THRESHOLD RESUMMATION OF THE DRELL-YAN RAPIDITY DISTRIBUTION

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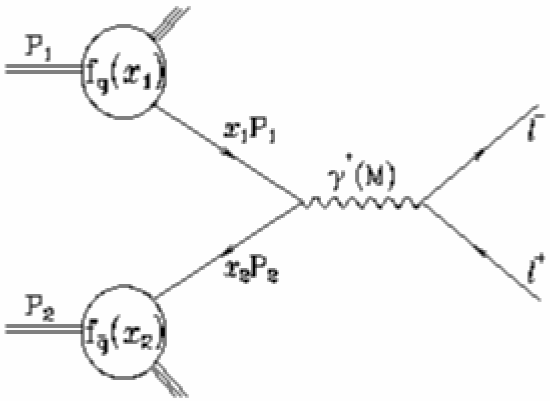
THE QCD CROSS SECTION OF DY

ACCORDING TO THE FACTORIZATION THEOREM THE DRELL-YAN CROSS SECTION CAN BE WRITTEN AS A CONVOLUTION OF THREE FUNCTIONS

$$\sigma(x, Q^2) = \sum_{a,b} \int_x^1 \frac{dx_1}{x_1} \int_{x/x_1}^1 \frac{dx_2}{x_2} f_a(x_1, \mu^2) f_b(x_2, \mu^2) C_{ab}\left(z, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right)$$

Q^2 IS THE SCALE OF THE PROCESS WHICH IN THIS CASE IS THE INVARIANT MASS OF THE DILEPTON PAIRS

μ^2 IS THE FACTORIZATION SCALE EQUAL TO THE RENORMALIZATION ONE



$$x = \frac{Q^2}{S}$$

FRACTION OF ENERGY THAT THE HADRONS TRANSFER TO THE PHOTON

$$z = \frac{x}{x_1 x_2}$$

FRACTION OF ENERGY THAT THE PARTONS TRANSFER TO THE PHOTON

WHAT IS RESUMMATION I

IN THE LIMIT $z \rightarrow 1$ ALL THE EXTRA RADIATION BECOMES SOFT
THE PERTURBATIVE EXPANSION RECIEVES
LARGE LOGARITHMIC CONTRIBUTIONS OF THE FORM

$$\alpha_s^n \left[\frac{\ln^k(1-z)}{1-z} \right]_+ , \qquad k \leq 2n - 1$$

IN ORDER TO HAVE RELIABLE PREDICTIONS, **THESE CONTRIBUTIONS MUST BE RESUMMED**

IT IS NATURAL TO WORK IN THE MELLIN MOMENTS "N":

$$\sigma(N, Q^2) = \int_0^1 x^{N-1} \sigma(x, Q^2)$$

1. THE CONVOLUTIONAL INTEGRAL IS TURNED INTO ORDINARY PRODUCT I.E. IT IS DIAGONALIZED
2. THE LARGE LOGS ln(1-z) ARE MAPPED INTO THE LARGE LOGS ln(N)

WHAT IS RESUMMATION II

THE RESUMMED STRUCTURE FUNCTION HAS A PARTICULAR SIMPLE FORM IN THE DIS FACTORIZATION SCHEME

$$\sigma^{res}(N,Q^2) = \sum_a f_a(N,\mu^2) f_b(N,Q^2) \exp\left\{ \int_{\mu^2}^{Q^2} \frac{dk^2}{k^2} \gamma_{ab}^{res} \right\}$$

$$\gamma^{res} = \gamma_{LL} + \gamma_{NLL} + \cdots + \gamma_{N^kLL} + \cdots$$

$$N^kLL \to \alpha_s^{k+n}(Q^2) \ln^n(N)$$

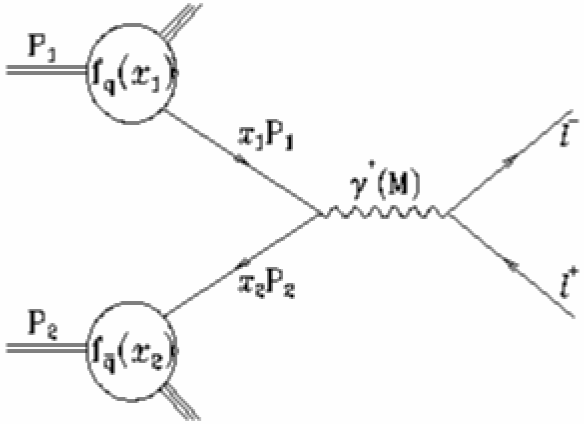
THERE IS A GENERAL PROCEDURE TO REWRITE THIS RESUMMED CROSS SECTION IN ANY OTHER FACTORIZATION SCHEME

