y\right) - G_{22}L^2 - G_{23}L^3\right) \\
&+ \bar{\alpha}_S^3\left(\mathcal{C}\left(y\right) - \mathcal{A}\left(y\right)\mathcal{B}\left(y\right) + \frac{1}{3}\mathcal{A}^3\left(y\right) - G_{33}L^3 - G_{34}L^4\right) .
\end{aligned}
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- fixed order
- resummation

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  - fixed order
  - resummation

- To ensure the vanishing of the matched expression at the kinematical boundary $y_{\max}$

$$L \longrightarrow \tilde{L} = \frac{1}{p}\ln\left(\left(\frac{y_0}{y\,x_L}\right)^p - \left(\frac{y_0}{y_{\max}\,x_L}\right)^p + 1\right),$$

with $y_0 = 6$ for $y = C$ and $y_0 = 1$ otherwise, ($x_L = p = 1$).

[Ford, Jones, Salam, Stenzel, Wicke.]

Renormalization scale dependence

- The full renormalization scale dependence is given by making the following replacements,

$$\alpha_s \quad \longrightarrow \quad \alpha_s(\mu)\ ,$$

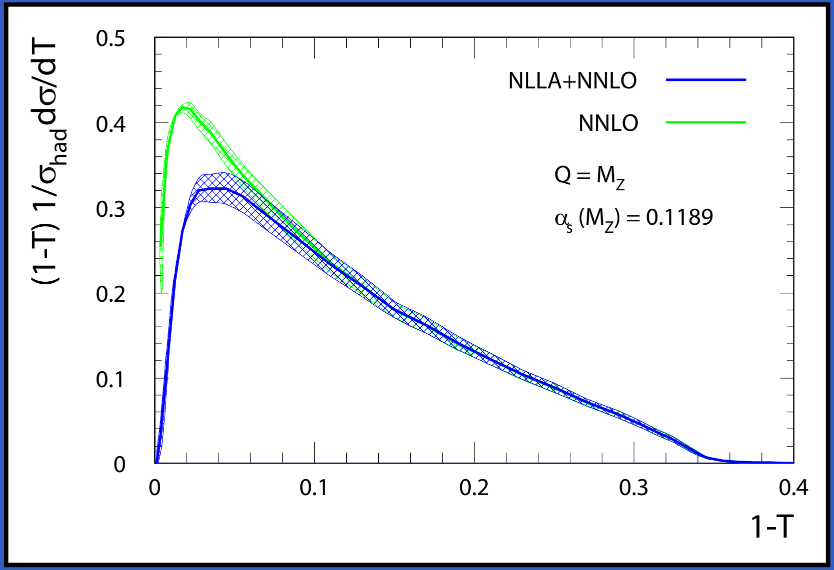
$$\begin{aligned} \mathcal{B}\,(y) \quad &\longrightarrow \quad \mathcal{B}\,(y,\mu) = 2\,\beta_0\,\ln x_\mu\,\mathcal{A}\,(y) + \mathcal{B}\,(y)\ , \\ \mathcal{C}\,(y) \quad &\longrightarrow \quad \mathcal{C}\,(y,\mu) = (2\,\beta_0\,\ln x_\mu)^2\,\mathcal{A}\,(y) + 2\,\ln x_\mu\,\left[2\,\beta_0\mathcal{B}\,(y) + 2\,\beta_1\,\mathcal{A}\,(y)\right] + \mathcal{C}\,(y)\ , \end{aligned}$$

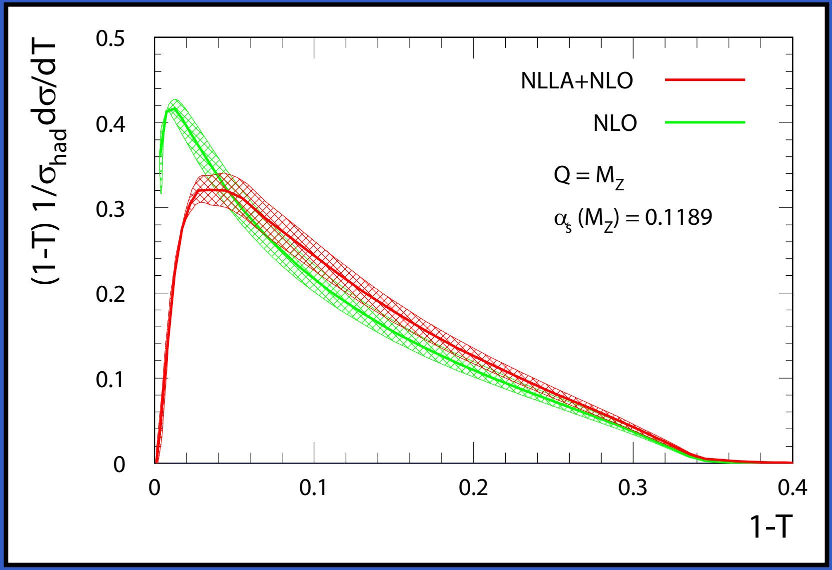
$$g_2\,(\alpha_S L) \quad \longrightarrow \quad g_2\left(\alpha_S L, \mu^2\right) = g_2\,(\alpha_S L) + \frac{\beta_0}{\pi}\,(\alpha_S L)^2\;g_1'\,(\alpha_S L)\,\ln x_\mu\ ,$$

