Parton Distributions from Lattice QCD

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Generalized Parton Distributions

- parton distributions and form factors - GPDs reduce to parton distributions and form factors in particular limits

$$q(x) \quad \text{and} \quad F_1(t),\ F_2(t)$$

- transverse structure - 3D distribution of quarks in a mixed representation: 2 transverse coordinates $\vec{b}_\perp$ and 1 longitudinal momentum x

$$q(x, \vec{b}_\perp)$$

- spin decomposition - decomposition of nucleon spin into quark helicity, quark orbital, and gluon contributions

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma_{u+d} + L_{u+d} + J_g$$

Lattice Calculation

- work with the Lattice Hadron Physics Collaboration (LHPC)

- lattice spacing: $a = 0.124$ fm ($a^{-1} = 1.6$ GeV)

- for nucleon structure, chiral extrapolation is the dominate error

L (fm)	m_π (MeV)				
2.52	354	498	594	693	761
3.53	353				

- one-loop perturbative renormalization at $\mu = 2$ GeV in $\overline{\text{MS}}$

- (for the experts: domain wall valence quarks on asqtad sea quarks)

Moments of Parton Distributions

- light-cone expansion generates twist-two operators

$$O_q^{\mu_1\cdots\mu_n} = \overline{q} iD^{\{\mu_1} \ldots iD^{\mu_{n-1}}\gamma^{\mu_n\}} q$$

- moments of parton distributions from forward matrix elements

$$\langle P|O_q^{\mu_1\cdots\mu_n}|P\rangle = 2\,\langle x^{n-1}\rangle_q\, P^{\{\mu_1} \ldots P^{\mu_n\}}$$

- unpolarized, helicity and transversity moments

$$\begin{aligned}
\langle x^n\rangle_q &= \int_0^1 dx\, x^n \left\{q(x) - (-1)^n \overline{q}(x)\right\} \\
\langle x^n\rangle_{\Delta q} &= \int_0^1 dx\, x^n \left\{\Delta q(x) + (-1)^n \Delta\overline{q}(x)\right\} \\
\langle x^n\rangle_{\delta q} &= \int_0^1 dx\, x^n \left\{\delta q(x) - (-1)^n \delta\overline{q}(x)\right\}
\end{aligned}$$

