Higher twists from the saturation model

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Overview

Motivation

Higher twist evolution in QCD at small x

From QCD to saturation model

Estimates of higher twists in F_2 and F_L

Summary

Based on results obtained with J. Bartels and K. Golec-Biernat

Motivation

At fixed Q^2 importance of higher twist operators in QCD increases quickly with $\sqrt{s}$

Reason: rapid higher twist evolution at large Q^2 and small x

Gluons: $$\frac{\text{Twist } 4}{\text{Twist } 2} \sim \frac{1}{Q^2 R^2} \exp\left(\sqrt{b \log(Q^2) \log(1/x)}\right)$$

Higher twists $\longrightarrow$ corrections to main HERA observables: F_2 and F_L

$\longrightarrow$ provide input to precision determination of PDF

Goal 1: Explicit determination of higher twist corrections to F_2 and F_L at small x

Goal 2: QCD analysis of Saturation Model

$\longrightarrow$ identify necessary approximations

$\longrightarrow$ interprete ingredients in terms of QCD

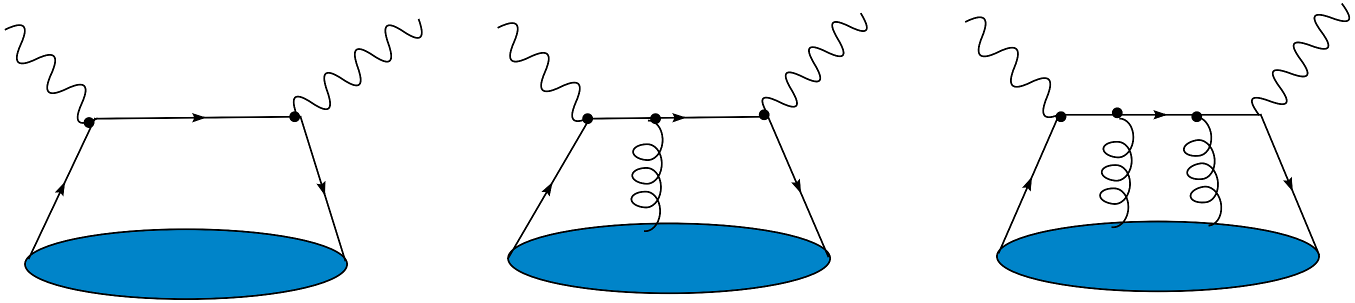
$\longrightarrow$ exploit the information on higher twists

Extension of twist analysis of GBW model **[Bartels, Golec-Biernat, Peters]**

Higher twist evolution: Quasi Partonic Operators (BFKLs)

DIS: OPE for hadronic tensor: $$W^{\mu\nu} = \sum_{\tau} \left(\frac{\Lambda}{Q}\right)^{\tau-2} \sum_{i} C^{\mu\nu}_{\tau,i} \otimes f_{\tau,i}(Q^2/\Lambda^2)$$

Complete twist 4 analysis o $q\bar{q}gg$ evolution **[Ellis, Furmanski and Petronzio, 1983]**



At small x gluon evolution expected to drive DIS cross-sections at all twists — complete analysis of twist 4 evolution for gluons does not exist

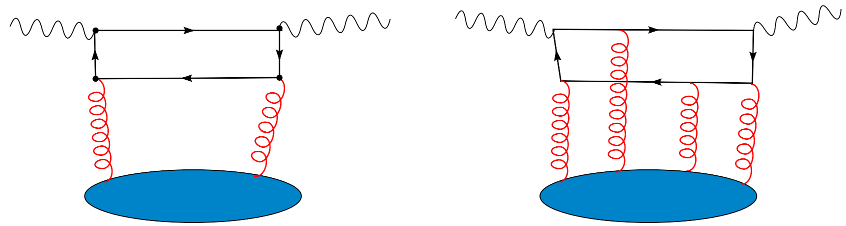
Known at all twists: evolution of Quasi-Partonic Operators **[Bokhvostov, Kuraev, Lipatov, Frolov, 1985]**

QPO hadronic matrix elements at twist $\tau \sim \tau$ free partons. Examples: $(\partial_{.} A^{\perp}_{\alpha})^2 (\partial_{.} A^{\perp}_{\beta})^2$, $\bar{\psi}\psi\bar{\psi}\psi$

Collinear evolution kernel for twist τ at LLA splits into disconnected pairwise parton interactions — non-forward (in x and p_T) DGLAP kernels

4-gluon evolution at twist 4

At small the dominant contribution should come from diagrams of the type:

