Heavy to Light
Transition Operator Matrix Elements at $O(a_s^2)$

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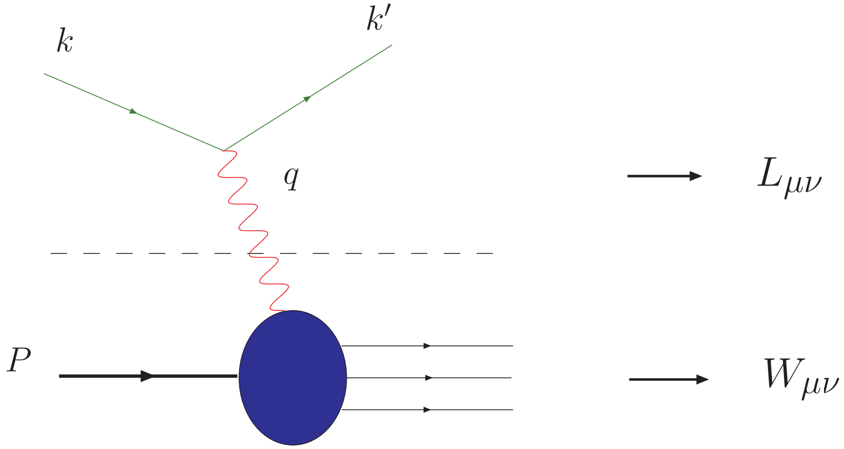
- Motivation
- HF parton distributions
- Heavy OMEs
- Beyond $O(a_s^2)$
- Conclusions

based on:

- I. Bierenbaum, J. Blümlein, S. K., and C. Schneider
  arXiv:0803.0273 [hep-ph].
- I. Bierenbaum, J. Blümlein, and S. K.,
  Phys. Lett. **B648** (2007) 195;
  Nucl. Phys. **B780** (2007) 40;
  Acta Phys. Polon. B **38** (2007) 3543;
- J. Blümlein, A. De Freitas, W.L. van Neerven, and S. K.,
  Nucl. Phys. **B755** (2006) 272.

1. Motivation

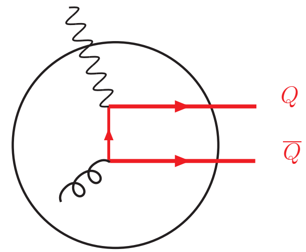
Deep–Inelastic Scattering (DIS):



$$Q^2 := -q^2, \quad x := \frac{Q^2}{2pq} \quad \text{Bjorken–x}$$

$$\frac{d\sigma}{dQ^2\, dx} \sim W_{\mu\nu} L^{\mu\nu}$$

LO:



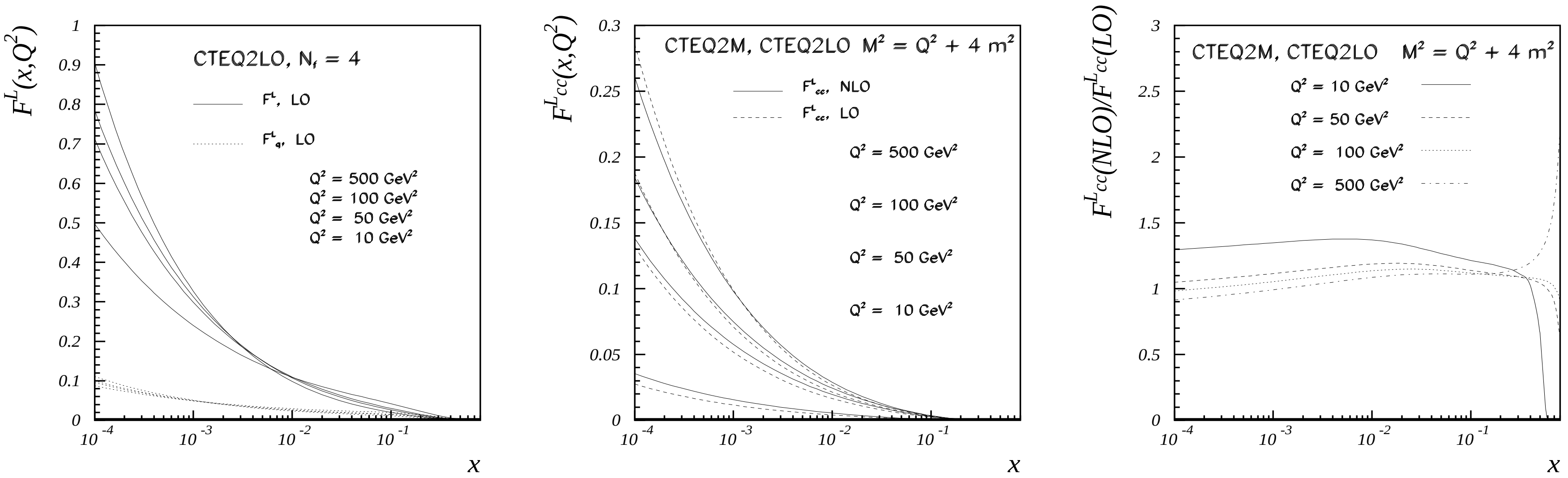
Hadronic tensor for heavy quark production via single photon exchange:

