Unintegrated QCD NLO kernel in the numerical form

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PDFs and Monte Carlo event generators

- Parton shower MC (psMC) revives phase sp. hidden in the PDFs – it represents by construction a “fully untintegrated PDF” (funPDF).
- However, it does that ONLY in the (improved) LL approximation.
- None of the existing MCs perform QCD evolution by themselves, they all use pretabulated PDFs from the external programs/libs.
- One exception... Krakow MCs (KrkMC): MMC/CMC/KrCMC.
- Would it be worth/possible to construct a NLO level MC model for funPDF based on QCD (Feyman diags.) in a rigorous way? Definitely YES!! But it is very difficult/impossible ??

This is the next goal of KrkMC project!!

KrkMC introductory exercise towards NLO psMC

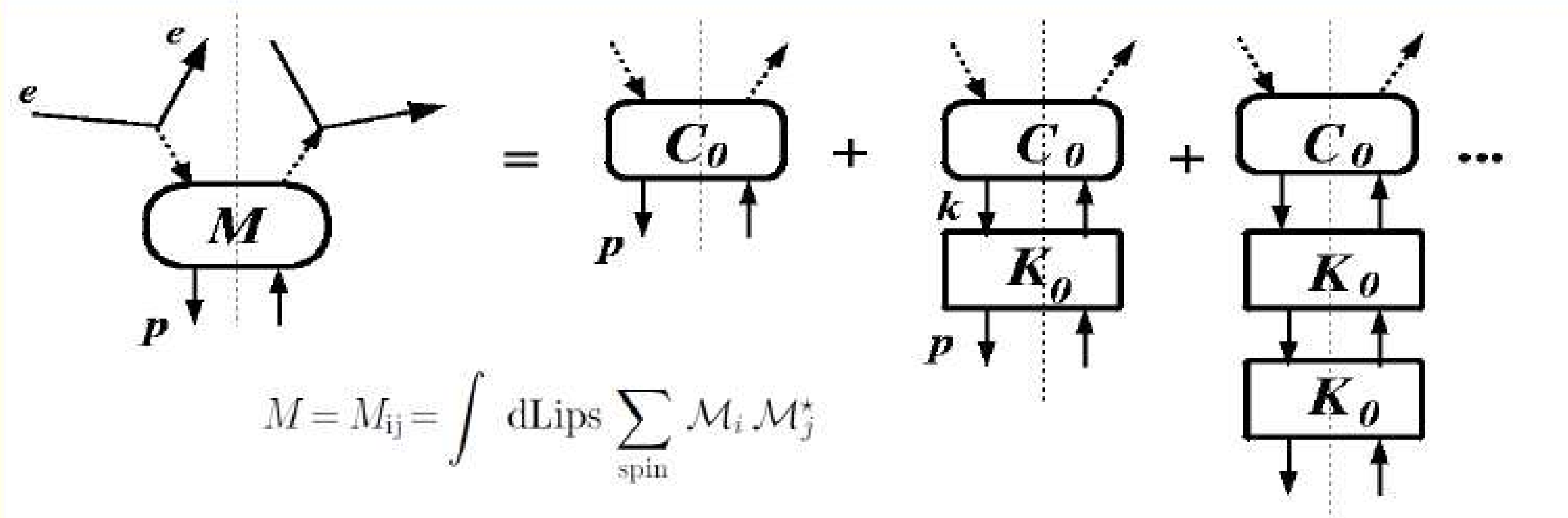
- DGLAP type of QCD evolution is an “industry standard” – we try to keep as close as possible to it, or keep the precise “bridge” to it.
- Recalculate NLO DGLAP kernels such that kernels come from numerical integration over the LIPS (already nontrivial!)
- Do the above using different parameterizations of the phase space
- Examine and remove the dimensional regularization from the game, MC has to be in $d = 4$, no way!
- Examine all internal IR singularities and various methods of regularizing/subtracting them
- Reinsert the above NLO unintegrated kernel into emission chain of the LL psMCs, such that MC performs NLO DGLAP evolution exactly (numerically 5-digit precision).
- Redo methodology of combining psMC and ME of hard process.

The starting point, the framework, the plan

- The NLO DGLAP QCD kernels were calculated directly from the Feynman diagrams by Curci, Furmanski and Pertronzio, 1979-82
- They did it using dimensional regularization ($\overline{MS}$), and axial gauge
- They exploited/extended "QCD factorization theorem" of EGMPR (Ellis, Georgi, Machacek, Politzer and Ross, PLB78, 1978).
  Later on CFP scheme refined by Collins, PRD58, 1998
- In the following we shall summarize briefly CFP/EGMPR scheme, and point out to its possible extensions/exploitations.
- Next, we shall demonstrate its (re)application to one particular contribution to the non-singlet kernel (double bremsstrahlung)
- "Numerical MC model" of unintegrated version of the NLO kernel (part of it) will be constructed, reproducing old integrated results
- Finally, numerical results visualizing singularity structure, change of the evolution variable, etc. will be shown

EGMPR scheme of collinear factorization (1978)

"Raw" factorization of the IR collinear singularities



$$M = M_{ij} = \int \mathrm{dLips} \sum_{\mathrm{spin}} \mathcal{M}_i \mathcal{M}_j^{\star}$$

- Cut vertex M: spin sums and Lips integrations over all lines cut across
- C_0 and K_0 and are 2-particle irreducible (2PI)
- C_0 is IR finite, while K_0 encapsulates all IR collinear singularities
- Use of the axial gauge essential for the proof
- Formal proof given in EGMPR NP B152 (1979) 285
- Notation next slide

$$M = C_0(1 + K_0 + K_0^2 + \cdots) = C_0 \frac{1}{1 - K_0} \equiv C_0 \Gamma_0$$

EGMPR scheme of collinear factorization (1978)

Factorization of EGMPR improved by Curci, Furmanski and Petronzio

$$F = C_0 \cdot \frac{1}{1-K_0} = C\left(\alpha, \frac{Q^2}{\mu^2}\right) \otimes \Gamma\left(\alpha, \frac{1}{\epsilon}\right),$$

$$= \left\{C_0 \cdot \frac{1}{1-(1-\mathbf{P})\cdot K_0}\right\} \otimes \left\{\frac{1}{1-\left(\mathbf{P}K_0 \cdot \frac{1}{1-(1-\mathbf{P})\cdot K_0}\right)}\right\}_\otimes$$

where

$$\Gamma\left(\alpha, \frac{1}{\epsilon}\right) \equiv \left(\frac{1}{1-K}\right)_\otimes = 1 + K + K \otimes K + K \otimes K \otimes K + ...,$$

$$K = \mathbf{P}K_0 \cdot \frac{1}{1-(1-\mathbf{P})\cdot K_0}, \quad C\left(\alpha, \frac{Q^2}{\mu^2}\right) = C_0 \cdot \frac{1}{1-(1-\mathbf{P})\cdot K_0}.$$

EGMPR scheme of collinear factorization (1978) cont.

Projection operator $\mathbf{P}$ consists of the kinematic (on-shell) proj. operator P_{kin}, spin proj. operator P_{spin} and pole part PP extracting $\frac{1}{\epsilon_{IR}^k}$ part

$$\mathbf{P} = P_{spin}\, P_{kin}\, PP.$$

Multiplication symbol $\cdot$ means full phase space integration $d^n k$ while convolution $\otimes$ only the integration over the lightcone variable.

After applying the method of Renormalisation Group one obtains the evolution equation

$$\frac{\partial}{\partial \ln Q^2} C\left(\alpha, \frac{Q^2}{\mu^2}\right) = C\left(\alpha, \frac{Q^2}{\mu^2}\right) \otimes P(\alpha)$$

$P(\alpha)$ is the evolution kernel, see next slide.

