Scaling laws from saturation with running coupling

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Outline

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Free partons *vs.* Saturation
Fixed coupling
Running coupling

- Introduction: gluon saturation.
- Geometric scaling from fixed coupling evolution equations with saturation.
- Scaling predictions from saturation at running coupling.

  based on G. B., arXiv:0803.2167

For the corresponding phenomenological study,
see David Šálek's talk,

based on G. B., R. Peschanski, C. Royon, D. Šálek, arXiv:0803.2186

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During the high-energy evolution (BFKL or saturation) of the proton's wave-function, low x partons are emitted by the larger x partons.

Low x gluons of transverse momentum k_T are radiated by a region of the transverse plane of size $R \propto (k_T)^{-1}$.

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Low x gluons of transverse momentum k_T are radiated by a region of the transverse plane of size $R \propto (k_T)^{-1}$.

For a dilute proton and/or for large k_T:
$R <$ typical transverse separation between partons.

$\Rightarrow$ the gluon is emitted by a single parton.

QCD parton model picture, associated with collinear and k_T factorizations.

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Low x gluons of transverse momentum k_T are radiated by a region of the transverse plane of size $R \propto (k_T)^{-1}$.

For a dense proton or nucleus and/or for small k_T:
$R >$ typical transverse separation between partons.

$\Rightarrow$ the gluon is emitted by a bunch of partons, screening partly each other.

Saturation $\Rightarrow$ need for arbitrary multi-parton distribution functions.

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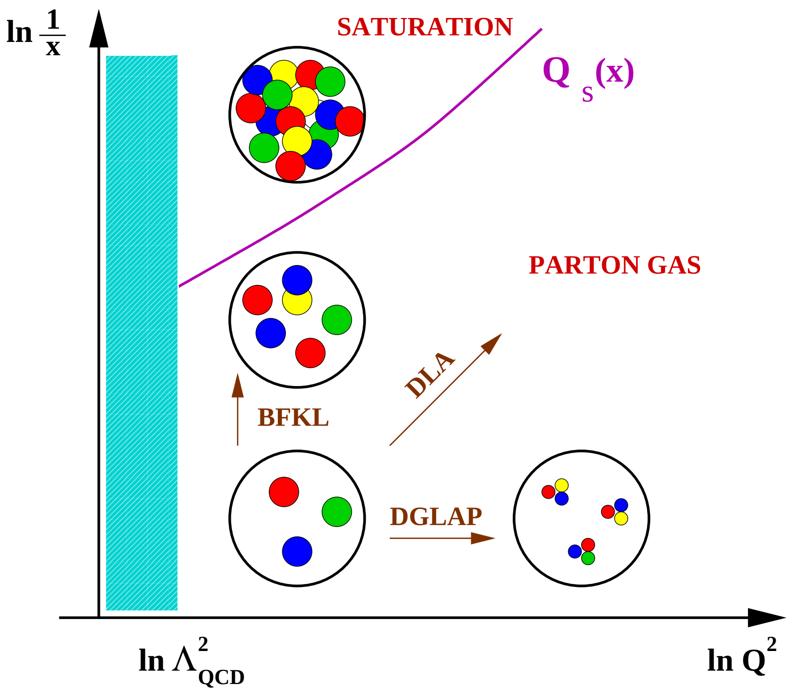

- Parton gas regime: linear, with incoherent free partons.
- Saturated regime: non-linear, with important parton correlations.