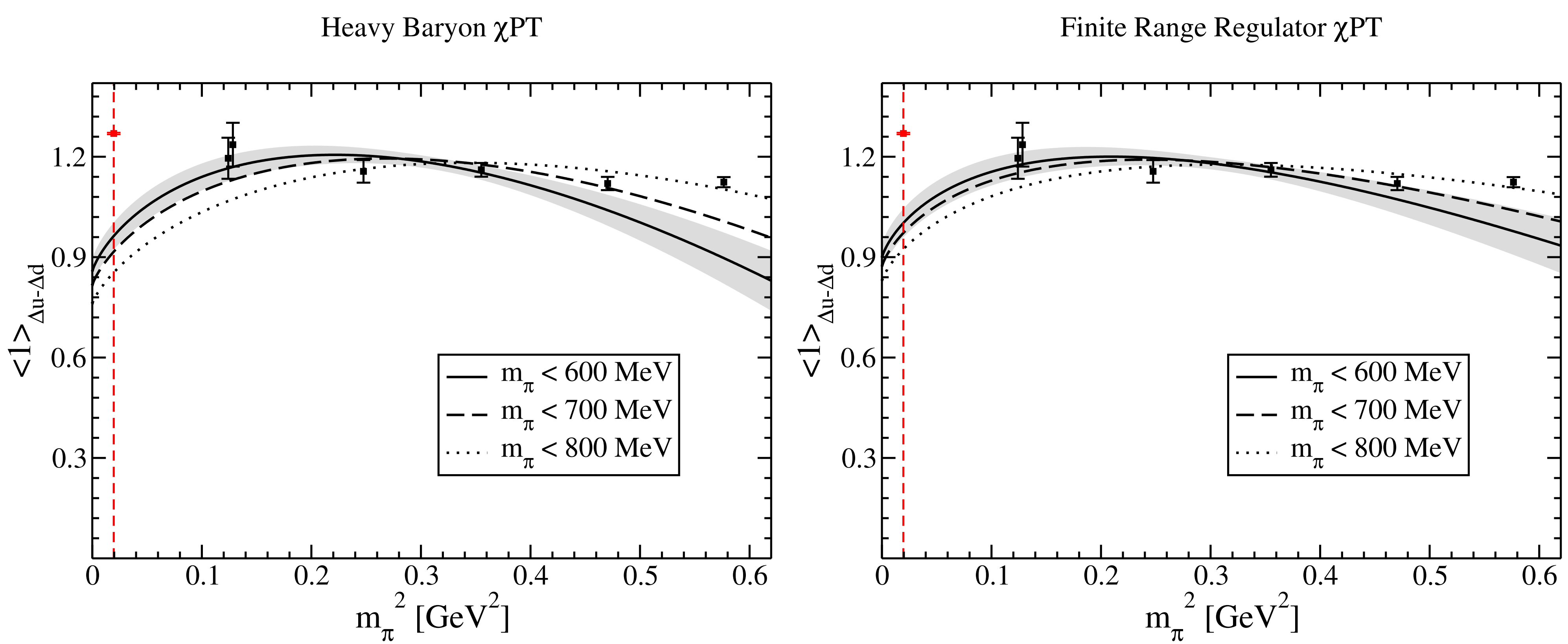
Simple Chiral Perturbation Theory

- standard one-loop heavy baryon chiral perturbation theory

$$\langle 1 \rangle_{\Delta u - \Delta d} = g = \mathring{g} \left(1 - \frac{(2\mathring{g}^2 + 1)}{(4\pi \mathring{f})^2} m^2 \ln \left(\frac{m^2}{\mu^2} \right) \right) + c_g(\mu) m^2$$

[Arndt, Savage, Chen, Ji]

- finite range regulator: $\ln(\mu^2) \to \ln(\Lambda^2 + m^2)$



Self-Consistent Chiral Perturbation Theory

- standard one-loop heavy baryon chipt for $\langle x \rangle$

$$\langle x \rangle = \langle \mathring{x} \rangle - \langle \mathring{x} \rangle (3\mathring{g}^2 + 1) m^2 / (4\pi \mathring{f})^2 \ln(m^2/\mu^2) + c_x(\mu) m^2$$

[Arndt, Savage, Chen, Ji]

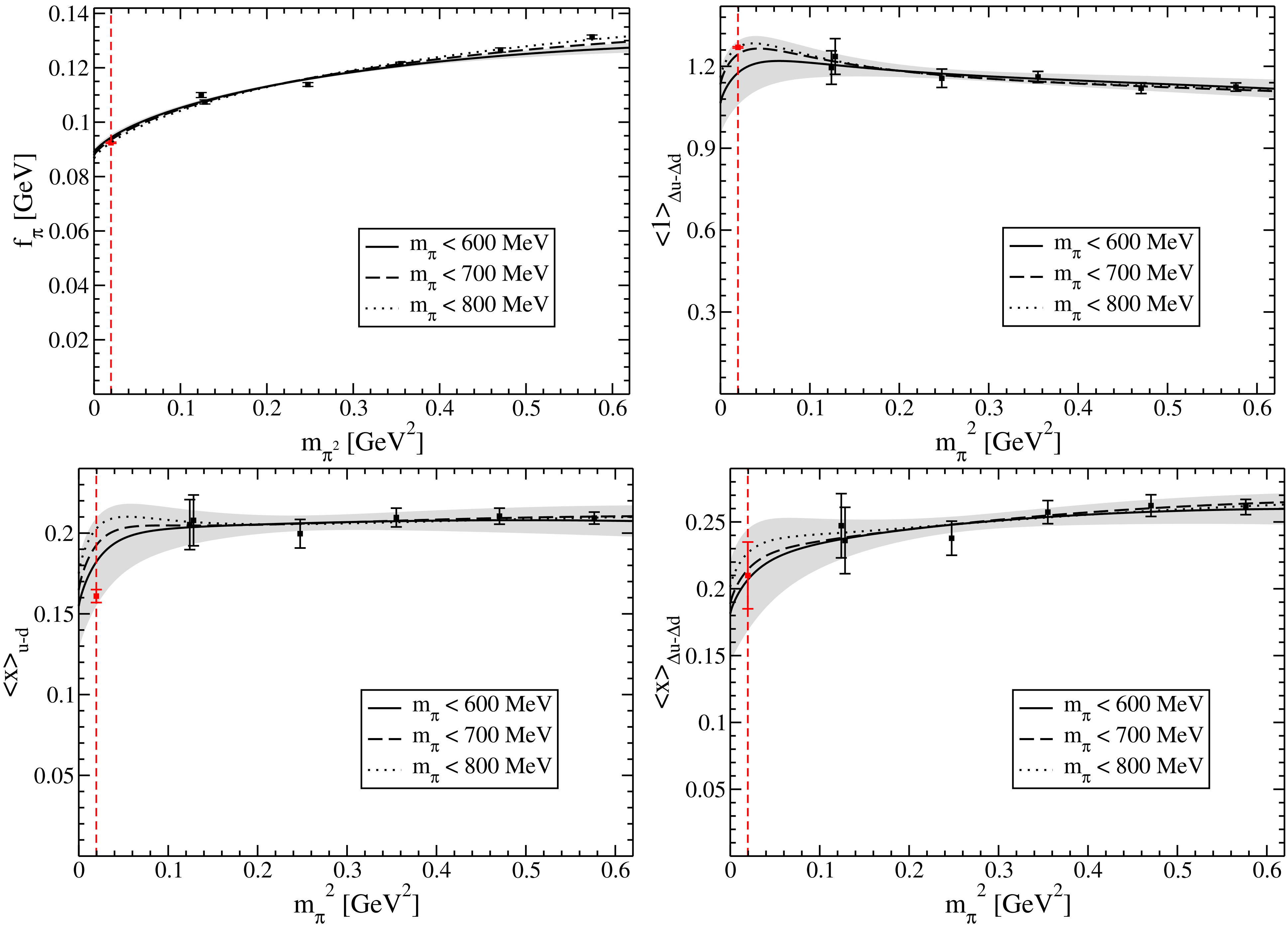
- choose $\mu = \alpha \mathring{f}$ and replace $\mathring{f}$, $\mathring{g}$, $\langle \mathring{x} \rangle$ with f, g, $\langle x \rangle$ in the NLO term