Two kernel extraction methods up to NLO

Pole-part method:

$$K = \mathbf{P}K_0 \cdot \frac{1}{1-(1-\mathbf{P})\cdot K_0} = \mathbf{P}K_0 + \mathbf{P}K_0 \cdot [(1-\mathbf{P})\cdot K_0] + \ldots,$$

$$\Gamma = \frac{1}{1-K} = 1 + \mathbf{P}K_0 + \mathbf{P}K_0 \cdot [(1-\mathbf{P})\cdot K_0] + [\mathbf{P}K_0] \otimes [\mathbf{P}K_0] + \cdots ,$$

The NLO part of the kernel comes from $\mathbf{P}K_0 \cdot [(1-\mathbf{P}) \cdot K_0]$ term with non-trivial subtraction $(1-\mathbf{P})$ of the internal IR divergency.

$$\frac{\alpha_R}{\pi} P(\alpha_R, x) = \frac{\alpha_R}{\pi} P^{(0)}(x) + \left(\frac{\alpha_R}{\pi}\right)^2 P^{(1)}(x) = \alpha_R \frac{\partial}{\partial \alpha_R} Res_1\, \Gamma(\alpha_R, x)$$

$$= Res_1 \Big\{ \mathbf{P}K_0 \Big\} + 2 Res_1 \Big\{ \mathbf{P}K_0 \cdot [(1-\mathbf{P})\cdot K_0] \Big\}.$$

Finite-part method:

$$C = C_0 \frac{1}{1-(1-\mathbf{P})K_0} = C_0 + C_0 \cdot (1-\mathbf{P}) \cdot K_0 + C_0 \cdot (1-\mathbf{P}) \cdot K_0 \cdot [(1-\mathbf{P}) \cdot K_0] +$$

$$P(\alpha_R, x) = -C^{-1}\left(\alpha, \frac{Q^2}{\mu^2}\right) \otimes \frac{\partial}{\partial \ln \mu^2} C\left(\alpha, \frac{Q^2}{\mu^2}\right)(x)$$

$$= \frac{-\partial}{\partial \ln \mu^2} F\big(\bar{C}_0(1-\mathbf{P})K_0\big)(x) + \frac{-\partial}{\partial \ln \mu^2} F\big(\bar{C}_0(1-\mathbf{P})K_0((1-\mathbf{P})K_0)\big)(x)$$

$$- F(\bar{C}_0(1-\mathbf{P})K_0) \otimes \frac{-\partial}{\partial \ln \mu^2} F\big(\bar{C}_0(1-\mathbf{P})K_0\big)(x).$$

LL example, single gluon emission

Single gluonstrahlung directly from Feynman diagram (axial gauge, spin projector):

$$F^{(1)}(\bar{C}_0 K_0)(x) = C_F \frac{\alpha}{\pi} \int_0^{Q^2} \frac{d(-q_1^2)}{-q_1^2} \left(\frac{-q_1^2 (1-x)}{\mu^2} \right)^{\epsilon} \frac{\Omega_{1+2\epsilon}}{2\pi} \mathcal{P}(x,\epsilon)$$

$$= C_F \frac{\alpha}{\pi} \frac{1}{\epsilon} \mathcal{P}(x,0) + C_F \frac{\alpha}{\pi} \left(\ln \frac{Q^2 (1-x)}{\mu^2} \mathcal{P}(x,0) + \omega' \mathcal{P}(x,0) + \mathcal{P}'(x) \right),$$

$$\mathcal{P}(z,\epsilon) \equiv \frac{1+z^2}{2(1-z)} + \frac{1}{2}\epsilon(1-z) = \mathcal{P}(z,0) + \mathcal{P}'(z), \qquad \frac{\Omega_{1+2\epsilon}}{2\pi} = 1 + \epsilon\, \omega',$$

True hard process part C_0 is replaced by simplifi ed $\bar{C}_0 \sim \Theta(Q^2)$ with $\Theta(Q^2) = \theta_{-q_1^2 < Q^2}$

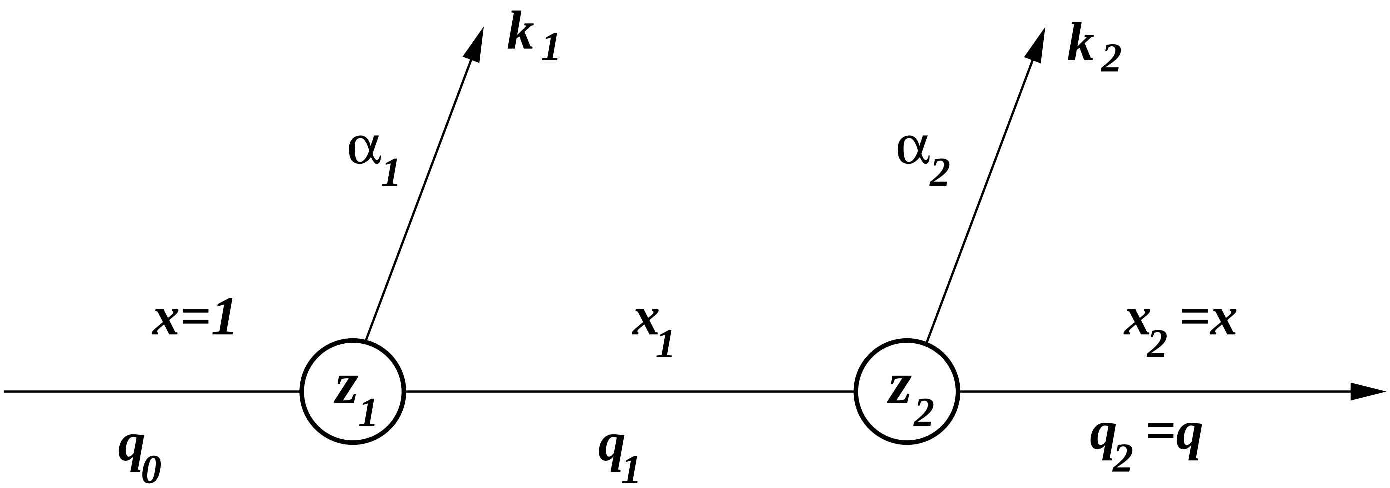
For $\Theta(Q^2) = \theta_{-k_{T1}^2 < Q^2}$ we obtain SIMPLER

$$F^{(1)}(\bar{C}_0 K_0)(x) = C_F \frac{\alpha}{\pi} \int_0^{Q^2} \frac{d(k_{T1}^2)}{k_{T1}^2} \left(\frac{k_{T1}^2}{\mu^2} \right)^{\epsilon} \frac{\Omega_{1+2\epsilon}}{2\pi} \mathcal{P}(x,\epsilon)$$

$$= C_F \frac{\alpha}{\pi} \frac{1}{\epsilon} \mathcal{P}(x,0) + C_F \frac{\alpha}{\pi} \left(\ln \frac{Q^2}{\mu^2} \mathcal{P}(x,0) + \omega' \mathcal{P}(x,0) + \mathcal{P}'(x) \right)$$

Both kernel extraction methods PP_1 and differentiation $\partial/\partial(\ln(Q^2/\mu^2))$ give the same $\mathcal{P}(z,0)$, HOWEVER it comes from different part of $F^{(1)}(\bar{C}_0 K_0)(x)$.

Kinematics of the double emission



Kinematics of the double/multiple real emission

$$x_i = \frac{\zeta \cdot q_i}{\zeta \cdot q_0}, \quad \alpha_i = \frac{\zeta \cdot k_i}{\zeta \cdot q_0}, \quad q_i = q_0 - \sum_{i=1}^{n} k_i,$$

$$q_0^2 = 0, \quad q_0 = (E, 0, 0, E),$$

$$\zeta = (1, 0, 0, -1), \quad \zeta^2 = 0,$$

$$k_i^2 = 0, \quad k_i = (k_i^0, \mathbf{k}_i, k_i^3).$$

Ladder (uncrossed) diagram

Start from expression for the ladder diagram

$$F^{(2)}(\bar{C}_0 K_0 K_0) = \mathcal{N} C_F \int \frac{d\alpha_1}{\alpha_1} \frac{d\alpha_2}{\alpha_2} \delta_{1-x=\alpha_1+\alpha_2} \int d^{2+2\epsilon}\mathbf{k}_1 d^{2+2\epsilon}\mathbf{k}_2 \; \mu^{-4\epsilon} \; \Theta(Q^2)$$

$$\times \left\{ \frac{T_1(\alpha_1,\alpha_2)}{\alpha_1\alpha_2} + \frac{T_2(\alpha_1,\alpha_2,\epsilon)}{\alpha_2^2} \frac{\mathbf{k}_2^2}{\mathbf{k}_1^2} + \frac{T_3(\alpha_1,\alpha_2)}{\alpha_2} \frac{2\mathbf{k}_1\cdot\mathbf{k}_2}{\mathbf{k}_1^2} \right\} \frac{1}{q^4(k_1,k_2)}$$

where γ-trace factors and the second propagator are

$$T_1(\alpha_1,\alpha_2) = (1+x^2+x_1^2)\alpha_1\alpha_2,$$

$$T_2(\alpha_1,\alpha_2,\epsilon) = (1+x_1^2)(x^2+x_1^2) + \epsilon T_2'(\alpha_1,\alpha_2),$$

$$T_2'(\alpha_1,\alpha_2) = (1-x_1)^2(x^2+x_1^2) + (x-x_1)^2(1+x_1^2),$$

$$T_3(\alpha_1,\alpha_2) = (1+x^2+x_1^2)x_1,$$

$$-q^2(k_1,k_2) = \frac{1-\alpha_2}{\alpha_1}\,\mathbf{k}_1^2 + \frac{1-\alpha_1}{\alpha_2}\,\mathbf{k}_2^2 + 2\mathbf{k}_1\cdot\mathbf{k}_2.$$

The above integral in $4+2\epsilon$ dimensions is finite for $\epsilon=\epsilon_{IR}>0$.