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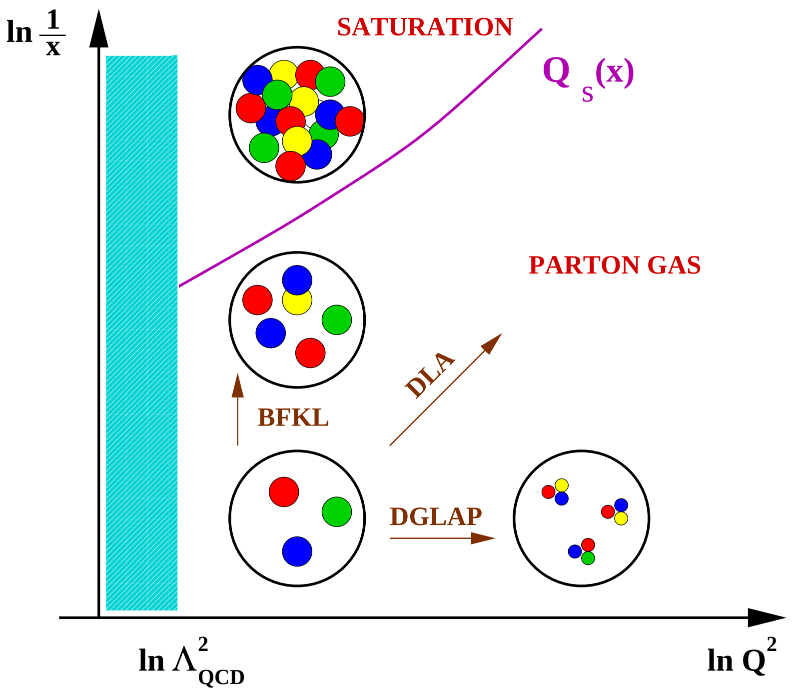

However, the very existence of saturation constrains the dynamics in the linear regime!

Dipole amplitude and BK equation

Dipole factorization of the DIS:

$$\sigma_{T,L}^{\gamma^* p}(Y,Q^2) = |\psi_{T,L}(Q^2,\boldsymbol{r})|^2 \otimes T(\boldsymbol{r},Y)$$

Fourier transform: $\boldsymbol{r} \mapsto \boldsymbol{k}$

$$T(\boldsymbol{r},Y) \mapsto N(L,Y)\text{ , with } L \equiv \log(\boldsymbol{k}^2/\Lambda_{QCD}^2)$$

Dipole amplitude and BK equation

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Balitsky-Kovchegov equation:

$$\partial_Y N(L, Y) = \bar{\alpha}\chi(-\partial_L) N(L, Y) - \bar{\alpha} N(L, Y)^2$$

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Generic saturation equation extending the BFKL equation:

$$\partial_Y N(L,Y) = \bar{\alpha}\chi(-\partial_L)N(L,Y) - \text{Non linear damping}$$

Universality

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Solution of the BFKL equation:

$$N(L,Y) = \int \frac{d\gamma}{2\pi i}\, e^{-(\gamma L - \chi(\gamma)\bar{\alpha} Y)}\, N_0(\gamma)$$

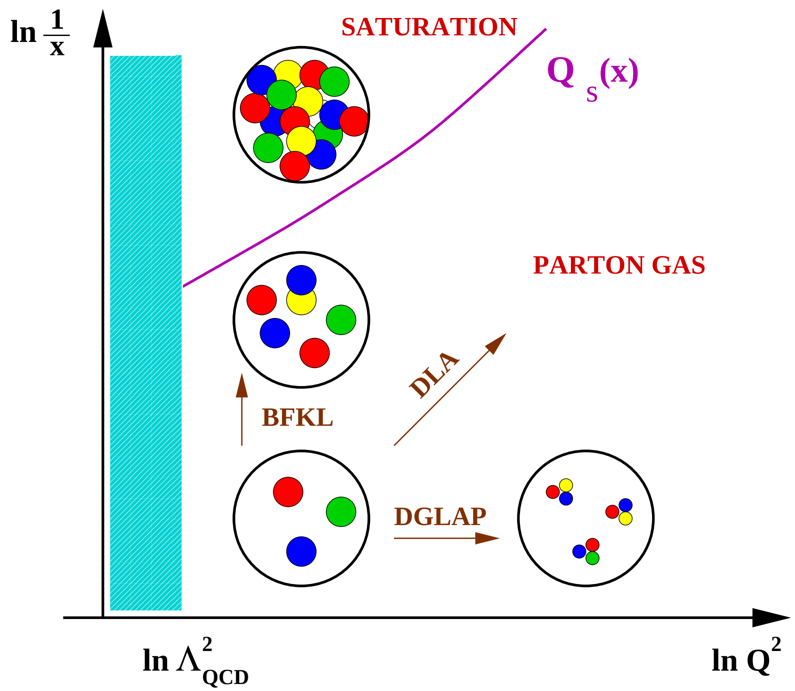
Sum of scaling solutions with different parameters: no scaling in general.