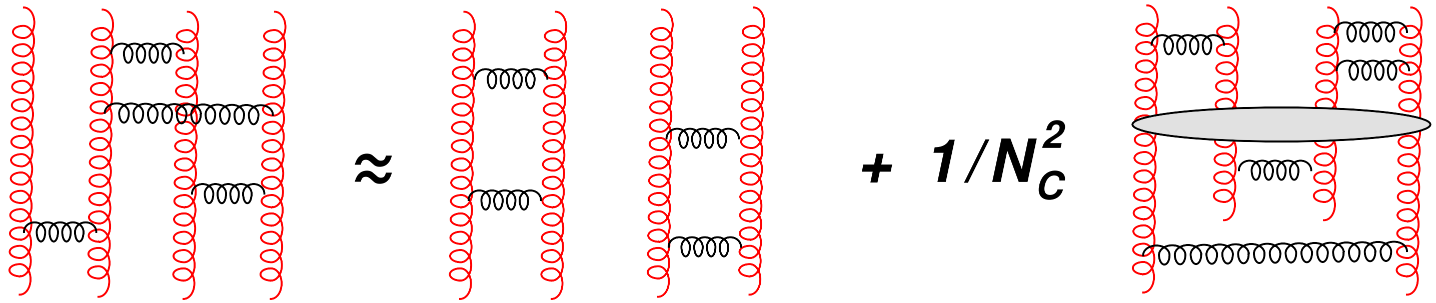


For twist-4, $N_c \to \infty$, in the leading $\alpha_s \log(Q^2) \log(1/x)$ approximation dominant singularity:

$$\gamma = \frac{4N_c\alpha}{\pi}\frac{1}{\omega}$$

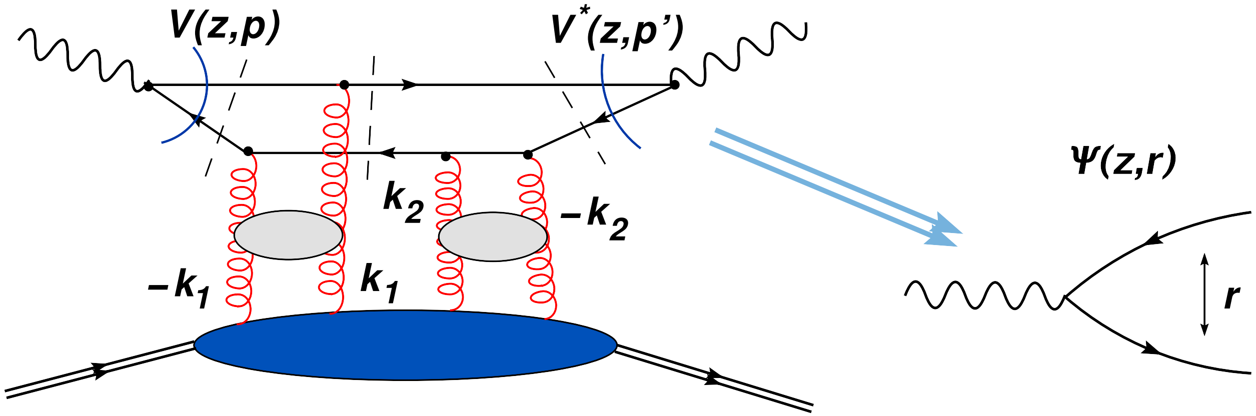
coming from two independent DGLAP evolutions

Corrections — color reconnections between ladders supressed by $\sim 1/N_c^2$ **[Bartels, Ryskin, 1993]**



γ^* cross section: quark box diagram

Structure: $$\Delta^{(2n)}\sigma_{\gamma^* p} \sim \int \prod_{i=1}^{2n} \frac{d^2 k_i}{k_i^4} \delta\left(\sum_i \boldsymbol{k}_i\right) \; G_{2n}^{\{a_i\}}(x, \{\boldsymbol{k}_i\}) \; \Phi_{2n}^{\{a_i\}}(\{\boldsymbol{k}_i\})$$



Multi-gluon coupling in high energy limit $\longrightarrow$ photon-gluon vertex fusion governs all couplings

$$\Phi_{2n} \sim \alpha_s^n \int d^2 p \int dz \sum_F \text{Color}(F) \; V^*(z, \boldsymbol{p}'(F)) \, V(z, \boldsymbol{p})$$

After projection on symmetric multiple color singlet, and the Fourier transform result is simple

$$\Phi_{2n} \sim \alpha_s^n \int d^2 r \int dz \, \Psi^*(z, \boldsymbol{r}) \prod_{i=1}^{n} \left[2 - e^{i\boldsymbol{k}_i \boldsymbol{r}} - e^{-i\boldsymbol{k}_i \boldsymbol{r}}\right] \Psi(z, \boldsymbol{r})$$

Resumming multi-gluon effects

Taking factorized and symmetric form of unintegrated multi-gluon density

$$G_{2n}^{\{a_i\}}(x, \{k_i^2\}) \sim \sum_\sigma \delta^{a_{\sigma(1)} a_{\sigma(2)}} \ldots \delta^{a_{\sigma(2n-1)} a_{\sigma(2n)}} f(x, \boldsymbol{k}_{\sigma(1)}, \boldsymbol{k}_{\sigma(2)}) \ldots f(x, \boldsymbol{k}_{\sigma(2n-1)}, \boldsymbol{k}_{\sigma(2n)})$$

