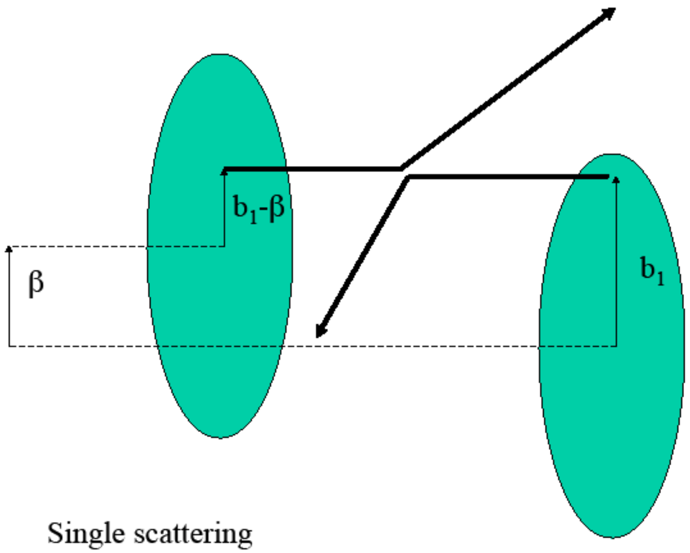
Multiparton scattering, diffraction and effective cross section

Daniele Treleani

A standard way to implement multi-parton collisions in Montecarlo simulations is to assume a **Poissonian distribution of multiple parton interactions** with average number depending on the impact parameter of the collision

The **Poissonian distribution** may be obtained introducing the three dimensional parton density Γ(x,b) and making the simplifying assumption Γ(x,b)=G(x)f(b). The single scattering inclusive cross section is hence express as:

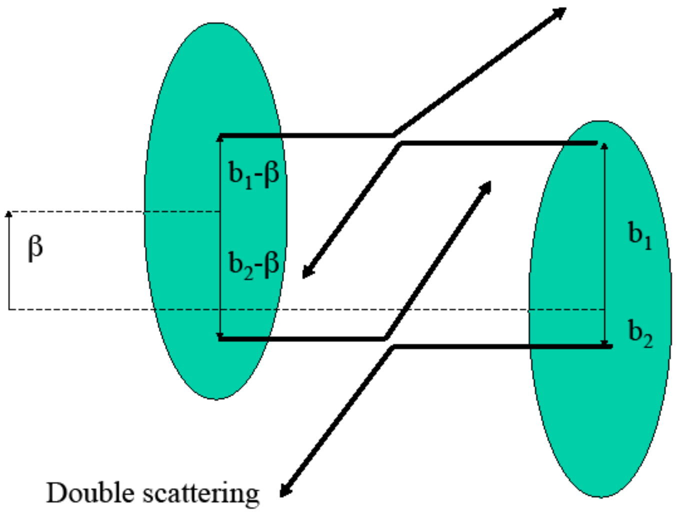


Single scattering

$$\sigma_S = \int_{p_t^c} G(x)\hat{\sigma}(x,x')G(x')dxdx'$$
$$= \int_{p_t^c} G(x)f(b)\hat{\sigma}(x,x')G(x')f(b-\beta)d^2b\,dx\,dx'd^2\beta$$

and the double scattering inclusive cross section is analogously given by

$$\begin{aligned}\sigma_D &= \frac{1}{2!}\int_{p_t^c} G(x_1)f(b_1)\hat{\sigma}(x_1,x_1')G(x_1')f(b_1-\beta)d^2b_1dx_1dx_1' \times \\ &\qquad \times G(x_2)f(b_2)\hat{\sigma}(x_2,x_2')G(x_2')f(b_2-\beta)d^2b_2dx_2dx_2'd^2\beta \\ &= \frac{1}{2!}\int\Big(\int_{p_t^c} G(x)f(b)\hat{\sigma}(x,x')G(x')f(b-\beta)d^2bdxdx'\Big)^2 d^2\beta \\ &= \frac{1}{2}\frac{\sigma_S^2}{\sigma_{eff}}\end{aligned}$$



Double scattering

where

$$F(\beta) = \int f(b)f(b-\beta)d^2b,$$

$$\sigma_{eff}^{-1} = \int d^2\beta\big[F(\beta)\big]^2,$$

The expression of the **inclusive cross section of a N parton collision process σ_N** is

$$\begin{aligned}\sigma_N &= \int \frac{1}{N!}\Big(\int_{p_t^c} G(x)f(b)\hat{\sigma}(x,x')G(x')f(b-\beta)d^2b\,dx\,dx'\Big)^N d^2\beta \\ &= \int \frac{1}{N!}\big(\sigma_S F(\beta)\big)^N d^2\beta\end{aligned}$$