$$W_{\mu\nu}^{Q}(q,P,s) = \frac{1}{4\pi} \int d^4\xi \exp(iq\xi) \langle P, s \mid [J_\mu^{em}(\xi), J_\nu^{em}(0)] \mid P, s\rangle_Q$$

$$\text{unpol.} \left\{ \quad = \frac{1}{2x}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) F_L^Q(x,Q^2) + \frac{2x}{Q^2}\left(P_\mu P_\nu + \frac{q_\mu P_\nu + q_\nu P_\mu}{2x} - \frac{Q^2}{4x^2} g_{\mu\nu}\right) F_2^Q(x,Q^2) \right.$$

$$\text{pol.} \left\{ \quad -\frac{M}{2Pq} \varepsilon_{\mu\nu\alpha\beta} q^\alpha \left[s^\beta g_1^Q(x,Q^2) + \left(s^\beta - \frac{sq}{Pq} p^\beta\right) g_2^Q(x,Q^2) \right] \; . \right.$$

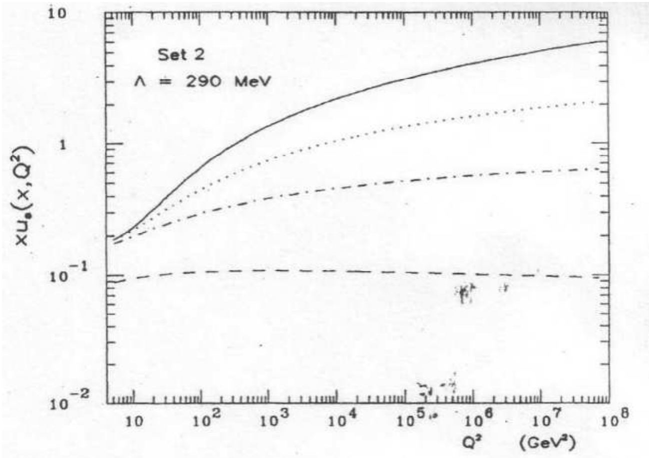
- Heavy flavor (charm) contributions to DIS structure functions are rather large [20–40 % at lower values of x] .



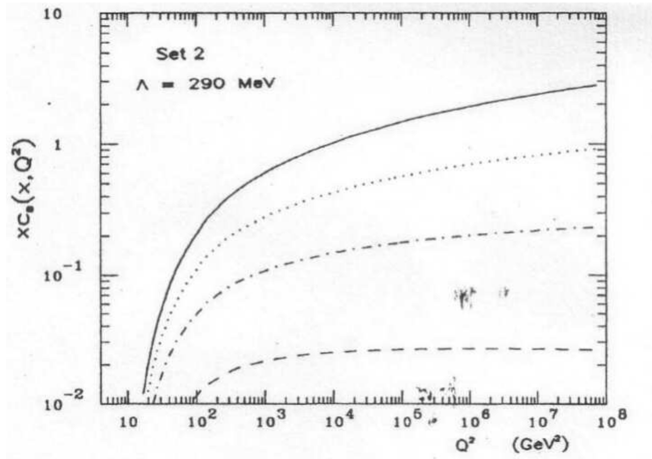
[Blümlein and Riemersma, 1996]