Invoking AGK rules one obtains the Glauber-Mueller form used by GBW

$$\Delta^{(2n)}\sigma \sim \frac{(-1)^{n+1}}{n!} R^2 \int d^2r\, dz\, |\Psi(z, \boldsymbol{r})|^2 \prod_{i=1}^{n} \underbrace{\left\{ \int \frac{d^2k_i}{k_i^4} \frac{\alpha_s f(x, \boldsymbol{k}_i^2)}{R^2} \left[2 - e^{i\boldsymbol{k}_i \boldsymbol{r}} - e^{-i\boldsymbol{k}_i \boldsymbol{r}} \right] \right\}}_{\text{single dipole scattering xs: } \sigma_1(x,r^2)/R^2}$$

In collinear limit ($k^2 \ll C/r^2$) dipole cross section coincides with DGLAP improved saturation model **[Bartels, Golec-Biernat, Kowalski]**

$$\sigma_1(x, r^2) \simeq \alpha_s(C/r^2) \int^{C/r^2} \frac{dk^2}{k^4} f(x, k^2)\, (k^2 r^2) \simeq r^2\, \alpha_s(C/r^2)\, xg(x, C/r^2)$$

Resummed cross section:

$$\sigma_d(x, r^2) \simeq R^2 \left[1 - \exp(-\sigma_1(x, r^2)/R^2)\right]$$

Recovering saturation model

Combining together QCD information on quark box diagram and multi-gluon density:

$$\sigma_{L,T}(x,Q^2) = \int_0^\infty \frac{dr^2}{r^2} \underbrace{\left[r^2 \int dz \left| \Psi_{L,T}(z,r^2) \right|^2 \right]}_{H_{T,L}(Q^2 r^2)} \sigma_d(x,r^2),$$

with multi gluon evolution in: $\sigma_d(x,r^2) = \sigma_0[1 - \exp(-\sigma_1(x,r^2)/\sigma_0)]$

Perturbative part of "dipole cross section" combined with modelled soft part ($\longrightarrow$ quark input):

$$\sigma_1 \;\propto\; \begin{cases} r^2\,\alpha_s(C/r^2)\,xg(x,C/r^2) & \text{for } C/r^2 < Q_0^2 \\ r^2\,\alpha_s(Q_0^2)\,xg(x,Q_0^2) & \text{for } C/r^2 > Q_0^2 \end{cases}$$

Twist analysis in the Mellin moment space:

$$\tilde{f}(s) = \int_0^\infty dr^2\, f(r^2)\,(r^2)^{s-1},$$

Decomposition will be performed using Parsival formula

$$\sigma_{T,L\,p}(x,Q^2) \;=\; \int_{\mathcal{C}_s} \frac{ds}{2\pi i}\, \tilde{\sigma}(x,s)\, \tilde{H}_{T,L}(-s,Q^2)$$

Single gluon density evolution

The gluon density obeys the LO DGLAP equation,

$$\mu^2 \frac{\partial xg(x,\mu^2)}{\partial \mu^2} = \frac{\alpha_s(\mu^2)}{2\pi} \int_x^1 dz\, P_{gg}(z)\, \frac{x}{z} g\left(\frac{x}{z},\mu^2\right)$$

In double Mellin space — $x \to \omega$, $Q^2 \to \gamma$:

$$\gamma \tilde{g}(\omega,\gamma) = \frac{1}{t_0 - \partial/\partial\gamma}\, \tilde{p}(\omega)\, \tilde{g}(\omega,\gamma), \qquad t_0 = \log(Q_0^2/\Lambda^2)$$

General solution

$$\tilde{g}(\omega,\gamma) = \hat{g}_0(\omega)\, e^{\gamma t_0}\, \gamma^{-1-\tilde{p}(\omega)}$$

In (ω, Q^2) representation $$xg(x,\mu^2) = \int \frac{d\omega}{2\pi i}\, \tilde{g}_0(\omega)\, x^{-\omega} \left[\frac{\log(\mu^2/\Lambda^2)}{\log(Q_0^2/\Lambda^2)}\right]^{\tilde{p}(\omega)}$$

$$\alpha_s(\mu^2)\tilde{g}(\omega,\mu^2) = \int \frac{d\omega}{2\pi i} x^{-\omega} \int \frac{d\gamma}{2\pi i} \left(\frac{\mu^2}{Q_0^2}\right)^{\gamma} \frac{b_0 \gamma\, \tilde{g}(\omega,\gamma)}{\tilde{p}(\omega)}$$