The cross sections are all divergent in the infrared region. To obtain a unitarized expression one may notice that the integrand is dimensionless and may be normalized. Replacing

$$\frac{1}{N!}\big(\sigma_S F(\beta)\big)^N$$

with

$$\frac{\big(\sigma_S F(\beta)\big)^N}{N!}e^{-\sigma_S F(\beta)} = \mathrm{P}_N(\beta)$$

the integrand may be understood as the **probability** to have N parton collisions in a hadronic interaction at impact paramer β

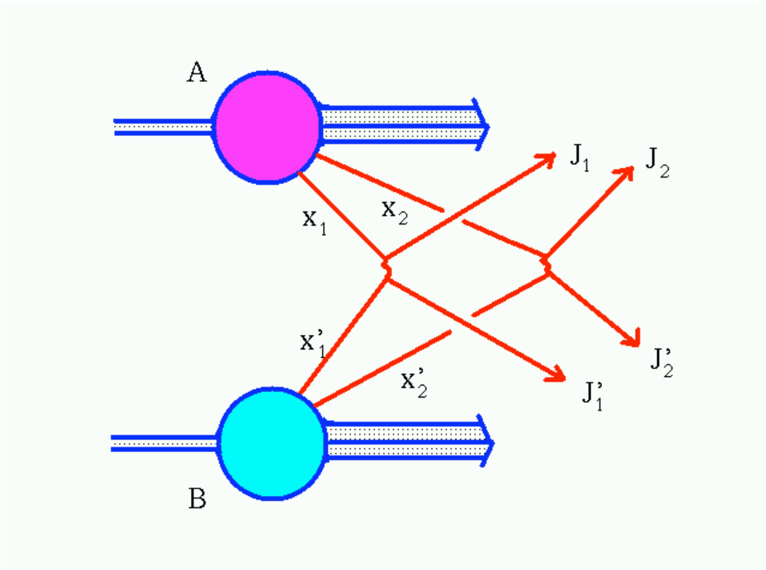
One may hence introduce the **hard cross section σ_{hard}** summing all events with **at least one large p_t parton collision**

$$\sigma_{hard} = \sum_{N=1}^{\infty} \int d^2\beta \frac{\big(\sigma_S F(\beta)\big)^N}{N!} e^{-\sigma_S F(\beta)} = \int d^2\beta \Big[1 - e^{-\sigma_S F(\beta)}\Big]$$

Notice that while σ_S is divergent when p_t^c goes to zero, σ_{hard} and all contributions to σ_{hard} with a given number N of parton collisions are, on the contrary, finite when p_t^c goes to zero and one may write:

$$\boxed{\sigma_{inel} = \sigma_{soft} + \sigma_{hard}}$$

Double parton collisions have been studied by CDF and the unexpected feature which was observed in the experiment is the **small value of the "effective cross section"**, the scale factor characterizing the process.



$$\sigma_D = \frac{1}{2}\frac{\sigma_S^2}{\sigma_{eff}}$$

In the Poissonian model the effective cross section is given by

$$\sigma_{eff}^{-1} = \int d^2\beta \left[F(\beta)\right]^2, \quad \text{with} \quad F(\beta) = \int f(b) f(b-\beta) d^2b,$$

where $f(b)$ is the density of partons in transverse space.

parton density	effective cross section			
rms radius		R=.6 fm	R=.7 fm	R=.86 fm
$e^{-\frac{3}{2}\left(\frac{r}{R}\right)^2}$	$= \frac{8}{3}\pi R^2$	30 mb	41 mb	62 mb
$e^{-\sqrt{12}\left(\frac{r}{R}\right)}$	$\simeq \frac{71}{12}R^2$	21 mb	29 mb	44 mb

rms charge radius

here R is the rooth mean square hadron radius

The experimental indication however is rather different

$$\sigma_{eff} \simeq 11 \quad \text{mb}$$

Notice that the **single** and the **double** parton **inclusive cross sections** are given by the **average** and by the **dispersion** of the distribution in the number of collisions

$$\langle N\rangle\sigma_{hard} = \int d^2\beta \sum_{N=1}^{\infty} \frac{N\big[\sigma_S F(\beta)\big]^N}{N!} e^{-\sigma_S F(\beta)} = \int d^2\beta\sigma_S F(\beta) = \sigma_S$$

$$\begin{aligned}\frac{\langle N(N-1)\rangle}{2}\sigma_{hard} &= \frac{1}{2}\int d^2\beta \sum_{N=2}^{\infty} \frac{N(N-1)\big[\sigma_S F(\beta)\big]^N}{N!} e^{-\sigma_S F(\beta)} \\ &= \frac{1}{2}\int d^2\beta \big[\sigma_S F(\beta)\big]^2 = \frac{1}{2}\frac{\sigma_S^2}{\sigma_{eff}} = \sigma_D\end{aligned}$$