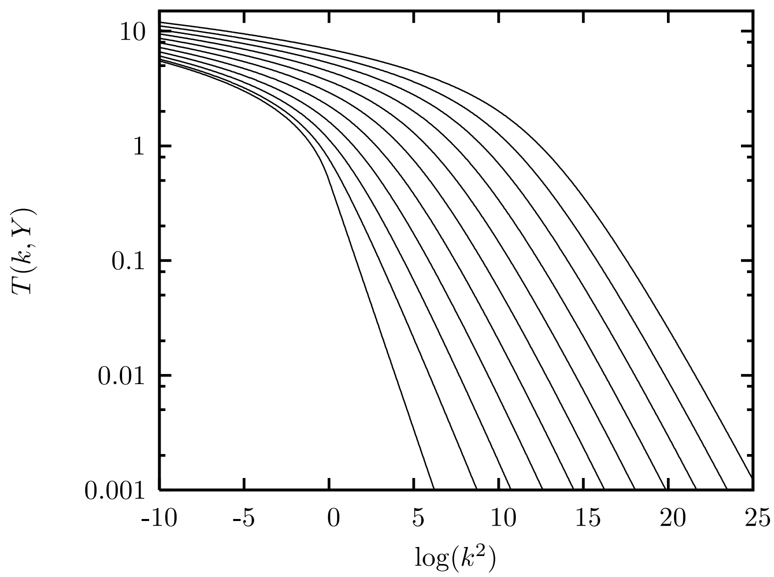
The existence of the nonlinear damping selects dynamically the wave solution with $\gamma = \gamma_c = 0.63$ (defined by $\chi(\gamma_c) = \gamma_c\, \chi'(\gamma_c)$):

$$N(L,Y) \propto e^{-(\gamma_c L - \bar{\alpha}\chi(\gamma_c) Y)}$$

$\Rightarrow$ Geometric scaling.

Traveling wave and geometric scaling





Geometric scaling ↔ traveling wave solution of fixed coupling evolution equation with saturation.

Saturation with running coupling

Running coupling prescription: $\bar{\alpha} \mapsto \bar{\alpha}(L) = 1/bL$

Saturation equation with running coupling:

$$\partial_Y N(L,Y) = \frac{1}{bL}\,\chi(-\partial_L)N(L,Y) - \text{Non linear damping}$$

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Saturation equation with running coupling:

$$\partial_Y N(L,Y) = \frac{1}{bL}\, \chi(-\partial_L) N(L,Y) - \text{Non linear damping}$$

With higher order contributions:

$$\begin{aligned}\partial_Y N(L,Y) \quad &= \quad \frac{1}{bL}\, \chi(-\partial_L)\, N(L,Y) \\ &+ \frac{1}{(bL)^2}\, \chi_{NLL}(-\partial_L)\, N(L,Y) + \cdots \\ &- \text{Non linear damping}\end{aligned}$$

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Impossible to diagonalize exactly the BFKL kernel with running coupling.

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Impossible to diagonalize exactly the BFKL kernel with running coupling.

However, in the relevant kinematical range $Y \propto L^2 \gg 1$:

Family of possible approximate scaling variables:

$$s(L,Y) = \frac{L}{2\delta} - \frac{1}{2\delta L^{2\delta-1}} \left(\frac{v(Y-Y_0)}{b} \right)^{\delta}$$

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Asymptotically, changing δ doesn't change the scaling behavior.

But at finite L and Y, different δ give different scaling properties.

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The nonlinear front formation mechanism will select one specific value of v, but leave δ and Y_0 undetermined.