For the definition of the cut-off function $\Theta(Q^2)$ see next slide.

The choice of the evolution scale variable

In practice we have examined 4 choices of the “cap function” $\Theta_q(Q^2) = \theta_{S(k_1,k_2)\le Q^2}$:

$$\Theta_q(Q^2) = \theta_{-q_2^2 \le Q^2},$$
$$\Theta_t(Q^2) = \theta_{\max(\mathbf{k}_1^2,\mathbf{k}_2^2)\le Q^2},$$
$$\Theta_v(Q^2) = \theta_{\max(q_0\cdot k_1, q_0\cdot k_2)\le Q^2},$$
$$\Theta_\eta(Q^2) = \theta_{\max(E_h e^{\eta_1}, E_h e^{\eta_2})\le Q^2},$$

The resulting NLO kernel from CFP recipe is the same, but . . .

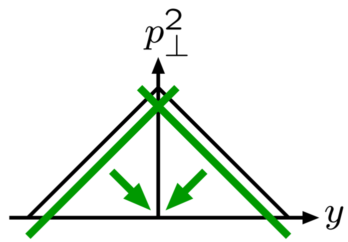
- In the fi nite-part method $\frac{\partial}{\partial \ln(Q^2/\mu^2)}\Theta_q(Q^2) = \delta_{Q^2=S(k_1,k_2)}$.
  Hence $S(k_1,k_2)$ defi nes a hypersphere in the LIPS where NLO kernels live!
- In the time-ordered solution of the evolution (also in the psMC) variable Q^2 will be the evolution variable of the NEXT emission (or repres. the hard process LIPS).
- In the actual analytical integration the choice of $S(k_1,k_2)$ determines the LIPS parametrization and the whole integration process.
- The form of the counterterm $\mathbf{P}K_0 \otimes \mathbf{P}\cdot K_0$ in the internal subtraction in $\mathbf{P}K_0\cdot[(1-\mathbf{P})\cdot K_0]$ depends strongly on the choice of $\Theta(Q^2)$.
  (In fact it compensates for it in the fi nal result.)

In the following we consider second choice $S(k_1,k_2) = \max(\mathbf{k}_1^2,\mathbf{k}_2^2)$.

Evolution time (transparency from T. Sjöstrand)

Ordering variables in final-state radiation

PYTHIA: $Q^2 = m^2$



large mass first

$\Rightarrow$ "hardness" ordered

coherence brute force

covers phase space

ME merging simple

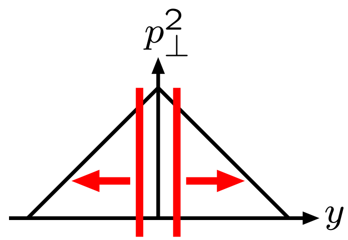
g $\to$ q$\overline{\text{q}}$ simple

not Lorentz invariant

no stop/restart

ISR: $m^2 \to -m^2$

HERWIG: $Q^2 \sim E^2\theta^2$



large angle first

$\Rightarrow$ **hardness not ordered**

coherence inherent

gaps in coverage

ME merging messy

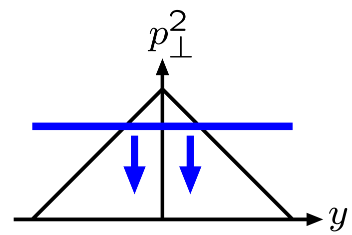
g $\to$ q$\overline{\text{q}}$ simple

not Lorentz invariant

no stop/restart

ISR: $\theta \to \theta$

ARIADNE: $Q^2 = p_\perp^2$



large $p_\perp$ first

$\Rightarrow$ "hardness" ordered

coherence inherent

covers phase space

ME merging simple

g $\to$ q$\overline{\text{q}}$ messy

Lorentz invariant

can stop/restart

ISR: more messy

LIPS parametrization for the ladder diagr.

For $S(k_1,k_2)=\max(\mathbf{k}_1^2,\mathbf{k}_2^2)$ the LIPS re-parametrized in dimensionless variables $y_1=\mathbf{k}_1^2/Q^2, y_2=\mathbf{k}_2^2/Q^2$ separately in two sectors $\mathbf{k}_1^2<\mathbf{k}_2^2<Q^2$ and $\mathbf{k}_2^2<\mathbf{k}_1^2<Q^2$.

$$F(\bar{C}_0K_0K_0)=\left(C_F\frac{\alpha}{\pi}\right)^2\frac{\Omega_{1+2\epsilon}}{2\pi}\int_0^1\frac{d\alpha_1}{\alpha_1}\frac{d\alpha_2}{\alpha_2}\delta_{1-x=\alpha_1+\alpha_2}\int_0^{Q_h^2}\frac{dQ^2}{Q^2}\left(\frac{Q^2}{\mu^2}\right)^{2\epsilon}$$

$$\Bigg\{\int_0^1 dy_1\; y_1^{\epsilon}\frac{1}{2\pi}\int_0^{\pi}d\varphi_{21}\;\Omega_{2\epsilon}\;\sin^{\epsilon}(\varphi_{21})\;\frac{1}{U^2(y_1,1,\varphi_{21})}$$

$$\times\left\{\frac{T_1(\alpha_1,\alpha_2)}{\alpha_1\alpha_2}+\frac{T_2(\alpha_1,\alpha_2,\epsilon)}{\alpha_2^2}\;\frac{1}{y_1}+\frac{T_3(\alpha_1,\alpha_2)}{\alpha_2}\;2\cos(\varphi_{21})\frac{1}{\sqrt{y_1}}\right\}$$

$$+\int_0^1 dy_2\; y_2^{\epsilon}\frac{1}{2\pi}\int_0^{\pi}d\varphi_{21}\;\Omega_{2\epsilon}\;\sin^{\epsilon}(\varphi_{21})\;\frac{1}{U^2(1,y_2,\varphi_{21})}$$

$$\times\left\{\frac{T_1(\alpha_1,\alpha_2)}{\alpha_1\alpha_2}+\frac{T_2(\alpha_1,\alpha_2,\epsilon)}{\alpha_2^2}\;y_2+\frac{T_3(\alpha_1,\alpha_2)}{\alpha_2}\;2\cos(\varphi_{21})\sqrt{y_2}\right\}\Bigg\},$$

$$U(y_1,y_2,\varphi_{21})=a_1y_1+a_2y_2+2\sqrt{y_1y_2}\cos(\varphi_{21}),\;\; a_1=\frac{1-\alpha_2}{\alpha_1},\;\; a_2=\frac{1-\alpha_1}{\alpha_2}.$$

The double pole is located in $\frac{1}{y_1}$ term and requires counterterm, see next slide.

Counterterm for the ladder diagr.

The action of the $\mathbf{P}$ operator of Furmanski-Petronzio:

- $\mathbf{P}$ sets $\mathbf{k}_1 = 0$ to its left,
- the integration $\int d\mathbf{k}_1^2$ is done on the part of the integrand to the right of $\mathbf{P}$; the pole part PP in term of $\frac{1}{\epsilon}$ poles is simultaneously extracted,
- the spin projection operator is applied

Following the above defi nition we obtain

$$F^{(2)}(\bar{C}_0 K_0 \mathbf{P} K_0) = \left(C_F \frac{\alpha}{\pi}\right)^2 \int_z \int_0^{Q_h^2} \frac{dQ^2}{Q^2} \left(\frac{Q^2}{\mu^2}\right)^\epsilon \frac{1}{2\pi} \int_0^{\pi} d\varphi_{21}\, \Omega_{2\epsilon}\, \sin^\epsilon(\varphi_{21})$$

$$\times\, 2\mathcal{P}(z_2,\epsilon)\, \frac{Q^4 a_2^2}{q^4(0,Q,0)}\, PP \int_0^{Q^2} d\mathbf{k}_1^2\, 2\mathcal{P}(z_1,\epsilon)\, \frac{\Omega_{1+2\epsilon}}{2\pi} \left(\frac{\mathbf{k}_1^2}{\mu^2}\right)^\epsilon \frac{1}{\mathbf{k}_1^2}.$$

where $z_1 = 1 - \alpha_1$ and $z_2 = 1 - \alpha_2/(1-\alpha_1)$.