It should be emphasized that the relations

$$\langle N\rangle\sigma_{hard} = \sigma_S \quad \text{and} \quad \frac{1}{2}\langle N(N-1)\rangle\sigma_{hard} = \sigma_D$$

are **not specific of the Poissonian** and can be derived on **much more general** grounds

The same relations can in fact be obtained also when considering the most general case of multiparton distributions, namely **including all possible multi-parton correlations** (in particular the correlations induced by conservation laws).
On the other hand, the direct link, between inclusive cross sections and moments of the distribution in multiplicity of collisions, gets spoiled when taking into account also connected multiparton interactions, namely 3->3 etc. parton collision processes which, nevertheless, should not give rise to major effects in pp collisions even at LHC energies.

Rather generally, **the effective cross section hence represents a measure of the dispersion** of the distribution in the number of collisions. A small effective cross section corresonds to a large value of the dispersion.

$$\langle N(N-1)\rangle = \langle N\rangle^2 \frac{\sigma_{hard}}{\sigma_{eff}}$$

In the Poissonian model one may obtain a small σ_{eff} by choosing a proper analytic form for the parton density.

A more natural possibility **to increase the dispersion** is perhaps to consider a different distribution in the number of collisions, which however implies that the **correlations in the multiparton distributions** play a non secondary role.

A source of correlations is the fluctuation of the hadron in different configurations, which is a phenomenon directly related to **hadron diffraction**

Hadron fluctuations may be implemented in a **multichannel eikonal model** of high energy hadronic interactions. As a result the approach gives a rather natural **generalization of the Poissonian model** of the hard cross section.

In the multichannel eikonal model the hadron state ψ_h is represented as a superposition of the eigenstates ϕ_i of the T matrix and the cross sections are combinations of the various cross sections σ_{ij}, which describe the interaction between the eigenstates ϕ_i and ϕ_j

$$\psi_h = \sum_i \alpha_i \phi_i$$

$$\begin{aligned} \sigma_{tot} &= \sum_{i,j} |\alpha_i|^2 |\alpha_j|^2 \sigma_{tot}^{ij} \\ \sigma_{el} + \sigma_{sd} + \sigma_{dd} &= \sum_{i,j} |\alpha_i|^2 |\alpha_j|^2 \sigma_{el}^{ij} \\ \sigma_{in} &= \sum_{i,j} |\alpha_i|^2 |\alpha_j|^2 \sigma_{in}^{ij} \end{aligned}$$

The hard cross section is analogously expressed as

$$\begin{aligned}\sigma_{hard} &= \sum_{i,j} |\alpha_i|^2 |\alpha_j|^2 \sigma^{ij}_{hard} = \sum_{i,j} |\alpha_i|^2 |\alpha_j|^2 \int d^2\beta \Big[1 - e^{-\sigma_S^{ij}(\beta)}\Big] \\ &= \sum_{i,j,N} |\alpha_i|^2 |\alpha_j|^2 \int d^2\beta \frac{\big(\sigma_S^{ij}(\beta)\big)^N}{N!} e^{-\sigma_S^{ij}(\beta)}\end{aligned}$$

which represents the obvious **generalization** of the expression obtained in the **Poissonian model**. The **increased dispersion**, with the consequent **decrease** of the value **of the effective cross section** as compared with the value obtained in the Poissonian model, is a natural feature of the multichannels.

For a qualitative understanding of the effect let us consider a simplest two channel toy model (**thanks to Mark Strikman**)