THE MOST COMMON $\longrightarrow$ $\overline{MS}$

DRELL-YAN RAPIDITY DISTRIBUTION

FERMILAB ⟶ EXTRACTION OF PARTON DENSITIES

LHC ⟶ USES THE PREVIOUS RESULTS TO STUDY THE HIGGS BOSON PRODUCTION AND THE ASYMMETRY $W^{\pm}$



WE DEFINE THE C.M. HADRONIC RAPIDITY:

$$Y \equiv \frac{1}{2} \log \left(\frac{Q^0 + Q^3}{Q^0 - Q^3} \right)$$

THE HADRONIC AND PARTONIC C.M. RAPIDITIES ARE RELATED BY A BOOST ⟹

$$Y = y + \frac{1}{2} \log\left(\frac{x_1}{x_2}\right)$$

FACTORIZATION PROPERTIES

WE RESUM ONLY THE QUARK-ANTI-QUARK CONTRIBUTION, WHICH ACCORDING TO THE FACTORIZATION THEOREM IS GIVEN BY

$$\sigma(x,Q^2,Y)=\int_x^1\frac{dx_1}{x_1}\int_{x/x_1}^1\frac{dx_2}{x_2}F_1^{H_1}(x_1,\mu^2)F_2^{H_2}(x_2,\mu^2)C(z,\frac{Q^2}{\mu^2},\alpha_s(\mu^2),y)$$

RESUMMATION IS USUALLY REALIZED IN MELLIN SPACE AS EXPONENTIATION OF THE LARGE LOGS OF THE N-MOMENTS BECAUSE THE CONVOLUTION BECOMES AN ORDINARY PRODUCT

IN OUR CASE THINGS ARE DIFFERENT, BECAUSE

$$y=Y-\frac{1}{2}\log(\frac{x_1}{x_2})$$

THE ORDINARY PRODUCT IN MELLIN SPACE CAN BE RECOVERED PERFORMING THE MELLIN TRANSFORM WITH RESPECT TO z OF THE FOURIER TRANSFORM WITH RESPECT TO Y

DOUBLE MELLIN-FOURIER MOMENTS

THE ORDINARY PRODUCT IN MELLIN SPACE CAN BE RECOVERED PERFORMING THE MELLIN TRANSFORM WITH RESPECT TO z OF THE FOURIER TRANSFORM WITH RESPECT TO Y

$$\sigma(Q^2,N,M) \equiv \int_0^1 dx x^{N-1} \int_{1/2\log(x)}^{1/2\log(1/x)} dY e^{iMY} \sigma(x,Q^2,Y)$$

$$\begin{aligned}\sigma(Q^2,N,M) &= F_1^{H_1}(N+iM/2,\mu^2) F_2^{H_2}(N-iM/2,\mu^2) \times \\ &\times C(N,M,Q^2/\mu^2,\alpha_s(\mu^2))\end{aligned}$$

$$C(N,M,Q^2/\mu^2,\alpha_s(\mu^2)) =$$

$$\int_0^1 dz z^{N-1} \int_{1/2\ln(z)}^{1/2\ln(1/z)} dy e^{iMy} C(z,Q^2/\mu^2,\alpha_s(\mu^2),y)$$

THE M-DEPPENDENCE IS DUE TO PARTON DENSITIES (TRANSLATED IN THE COMPLEX PLANE BY $\pm$ iM/2 AND TO THE EXPONENTIAL OF iMy