Ladder diagr. subtracted

In the subtracted expression the NLO kernels comes form the integral in which we may set $\epsilon \to 0$

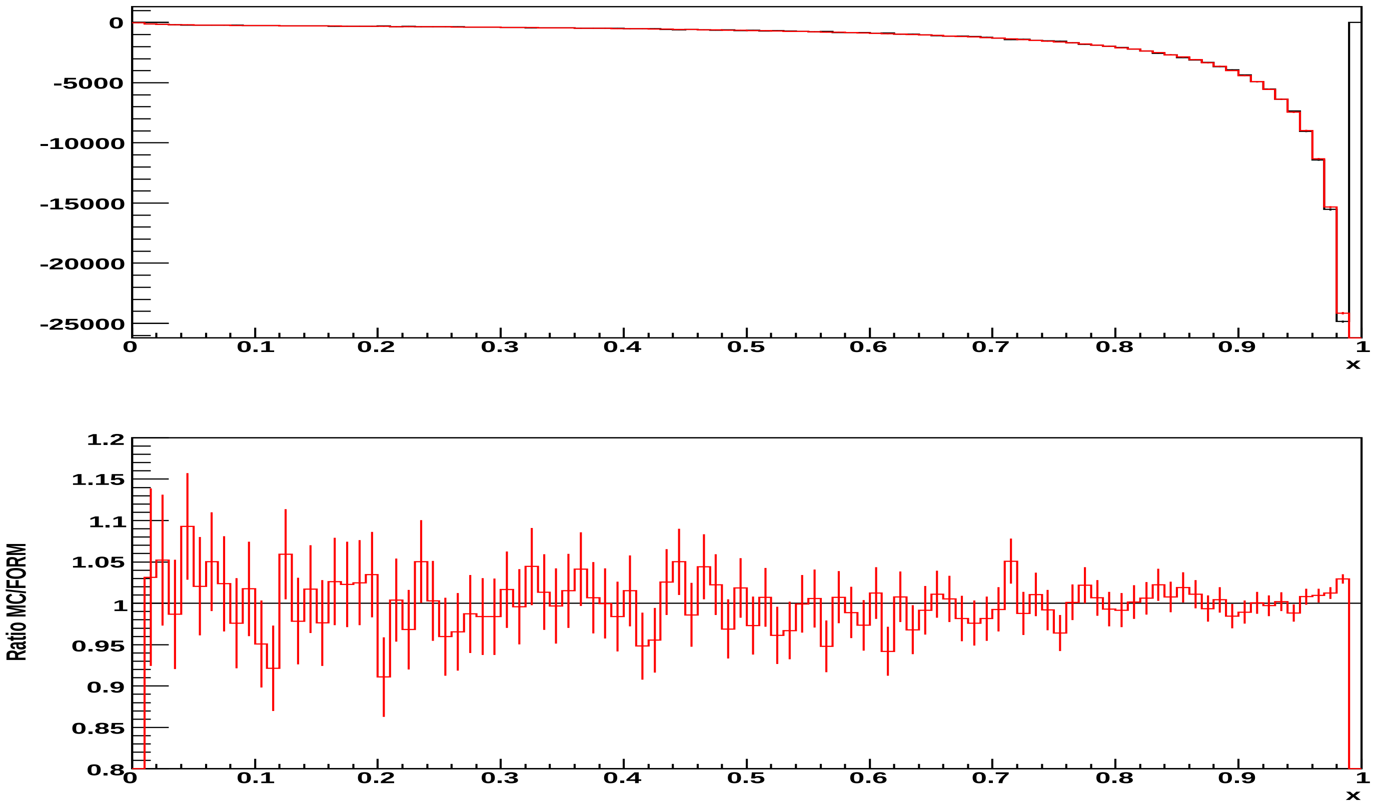
$$
\begin{aligned}
F^{(2)}(\bar{C}_0 \mathbf{P} K_0 (1-\mathbf{P}) K_0) &= \left(C_F \frac{\alpha}{\pi} \right)^2 \frac{1}{2\epsilon} \int_\alpha \frac{\alpha_1}{\alpha_1^2+\delta^2} \frac{\alpha_2}{\alpha_2^2+\delta^2} \int_0^1 dy \int_0^{2\pi} \frac{d\varphi_{21}}{2\pi} \\
&\times \left[\frac{1}{U^2(y,1,\varphi_{21})} \left\{ \frac{T_1(\alpha_1,\alpha_2)}{\alpha_1\alpha_2} + \frac{T_2(\alpha_1,\alpha_2,0)}{\alpha_2^2} \left(\frac{1}{y} \right)_+ + \frac{T_3(\alpha_1,\alpha_2)}{\alpha_2} \, 2\cos(\varphi_{21}) \frac{1}{\sqrt{y}} \right\} \right. \\
&\quad + \frac{1}{U^2(1,y,\varphi_{21})} \left\{ \frac{T_1(\alpha_1,\alpha_2)}{\alpha_1\alpha_2} + \frac{T_2(\alpha_1,\alpha_2,0)}{\alpha_2^2} \, y + \frac{T_3(\alpha_1,\alpha_2)}{\alpha_2} \, 2\cos(\varphi_{21}) \sqrt{y} \right\} \\
&\quad \left. + \delta(y) \, \frac{T_2'(\alpha_1,\alpha_1) - 2T_2'''(\alpha_1,\alpha_1)}{x_1^2} \right] + \mathcal{O}\left(\frac{1}{\epsilon^2} \right).
\end{aligned}
$$

Explicit regularization of the IR singularity in the lightcon variables α_i using $\delta \ll 1$. (Dim. reg. is not used here!). $T_2' - 2T_2'''$ comes from ϵ terms in γ-traces. The above is ready to go for analytical and/or numerical 3-dim. integration.

Ladder final result, agreers with CFP

After integrating over all variables we get full agreement with CFP paper

$$F^{(2)}(\bar{C}_0 \mathbf{P} K_0 (1-\mathbf{P}) K_0) = \Big(C_F \frac{\alpha}{\pi}\Big)^2 \frac{1}{2\epsilon}$$
$$\times \Big\{ -4 \frac{1+x^2}{1-x} [\ln \frac{1}{\delta} + \ln(1-x)] + \frac{-1+\frac{5}{2}(1+x^2)}{1-x} \ln^2 x + (2-4x)\ln x + 3(1-x) \Big\}.$$



Contribution to NLO kernel from numerical MC integration by FOAM and analytical integration and their ratio. The range of x limited to $(0.01, 0.99)$. IR cut-offs $\delta = 10^{-5}$ and $\Delta = 10^{-10}$. ϵ-part of T_2 omitted.