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$$bL\,\partial_Y N(L,Y) = \chi(-\partial_L)N(L,Y) - N(L,Y)^2$$

Scaling solution: $N(L,Y) \equiv N_s(s(L,Y))$

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Sufficient conditions:

$$\begin{aligned} bL\,\partial_Y s(L,Y) &= f_1(s) \\ \partial_L s(L,Y) &= f_2(s) \end{aligned}$$

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$\rightarrow$ Incompatible conditions, as they imply

$$\partial_L \partial_Y s(L,Y) \neq \partial_Y \partial_L s(L,Y)\,.$$

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Scaling laws

$$bL\,\partial_Y N(L,Y) = \chi(-\partial_L)N(L,Y) - N(L,Y)^2$$

Scaling solution: $N(L,Y) \equiv N_s(s(L,Y))$

The variable with generic δ verifies approximately the conditions:

$$\begin{aligned} bL\,\partial_Y s(L,Y) &\simeq f_1(s) \\ \partial_L s(L,Y) &\simeq f_2(s) \end{aligned}$$

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For $\delta = 1/2$:

$$
\begin{aligned}
bL\, \partial_Y s(L,Y) &\simeq f_1(s) \\
\partial_L s(L,Y) &= f_2(s)
\end{aligned}
$$

$\longrightarrow$ RC geometric scaling: $s(L,Y) = L - \sqrt{v\, \frac{(Y-Y_0)}{b}}$

Called *RC1* scaling in David Šálek's talk.

Scaling laws

For $\delta = 1$:

$$\begin{aligned} bL\,\partial_Y s(L,Y) &= f_1(s) \\ \partial_L s(L,Y) &\simeq f_2(s) \end{aligned}$$

$\rightarrow$ other RC scaling variable: $s(L,Y) = \frac{L}{2} - v\,\frac{(Y-Y_0)}{2bL}$

Called *RC2* scaling in David Šálek's talk.

RC traveling wave solution

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Approximate solution in the range $L \gg 1$ and $1 \lesssim \bar{s} \ll \sqrt{L}$:

$$N(L,Y) \propto e^{-\gamma_c \bar{s} + \mathcal{O}(\log L)} \, \mathrm{Ai}\left(\xi_1 + \frac{\bar{s}}{(DL)^{1/3}}\right) \left[1 + \mathcal{O}\left(L^{-1/3}\right)\right]$$

$$\bar{s} = \frac{L}{2\delta} - \frac{1}{2\delta L^{2\delta-1}} \left(\frac{v_c Y}{b}\right)^{\delta} - \frac{3\xi_1}{4}(DL)^{1/3}$$

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Leading behavior: RC scaling law with δ.

Saturation critical exponent: γ_c, solution of $\chi(\gamma_c) = \gamma_c\, \chi'(\gamma_c)$.

Critical velocity: $v_c = \frac{2\chi(\gamma_c)}{\gamma_c}$.

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Universal scaling violations:

- Partly absorbed by a redefinition of the scaling law.
- Scaling violations from BFKL diffusion remains, with: $D = \frac{\chi''(\gamma_c)}{2\gamma_c \chi'(\gamma_c)}$

RC traveling wave solution

Approximate solution in the range $L \gg 1$ and $1 \lesssim \bar{s} \ll \sqrt{L}$:

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$$\bar{s} = \frac{L}{2\delta} - \frac{1}{2\delta L^{2\delta-1}}\left(\frac{v_c Y}{b}\right)^{\delta} - \frac{3\xi_1}{4}(DL)^{1/3}$$

For any value of δ, one gets the same asymptotic expression for the saturation scale:

$$\log\left(\frac{Q_s^2(Y)}{\Lambda_{QCD}^2}\right) = \sqrt{\frac{v_c Y}{b}} + \frac{3\xi}{4}\left(D\sqrt{\frac{v_c Y}{b}}\right)^{1/3} + \mathcal{O}(\log Y).$$

Summary

The high density regime is dominated by saturation, but saturation also influences the dilute regime.

The nonlinear wave front formation provides a natural explanation to scaling properties in low-x DIS, and works also with running coupling.

