Semi-inclusive Drell-Yan at LHC and Fracture Functions

Federico Alberto Ceccopieri

Dipartimento di Fisica, Universitá di Parma,

INFN Gruppo Collegato di Parma, Italy

Federico.Alberto.Ceccopieri@cern.ch

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Inclusive Drell-Yan

Drell-Yan cross-section: $P_1 + P_2 \rightarrow \gamma^* + X$

- is described by the parton model formula

$$\frac{d\sigma^{DY}(\tau)}{dQ^2} = \frac{4\pi\alpha^2}{9SQ^2} \int \frac{dx_1}{x_1} \int \frac{dx_2}{x_2} \sum_q e_q^2 \Big[f_q(x_1) f_{\bar{q}}(x_2) + (x_1 \leftrightarrow x_2) \Big] \delta\Big(1 - \frac{\tau}{x_1 x_2}\Big)$$

- **S. D. Drell, T. Yan *Phys. Rev. Lett.* 25, 316 (1970)**

$\Rightarrow$ Definitions and kinematics:

1. x_i fractional momentum of the parton entering the hard scattering;
2. Q^2 is the invariant mass of the lepton pair;
3. $S = (P_1 + P_2)^2$, $s = (p_1 + p_2)^2$, $s = x_1 x_2 S$, $\tau = Q^2/S$

- **G. Altarelli, R. K. Ellis, G. Martinelli, *Nucl. Phys.* B157, 461 (1979)**

Motivations

Drell-Yan cross-section: $P_1 + P_2 \rightarrow \gamma^* + X$

- Factorization of the process at "soft" level;
  - **W. W. Lindsay, D .A. Ross, C. T. Sachrajda, *Nucl. Phys.* B222, 189 (1983)**
  - **J. C. Collins, D. E. Soper, G. Sterman, *Phys. Lett.* B134, 263 (1984)**

- Moreover:

1. Invariant mass Q^2 of the lepton pair can be accurately reconstructed;
2. The process is free of final state QCD corrections;
3. Higher order corrections known.

$\Downarrow$

Prototype process

for factorization studies in hadronic collisions.

Interlude on fracture functions

- Fracture functions parametrize soft QCD dynamics in forward semi-inclusive processes.

- $M^{i}_{h/P}(x,z)$ give the conditional probability of finding a parton i with a fractional momentum x while an hadron h, with fractional momentum z of the incoming hadron momentum P, is detected in the target fragmentation region of P.

  - **L. Trentadue, G. Veneziano, *Phys. Lett.* B323, 201 (1994)**

  $\Rightarrow$ Factorization for FF in SIDIS has been proven at collinear and soft level

  - **M. Grazzini, L. Trentadue, G. Veneziano, *Nucl. Phys.* B519, 394 (1998)**
  - **J.C. Collins, *Phys. Rev.* D57, 3051 (1998)**

  $\Rightarrow$ an explicit pQCD $\mathcal{O}(\alpha_s)$ calculation was performed in

  - **D. Graudenz, *Nucl. Phys.* B432, 351 (1994)**

The extraction of diffractive PDF (spacial case of fracture functions) from HERA data is supported by factorization.

Semi-Inclusive DY

- Consider the Semi-inclusive Drell-Yan process:

$$P_1 + P_2 \longrightarrow \gamma^* + h + X$$

in which an additional hadron h is detected in the final state.

- The factorization property of the corresponding cross-section should depend on the region of phase space in which h is detected:

• **C. E. DeTar, S. D. Ellis, P. V. Landshoff, *Nucl. Phys.* B87, 176 (1975)**

1. at high $p_{h\perp}^2$ (central fragmentation region) pQCD should be applicable;
2. at low $p_{h\perp}^2$ (target fragmentation region) arguments against factorization have been given.

• **J. C. Collins, L. Frankfurt, M. Strikman, *Phys.Lett.* B307, 161 (1993)**
• **A. Berera, D. E. Soper, *Phys. Rev.* D50, 4328 (1994)**
• **J.C. Collins, *Phys. Rev.* D57, 3051 (1998)**

Parton model formula for Semi-Inclusive DY

- We consider the next-to-simple differential cross-sections

  1. for producing a lepton pair of invariant mass $Q^2 \gg \Lambda^2_{QCD}$, accompained by an additional hadron h with fractional energy $z = 2E_h/\sqrt{S}$;
  2. and integrated over all $p^2_{h\perp}$ of the detected hadron.

- However, QCD radiation at parton level is absent in lowest order.

  We assume that, at $\mathcal{O}(\alpha_s^0)$, h is non-perturbatively produced by a fracture functions.