Details of “Numerical Model” for NLO integral

Ultimate goal: Monte Carlo simulation

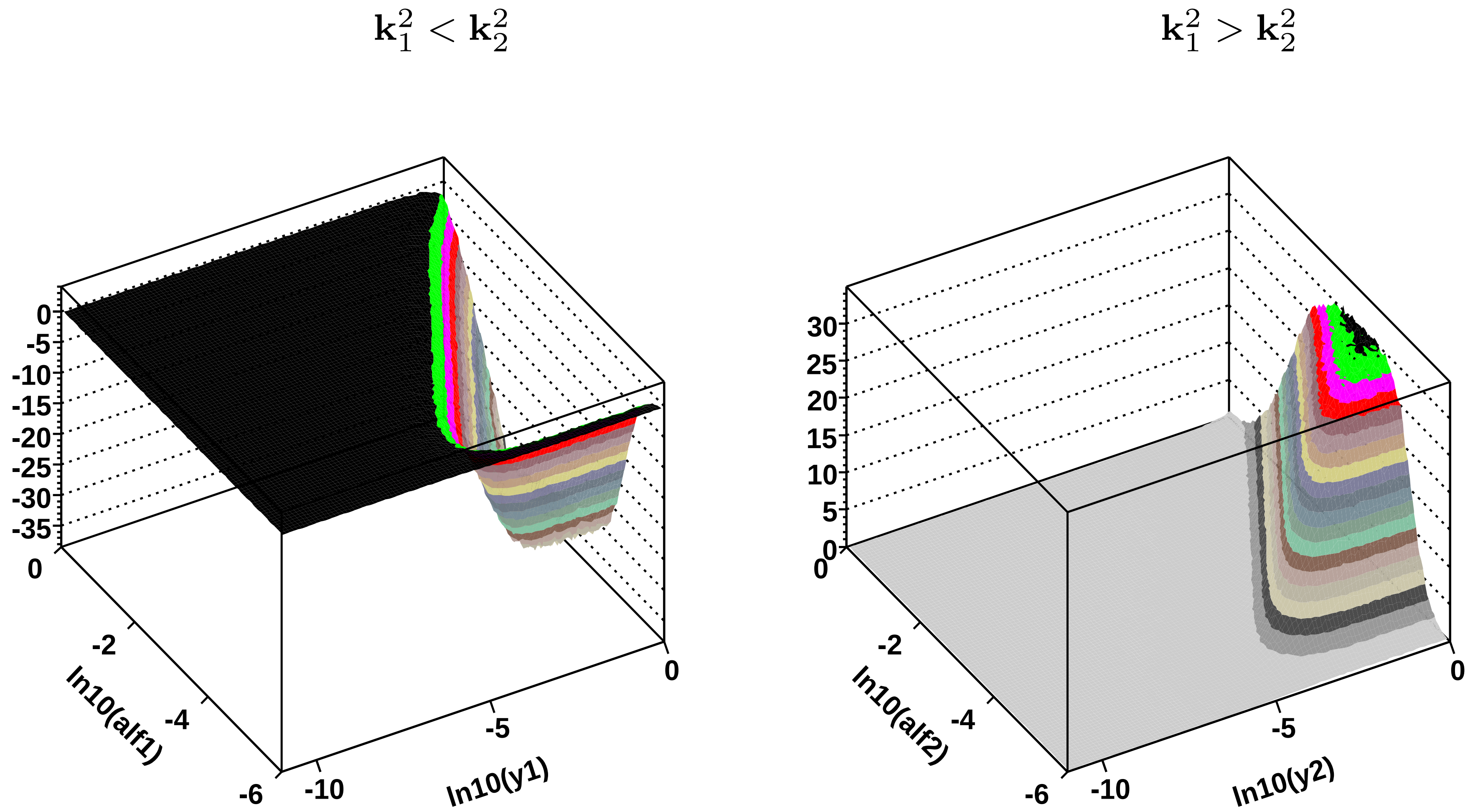
If we move towards NLO MC then 2 important introductory steps are mandatory:

1. Develop "numerical model" for the LIPS integral representing NLO kernel.
2. Examine thoroughly the structure of IR singularities in the LIPS.

- Omit temporarily part $\sim \delta(y)$ of the integrand due to ϵ-term of the γ-trace T_2.
- NB. The present choice of $\Theta(Q^2)$ is special, the terms $\sim \delta(y) \ln \frac{\alpha_1}{\alpha_2}$ are ABSENT!
- Use $\int_0^1 dy f(y) \left(\frac{1}{y}\right)_+ = \int_\Delta^1 dy \left(\frac{f(y)}{y} - \frac{f(0)}{y}\right)$, with $\Delta \ll \delta \ll 1$.
- The resulting integrand is programmed for the FOAM simulator/integrator. FOAM generated weighted MC events in ALL variables: $\varphi_{21}, y = y_1, y_2$ and α_i obeying $\alpha_1 + \alpha_2 = 1 - x$.
- The average MC weight provides the value of the integral.

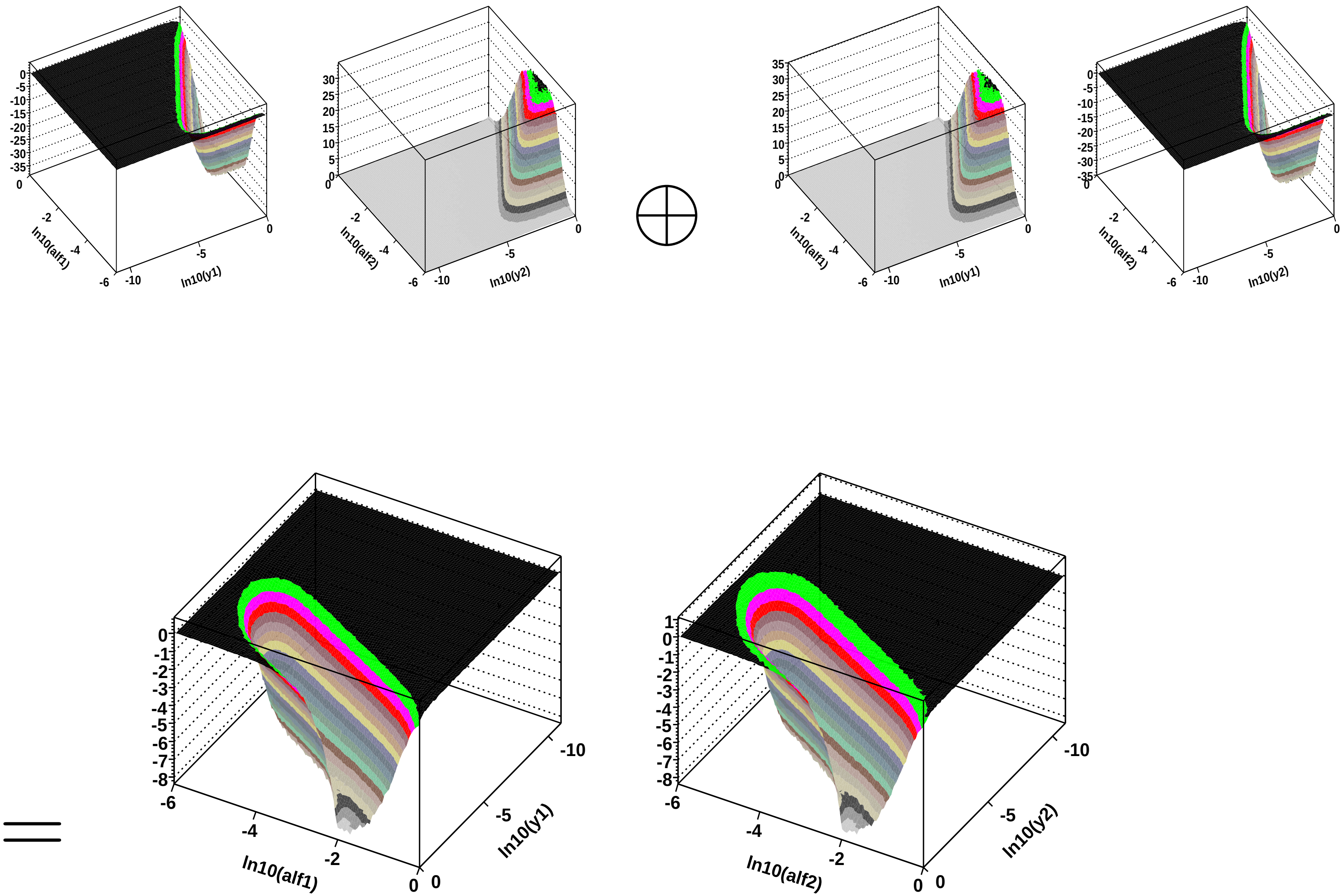
Numerical model by FOAM, closer look inside LIPS

Cancellations (ladder diagram)



HUGE cancellations between the contribs. from two LIPS sectors $\mathbf{k}_1^2 < \mathbf{k}_2^2$ and $\mathbf{k}_1^2 > \mathbf{k}_2^2$!!! Each of them of the double-log size $\sim \ln\frac{1}{\Delta}\ln\frac{1}{\delta}$. Forget about NLO MC??? Not yet! Apply simple procedure of Bose symmetrization, see next slide...

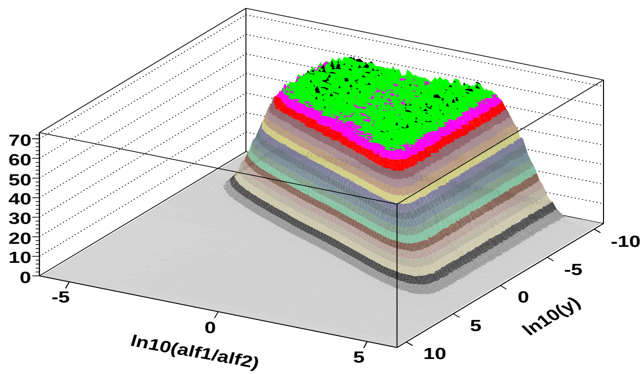
Look inside LIPS: Bose symmetrization



The single log contr. $\sim \ln\frac{1}{\delta}$ of the analyt. formula is now manifest!