$$\begin{aligned} G_{22} \quad &\longrightarrow \quad G_{22}\,(\mu) = G_{22} + 2\beta_0 G_{12}\ln x_\mu\ , \\ G_{33} \quad &\longrightarrow \quad G_{33}\,(\mu) = G_{33} + 4\beta_0 G_{23}\ln x_\mu\ . \end{aligned}$$

Results: renormalization scale dependence

- Thrust T: consider $\tau = 1 - T$





- difference between NLLA+NNLO and NNLO restricted to the two-jet region, whereas NLLA+NLO differ in normalisation throughout the full kinematical range.

Results: renormalization scale dependence

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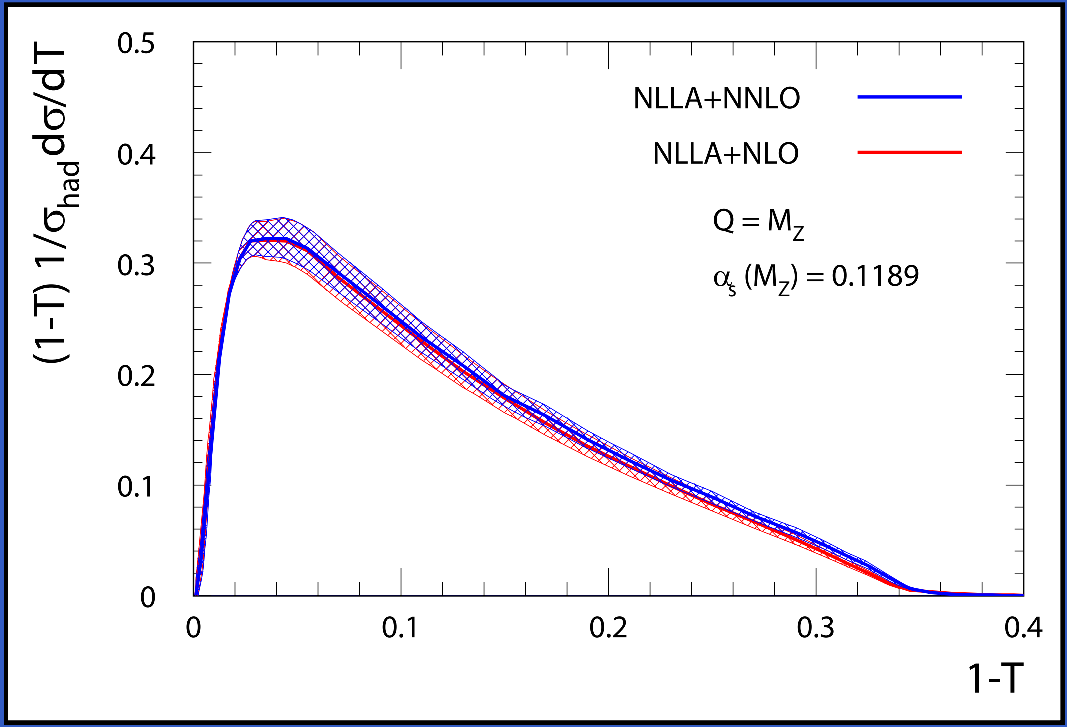

- difference between NLLA+NNLO and NLLA+NLO moderate in the three-jet region.
- renormalization scale dependence reduced in three-jet region.