Let us assume that the hadron may appear with equal weights in two different configurations :

$$\psi_h = \frac{1}{\sqrt{2}}\phi_1 + \frac{1}{\sqrt{2}}\phi_2$$

The hard cross section σ_{hard} is hence given by

$$\sigma_{hard} = \frac{1}{4}\sigma_{hard}^{11} + \frac{1}{2}\sigma_{hard}^{12} + \frac{1}{4}\sigma_{hard}^{22}$$

and the expression of the N-parton scattering inclusive cross section σ_N is

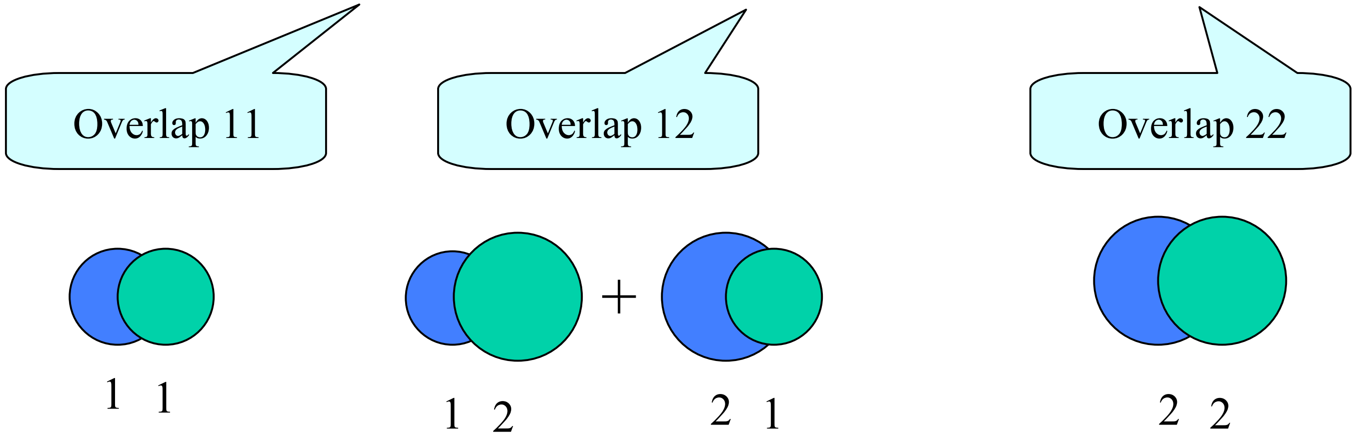
$$\sigma_N = \frac{\sigma_S^N}{N!}\left\{\frac{1}{4}\int \left[F_{11}(\beta)\right]^N d^2\beta + \frac{1}{2}\int \left[F_{12}(\beta)\right]^N d^2\beta + \frac{1}{4}\int \left[F_{22}(\beta)\right]^N d^2\beta\right\}$$

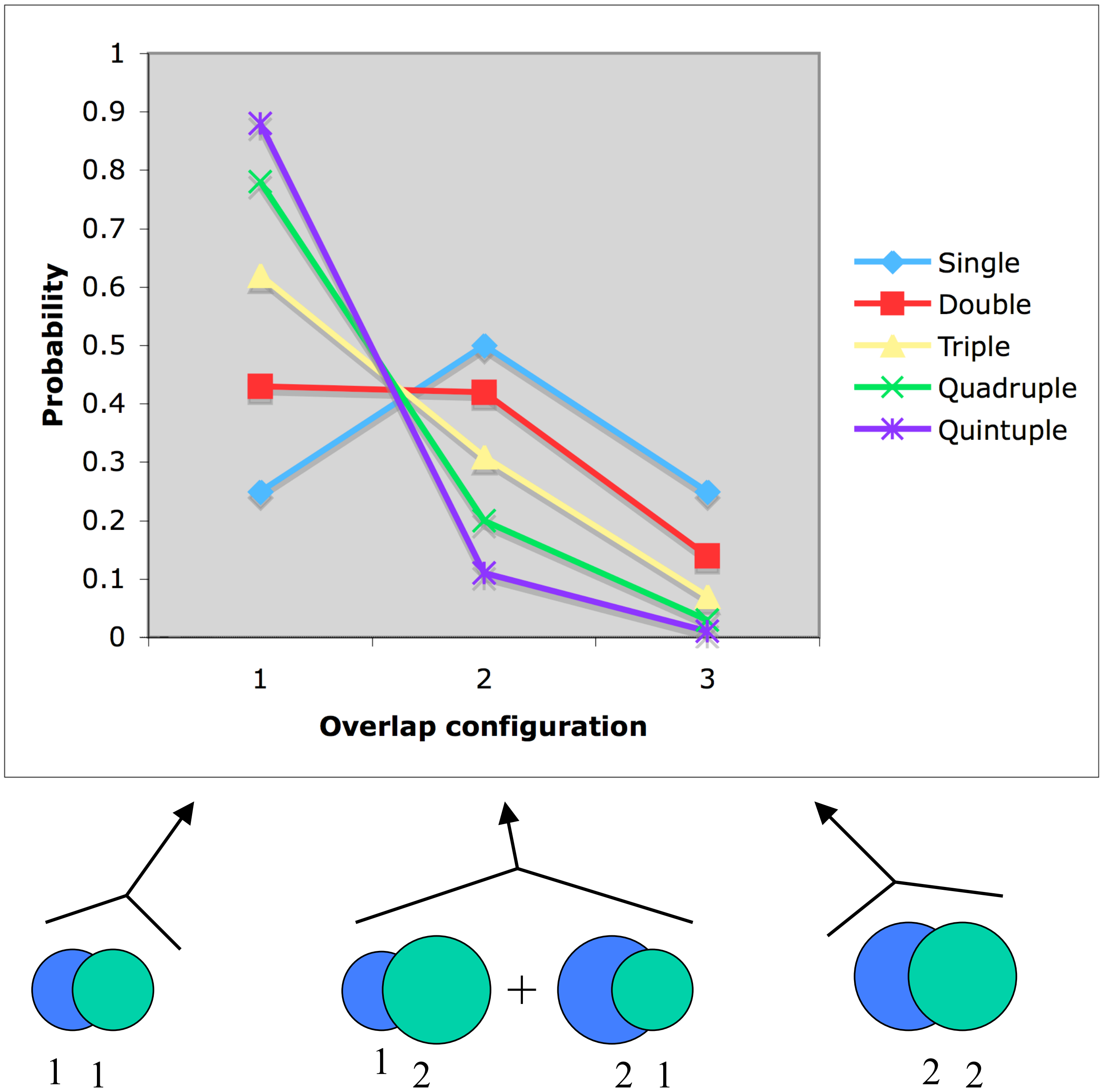
A simplest possibility to evaluate $\int [F_{ij}(\beta)]^N d^2\beta$ is to assume a Gaussian shape