THE RESUMMATION FORMULA

IT CAN BE SHOWN THAT UP TO TERMS SUPPRESSED BY FACTORS 1/N, WE HAVE

$$\sigma(Q^2, N, M) = F_1^{H_1}(N + iM/2, \mu^2) F_2^{H_2}(N - iM/2, \mu^2) \times \times \underbrace{C(N, Q^2/\mu^2, \alpha_s(\mu^2))}$$

THIS IS THE RAPIDITY-INTAGRATED DY COEFFICIENT FUNCTION

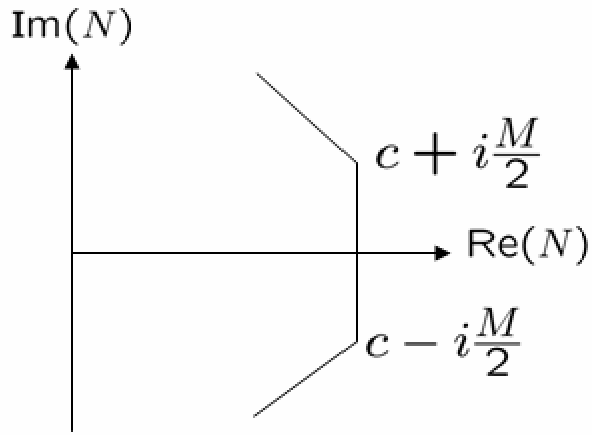
TO GET THE RESUMMED CROSS SECTION, WE NEED ONLY TO USE THE WELL KNOWN RESUMMED RAPIDITY-INTEGRATED DRELL-YAN COEFFICIENT FUNCTION

$$\sigma^{res}(Q^2, N, M) = F_1^{H_1}(N + iM/2, \mu^2) F_2^{H_2}(N - iM/2, \mu^2) \times \times C^{res}(N, Q^2/\mu^2, \alpha_s(\mu^2))$$

THE RESUMMED PART HAS SOURCES OF RAPIDITY DEPENDENCE ONLY FROM THE PARTON DISTRIBUTIONS AND IS VALID FOR ALL VALUES OF RAPIDITY

IMPLEMENTATION

WE PERFORM THE MELLIN INVERSE TRANSFORM USING THE MINIMAL PRESCRIPTION AND DEFORMING THE BOUNDARY IN ORDER TO AVOID THE SINGULARITIES



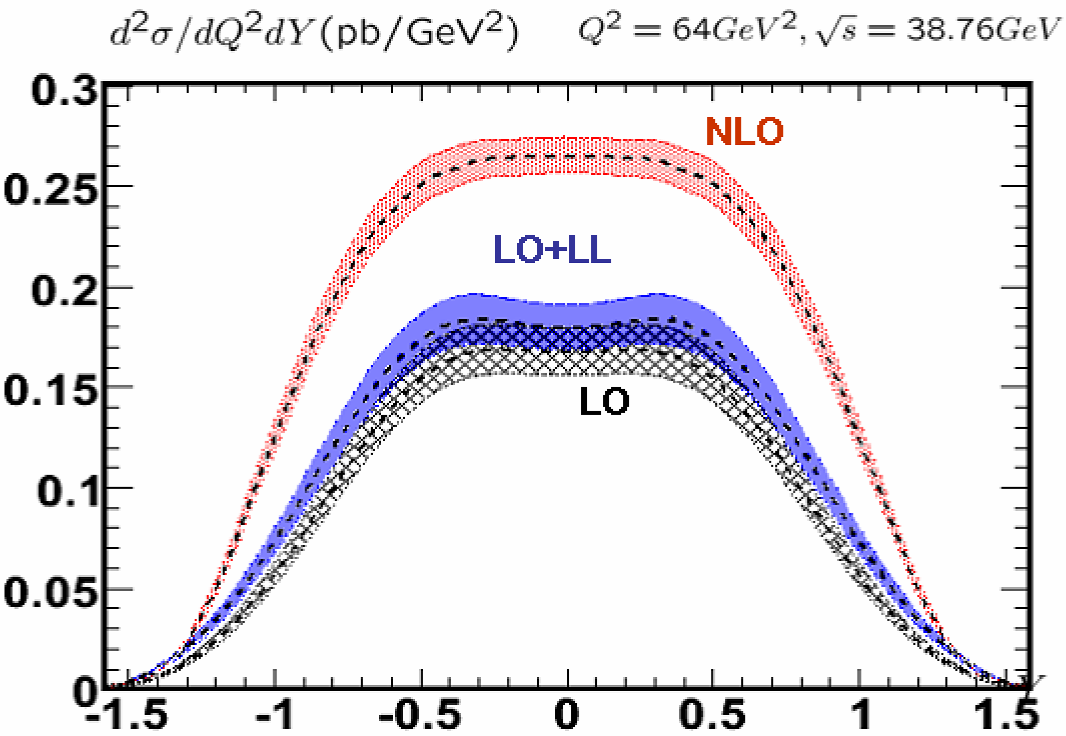
PARTON DISTRIBUTIONS OBTAINED SOLVING THE AP EQS.
EVOLVING UP THE PARTON SET OF MRST'01 AT 1GeV^2 IN ANALOGY WITH