Double and multiple gluon density evolution

Double gluon density given by convolution in Mellin space:

$$[\alpha_s(\mu^2) x g(x,\mu^2)]^2 = \int \frac{d\omega}{2\pi i} \int \frac{d\omega'}{2\pi i} x^{-\omega} \tilde{G}_{4,0}(\omega', \omega-\omega')$$

$$\times \int \frac{d\gamma}{2\pi i} \int \frac{d\gamma'}{2\pi i} \left(\mu^2/\Lambda^2\right)^{\gamma} (\gamma')^{-\tilde{p}(\omega')} \left(\gamma-\gamma'\right)^{-\tilde{p}(\omega-\omega')}$$

$$[\alpha_s(\mu^2) x g(x,\mu^2)]^2 = \int \frac{d\omega}{2\pi i} \int \frac{d\omega'}{2\pi i} x^{-\omega} \hat{G}_{4,0}(\omega', \omega-\omega') \int \frac{d\gamma}{2\pi i} \left(\mu^2/\Lambda^2\right)^{\gamma} \gamma^{1-\tilde{p}(\omega')-\tilde{p}(\omega-\omega')}$$

In general: $$[\alpha_s(\mu^2) x g(x,\mu^2)]^n = \int \frac{d\omega}{2\pi i} x^{-\omega} \int \prod_{i=1}^{n} \frac{d\omega_i}{2\pi i} \hat{G}_{n,0}(\omega_i)\, \delta\left(\sum_i \omega_i - \omega\right)$$

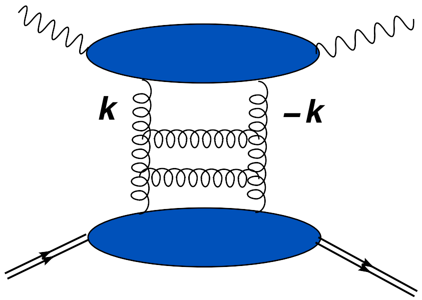
$$\times \int \frac{d\gamma}{2\pi i} \gamma^{n-1-\sum_i \tilde{p}(\omega_i)} \left(\frac{\mu^2}{\Lambda^2}\right)^{\gamma}$$

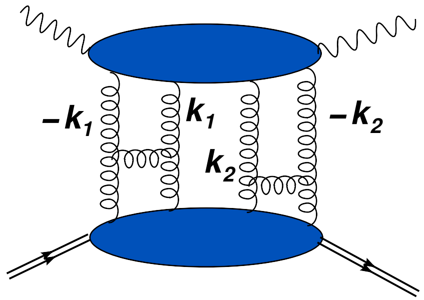
Twist structure of γ^* impact factor

2 gluon impact factor $D(k) = \Phi_2(k^2/Q^2)$ $\longrightarrow$ Mellin transform: $\tilde{D}(\gamma)$

Double poles of $\tilde{D}_{T,L}(\gamma)$: for integer γ except of single poles for $\gamma = 1$ ($\tilde{D}_L$) and $\gamma = 2$ ($\tilde{D}_T$)

Twist-2 double pole of $\tilde{D}_T(\gamma)$ $\longrightarrow$ $D(k) \sim (k^2/Q^2)\left[a^{(2)}\log(Q^2/k^2) + b^{(2)}\right]$ $\longrightarrow$ $\tilde{P}_{q/g} \otimes g$





Coupling of 4 gluons in two forward ladders:

$$\Phi_4 \sim \int \frac{d\gamma}{2\pi i} \left\{ \tilde{D}(\gamma)\,(1/Q^2)^\gamma \left[2(\boldsymbol{k}_1^2)^\gamma + 2(\boldsymbol{k}_2^2)^\gamma - ((\boldsymbol{k}_1 + \boldsymbol{k}_2)^2)^\gamma - ((\boldsymbol{k}_1 - \boldsymbol{k}_2)^2)^\gamma\right]\right\}$$

At twist 4: $\Phi_4 \sim k_1^2 k_2^2 \log(Q^2/k_i^2)$ $\longrightarrow$ twist-4 evolution

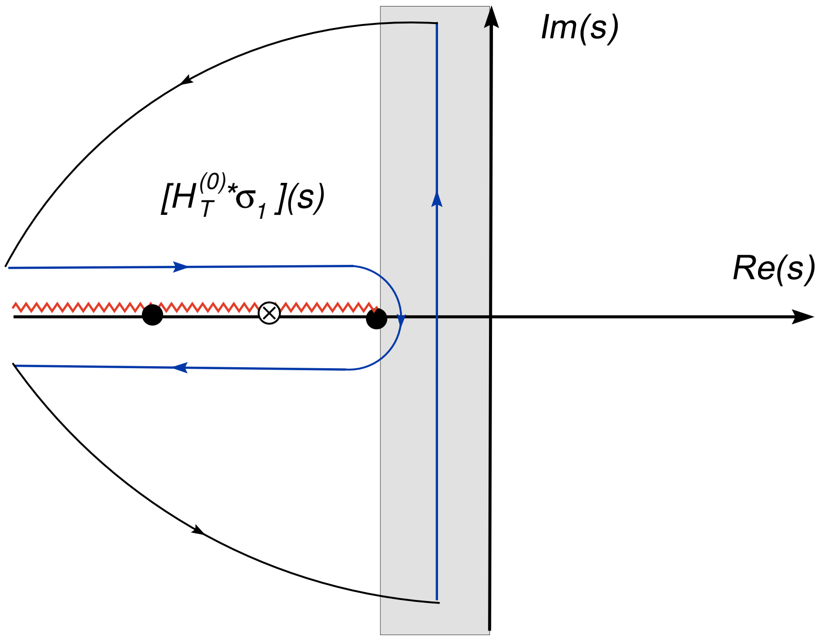
$$\int_{Q_0^2}^{Q^2} \frac{d^2k_1}{k_1^4} f(x,k_1^2) \int_{Q_0^2}^{Q^2} \frac{d^2k_2}{k_2^4} f(x,k_2^2)\; \Phi_4(\boldsymbol{k}_1, \boldsymbol{k}_2, Q) \;\sim\; \alpha_s^2(Q^2)[xg(x,Q^2)]^2 \log(Q^2/Q_0^2)$$