$$F_{ij}(\beta) = \frac{1}{\pi(R_i^2 + R_j^2)} \times \exp\left(\frac{-\beta^2}{R_i^2 + R_j^2}\right)$$

while for the sizes of the two different configurations one may take $(R_1)^2 = R^2/2$ and $(R_2)^2 = 3R^2/2$, in such a way that the average hadron size is R^2.
The inclusive cross section σ_N is hence given by:

$$\sigma_N = \frac{\sigma_S^N}{N N!(\pi R^2)^{N-1}} \left\{ \frac{1}{4}\left(\frac{1}{\frac{1}{2}+\frac{1}{2}}\right)^{N-1} + \frac{1}{2}\left(\frac{1}{\frac{1}{2}+\frac{3}{2}}\right)^{N-1} + \frac{1}{4}\left(\frac{1}{\frac{3}{2}+\frac{3}{2}}\right)^{N-1} \right\}$$





Probability
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1
2
3
Overlap configuration
Single
Double
Triple
Quadruple
Quintuple
1 1
1 2
+
2 1
2 2

A simplest **eikonal model**, where all hadronic diffractive states are included in a single channel and which is capable of reproducing the total, elastic, single and double diffraction cross sections in high energy hadronic interactions is due to **Gotsman, Levin and Maor**

Utilizing the weights of the different channels, fitted to reproduce the total, elastic, single and double diffractive cross sections, one may estimate the value of the effective cross section at TeVatron and at the LHC

$$\sqrt(s) = 14\text{TeV} \quad \sigma_{tot} = 114\text{mb} \quad \sigma_{inel} = 71\text{mb} \quad \sigma_{eff} = 12\text{mb}$$
$$\sqrt(s) = 1.8\text{TeV} \quad \sigma_{tot} = 81\text{mb} \quad \sigma_{inel} = 50\text{mb} \quad \sigma_{eff} = 10\text{mb}$$

which is rather close to the experimental indication (at TeVatron about 11 mb)

Concuding summary

The **effective cross section** is related to the **typical transverse distance between partons** in the hadron structure. **A naïve estimate**, based on the Poissonian model of hadronic interactions, **gives a too large value** of the effective cross section, in comparison with the experimental indication.

The **inclusive cross sections** are given by **the moments of the distribution of multiple parton collisions**. The experiemental indication is hence that the **dispersion of the distribution is larger than expected naïvely**.

Keeping into account the **fluctuations of the hadron structure** one will obtain a **natural increase of the dispersion** of the distribution of multiple parton collisions.

Concuding summary II

While the form factor is an average over many **different hadronic configurations**, the experimental indication is hence that in a high energy hadronic collision different hadronic configurations **interact with different strengths**

Multiple parton interactions are more likely when the hadron fluctuates in a compact configuration. When looking at events with an increasingly large number of multi-parton interactions, the configurations with relatively small transverse dimensions are increasingly important.