C. Anastasiou, L.J. Dixon, D.K. Melnikov and F. Petriello Phys. Rev. Lett. 91:182002 (2003)

FINALLY WE MUST SUBTRACT THE DOUBLE COUNTING TERMS

$$\frac{\sigma(Q^2,x,\eta)}{\sigma_0(Q^2,x)} = \sigma_{FO}(Q^2,x,\eta) + \sigma^{res}(Q^2,x,\eta) - -\sigma^{res}(Q^2,x,\eta)|_{\alpha_s=0} - \alpha_s \frac{\partial}{\partial \alpha_s}\sigma^{res}(Q^2,x,\eta)|_{\alpha_s=0}$$

LL RESUMMATION AND NLO



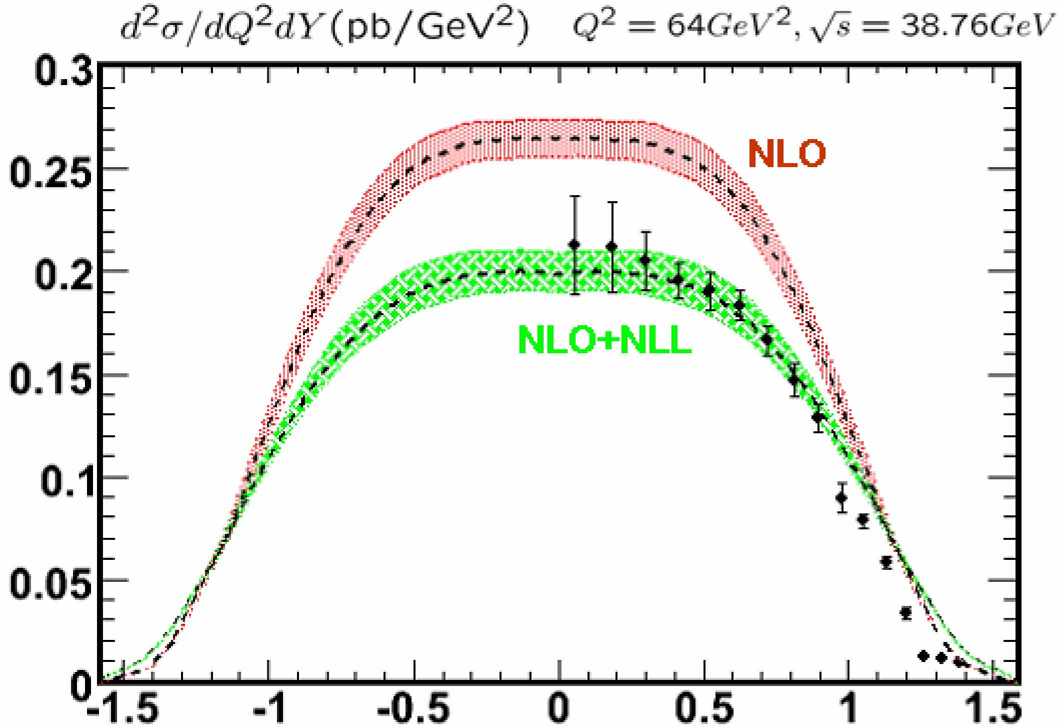
RAPIDITY DISTRIBUTION IN pp COLLISIONS AT LO NLO AND LO+LL WITH THE PARAMETER CHOICES FOR FIXED TARGET EPERIMENT E866/NuSea