The conjectured parton model formula is thus (M in principle not related to DIS ones):

$$\frac{d\sigma^{DY}(\tau)}{dQ^2dz} = \frac{4\pi\alpha^2}{9SQ^2}\int_\tau^{1-z}\frac{dx_1}{x_1}\int_{\frac{\tau}{x_1}}^1\frac{dx_2}{x_2}\sum_q e_q^2\Big[M_q^h(x_1,z)f_{\bar{q}}(x_2)+(x_1\leftrightarrow x_2)\Big]\,\delta\Big(1-\frac{\tau}{x_1x_2}\Big)$$

with

$$M_q^h(x,z) = M_q^{h/P_1}(x,z) + M_q^{h/P_2}(x,z)\,.$$

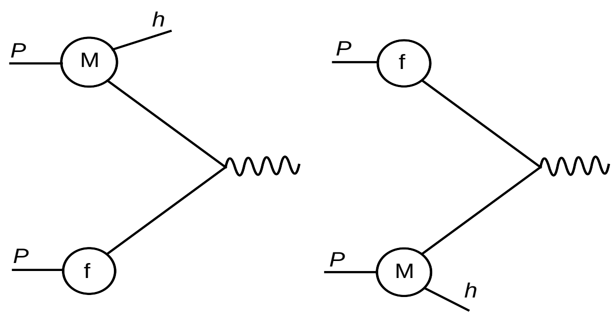
Parton model formula for Semi-Inclusive DY

- "Bare" fracture functions describe hadron production

$\Rightarrow$ in the target fragmentation region $\mathcal{R}_{T_1}$ of P_1 $\Longleftrightarrow$ $\theta_{\rm cm} = 0$

$\Rightarrow$ in the target fragmentation region $\mathcal{R}_{T_2}$ of P_2 $\Longleftrightarrow$ $\theta_{\rm cm} = \pi$

$\Rightarrow$ $\theta_{\rm cm}$ is the relative angle between h and P_1 in the hadronic center of mass.



- momentum conservation implies $1 - z \geq x_1 x_2 \geq \tau$ and

$$x_1 + z \leq 1 \text{ in } \mathcal{R}_{T_1} \text{ and } x_2 + z \leq 1 \text{ in } \mathcal{R}_{T_2}\,,$$

$$x_2 + z \leq 1 \text{ in } \mathcal{R}_{T_1} \text{ and } x_1 + z \leq 1 \text{ in } \mathcal{R}_{T_2}\,.$$

$\mathcal{O}(\alpha_s)$ corrections, $q\bar{q}$ channel

- If h is observed in $\mathcal{R}_{T_1}$ or $\mathcal{R}_{T_2}$ we assume that it has been non perturbatively produced by fracture functions.
- All the perturbative real NLO radiation $q + \bar{q} \to \gamma^* + g$ and $q + g \to \gamma^* + q$ thus must be integrated over and virtual corrections added. The singular contributions are:

$$\frac{d\sigma_t^{DY}(\tau)}{dQ^2dz} = \frac{4\pi\alpha^2}{9SQ^2} \int_{\tau}^{1-z} \frac{dx_1}{x_1} \int_{\frac{\tau}{x_1}}^{1} \frac{dx_2}{x_2} \sum_q e_q^2 \Big[M_q^h(x_1, z) f_{\bar{q}}(x_2) + (x_1 \leftrightarrow x_2) \Big] \cdot$$

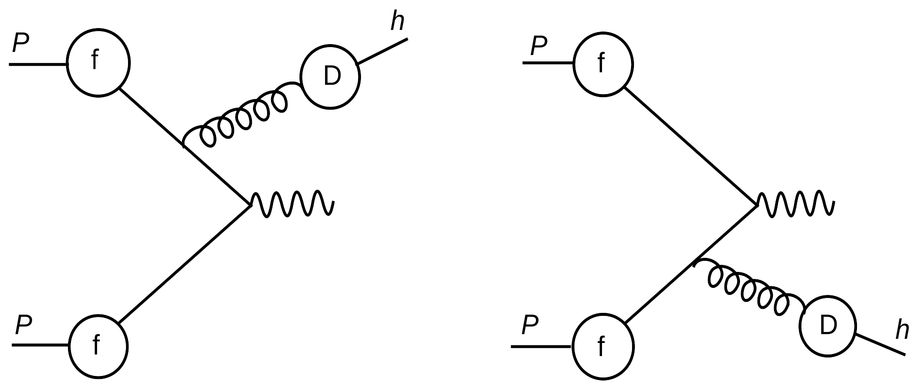
$$\cdot \left[\delta(1-w) - \frac{2}{\epsilon} \frac{\alpha_s(\mu_r^2)}{2\pi} P_{qq}(w) \left(\frac{4\pi\mu_r^2}{Q^2} \right)^{\epsilon} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \right]$$

- All dependence on hadron h variables is contained in fracture functions;
- $n = 4 - 2\epsilon$, $w = \tau/x_1 x_2$ and $P_{ij}(w)$ are the Altarelli-Parisi splitting functions.