Goals

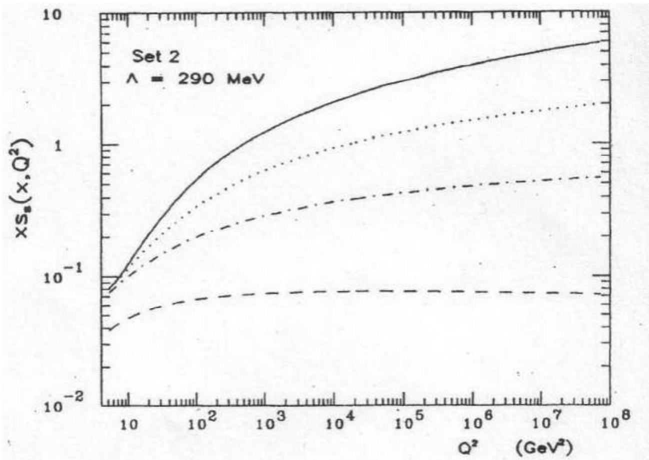
- Derivation of variable flavor number scheme (VFNS) for heavy quarks to $O(a_s^3)$.
  - Calculation of the heavy to light transition functions to define parton densities in a VFNS for heavy quarks [needed for HERA and LHC] .
- Calculation of the heavy flavor Wilson coefficients to higher orders for $Q^2 \geq 25\,\mathrm{GeV}^2$:
  - Increase in accuracy of the perturbative description of DIS structure functions.
  - $\Longleftrightarrow$ QCD analysis and determination of Λ_{QCD} .
  - $\Longleftrightarrow$ Precise determination of the gluon and sea quark distributions.



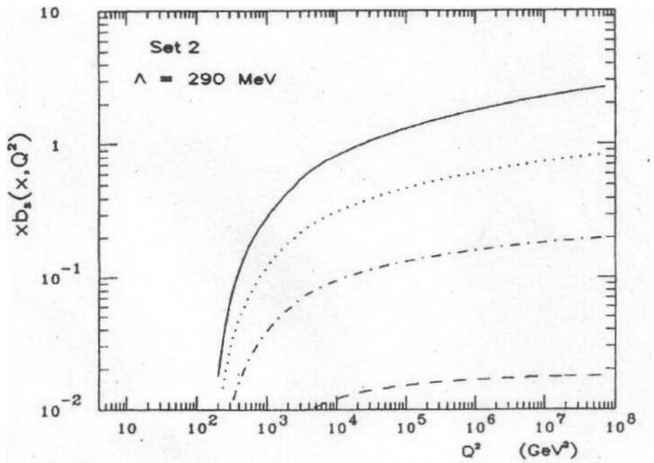
Up-sea



Charm-sea



Strange-sea



Bottom-sea

E.g. LO–QCD: [Eichten, Hinchliffe, Lane and Quigg, 1984]

Need:

- Heavy Flavor initial state parton densities for the LHC. E.g. for $c\,\overline{s} \rightarrow W^+$, etc.
- Monte–Carlo generators at the LHC.
- Physics beyond the Standard Modell (SUSY–processes).

- Heavy quark contributions given by heavy quark Wilson coefficient:

$$H_{(2,L),i}^{\rm S,NS}\left(\frac{Q^2}{\mu^2},\frac{m^2}{Q^2}\right)\quad(\gamma^*+Q),\qquad L_{(2,L),i}^{\rm S,NS,PS}\left(\frac{Q^2}{\mu^2},\frac{m^2}{Q^2}\right)\quad(\gamma^*+q)\ .$$

- In the limit $Q^2 \gg m_Q^2$ [$Q^2 \approx 10\ m_Q^2$ for F_2]:
massive RGE, derivative $m^2\partial/\partial m^2$ acts on Wilson coefficients only: all terms but power corrections calculable through partonic operator matrix elements, $\langle i|A_l|j\rangle$, which are process independent objects! E.g. for H one obtains:

$$H_{(2,L),i}^{\rm S,NS}\left(\frac{Q^2}{\mu^2},\frac{m^2}{\mu^2}\right) = \underbrace{A_{k,i}^{\rm S,NS}\left(\frac{m^2}{\mu^2}\right)}_{\text{massive OMEs}} \otimes \underbrace{C_{(2,L),k}^{\rm S,NS}\left(\frac{Q^2}{\mu^2}\right)}_{\text{light–parton–Wilson coefficients}}\ ,$$