Results: renormalization scale dependence

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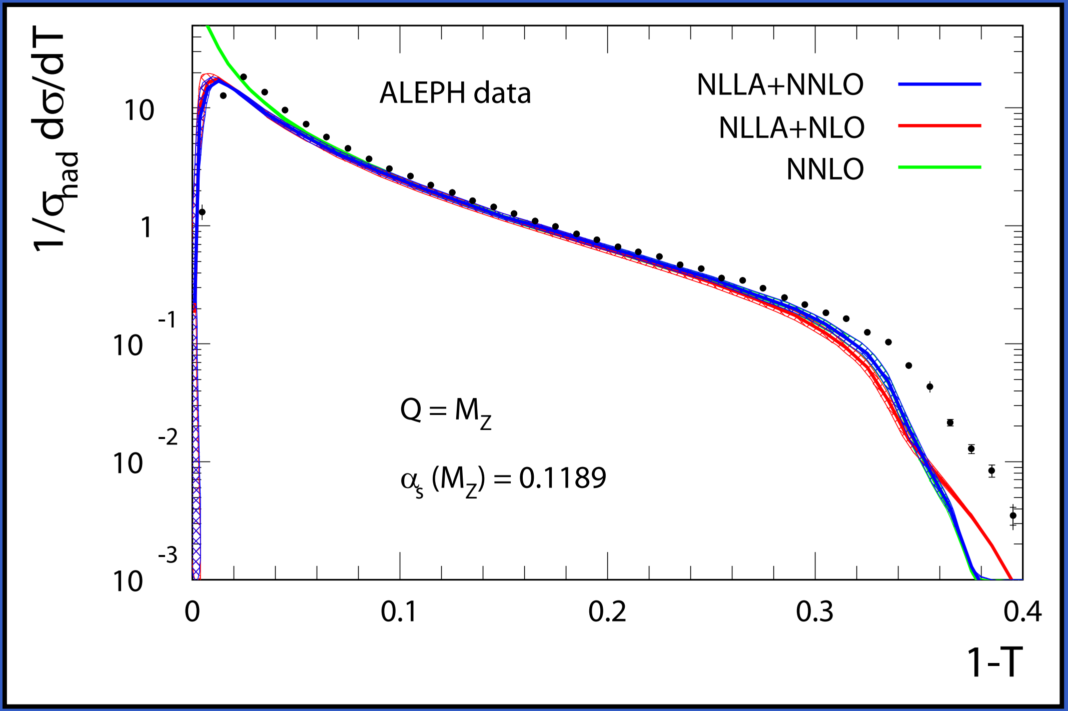

- description of the hadron-level data improves between parton-level NLLA+NLO and parton-level NLLA+NNLO, especially in the three-jet region.

Results

- Full set of event shape variables:
  - Heavy jet mass: $\rho = \frac{M_H^2}{s} = \max_i \frac{1}{E_{\text{vis}}} \left(\sum_{k \in H_i} p_k \right)^2$
  - C-parameter: $\Theta^{\alpha\beta} = \frac{1}{\sum_k |\vec{p}_k|} \frac{\sum_k p_k^\alpha p_k^\beta}{\sum_k |\vec{p}_k|}$,

$$C = 3 \left(\Theta^{11}\Theta^{22} + \Theta^{22}\Theta^{33} + \Theta^{33}\Theta^{22} - \Theta^{12}\Theta^{12} - \Theta^{23}\Theta^{23} - \Theta^{31}\Theta^{31} \right)$$

  - Total jet broadening: $B_T = B_1 + B_2$
  - Wide jet broadening: $B_W = \max(B_1, B_2)$,

$$B_i = \frac{\sum_{k \in H_i} |\vec{p}_k \times \vec{n}_T|}{2 \sum_k |\vec{p}_k|}$$

  - Two-to-three jet parameter for Durham jet algorithm:

$$y_{ij,D} = \frac{2 \min\left(E_i^2, E_j^2\right)(1 - \cos\theta_{ij})}{E_{\text{vis}}^2}$$

Determination of α_S

- Two separated determinations:
  - using only theoretical NNLO predictions,

    [Dissertori, Gehrmann-De Ridder, Gehrmann, Glover, Heinrich, Stenzel.]