In the running coupling case, a whole family of scaling variables are possible, indexed by a parameter δ. δ is irrelevant asymptotically and theoretically undetermined. It may be relevant in the experimental kinematical range.

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Conclusion

If you have to remember one thing:

Even if the geometric scaling properties of the HERA data are due to saturation, it does not mean that the non-linear saturated regime has been probed at HERA and that the pdf determination at low-x suffers from saturation uncertainties.

Shape of the front

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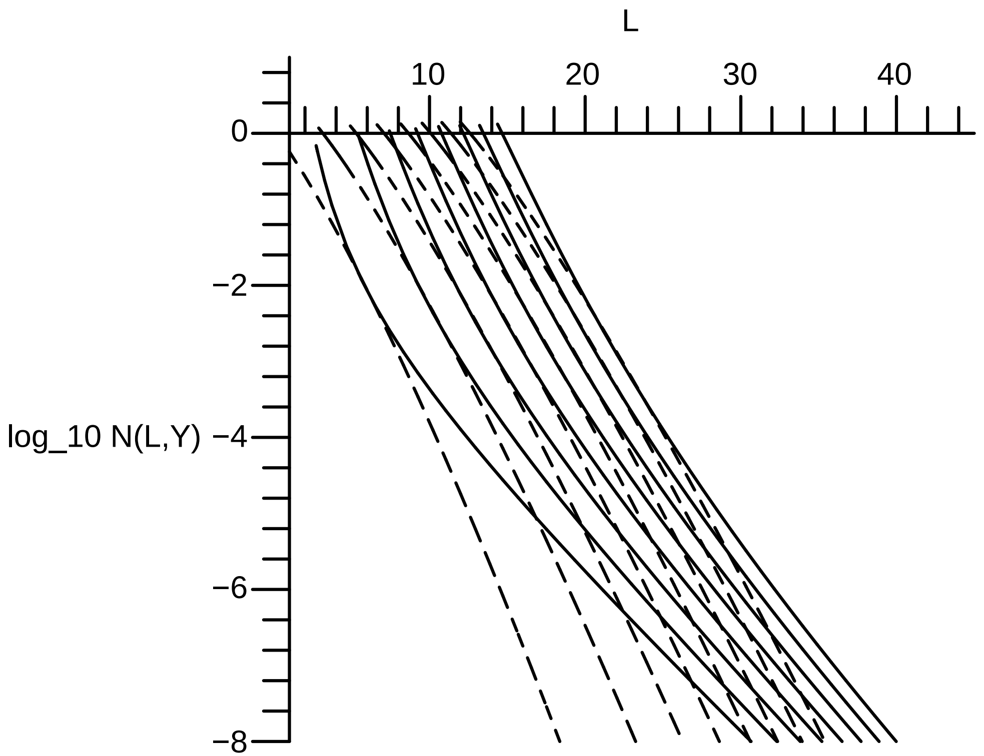