A negative ridge along $\ln y_i = \ln \alpha_i$ has volume proportional to its length $\sim \ln\frac{1}{\delta}$.

Look inside LIPS: How the counterterm works?



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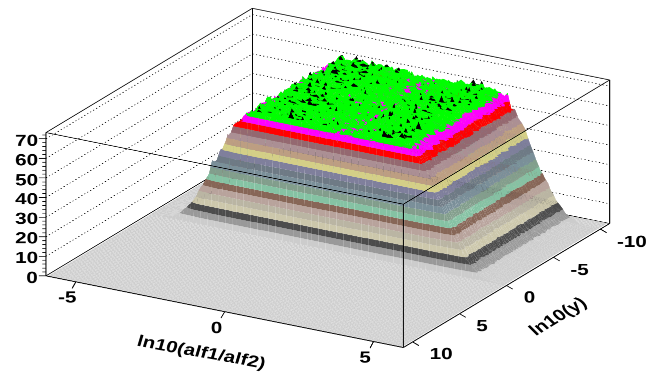

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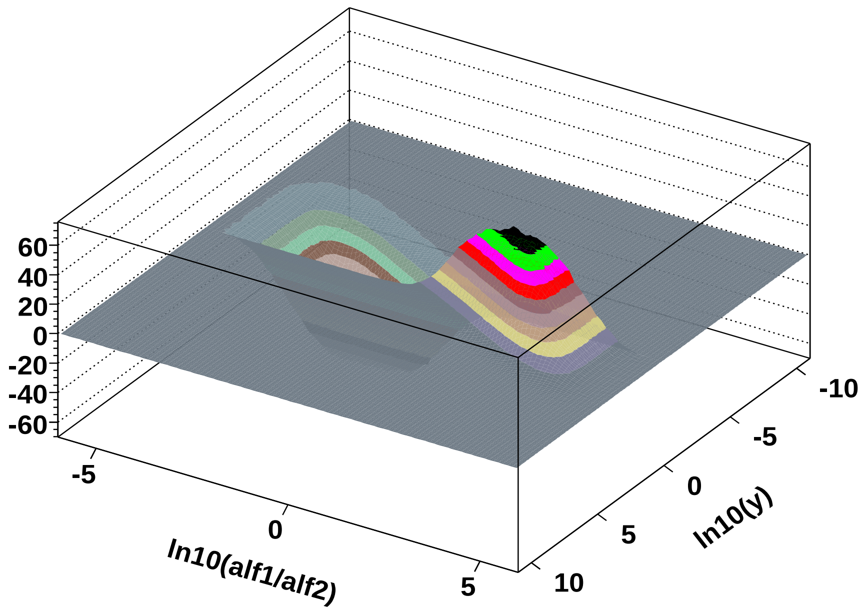
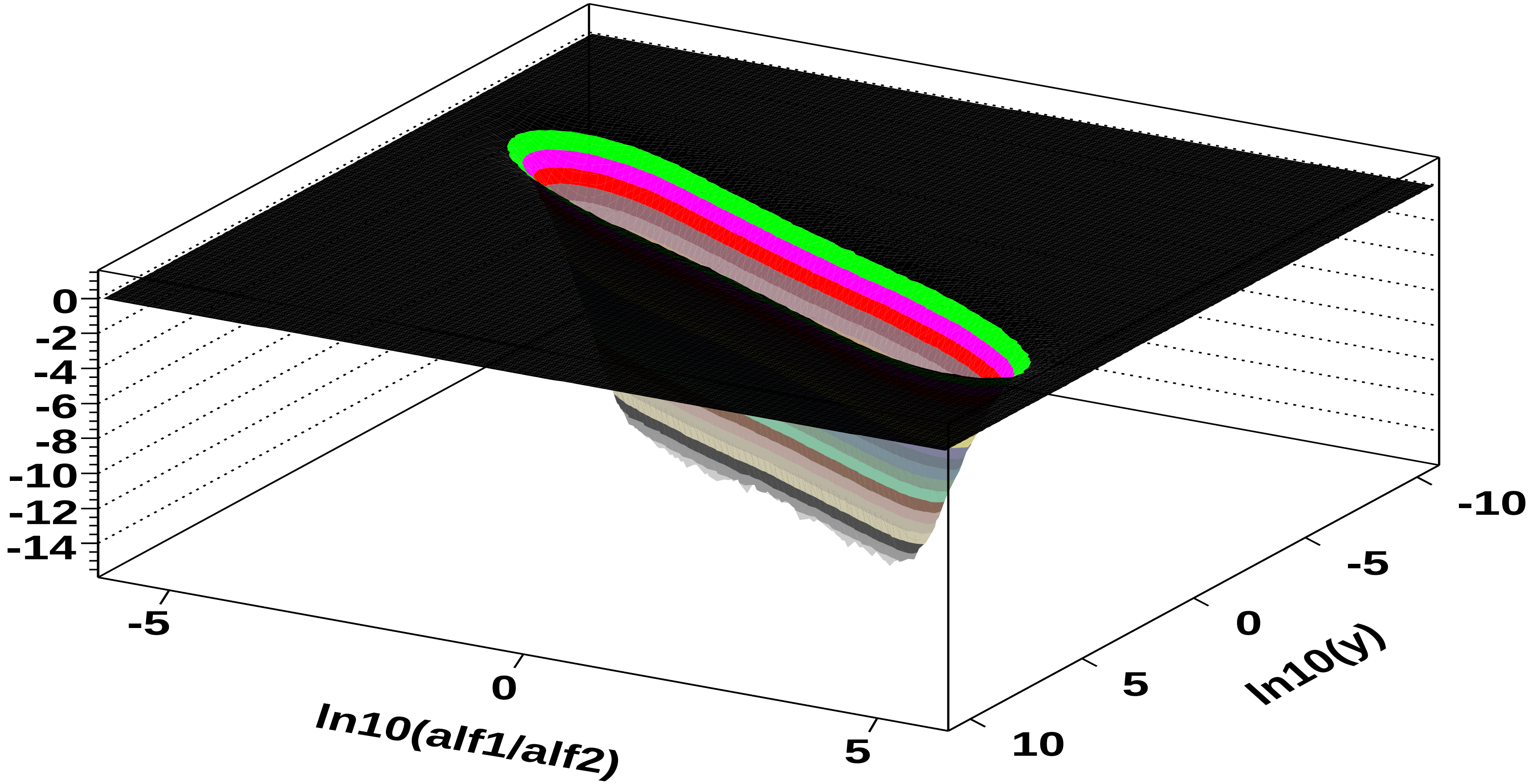


The "sum" of top left plots from prev. slide ($\ln y < 0$ - left; $\ln y > 0$ - right)

$$y = \begin{cases} \mathbf{k}_1^2/Q^2 \equiv y_1, & y_1 < y_2 \\ Q^2/\mathbf{k}_2^2 \equiv 1/y_2, & y_2 < y_1 \end{cases}$$

This is better seen in the "composite plot" of $\ln(\alpha_1/\alpha_2)$ and $\ln y = \ln y_1, -\ln y_2$ for two sectors $\mathbf{k}_1^2 < \mathbf{k}_2^2$ and $\mathbf{k}_1^2 > \mathbf{k}_2^2$ together.
The two sectors are "glued" in the plot along the line $y_1 = y_2 = 1$, i.e. $\ln y = 0$.