$O(a_s^3)$: [Moch, Vermaseren, Vogt, 2005.]

$$A_{k,i}^{\rm S,NS}\left(\frac{m^2}{\mu^2}\right) = \langle i|O_k^{\rm S,NS}|i\rangle = \delta_{k,i} + \sum_{l=1}^{\infty} a_s^l A_{k,i}^{{\rm S,NS},(l)}\left(\frac{m^2}{\mu^2}\right),\qquad i=q,g\ .$$

[Buza, Matiounine, Migneron, Smith, van Neerven, 1996; Buza, Matiounine, Smith, van Neerven, 1997.]

- Heavy OMEs are the transition functions to define a VFNS starting from a fixed flavor number scheme(FFNS).

[Buza, Matiounine, Smith, van Neerven, 1998; Chuvakin, Smith, van Neerven, 1998.]

(Some) Previous Works:

Fixed–flavor number scheme: General starting point.

Variable flavor number scheme at LO: [Aivazis, Collins, Olness, Tung 1994.]

Variable flavor number scheme to all orders:

[Buza, Matiounine, Smith, van Neerven, 1998; Chuvakin, Smith, van Neerven 1998.]

Unpolarized heavy OMEs:

- NLO : [Buza, Matiounine, Smith, Migneron, van Neerven, 1996; Bierenbaum, Blümlein, S.K, 2007]

Polarized heavy OMEs:

- NLO : [Buza, Matiounine, Smith, van Neerven, 1997; Bierenbaum, Blümlein, S.K, 2008, to appear]

Mellin–space expressions: [Alekhin, Blümlein, 2003].

In the following, we report on results for a further step towards the asymptotic description of HQ effects in parton densities beyond NLO.

2. VFNS vs. FFNS

- In the QCD–improved parton model, the heavy flavor contributions to the structure functions can be written as

$$
\begin{aligned}
F_{i,Q}(n_f, Q^2, m^2) \quad = \quad & \sum_{i=1}^{n_f} e_k^2 \Biggl[\Sigma(n_f, \mu^2) \otimes \tilde{L}_{i,q}^{\mathsf{PS}}\left(n_f, \frac{Q^2}{m^2}, \frac{m^2}{\mu^2}\right) + G(n_f, \mu^2) \otimes \tilde{L}_{i,g}^{\mathsf{S}}\left(n_f, \frac{Q^2}{m^2}, \frac{m^2}{\mu^2}\right) \\
& + \Bigl\{ f_k(n_f, \mu^2) + f_{\overline{k}}(n_f, \mu^2) \Bigr\} \otimes L_{i,q}^{\mathsf{NS}}\left(n_f, \frac{Q^2}{m^2}, \frac{m^2}{\mu^2}\right) \Biggr] \\
& + e_Q^2 \Biggl[\Sigma(n_f, \mu^2) \otimes H_{i,q}^{\mathsf{PS}}\left(n_f, \frac{Q^2}{m^2}, \frac{m^2}{\mu^2}\right) + G(n_f, \mu^2) \otimes H_{i,g}^{\mathsf{S}}\left(n_f, \frac{Q^2}{m^2}, \frac{m^2}{\mu^2}\right) \Biggr] , \\
(L \quad = \quad & n_f \tilde{L} \; .)
\end{aligned}
$$

- The singlet light flavor densitity is:

$$
\Sigma(n_f, \mu^2) \quad = \quad \sum_{i=1}^{n_f} \left(f_i(n_f, \mu^2) + \bar{f}_i(n_f, \mu^2) \right) ,
$$

FFNS:

- One works in fixed order perturbation theory and assumes a fixed number of light partons in the proton (g,u,d,s).
- The heavy quarks (charm) are produced extrinsically only.
- In the asymptotic limit $Q^2 \gg m_c^2$, the heavy flavor coefficient functions can be expressed as

$$H_{k,i}\left(n_f, \frac{Q^2}{m_c^2}, \frac{m_c^2}{\mu^2}\right) = A_{ji}\left(n_f, \frac{m_c^2}{\mu^2}\right) \otimes C_{k,j}\left(n_f + 1, \frac{Q^2}{\mu^2}\right) .$$

- Gives a good description of F_2 for $Q^2 \geq 10m_c^2$.
- The large logarithmic terms in the heavy quark coefficient functions entirely determine the charm component of the structure function for large values of Q^2.
- Since these corrections spoil the perturbation series when Q^2 gets large they should be resummed in all orders of perturbation theory.