Mellin structure of DGLAP improved saturation model

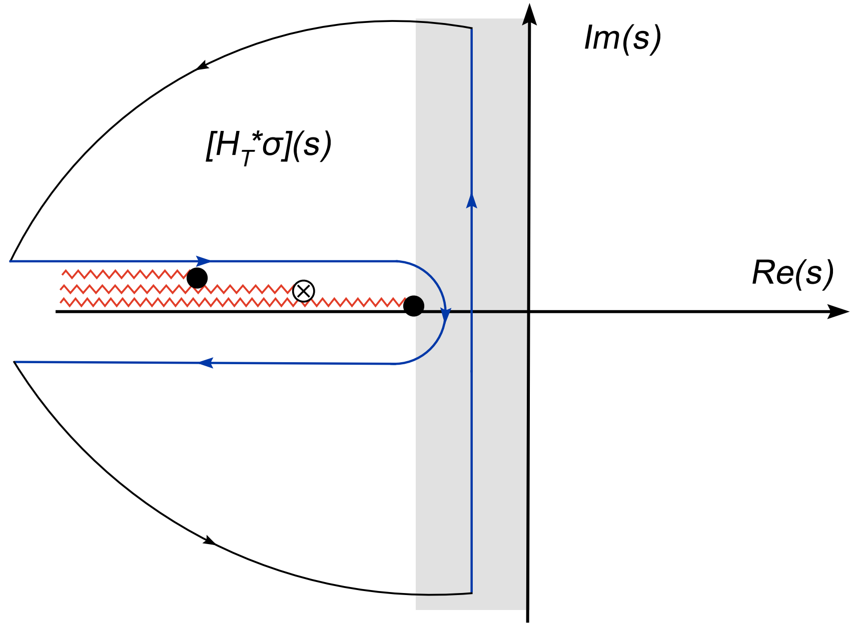
Mellin transform of perturbative part od dipole cross section – term by term

$$\mathcal{M}[\sigma](x,s) \;=\; \mathcal{M}\left[\sum_{n=1}^{\infty}\frac{(-1)^n}{n!}\sigma_1^n\right](x,s) \;=\; \sum_{n=1}^{\infty}\mathcal{M}[\sigma_n](x,s)$$

$$\mathcal{M}_{r^2}[\sigma_1^n](x,s) \;\propto\; \mathcal{M}_{\mu^2}[(\alpha_s^n(xg)^n](x,s+n)$$



Single ladder exchanges



Multiple ladder exchange

QCD decomposition of B-GB-K

At twist $2n$, n-ladder exchange:

$$\frac{\Delta_{ab}\sigma^{\tau=2n}(x,Q^2)}{\sigma_0} = \frac{1}{R^{2n}} \int_{\mathcal{C}_\omega} \frac{d\omega}{2\pi i} \tilde{G}_{n,0}(\omega) \int_{\mathcal{C}_s^{(n)}} \frac{ds}{2\pi i} \, [\, s+n\,]^{-\tilde{p}(\omega)} \; \tilde{H}_{T,L}(-s,Q^2),$$

LL twist-$2n$ evolution (gluon to quark splitting): $\tilde{\sigma}_n(s)a_n/(s+n)$

$$\Delta_a\sigma^{\tau=2n}(x,Q^2) \simeq \sigma_0 \frac{a_n}{(Q^2R^2)^n} \int_{Q_0^2}^{Q^2} \frac{dQ'^2}{Q'^2} \left[\alpha_s(Q'^2)\, xg(x,Q'^2)\right]^n$$

NLO corrections to twist-$2n$ coefficient function *of kinematic origin*: $\tilde{\sigma}_n(s)[b_n + c_n(s+n) + \ldots]$

$$\Delta_b\sigma^{\tau=2n}(x,Q^2) = \sigma_0 \frac{b_n}{(Q^2R^2)^n} \left[\alpha_s(Q^2)\, xg(x,Q^2)\right]^n (1 + \mathcal{O}(\alpha_s))$$

In addition: soft contribution: *twist-$2n$ quark input* from higher scatterings: $\tilde{\sigma}^{(n)}(s)a_n/(s+n)$

$$\Delta_c\sigma^{\tau=2n}(x,Q^2) = \int_{\mathcal{C}_s} \frac{ds}{2\pi i} \tilde{\sigma}^{(m>n)}(s) \; \frac{a_n}{s+n} (Q^2)^s$$