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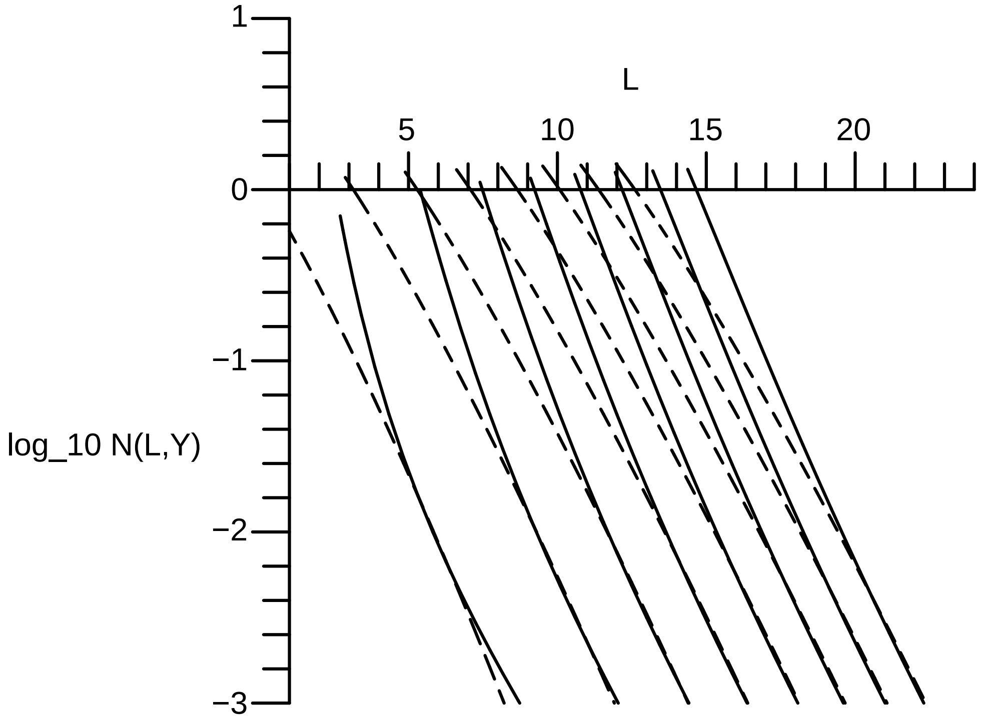
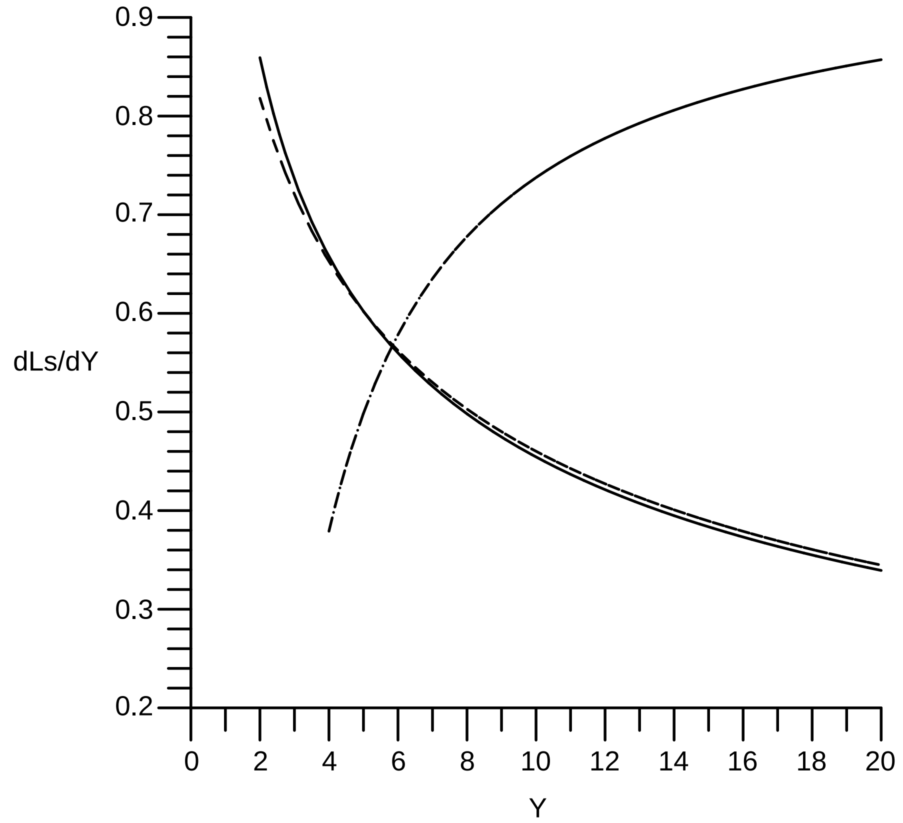


Saturation scale



$$\frac{\mathrm{d}\ L_s(Y)}{\mathrm{d}Y} = \frac{\mathrm{d}}{\mathrm{d}Y}\ \log\left(\frac{Q_s^2(Y)}{\Lambda_{QCD}^2}\right)$$