  - using the the matched NLLA+NNLO predictions. [work in progress]

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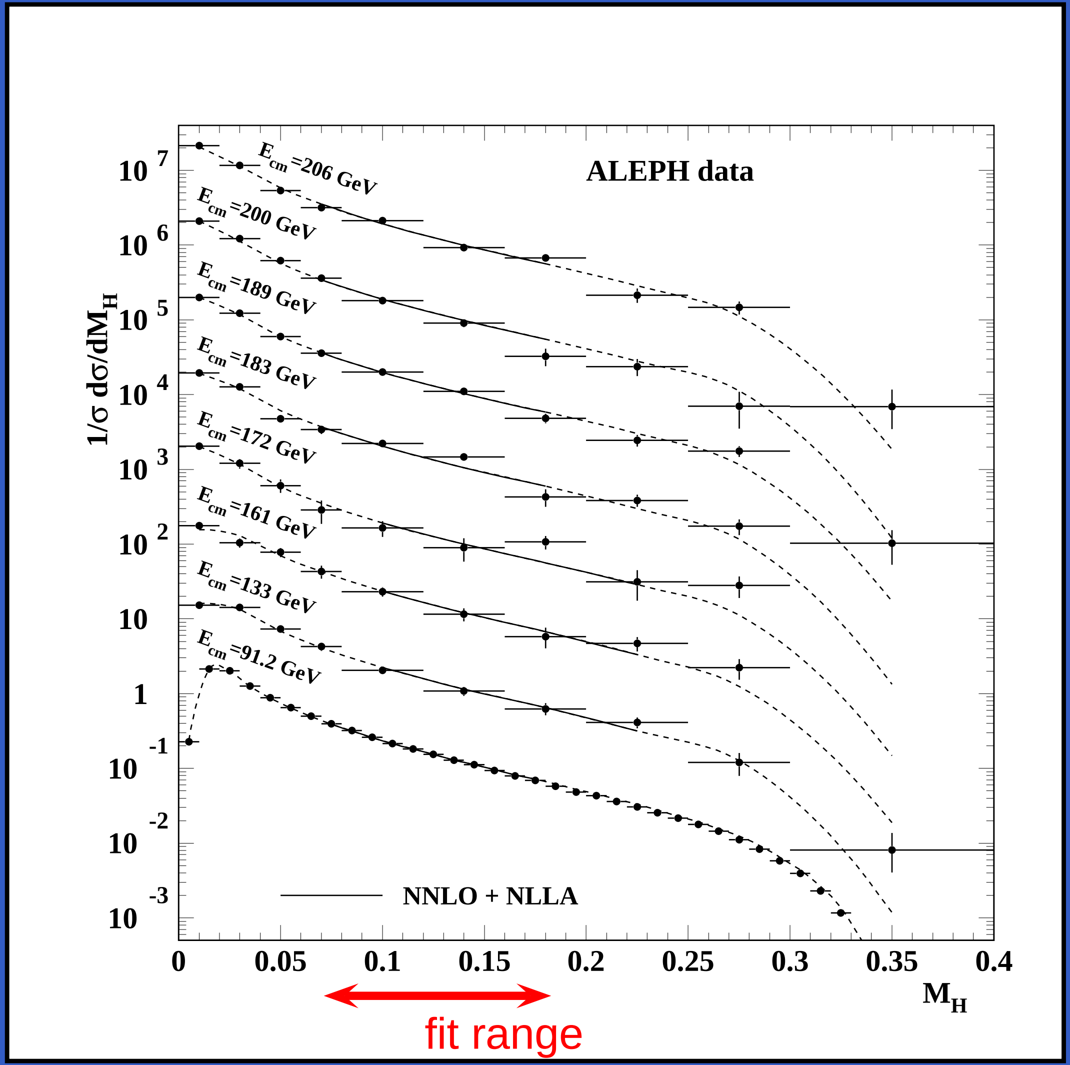
- Analysis outline
  - use public ALEPH data on event shapes, [Heister et al.]
  - data are corrected to hadron level using MC corrections accounting for ISR/FSR and background,
  - data are fit by NNLO or NLLA+NNLO predictions, including NLO quark mass correction, folded to hadron level by MC generators,
  - combine 6 variables and 8 data sets (LEPI + LEPII)