Generic features on twists 2 and 4 in F_2, F_T and F_L

Key information: twist 2 and twist 4 poles of the box diagram

$$\tilde{\Phi}_T(\gamma) \sim \frac{+a_T^{(2)}}{(\gamma-1)^2} \quad \Rightarrow \quad \sigma_T^{(2)} \sim \frac{a_T^{(2)}}{Q^2} \sum_f e_f^2 \frac{\alpha_{em}}{\pi} \int_{Q_0^2}^{Q^2} \frac{dQ'^2}{Q'^2} \alpha_s(Q'^2) x g(x, Q'^2)$$

$$\tilde{\Phi}_L(\gamma) \sim \frac{+b_L^{(2)}}{\gamma-1} \quad \Rightarrow \quad \sigma_L^{(2)} \sim \frac{b_L^{(2)}}{Q^2} \sum_f e_f^2 \frac{\alpha_{em}}{\pi} \alpha_s(Q^2) x g(x, Q^2)$$

$$\tilde{\Phi}_T(\gamma) \sim \frac{+b_T^{(4)}}{\gamma-2} \quad \Rightarrow \quad \sigma_T^{(4)} \sim \frac{+b_T^{(4)}}{Q^4} \sum_f e_f^2 \frac{\alpha_{em}}{\pi} [\alpha_s(Q^2) x g(x, Q^2)]^2$$

$$\tilde{\Phi}_L(\gamma) \sim \frac{-a_L^{(4)}}{(\gamma-2)^2} \quad \Rightarrow \quad \sigma_L^{(4)} \sim \frac{-a_L^{(4)}}{Q^4} \sum_f e_f^2 \frac{\alpha_{em}}{\pi} \int_{Q_0^2}^{Q^2} \frac{dQ'^2}{Q'^2} [\alpha_s(Q'^2) x g(x, Q'^2)]^2$$

F_T: twist-2: $\alpha_s\, x^{-\lambda} \log(Q^2)/Q^2$ — large, twist-4 : $\alpha_s^2\, x^{-2\lambda}/Q^4$ — suppressed correction

F_L: twist-2: $\alpha_s\, x^{-\lambda}/Q^2$ — small, twist-4: $-\alpha_s^2 x^{-2\lambda} \log(Q^2)/Q^4$ — enhanced correction

F_2: twist-2: $\alpha_s\, x^{-\lambda} \log(Q^2)/Q^2$

F_2: twist-4 : $[b_T^{(4)} - a_T^{(4)} \log(Q^2)]\, \alpha_s^2\, x^{-2\lambda}/Q^4$ — correction supressed by the sign structure

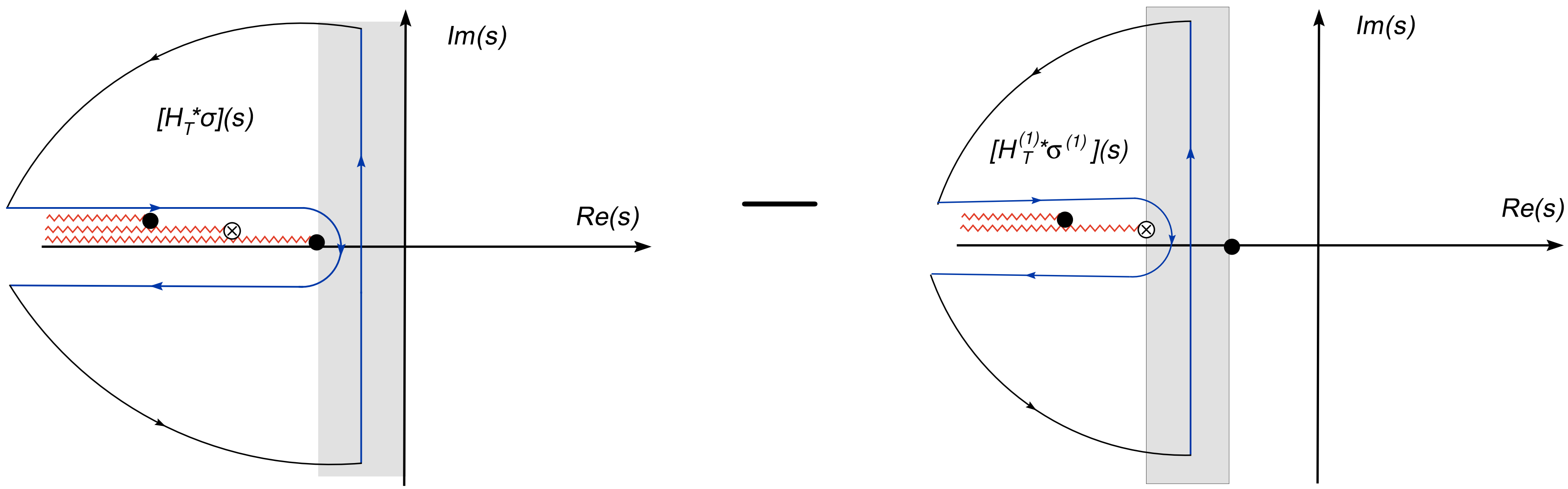
Direct twist extraction in position space: B-GB-K