The $\mathcal{O}(\alpha_s)$ pQCD corrections in the target fragmentation region of SIDY are the same as for inclusive DY.

$\mathcal{O}(\alpha_s)$ corrections, $q\bar{q}$ channel

- Next we consider the production of the observed hadron h by the fragmentation of a real final state parton (*i.e.* a gluon) in the partonic sub-process.



$$\frac{d\sigma_c^{DY}(\tau)}{dQ^2dz} = \frac{4\pi\alpha^2}{9SQ^2}\sum_q e_q^2 \int \frac{d\rho}{\rho}\int \frac{dx_1}{x_1}\int \frac{dx_2}{x_2} f_q(x_1) f_{\bar{q}}(x_2) D_g^h(z/\rho)\frac{d\hat{\sigma}_{q\bar{q}}^g}{dQ^2dx_1dx_2d\rho}\,.$$

$\rho = 2E_k/\sqrt{S}$ is the partonic analogue of z, being E_k the energy of the outgoing gluon.

- **F. Aversa, P. Chiappetta, M. Greco, J. P .Guillet, *Nucl. Phys.* B327, 105 (1989)**

$\mathcal{O}(\alpha_s)$ corrections, $q\bar{q}$ channel

- Final state gluon depends on its fractional energy ρ and on $y = (1 - \cos\theta_{cm})/2$.
- These variables are however not independent and constrained by phase space:

$$\rho(y) = \frac{2(x_1 x_2 - \tau)}{x_1 + x_2 + (2y - 1)(x_1 - x_2)}$$

- We rearrange the convolution integrals as

$$\frac{d\sigma_c^{DY}(\tau)}{dQ^2 dz} = \frac{4\pi\alpha^2}{9SQ^2}\sum_q e_q^2 \int_0^1 dy \int_{r_1}^1 \frac{dx_1}{x_1}\int_{r_2}^1 \frac{dx_2}{x_2} \cdot$$
$$\cdot f_q(x_1) f_{\bar{q}}(x_2) D_g^h(z/\rho)\frac{d\hat{\sigma}_{q\bar{q}}^g}{dQ^2 dx_1 dx_2 dy}\frac{1}{\rho}$$

$\Rightarrow$ with the function r_1 and r_2 given by

$$r_1(\tau, z; y) = \frac{\tau + z(1-y)}{1 - zy}, \qquad r_2(\tau, z; y) = \frac{\tau + x_1 z y}{x_1 - z(1-y)}.$$

$\mathcal{O}(\alpha_s)$ corrections, $q\bar{q}$ channel

- The emitted gluon in this region it is not allowed to be soft ($\rho > z$) since it is required to produce the observed hadron.
- we perform an ϵ-expansion in two disjoint singular limits, *i.e.* for $y \to 0, 1$.

$\Rightarrow$ The singular contributions to the cross-sections in this region of phase space read:

$$\frac{d\sigma_c^{DY}(\tau)}{dQ^2 dz} = \frac{4\pi\alpha^2}{9SQ^2}\int_{\tau+z}^{1}\frac{dx_1}{x_1}\int_{\tau/(x_1-z)}^{1}\frac{dx_2}{x_2} f_q(x_1)\, f_{\bar{q}}(x_2)\, D_g^h\Big(\frac{z x_2}{x_1 x_2-\tau}\Big)\frac{\alpha_s(\mu_r^2)}{2\pi}\,\cdot$$
$$\cdot\Big(-\frac{1}{\epsilon}\Big)\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}\Big(\frac{4\pi\mu_r^2}{Q^2}\Big)^{\epsilon}\hat{P}_{(g)q\leftarrow q}\Big(\frac{\tau}{x_1 x_2}\Big)\frac{x_2}{x_1 x_2-\tau} + (y \to 1)$$

Factorization of collinear singularities

- The subtraction of singular terms in the partonic cross-sections is performed by lumping the divergences into bare distributions.