$$\langle x \rangle = \langle \mathring{x} \rangle - \langle x \rangle (3g^2 + 1) m^2 / (4\pi f)^2 \ln(m^2/(\alpha f)^2) + c_x(\alpha) m^2$$

- rearrange and isolate the unknown low energy constants: $\langle \mathring{x} \rangle$, $c_x(\alpha)$

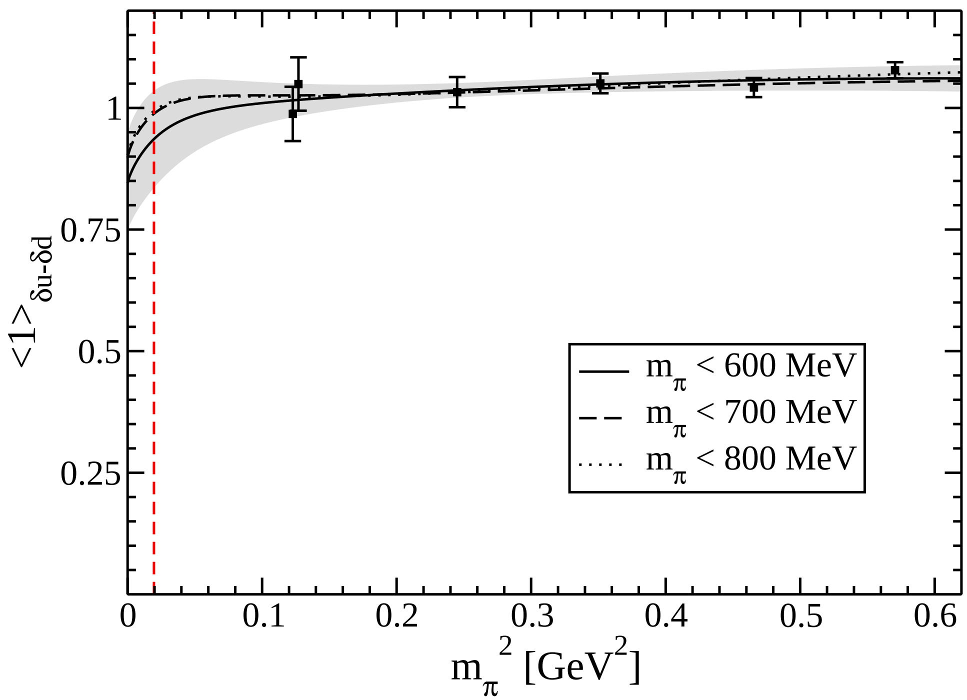
$$\langle x \rangle (1 + (3g^2 + 1) m^2 / (4\pi f)^2 \ln(m^2/(\alpha f)^2)) = \langle \mathring{x} \rangle + c_x(\alpha) m^2$$