Inside LIPS: Bose symmetrization again



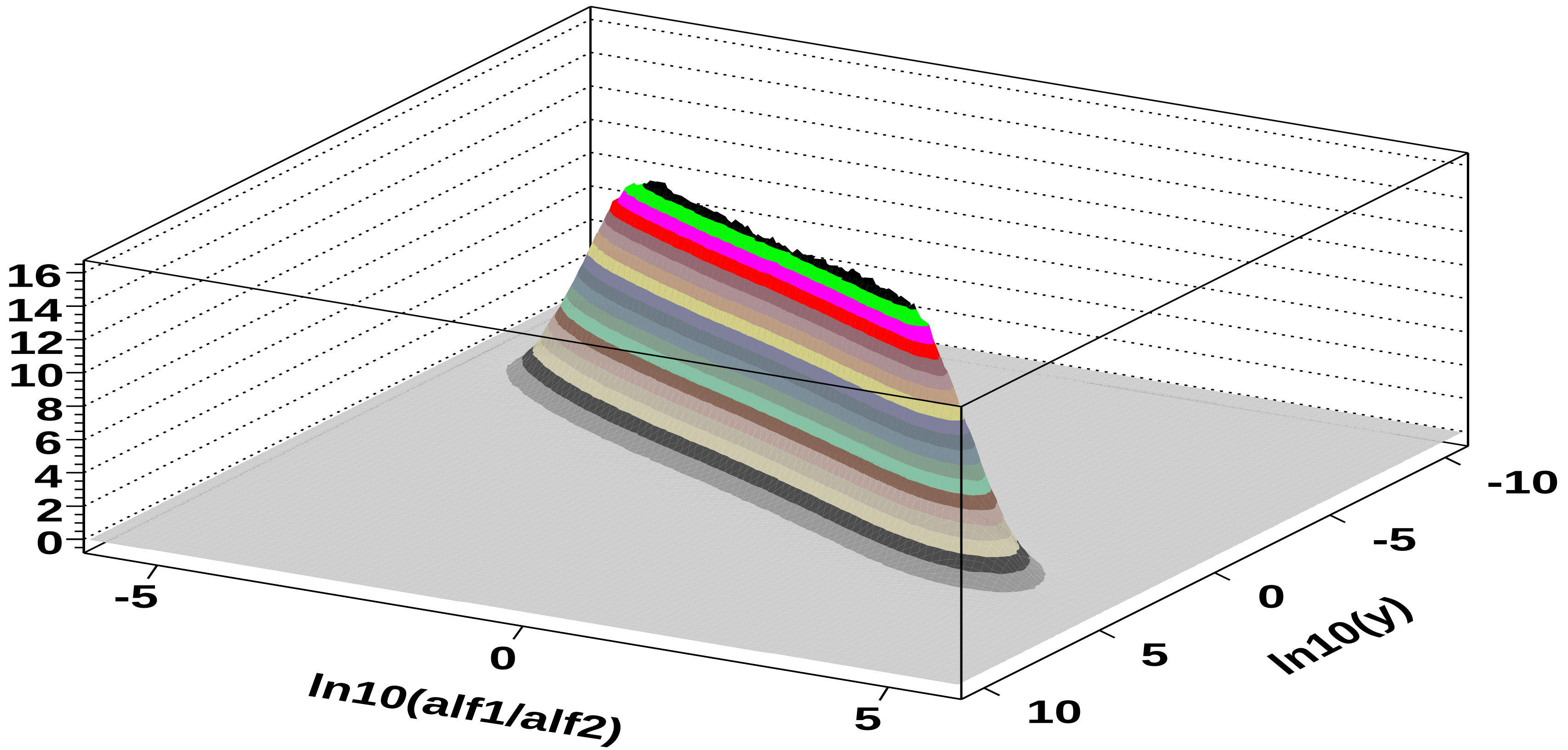
The bose-symmetrized distr. again in the “composite plot”.

However, IR singularity of the type $\ln \frac{1}{\delta}$ will be absent in the end!

Who cancells it? The interference (crossed ladder) graph!

Can this cancellation be seen at the integrand (exclusive) level? YES!!!

Inside LIPS: Crossed-ladder interference graph, C_F part



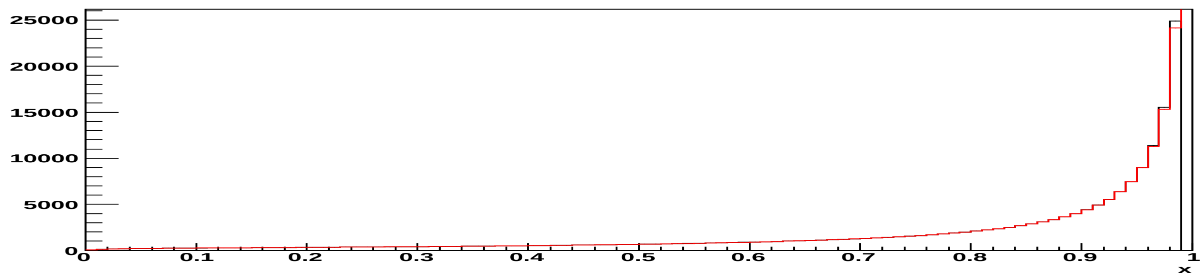
The contr. of the interference graph looks almost as a mirror image of the previous!
The new distribution added in the Numerical MC model

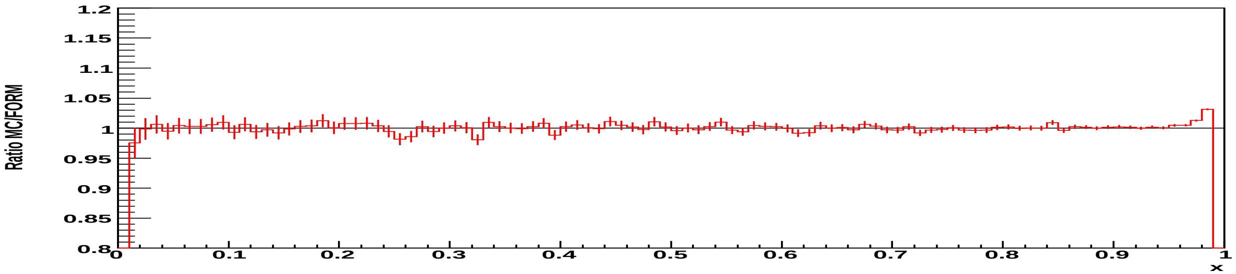
$$F(\bar{C}_0 K_0 K_0) = N \int \frac{d\alpha_1}{\alpha_1} \frac{d\alpha_2}{\alpha_2} \delta_{1-x=\alpha_1+\alpha_2} \int d^{2+2\epsilon}\mathbf{k}_1 d^{2+2\epsilon}\mathbf{k}_2 \; \mu^{-4\epsilon} \; \theta_{\max(\mathbf{k}_1^2,\mathbf{k}_2^2)\le Q^2} \; \frac{1}{q^4(k_1,k_2)}$$

$$\times \left\{ \frac{2T_1^x(\alpha_1,\alpha_2)}{\alpha_1\alpha_2} - T_{2a}^x(\alpha_1,\alpha_2)\frac{2\mathbf{k}_1\cdot\mathbf{k}_2}{\alpha_1\mathbf{k}_2^2} - T_{2b}^x(\alpha_1,\alpha_2)\frac{2\mathbf{k}_1\cdot\mathbf{k}_2}{\alpha_2\mathbf{k}_1^2} + T_3^x(\alpha_1,\alpha_2)\frac{(\mathbf{k}_1\cdot\mathbf{k}_2)^2}{\mathbf{k}_1^2\mathbf{k}_2^2} \right\}$$

and integrated using THE SAME parametrization of LIPS.

Interference graph integrated analytically and numerically

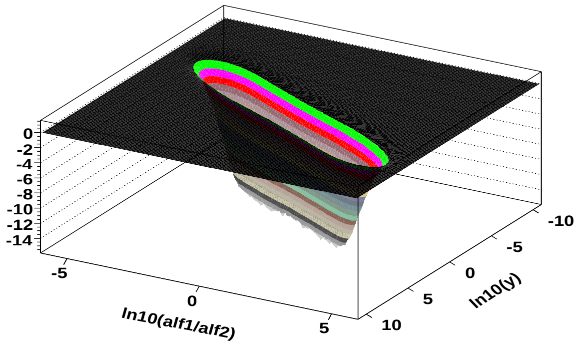




Contribution from the interference diagram to NLO kernel (C_F part) from numerical MC integration by FOAM and analytical formula (below) is shown in the upper plot. Their ratio in the lower plot.

$$F(\bar{C}_0 \mathbf{P} K_0 (1-\mathbf{P}) K_0)_{1P} = \left(C_F \frac{\alpha}{\pi} \right)^2 \frac{1}{2\epsilon}$$
$$\times \left\{ 4 \frac{1+x^2}{1-x} \left[\ln \frac{1}{\delta} + \ln(1-x) \right] - \frac{(1+x^2)}{1-x} \ln^2 x + 2(1+x) \ln x \right\}.$$

Finally Ladder + Interference! And IR goes away!!!