Determination of α_S: NLLA+NNLO fits to data

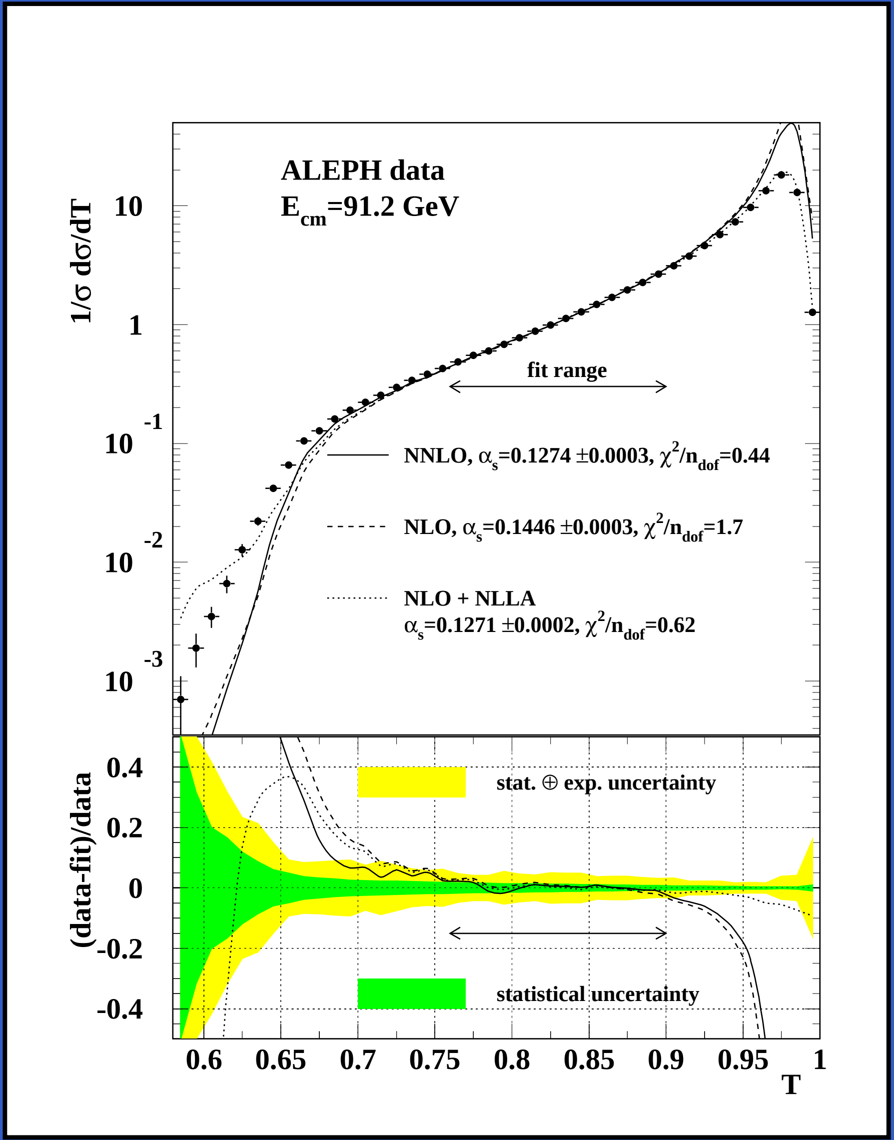
PRELIMINARY

- data are fit in the central part of the event shape distribution,
- only statistical uncertainties are included in the χ^2.



Determination of α_S: NLLA+NNLO fits to data

- NNLO vs NLLA+NLO
  - clear improvement of NNLO over NLO,
  - good fit quality (but includes still large statistical uncertainties of C-coefficient),
  - matched NLLA+NLO still yields a better prediction in the 2-jet region,