We define series of subtracted functions

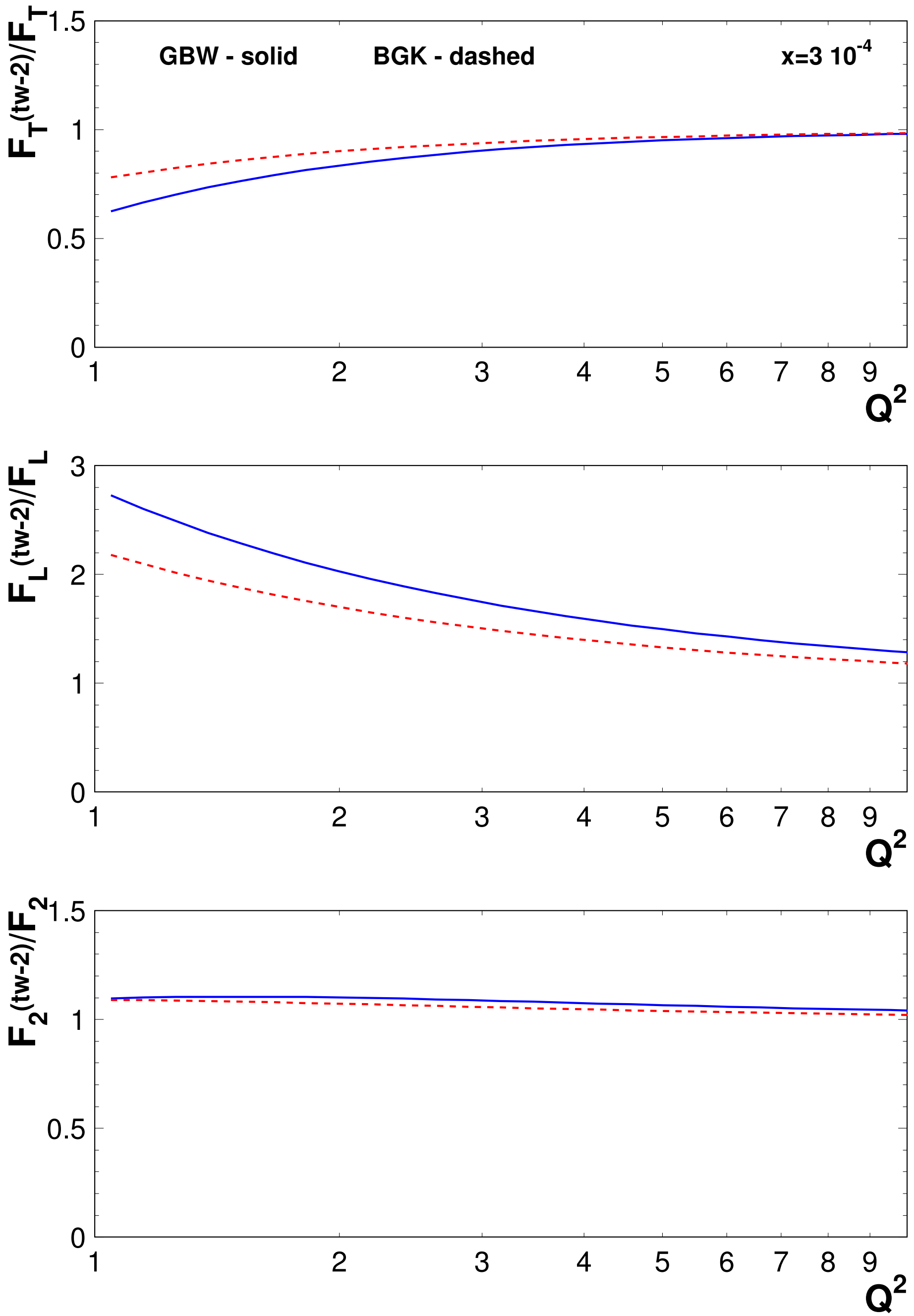
$$\sigma^{(n)}(x,r^2) = \sigma_d(x,r^2) \;-\; \sum_{k=1}^{n} \sigma_k(x,r^2)$$

$$H^{(n)}_{T,L}(Q^2r^2) = H_{T,L}(Q^2r^2) \;-\; \sum_{k=1}^{n} \frac{h^{T,L}_k}{(Q^2r^2)^k},$$

$$\Delta^{(n)}[\sigma H_{T,L}] = \int_{\mathcal{C}_s^{(n-1)}} \frac{ds}{2\pi i}\, \tilde{\sigma}^{(n-1)}(x,s)\, \tilde{H}^{(n-1)}_{T,L}(-s,Q^2) \;-\; \int_{\mathcal{C}_s^{(n)}} \frac{ds}{2\pi i}\, \tilde{\sigma}^{(n)}(x,s)\, \tilde{H}^{(n)}_{T,L}(-s,Q^2),$$