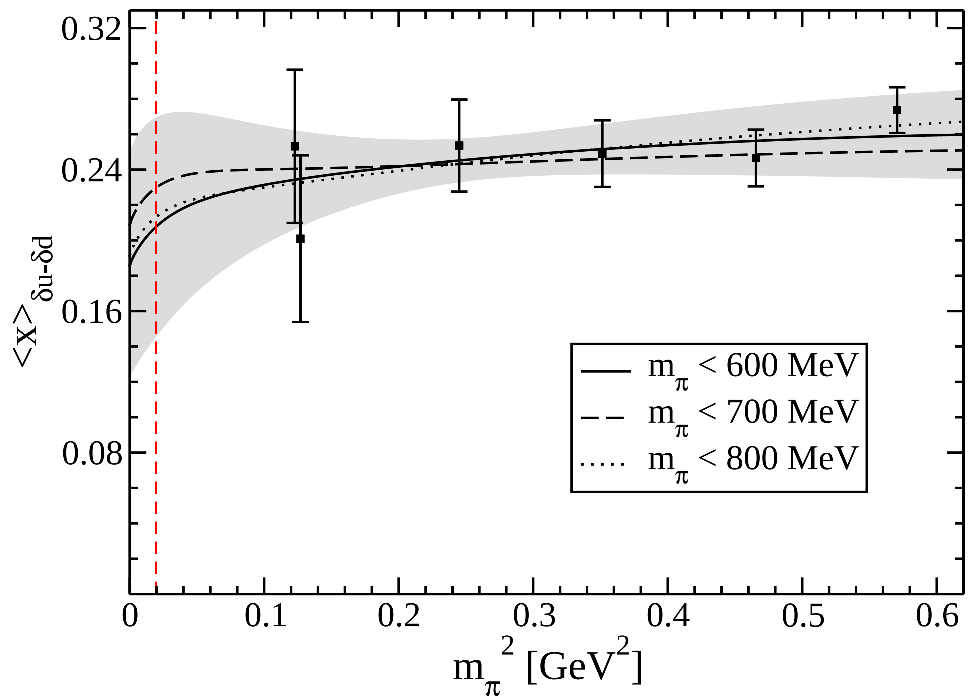
Benchmark Observables



Transversity Distribution

- statistical errors, finite size effects and the systematic error from extrapolation in m_π can been estimated below
- a study of lattice artifacts and renormalization will yield genuine predictions for transversity





Generalized Parton Distributions

- moments of parton distributions from forward matrix elements

$$\langle P | O_q^{\mu_1 \cdots \mu_n} | P \rangle = 2 \, \langle x^{n-1} \rangle_q \, P^{\{\mu_1} \ldots P^{\mu_n\}}$$