VFNS:

- Resum the large logarithms using mass factorization.
- One requires that the following relation holds

$$F_i(n_f, Q^2) + \lim_{Q^2 \gg m^2} \Big[F_{i,Q}(n_f, Q^2, m^2) \Big] = F_i^{VFNS}(n_f + 1, Q^2) \ .$$

- Remove the mass singular terms from the asymptotic heavy quark coefficient functions and absorb them into parton densities.
- New parton density appears corresponding to the heavy quark, which is now treated as light (massless).
  $\Longrightarrow$ Relations between parton densities for n_f and $n_f + 1$ flavors.
- Is different to the picture of the charm being produced intrinsically.

- The original light flavor densities get modified such that

$$
\begin{aligned}
f_k(n_f+1,\mu^2)+f_{\overline{k}}(n_f+1,\mu^2) &= A_{qq,Q}^{\mathsf{NS}}\left(n_f,\frac{m^2}{\mu^2}\right)\otimes\left[f_k(n_f,\mu^2)+f_{\overline{k}}(n_f,\mu^2)\right] \\
&\quad +\tilde{A}_{qq,Q}^{\mathsf{PS}}\left(n_f,\frac{m^2}{\mu^2}\right)\otimes\Sigma(n_f,\mu^2) \\
&\quad +\tilde{A}_{qg,Q}^{\mathsf{S}}\left(n_f,\frac{m^2}{\mu^2}\right)\otimes G(n_f,\mu^2)\ .
\end{aligned}
$$

- The new charm density is given by

$$
\begin{aligned}
f_{Q+\bar{Q}}(n_f+1,\mu^2) &\equiv f_{n_f+1}(n_f+1,\mu^2)+f_{\overline{n_f+1}}(n_f+1,\mu^2) \\
&= \tilde{A}_{Qq}^{\mathrm{PS}}\left(n_f,\frac{\mu^2}{m^2}\right)\otimes\Sigma(n_f,\mu^2)+\tilde{A}_{Qg}^{\mathrm{S}}\left(n_f,\frac{\mu^2}{m^2}\right)\otimes G(n_f,\mu^2)\ .
\end{aligned}
$$

- The singlet combination of the quark densities becomes

$$\begin{aligned}\Sigma(n_f+1,\mu^2) &= \sum_{k=1}^{n_f+1}\Big[f_k(n_f+1,\mu^2)+f_{\bar{k}}(n_f+1,\mu^2)\Big] \\ &= \left[A_{qq,Q}^{\rm NS}\left(n_f,\frac{\mu^2}{m^2}\right)+n_f\tilde{A}_{qq,Q}^{\rm PS}\left(n_f,\frac{\mu^2}{m^2}\right)+\tilde{A}_{Qq}^{\rm PS}\left(n_f,\frac{\mu^2}{m^2}\right)\right]\otimes\Sigma(n_f,\mu^2) \\ &\quad +\left[n_f\tilde{A}_{qg,Q}^{\rm S}\left(n_f,\frac{\mu^2}{m^2}\right)+\tilde{A}_{Qg}^{\rm S}\left(n_f,\frac{\mu}{m^2}\right)\right]\otimes G(n_f,\mu^2)\ .\end{aligned}$$

- The non-singlet combination $\Delta_k(n_f+1)$ reads

$$\Delta_k(n_f+1,\mu^2)=f_k(n_f+1,\mu^2)+f_{\bar{k}}(n_f+1,\mu^2)-\frac{1}{n_f+1}\Sigma(n_f+1,\mu^2)\ .$$

- The gluon density for n_f+1 light flavours is

$$\begin{aligned}G(n_f+1,\mu^2) &= A_{gq,Q}^{\rm S}(n_f,\mu^2)\otimes\Sigma(n_f,\mu^2) \\ &\quad +A_{gg,Q}^{S}(n_f,\mu^2)\otimes G(n_f,\mu^2)\ .\end{aligned}$$