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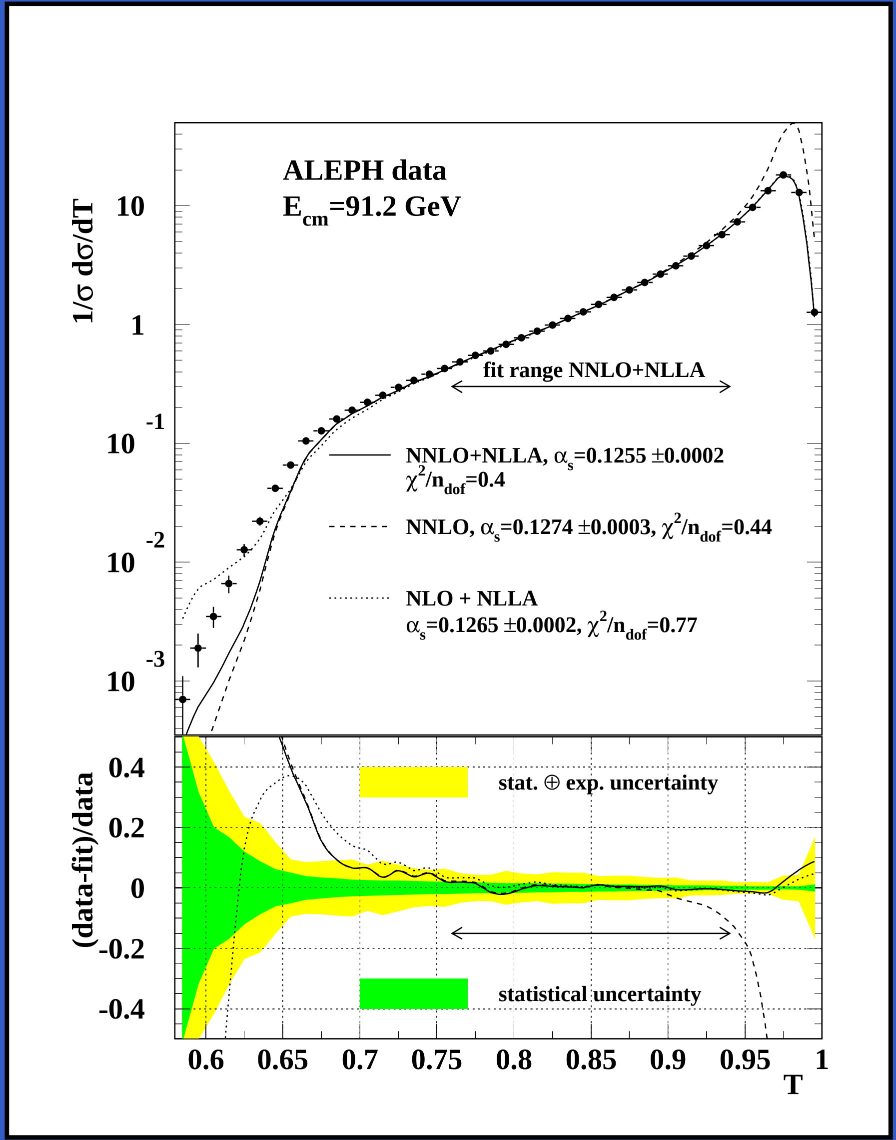
- NLLA+NNLO vs NNLO
  - better predictions in two jet region,
  - extended fit range,
- fit to fixed order calculations gives higher values for α_S,
- tendency to decrease from NLO to NNLO.

PRELIMINARY



Determination of α_S: perturbative uncertainty

- Uncertainty band method to estimate missing higher orders

  [Ford, Jones, Salam, Stenzel, Wicke.]

  - evaluate distribution of event shape y for a given value of α_S with a reference theory,
  - calculate theoretical uncertainties for $y \longrightarrow$ uncertainty band,
  - fill the uncertainty band with the nominal prediction by varying α_s,
  - corresponding variation range for α_S is assigned as systematic uncertainty.

- Parameter taken into account

  - for NNLO fit: only x_μ variation,
  - for NLLA+NNLO fit: variation of x_μ, x_L, y_{max}, p and matching scheme.

Determi... uncertainty

- Uncertaint... rders

  [Ford, Jones, Salam, ...