HERE, THE BANDS ARE OBTAINED VARYING THE FACTORIZATION SCALE BETWEEN $\mu^2=2Q^2$ AND $\mu^2=1/2Q^2$

WE REALIZE THAT LL RESUMMATION IS SMALL COMPARED TO THE EFFECT OF THE FULL NLO CORRECTION

THIS MEANS THAT THE LL RESUMMATION IS NOT ENOUGH

NLL RESUMMATION AND E866 DATA



RAPIDITY DISTRIBUTION IN pp COLLISIONS AT NLO AND NLO+NLL WITH THE PARAMETER CHOICES AND DATA OF Anastasiou et al. (hep-ph/0306192)

NLL RESUMMATION REDUCES THE CROSS SECTION INSTEAD OF ENHANCING IT FOR NOT LARGE VALUES OF RAPIDITY

THE AGREEMENT WITH THE DATA IS GOOD

A GREAT IMPROVEMENT FOR NOT LARGE RAPIDITY IS OBTAINED WITH RESPECT TO THE NLO CALCULATION

CONCLUSIONS...

1. OUR RESUMMATION FORMULA IS VALID FOR ALL VALUES OF RAPIDITY AND TO ALL LOGARITHMIC ORDERS
2. IT USES THE WELL KNOWN RESUMMATION OF THE RAPIDITY INTEGRATED DRELL-YAN
3. NLL RESUMMATION HAS AN IMPORTANT EFFECT ON PREDICTIONS AT RELATIVELY LOW ENERGIES
4. AT FIXED-TARGET EXPERIMENT NLL RESUMMATION IS MORE IMPORTANT THAN HIGH-FIXED-ORDER CALCULATIONS

...A LONG AND ALREADY OPEN STORY

THIS RESUMMATION WAS FIRST CONSIDERED AT FERMILAB (1992)

(E. Laenen, G. Sterman, Fermilab-Conf-92/359-T)

THE PHENOMENOLOGY FOR W-BOSON AT RHIC HAS BEEN STUDIED

(A. Mukherjee, W. Vogelsang, Phys.Rev. D73:074005,2006)

ALSO THE PHENOMENOLOGY AT HIGH ENERGY HADRON COLLIDERS HAS BEEN CONSIDERED

(V. Ravindran, J. Smith, Phys.Rev. D76:114004,2007)

AT LOWER ENERGIES THE FIXED-TARGET EXPERIMENT E866/NuSea HAS ALSO BEEN STUDIED WITH INTERESTING OPEN QUESTIONS

(P. Bolzoni, Phys.Lett. B643:325-330,2006)
(T. Becher, M. Neubert, G. Xu, arXiv:0710.0680)

THE CONVOLUTION BECOMES A PRODUCT

$$f(x) = \int_x^1 \frac{dy}{y} f_1(y) f_2\left(\frac{x}{y}\right) = \int_0^1 dy dw\, f_1(y) f_2(w) \delta(x-yw)$$

$$\begin{aligned}
f(N) &\equiv \int_0^1 dx\, x^{N-1} f(x) \\
&= \int_0^1 dx dy dw\, f_1(y) f_2(w) x^{N-1} \delta(x - yw) \\
&= \int_0^1 dy dw\, y^{N-1} x f_1(y) w^{N-1} f_2(w) = f_1(N) f_2(N)
\end{aligned}$$