- The old as well as the new parton densities have to satisfy the momentum sum rule

$$\Sigma(n_f, \mu^2, N = 2) + G(n_f, \mu^2, N = 2) = 1 \ .$$

- Thus one obtains the following sum rules for the OMEs A_{Qk}, $A_{kl,Q}$:

$$\begin{aligned} A_{qq,Q}^{\rm NS}\Big(N=2\Big) + n_f \tilde{A}_{qq,Q}^{\rm PS}\Big(N=2\Big) + \tilde{A}_{Qq}^{\rm PS}\Big(N=2\Big) + A_{gq,Q}^{\rm S}\Big(N=2\Big) &= 1 \ , \\ n_f \tilde{A}_{qg,Q}^{\rm S}\Big(N=2\Big) + \tilde{A}_{Qg}^{\rm S}\Big(N=2\Big) + A_{gg,Q}^{\rm S}\Big(N=2\Big) &= 1 \ . \end{aligned}$$

$\Longrightarrow$ Sum rules used as check on our results.

3. Renormalization

$$\hat{\hat{A}}_{ij} = \delta_{ij} + \sum_{k=0}^{\infty} \hat{a}_s^k \hat{\hat{A}}_{ij}^{(k)}$$

need for:

- Mass renormalization (on–mass shell scheme)
- Charge renormalization
- Renormalization of ultraviolet singularities
  $\Longrightarrow$ are absorbed into Z–factors given in terms of anomalous dimensions γ_{ij}.
- Factorization of collinear singularities
  $\Longrightarrow$ are factored into Γ–factors Γ_{NS}, $\Gamma_{ij,S}$ and $\Gamma_{qq,PS}$.
  For massless quarks it would hold: $\Gamma = Z^{-1}$.
  Here: Γ–matrices apply to parts of the diagrams with massless lines only .

$\rightarrow$ use $\overline{\text{MS}}$ scheme and decoupling formalism [Ovrut, Schnitzer 1981; Bernreuther, Wetzel 1982].

Since the light-cone expansion is used, external legs obtain self-energy insertions due to heavy quarks.

The renormalized operator matrix elements are obtained removing the ultraviolet singularities and collinear singularities of the operator matrix elements,

$$\begin{pmatrix} A_{Qq}^{\mathsf{PS}} & A_{Qg} \\ A_{gq,Q} & A_{gg,Q} \end{pmatrix} = \begin{pmatrix} Z_{qq} & Z_{qg} \\ Z_{gq} & Z_{gg} \end{pmatrix}^{-1} \begin{pmatrix} \hat{A}_{Qq}^{\mathsf{PS}} & \hat{A}_{Qg} \\ \hat{A}_{gq,Q} & \hat{A}_{gg,Q} \end{pmatrix} \begin{pmatrix} \Gamma_{qq} & \Gamma_{qg} \\ \Gamma_{gq} & \Gamma_{gg} \end{pmatrix}^{-1}$$

$$A_{qq,Q}^{\mathsf{NS}} = Z_{qq}^{-1,\mathsf{NS}} \hat{A}_{qq,Q}^{\mathsf{NS}} \Gamma_{qq}^{-1,\mathsf{NS}} .$$

- For example:

$$A_{Qg}^{(2)} = \hat{A}_{Qg}^{(2)} + Z_{qq}^{-1,(1)} \hat{A}_{Qg}^{(1)} + Z_{qg}^{-1,(1)} \hat{A}_{gg,Q}^{(1)} + Z_{qg}^{-1,(2)} + \left(\hat{A}_{Qg}^{(1)} + Z_{qg}^{-1,(1)} \right) \Gamma_{gg}^{-1,(1)} .$$

- Mixing in the singlet sector.
- From terms

$$Z_{ij}^{-1,(1)} \hat{A}_{ij}^{(2)}$$

stem contributions of higher orders in ε to the renormalization of the 3–Loop heavy OMEs.
$\Longrightarrow$ 2–Loop OMEs have to be calculated up to $O(\varepsilon)$.