Twist ratios: tw-2/exact



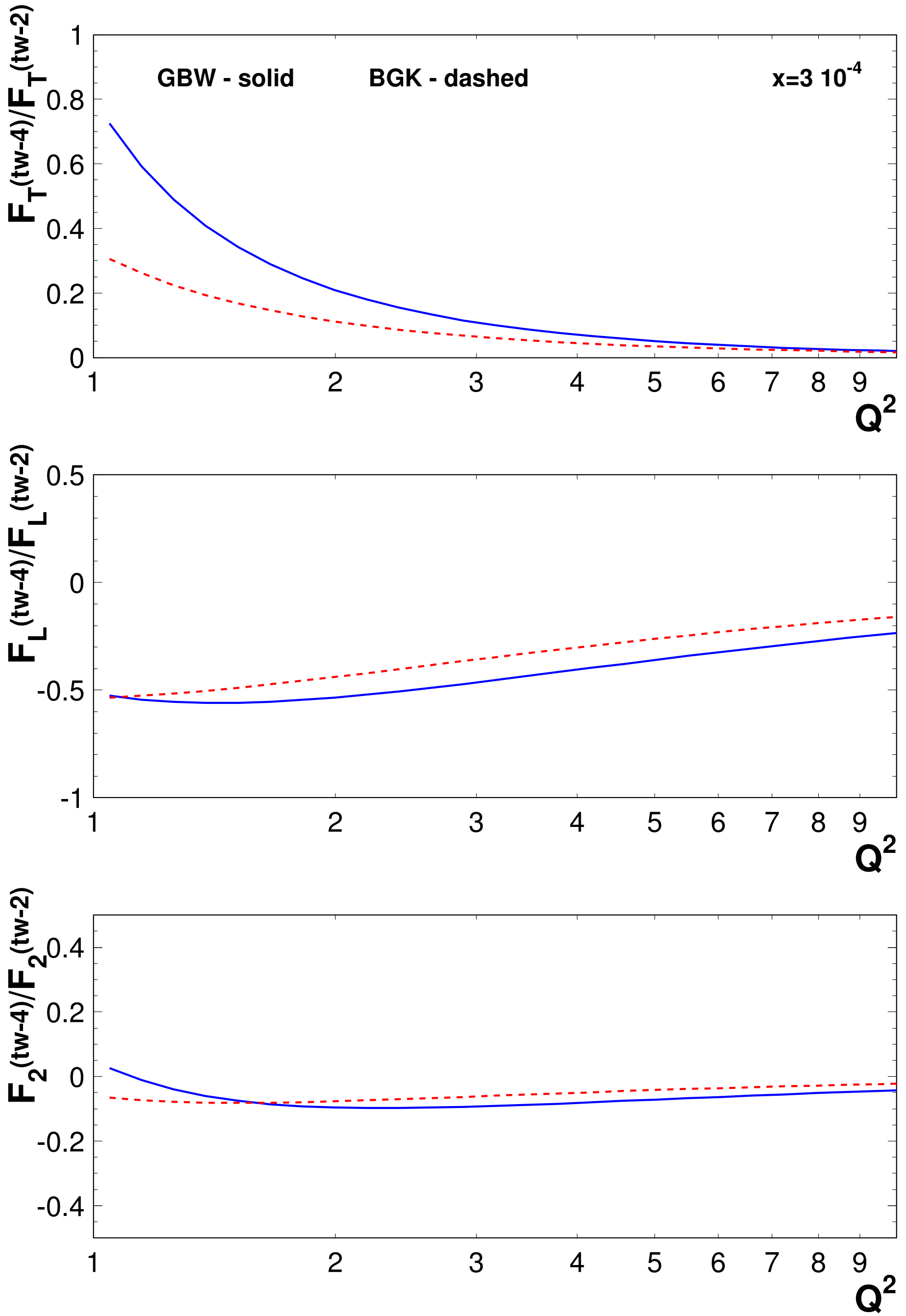
Higher twist contribution at $x = 3 \cdot 10^{-4}$ and $Q^2 = 10$ GeV^2:

F_T: $\sim 1\%$

F_L: $\sim 20\%$

F_2: $\sim 1\%$

Twist ratios: tw-4/tw-2



Summary

- Approximations leading from evolution of higher twist gluon correlators to DGLAP improved saturation model were described

- Twist content of saturation model was analyzed

- Twist 2 and twist 4 contributions to F_L and F_T following from saturation model were explicitly determined

- Twist-4 relative corrections at small x were found to be enhanced for F_L and suppressed for F_T and F_2. This conclusion is generic and does not rely on model details

- Saturation model gives surprisingly fine cancellations of higher twist corrections in F_2 at all Q^2: higher twist effects in F_2 found to be $\mathcal{O}(5\%)$ down to $Q^2 = 1$ GeV^2!

THANKS