- In the $\overline{\mathrm{MS}}$ scheme the subtraction, for f, reads:

$$f_i(\xi) = \int_{\xi}^{1} \frac{du}{u} \Big[\delta_{ij}\delta(1-u) + \frac{1}{\epsilon}\frac{\alpha_s(\mu_r^2)}{2\pi}\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}\Big(\frac{4\pi\mu_r^2}{\mu^2}\Big)^{\epsilon} P_{ij}(u)\Big] f_j\Big(\frac{\xi}{u},\mu^2\Big)$$

- An analogous $\overline{\mathrm{MS}}$ the subtraction formula holds also for fracture functions:

$$M_i^h(\xi,\zeta) = \int_{\frac{\xi}{1-\zeta}}^{1} \frac{du}{u} \Big[\delta_{ij}\delta(1-u) + \frac{1}{\epsilon}\frac{\alpha_s(\mu_r^2)}{2\pi}\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}\Big(\frac{4\pi\mu_r^2}{\mu^2}\Big)^{\epsilon} P_{ij}(u)\Big] M_j^h\Big(\frac{\xi}{u},\zeta,\mu^2\Big)$$

$$+ \int_{\xi}^{\frac{\xi}{\xi+\zeta}} \frac{du}{u}\frac{1}{1-u}\frac{u}{\xi}\frac{1}{\epsilon}\frac{\alpha_s(\mu_r^2)}{2\pi}\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}\Big(\frac{4\pi\mu_r^2}{\mu^2}\Big)^{\epsilon} \hat{P}_{(k)i\leftarrow j}(u)\, f_j\Big(\frac{\xi}{u}\Big)\, D_k^h\Big(\frac{\zeta u}{\xi(1-u)}\Big)$$

- **D. Graudenz, *Nucl. Phys.* B432, 351 (1994)**

Factorization of collinear singularities

- Rewrite the SIDY parton model formula in terms of renormalized quantity;

√ The singularities in the target fragmentation region are cancelled by the f and the homogeneous M renormalization term.

√ The singularities in the central fragmentation region are cancelled by the inhomogeneous M renormalization term.

$$
\begin{aligned}
\frac{d\sigma^{DY}(\tau)}{dQ^2dz} = & \frac{4\pi\alpha^2}{9SQ^2}\sum_q e_q^2 \int_\tau^{1-z}\frac{dx_1}{x_1}\int_{\frac{\tau}{x_1}}^1 \frac{dx_2}{x_2}\cdot\left[M_q^h(x_1,z,\mu_F^2)f_{\bar q}(x_2,\mu_F^2)+(x_1\leftrightarrow x_2)\right]\cdot \\
& \cdot\left[\delta\left(1-\frac{\tau}{x_1x_2}\right)+\frac{\alpha_s(Q^2)}{2\pi}C_{qq}\left(\frac{\tau}{x_1x_2},\frac{\mu_F^2}{Q^2}\right)\right]+ \\
& +\frac{4\pi\alpha^2}{9SQ^2}\sum_q e_q^2\int_0^1 dy\int_{r_1}^1\frac{dx_1}{x_1}\int_{r_2}^1\frac{dx_2}{x_2}\left[f_q(x_1,\mu_F^2)\,f_{\bar q}(x_2,\mu_F^2)+(x_1\leftrightarrow x_2)\right]\cdot \\
& \cdot D_g^h\left(\frac{z}{\rho},\mu_F^2\right)\frac{\alpha_s(Q^2)}{2\pi}K_{q\bar q}^g\left(z,y,\frac{\tau}{x_1x_2},\frac{\mu_F^2}{Q^2}\right)
\end{aligned}
$$

- SIDY formula in NLO pQCD

Factorization of collinear singularities

- $C_{q\bar{q}}$ and $K^{g}_{q\bar{q}}$ are **infrared finite**;
- All bare distributions are replaced by renormalized ones;
- $C_{q\bar{q}}$ is the same as in the inclusive Drell-Yan case;
- $K^{g}_{q\bar{q}}$ is specific of the semi-inclusive process;
- if we set $\mu_F^2 = Q^2$, we can remove potentially large $\ln(\mu_F^2/Q^2)$ from coefficient functions and resum them by using the appropriate evolution equations for f and M;
- Explicit expressions for $C_{q\bar{q}}$ and C_{gq} as well as $K^{g}_{q\bar{q}}$ and K^{q}_{gq} are already calculated and presently under check.

Conclusions

- We have proven in $\mathcal{O}(\alpha_s)$ pQCD a collinear factorization formula for the Semi-Inclusive Drell-Yan process.
  1. Factorization is likely to be $p_{h\perp}^2$-dependent;
  2. But phase space slicing in target and central fragmentation region is unphysical:
- Only the sum of both contributions makes sense, covering all phase space $\Rightarrow$ on this argument relies the need for a NLO calculation.
- collinear factorization does work fine at this order in PT
- The formalism is blind to soft exchanges between active/spectators partons.
- SIDY@NLO is **quantitative phenomenological factorization analyzer**.
- Factorization tests in leading baryon and light mesons production.
- Generalization to double hadron production at NLO (*e.g.* DPE) is straightforward if one considers two hadron at low $p_{h\perp}^2$ observed in opposite fragmentation regions. (Better if differential in t_1 and t_2)

• **F. A. Ceccopieri, L. Trentadue, *Phys. Lett.* B655, 15 (2007)**