4. Results

- Calculation in Mellin-space for space–like q^2, $Q^2 = -q^2$: $\quad 0 \le x \le 1$
- use of generalized hypergeometric functions for general analytic results
- Summation of new infinite one-parameter sums into harmonic sums. Use of integral techniques and the Mathematica package SIGMA [C. Schneider, 2007], [I. Bierenbaum, J. Blümlein, S. K., C. Schneider, arXiv:0707.4659 [math-ph]; arXiv:0803.0273 [hep-ph]]
- Partial checks for fixed values of N using SUMMER, [Vermaseren, Int. J. Mod. Phys. A **14** (1999)].
- Check using sum rules for $N = 2$. $\Longrightarrow$ Agreement. (Even on the unrenormalized level to higher orders in ε).
- Algebraic and structural simplification of the harmonic sums [J. Blümlein, 2003, 2007].
- Partial checks for fixed values of N using MATAD [Steinhauser, 2001].

$\hat{A}_{gg,Q}^{(2)}$ unpolarized

$$
\begin{aligned}
\hat{A}_{gg,Q}^{(2)} &= \\
& T_F C_A \Biggl\{ \frac{1}{\varepsilon^2}\left(-\frac{32}{3}S_1 + \frac{64(N^2+N+1)}{3(N-1)N(N+1)(N+2)}\right) + \frac{1}{\varepsilon}\left(-\frac{80}{9}S_1 + \frac{16P_1}{9(N-1)N^2(N+1)^2(N+2)}\right) \\
& + \left(-\frac{8}{3}\zeta_2 S_1 + \frac{16(N^2+N+1)\zeta_2}{3(N-1)N(N+1)(N+2)} - 4\frac{56N+47}{27(N+1)}S_1 + \frac{2P_3}{27(N-1)N^3(N+1)^3(N+2)}\right) \\
& + \varepsilon\Biggl(-\frac{8}{9}\zeta_3 S_1 - \frac{20}{9}\zeta_2 S_1 + \frac{16(N^2+N+1)}{9(N-1)N(N+1)(N+2)}\zeta_3 + \frac{2N+1}{3(N+1)}S_2 - \frac{S_1^2}{3(N+1)} \\
& + \frac{4P_1\zeta_2}{9(N-1)N^2(N+1)^2(N+2)} - 2\frac{328N^4+256N^3-247N^2-175N+54}{81(N-1)N(N+1)^2}S_1 + \frac{P_5}{81(N-1)N^4(N+1)^4(N+2)}\Biggr)\Biggr\} \\
& + T_F C_F \Biggl\{ \frac{1}{\varepsilon^2}\left(\frac{16(N^2+N+2)^2}{(N-1)N^2(N+1)^2(N+2)}\right) + \frac{1}{\varepsilon}\left(\frac{4P_2}{(N-1)N^3(N+1)^3(N+2)}\right) \\
& + \left(\frac{4(N^2+N+2)^2\zeta_2}{(N-1)N^2(N+1)^2(N+2)} - \frac{P_4}{(N-1)N^4(N+1)^4(N+2)}\right) \\
& + \varepsilon\left(\frac{4(N^2+N+2)^2\zeta_3}{3(N-1)N^2(N+1)^2(N+2)} + \frac{P_2\zeta_2}{(N-1)N^3(N+1)^3(N+2)} + \frac{P_6}{4(N-1)N^5(N+1)^5(N+2)}\right)\Biggr\}
\end{aligned}
$$

→ Result obtained in terms of Γ and Ψ functions to all orders in ε.

→ to $O(\varepsilon^0)$ agreement with van Neerven et al.

→ $O(\varepsilon)$ new.

$\hat{A}_{gq,Q}^{(2)}$ unpolarized

$$\hat{A}_{gq,Q}^{(2)} =$$

$$T_F C_F \Bigg\{ \frac{1}{\varepsilon^2} \left(\frac{32}{3} \frac{N^2+N+2}{(N-1)N(N+1)} \right) + \frac{1}{\varepsilon} \left(-\frac{16}{3} \frac{N^2+N+2}{(N-1)N(N+1)} S_1 + \frac{16}{9} \frac{13N^2+27N+8N^3+16}{(N-1)N(N+1)^2} \right)$$