THE LARGE LOGS OF (1-z) ARE MAPPED INTO LARGE LOGS OF N

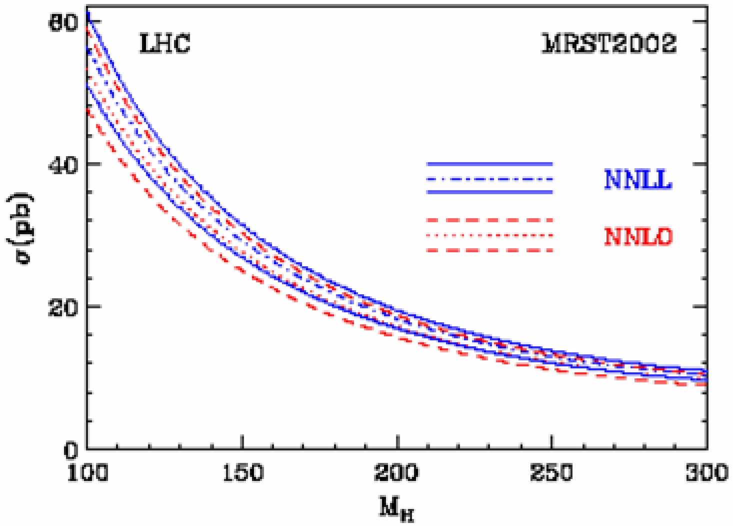
$$
\begin{aligned}
\int_0^1 dz\, z^{N-1} \left[\frac{\ln(1-z)}{1-z}\right]_+ &= \int_0^1 dz\, \frac{z^{N-1}-1}{1-z} \ln(1-z) \\
&= \left[\frac{d}{d\eta}\int_0^1 dz\,(z^{N-1}-1)e^{(\eta-1)\ln(1-z)}\right]_{\eta=0} \\
&= \left[\frac{d}{d\eta}\int_0^1 dz\,(z^{N-1}-1)(1-z)^{\eta-1}\right]_{\eta=0} \\
&= \left[\frac{d}{d\eta}\left(\frac{\Gamma(N)\Gamma(\eta)}{\Gamma(N+\eta)} - \frac{1}{\eta}\right)\right]_{\eta=0} \\
&= \left\{\frac{d}{d\eta}\left[\frac{1}{\eta}(\Gamma(1+\eta)N^{-\eta}-1)\right]\right\}_{\eta=0} \\
&= \frac{1}{2}\ln^2\left(\frac{1}{N}\right) - \gamma_E \ln\left(\frac{1}{N}\right) + O\left(\left(\frac{1}{N}\right)^0\right)
\end{aligned}
$$

IN THE FIFTH STEP THE STIRLING FORMULA HAS BEEN USED

$$\Gamma(N+1) = \sqrt{2\pi N}e^{N\ln N - N} + O(1/N)$$

IMPORTANCE OF RESUMMATION

TOTAL CROSS SECTION FOR THE HIGGS BOSON PRODUCTION AT LHC IN THE GLUON FUSION CHANNEL



Soft gluon resummation for Higgs boson production at hadron colliders.

Stefano Catani (INFN, Florence) , Daniel de Florian (Buenos Aires U.) , Massimiliano Grazzini (CERN) , Paolo Nason (INFN,Milan) .

BICOCCA-FT-03-12, CERN-TH-2003-117, Jun 2003. 44pp.

Published in **JHEP 0307:028,2003**.

e-Print Archive: **hep-ph/0306211**

RESUMMATION HAS AN IMPACT OF ABOUT 6% AND THE SCALE UNCERTAINTY IS REDUCED FROM 15% TO 10%

KINEMATICS OF DRELL-YAN RAPIDITY DISTRIBUTION

WE DEFINE THE C.M. HADRONIC RAPIDITY: $Y \equiv \frac{1}{2}\log\left(\frac{Q^0+Q^3}{Q^0-Q^3}\right)$

THE CROSS SECTION IS PARAMETRIZED IN TERMS OF PARTON RAPIDITY AND IN TERMS OF

Q^2 VIRTUALITY OF THE PHOTON OR MASS OF THE GAUGE BOSON

$x \equiv \frac{Q^2}{s}$ FRACTION OF ENERGY THAT THE HADRONS TRANSFER TO THE PHOTON OR VECTOR BOSON

THE HADRONIC AND PARTONIC C.M. RAPIDITIES ARE RELATED WITH A BOOST ALONG THE COLLISIONAL AXIS $Y = y + \frac{1}{2}\log(\frac{x_1}{x_2})$