  - evaluate ... th a reference theory,
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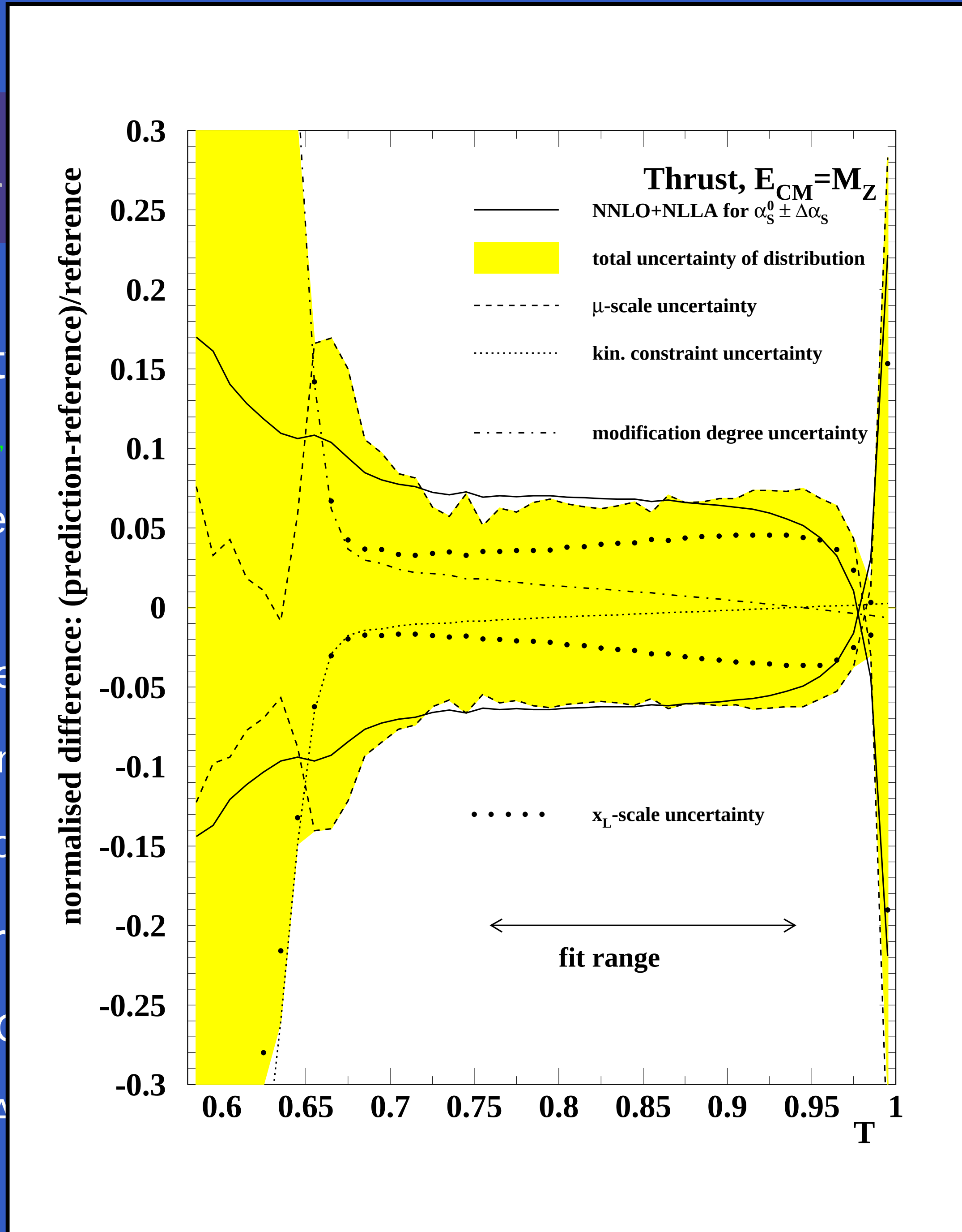
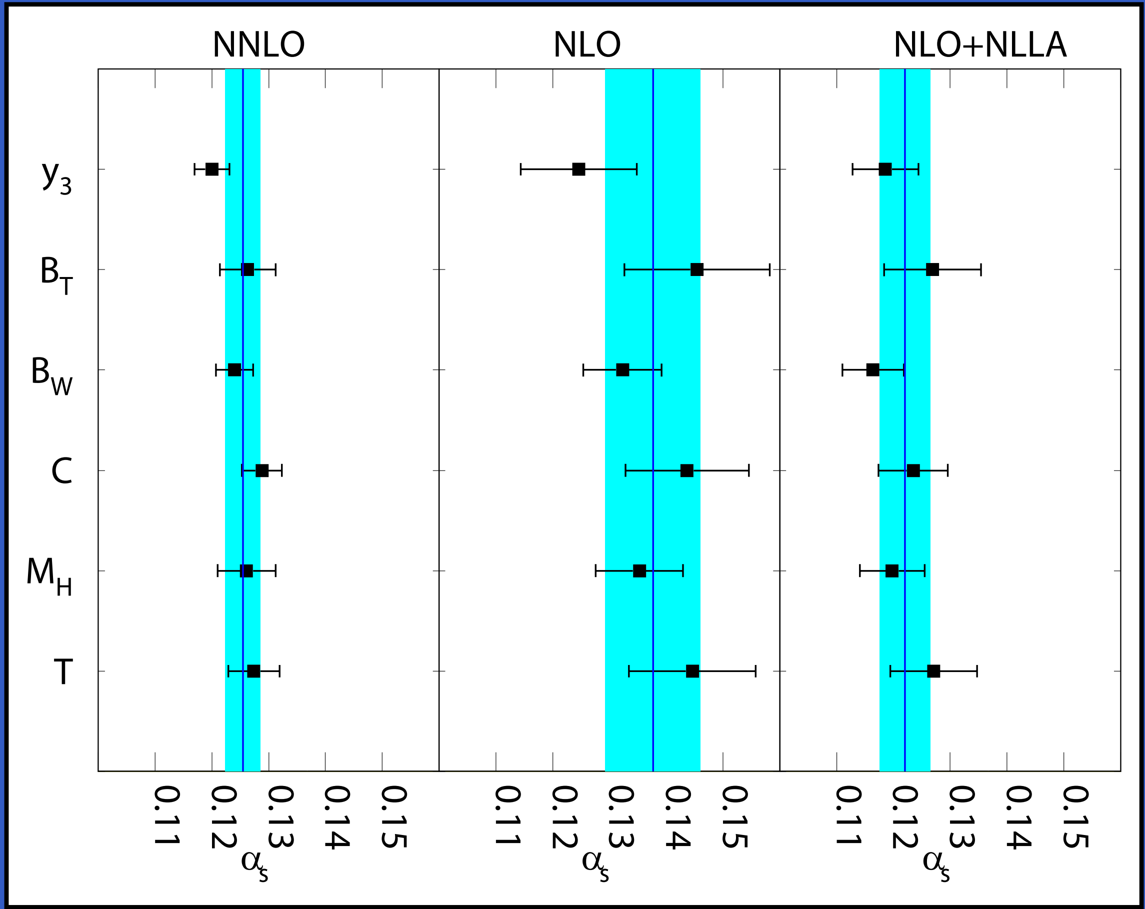
  - correspo ... uncertainty.
- Parameter...
  - for NNLO...
  - for NLLA ... heme.