$$+\frac{4}{3} \frac{N^2+N+2}{(N-1)N(N+1)} \left(2\zeta_2 + S_2 + {S_1}^2 \right) - \frac{8}{9} \frac{8N^3+13N^2+27N+16}{(N-1)N(N+1)^2} S_1 + \frac{8}{27} \frac{P_1}{(N-1)N(N+1)^3}$$

$$+\varepsilon \left(\frac{2}{9} \frac{N^2+N+2}{(N-1)N(N+1)} \left(-2S_3 - 3S_2 S_1 - {S_1}^3 + 4\zeta_3 - 6\zeta_2 S_1 \right) - \frac{4}{27} \frac{P_1 S_1}{(N-1)N(N+1)^3} \right.$$

$$\left. + \frac{2}{9} \frac{8N^3+13N^2+27N+16}{(N-1)N(N+1)^2} \left(2\zeta_2 + S_2 + {S_1}^2 \right) + \frac{4}{81} \frac{P_2}{(N-1)N(N+1)^4} \right) \Bigg\} .$$

→ Result obtained in terms of Γ functions to all orders in ε.

→ to $O(\varepsilon^0)$ agreement with van Neerven et al.

→ $O(\varepsilon)$ new .

5. Towards the calculation of $A_{ij,Q}^{(3)}$

Contributing OMEs:

Singlet	A_{Qg}	$A_{qg,Q}$	$A_{gg,Q}$	$A_{gq,Q}$	mixing
Pure–Singlet		$A_{Qq}^{\rm PS}$	$A_{qq,Q}^{\rm PS}$		
Non–Singlet	$A_{qq,Q}^{\rm NS,+}$	$A_{qq,Q}^{\rm NS,-}$	$A_{qq,Q}^{\rm NS,v}$		

- All 2–loop $O(\varepsilon)$–terms in the unpolarized case are known:

$$\overline{a}_{Qg}^{(2)}, \quad \overline{a}_{Qq}^{(2),\mathbf{PS}}, \quad \overline{a}_{gg,Q}^{(2)}, \quad \overline{a}_{gq,Q}^{(2)}, \quad \overline{a}_{qq,Q}^{(2),\mathbf{NS}}.$$

- Unpolarized anomalous dimensions are known up to $O(a_s^3)$ [Moch, Vermaseren, Vogt, 2004.]

  $\Longrightarrow$ All terms needed for the renormalization of unpolarized 3–Loop heavy OMEs are present.

  $\Longrightarrow$ Calculation will provide first independent checks on $\gamma_{qg}^{(2)}$, $\gamma_{qq}^{(2),\rm PS}$ and on respective color projections of $\gamma_{qq}^{(2),{\rm NS}\pm,{\rm v}}$, $\gamma_{gg}^{(2)}$ and $\gamma_{gq}^{(2)}$.

- Calculation proceeds in the same way in the polarized case. Known so far :

$$\Delta\overline{a}_{Qg}^{(2)}, \quad \Delta\overline{a}_{Qq}^{(2),\mathbf{PS}}, \quad \Delta\overline{a}_{qq,Q}^{(2),\mathbf{NS}} = \overline{a}_{qq,Q}^{(2),\mathbf{NS}} .$$

- Calculation of first moments using MATAD will soon be possible.

6. Conclusions

- QCD precision analyses require the description of the heavy quark contributions to 3–loop order.
- We calculated the massive operator matrix elements and newly presented first contributions to the $O(a_s^3)$ terms:
  - $\bar{a}_{Qg}^{(2)}$, $\bar{a}_{Qq}^{PS,(2)}$, $\bar{a}_{gg,Q}^{(2)}$, $\bar{a}_{qq}^{NS,(2)} = \Delta\bar{a}_{qq,Q}^{NS,(2)}$, $\bar{a}_{gq,Q}^{(2)}$
  - $\Delta\bar{a}_{Qg}^{(2)}$, $\Delta\bar{a}_{Qg}^{PS,(2)}$

  in the unpolarized and polarized case for general values of the Mellin variable.
- These terms contribute to the three–loop heavy flavor Wilson coefficients and the heavy to light transitions functions in a VFNS.
- We develop a programme–chain to calculate the massive operator matrix elements $A_{ij}^{(3)}$ for fixed Mellin moments based on QGRAF and MATAD
- We will report on first complete 3–loop results in the near future.