- off-forward matrix elements of twist-two operators, $t = (P' - P)^2$

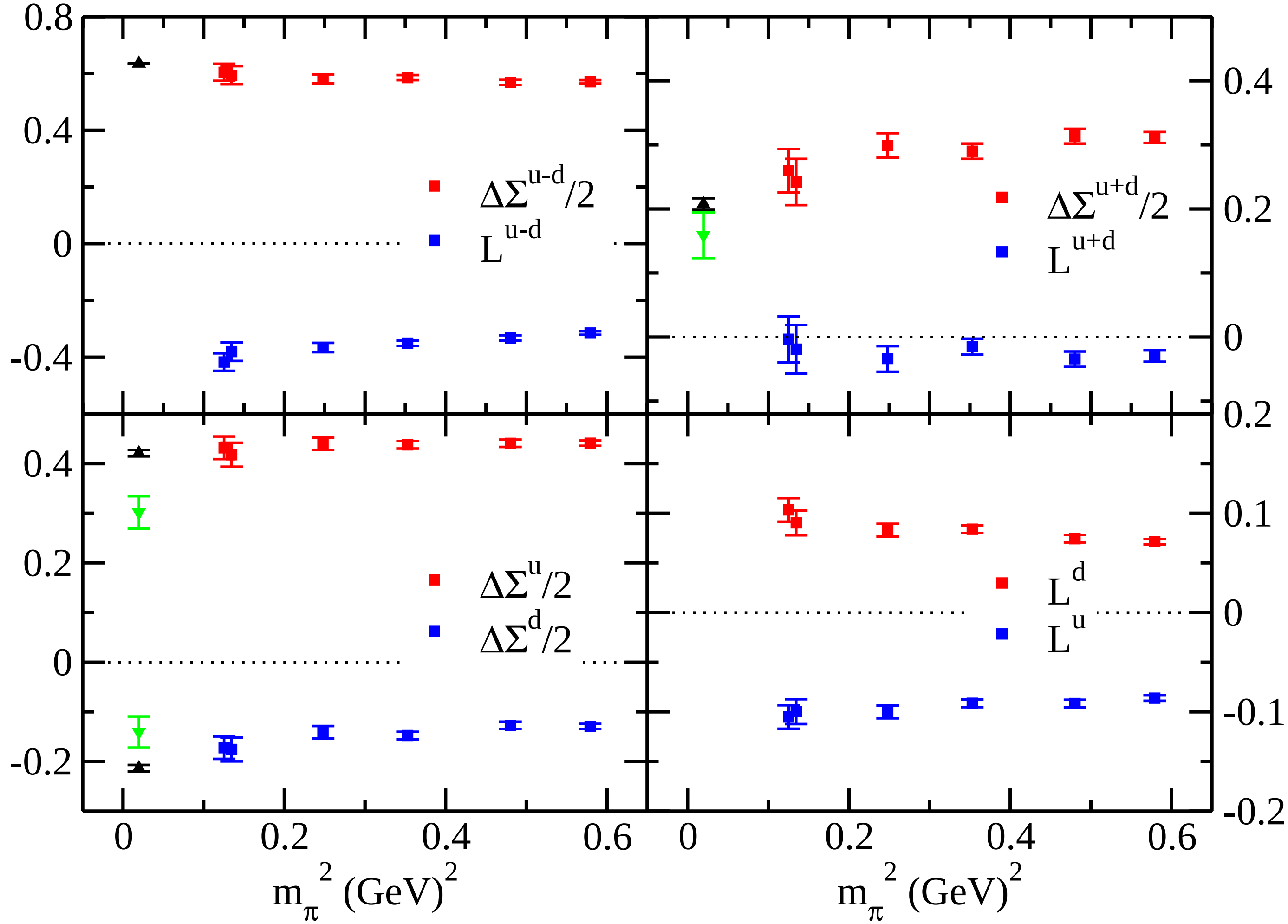
$$\langle P' | O_q^{\mu_1 \cdots \mu_n} | P \rangle = \sum_{\substack{i=0 \\ \text{even}}}^{n-1} \left\{ A_{ni}^q(t) K_{ni}^A + B_{ni}^q(t) K_{ni}^B \right\} + \delta_{\text{even}}^n C_n^q(t) K_n^C$$

- nucleon spin decomposition [X.D. Ji]

$$\frac{1}{2} = \frac{1}{2} \Delta\Sigma^{u+d} + L^{u+d} + J^g$$

$$\Delta\Sigma^q = \langle 1 \rangle_{\Delta q} \qquad L^q = \frac{1}{2} \Big(\langle x \rangle_q + B_{20}^q(0) - \langle 1 \rangle_{\Delta q} \Big)$$

Spin Content



- caveat: disconnected diagrams are omitted in $u + d$, u, d but not $u - d$

HERMES, PRD 75, 012007 (2007)

Conclusions

- replacing $\overset{\circ}{f}$, $\overset{\circ}{g}$, $\langle \overset{\circ}{x} \rangle$ with f, g, $\langle x \rangle$ in NLO terms of chipt gives results that agree with experimental measurements of f_π, g_A, $\langle x \rangle_{u-d}$, $\langle x \rangle_{\Delta u - \Delta d}$

- careful study of systematic errors should lead to genuine predictions for $\langle 1 \rangle_{\delta u - \delta d}$ and $\langle x \rangle_{\delta u - \delta d}$ and other moments of parton distributions

- the nucleon's spin decomposition agrees much better with the latest HERMES results

- a wide range of observables can be examined by calculating moments of not only the parton distributions but also the generalized parton distributions