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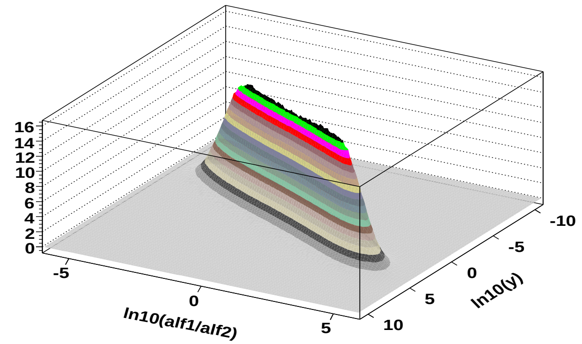

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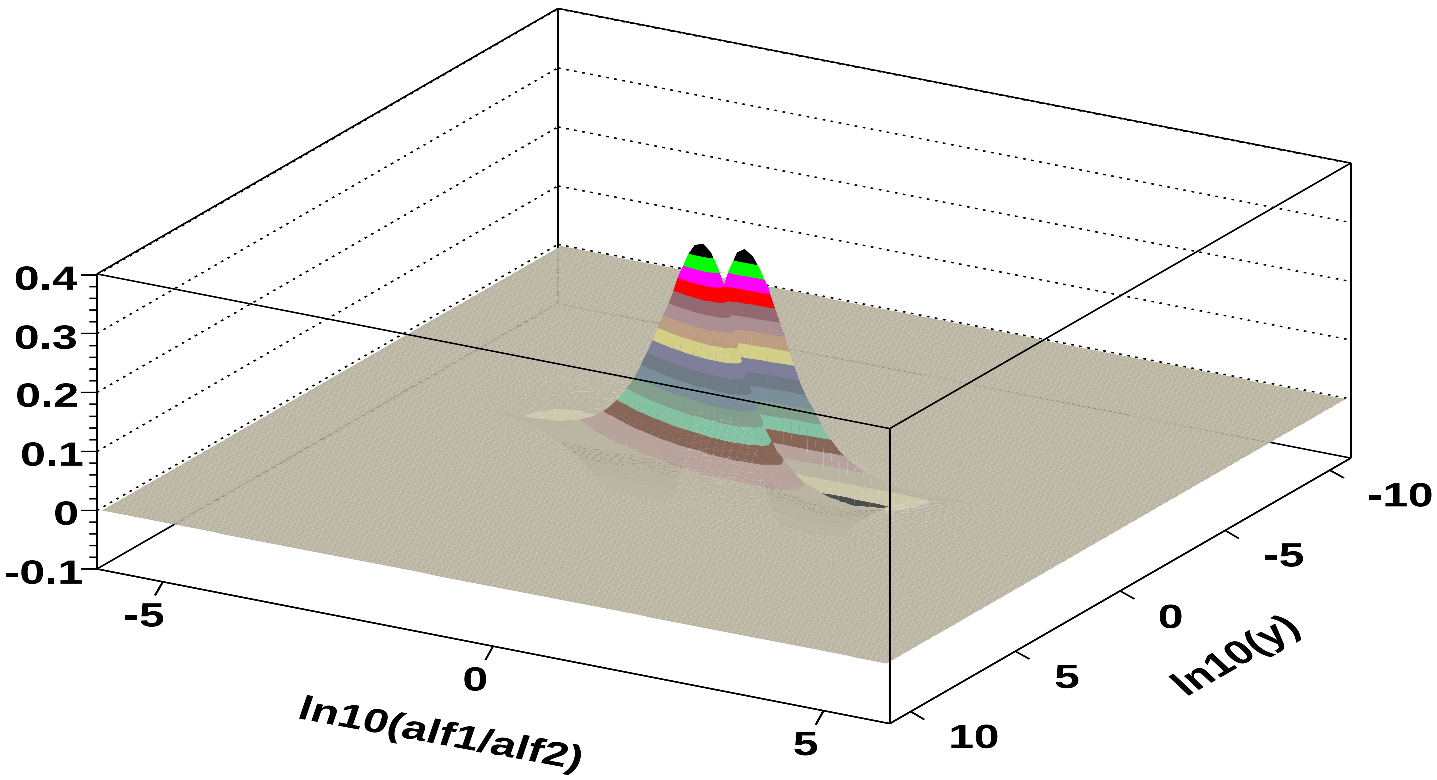
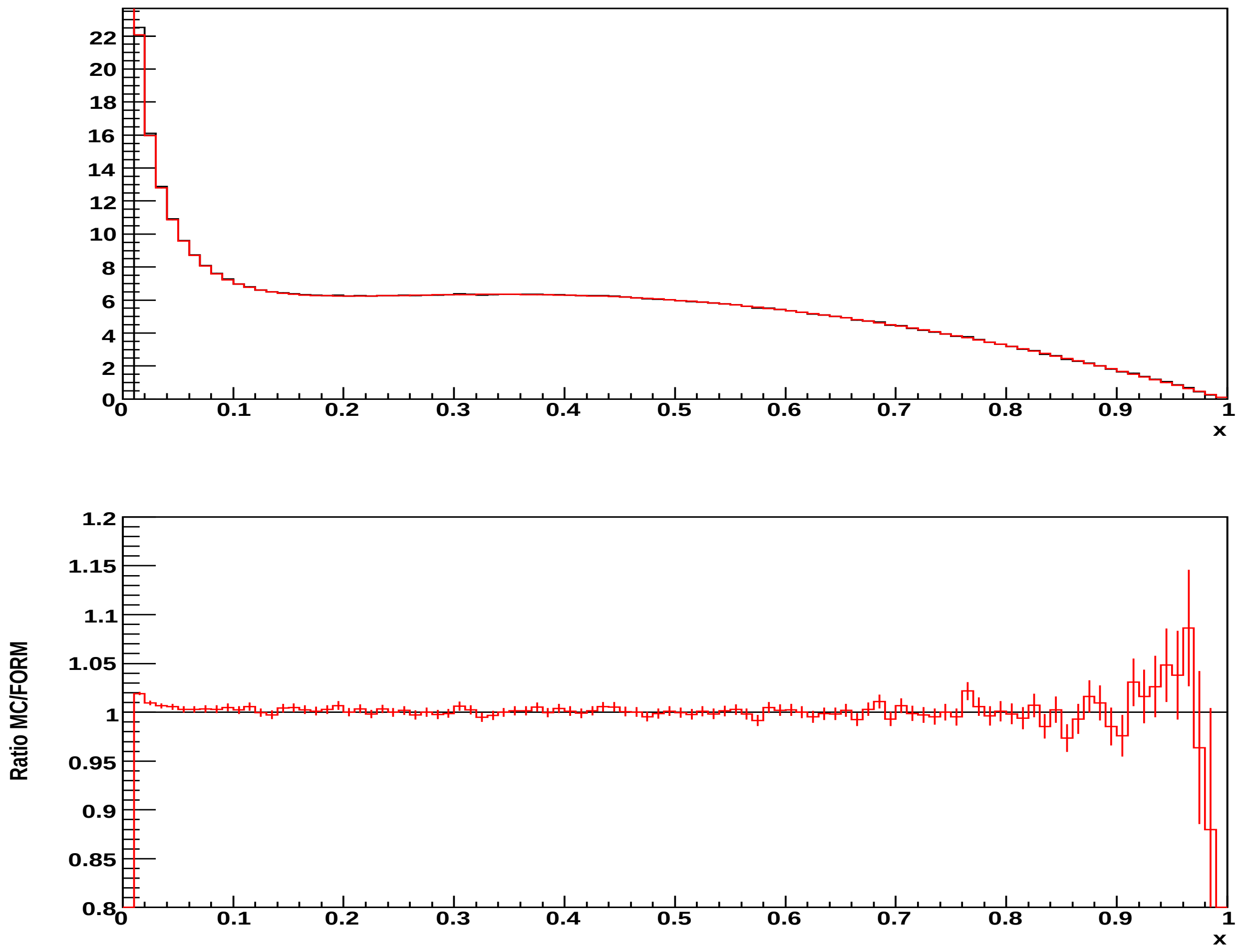


This is the most important result of this talk!

Ladder + Interference integrated analytically and numerically



The upper plot: Contribution from the ladder AND interference diagrams to NLO kernel (C_F part) from numerical MC integration by FOAM and analytical formula. Their ratio in the lower plot. ϵ-part of T_2 still omitted. (1G MC events, 2h on the laptop).

Discussion and Conclusions

- Numerical model for unintegrated NLO kernel within full 2-particle LIPS is feasible!
- Dimensional regularization can be removed from the game, especially for the “finite-part” extraction method
- The integrand of the NLO kernel features nice IR cancellations, such that only large y_i and α_i do matter; No long tails! No cancellations between distant regions in the LIPS!
- The above integrand looks just like short-range correlation function!
- This is a very promising property for the NEXT step of re-integration of the NLO unintegrated kernel into LL MC model representing LL+NLO (DGLAP) evolution.
- All numerical results shown are averaged in φ and $x \in (0.01, 0.99)$. We have checked by fixing/restricting the values of φ and x that main features/patterns remain the same.