MOMENTUM CONSERVATION IMPLIES THAT

$$\frac{1}{2}\log(x) \le Y \le \frac{1}{2}\log(\frac{1}{x})$$

$$\frac{1}{2}\log(z) \le y \le \frac{1}{2}\log(\frac{1}{z})$$

$$z = \frac{x}{x_1 x_2}$$

FRACTION OF ENERGY THAT THE PARTONS TRANSFER TO THE PHOTON

THE RESUMMED COEFFICIENT FUNCTION IN MSBAR SCHEME

$$C^{res}(N, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2)) =$$

$$\exp\left\{-\int_1^{N^2} \frac{dn}{n}\left[\int_{n\mu^2}^{Q^2} \frac{dk^2}{k^2} A(\alpha_s(k^2), \alpha_s(\frac{k^2}{n})) + B(\alpha_s(k^2), \alpha_s(\frac{k^2}{n}))\right]\right\}$$

THIS EXPRESSION CAN BE OBTAINED WITH THE RENORMALIZATION GROUP APPROACH DEVELOPED BY S. FORTE AND G. RIDOLFI (Nucl. Phys. B 650 (2003) 229)

$$A = \sum_{i=0}^{\infty}\sum_{j=1}^{\infty} A_{ij}\alpha_s^i(k^2)\alpha_s^j(k^2/n), \qquad B = \sum_{i=0}^{\infty}\sum_{j=1}^{\infty} B_{ij}\alpha_s^i(k^2)\alpha_s^j(k^2/n)$$

THE COEFFICIENTS ARE OBTAINED MATCHING THE FIXED ORDER CALCULATION OF THE COEFFICIENT FUNCTION WITH THE FIXED ORDER EXPANSION OF THE RESUMMED FORMULA

NLL RESUMMATION COEFFICIENTS FOR DRELL-YAN

$$A_{02} = \frac{C_F}{2\pi^2}\left[C_A\left(\frac{67}{18} - \frac{\pi^2}{6}\right) - \frac{5}{9}N_f\right],$$

$$A_{11} = 0, \quad B_{01} = -\frac{\gamma_E A_{01}}{2\pi}, \quad A_{01} = \frac{C_F}{\pi}$$

PHYSICAL ANOMALOUS DIMENSION

$$F(N,Q^2) = \sum_a f_a(N,\mu^2) C_a \left(N, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2) \right)$$

WE WILL TREAT EACH SUBPROCESS INDEPENDENTLY BECAUSE
IT CAN BE SHOWN THAT THEY ARE μ^2-INDEPENDENT
UP TO POWERS OF 1/N

$$Q^2 \frac{\partial F(N,Q^2)}{\partial Q^2} = \gamma(N, \alpha_s(Q^2)) F(N,Q^2)$$

$\Downarrow$

$$\begin{aligned} \gamma(N,\alpha_s(Q^2)) &= \frac{\partial \ln C(N, Q^2/\mu^2, \alpha_s(\mu^2))}{\partial \ln Q^2} \\ &= \gamma^{AP}(N,\alpha_s(Q^2)) + \frac{\partial \ln C(N,1,\alpha_s(Q^2))}{\partial \ln Q^2} \end{aligned}$$

WHAT IS THE LIMIT z→1

THE LIMIT z→1 CORRESPONDS TO THE THRESHOLD LIMIT FOR THE PRODUCTION OF A VECTOR BOSON OF INVARIANT MASS Q^2:

THE SQUARING THE FOUR MOMENTUM CONSERVATION LAW IMPLIES

$$
\begin{aligned}
x_1 x_2 s(1-z) \;=\; & \sum_{i,j=1}^{n} k_i^0 k_j^0 (1-\cos\theta_{ij}) \\
& +2\sum_{i=1}^{n} k_i^0 \left(\sqrt{Q^2+|\vec{Q}|^2} - |\vec{Q}|\cos\theta_i\right)
\end{aligned}
$$

SINCE THE TERMS IN THE FIRST SUM CANNOT BE NEGATIVE AND THE TRMS IN THE SECOND ONE ARE POSITIVE, WE HAVE THE LIMIT z=1 IS ACHIEVD ONLY IF ALL THE n EXTRA EMISSIONS ARE SOFT