Determination of α_S: NNLO results

- $\alpha_S\left(M_Z\right)$
- consistent results at NNLO,
- scattering between variables much reduced.
- calculate weighted average for $\alpha_S\left(Q\right)$ from 6 variables

$$\bar{\alpha}_S = \sum_{i=1}^{6} w_i\, \alpha_S^i, \quad w_i \propto \frac{1}{\sigma_i^2}$$

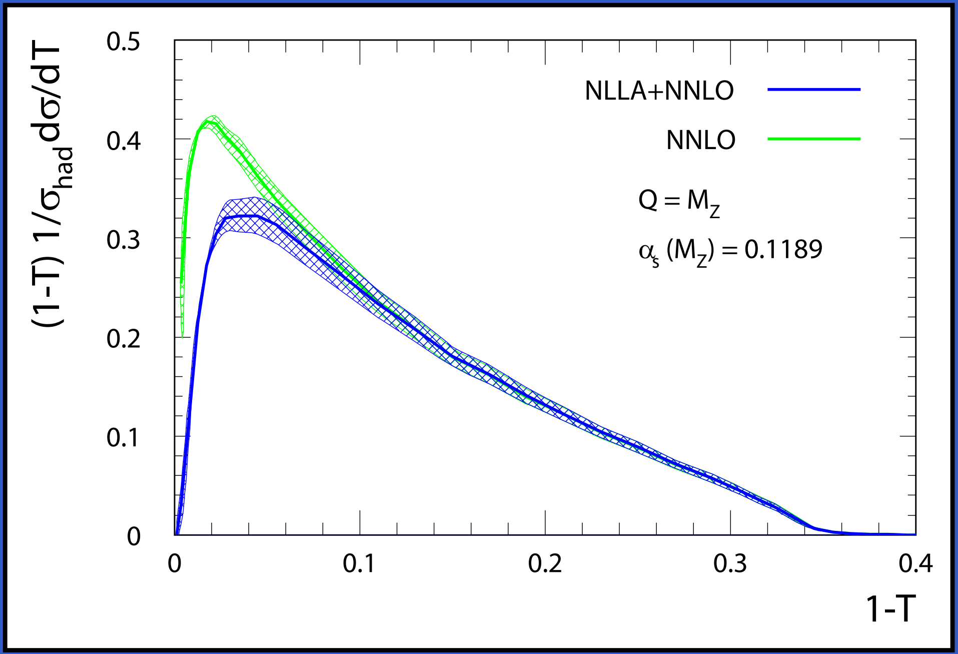


$$\Rightarrow \bar{\alpha}_S\left(M_Z\right) = 0.1240 \pm 0.0033$$

Determination of α_S: NLLA+NNLO results

PRELIMINARY

- Not yet, but still some hints:
  - improvement from NNLO to NLLA+NNLO smaller than from NLO to NLLA+NLO: two loop running terms not compensated in NLLA.



Conclusions and outlook

- Matched NLLA+NNLO distributions in the log-R matching scheme,
  - NLLA+NNLO results improved wrt. NLLA+NLO especially in 3-jet region,
  - in 2-jet region better prediction than NNLO, but far from perfect.

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  - competitive result, but "somewhat" high compared to other results.

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  - substantial improvement over NLO and NLLA+NLO,
  - competitive result, but "somewhat" high compared to other results.
- Further steps and improvements:
  - combinations of α_S measurements using NLLA+NNLO, [work in progress]
  - resummation of subleading logarithms (for all observables),
  - inclusion of EW-corrections and hadronization corrections from modern NLO+PS MC.