"QCD Resummation": comparing "direct" and "effective" methods

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2010 Jets Workshop @ UW : GS QCD Resummation
CL SCET

Sal Rappoccio : "Are they the same?"

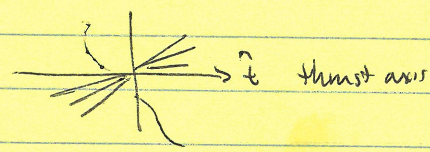
~~Why another~~ 2014 Boston : "Yes!" in principle
~~GS~~ but not always in practice (truncation)
↓
demand equality!
clarity of "N^kLL" meaning

I comparing angularity distributions $\sigma(\tau) = \frac{1}{\sigma_0}\frac{d\sigma}{d\tau_a}$
II counting logs
III options for computing $\sigma(\tau)$ in momentum space

I. angularities $e^+e^- \to X$ $\tau_a = \frac{1}{Q}\sum_{i\in X} |\vec{p}_i| e^{-|\eta_i|(1-a)}$ $-\infty < a < 2$
($a<1$ here)

$\sigma(\tau) \equiv \frac{1}{\sigma_0}\frac{d\sigma}{d\tau}$ $R(\tau) = \int_0^\tau \sigma(\tau')d\tau'$

$\tilde\sigma(\nu) = \int_0^\infty d\tau\, e^{-\nu\tau}\sigma(\tau)$



At "NLL", ~~compared~~ in 2009 compared

SCET

$$\sigma(\tau) = \frac{1}{\tau}\frac{e^{K+\gamma_E\Omega}}{\Gamma(-\Omega)}$$

QCD

$$\sigma(\tau) = \frac{e^{-E(\ln 1/\tau)}}{\Gamma(-E'(\ln 1/\tau))}$$

similar ... $K, \Omega \Leftrightarrow E, E'$?

~~$E(\ln\nu) = \sim 2$~~ [open SCET evolution boards]
+ QCD "E" board

all describe evolution

H ___ Q
J ___ $\sim Q\tau^{\frac{1}{2-a}}$ (e^{γ_E})?
S ___ $\sim Q\tau$

II. $\tilde{\sigma}(\nu) = H_2(Q^2,\mu)\,\tilde{J}(0,\mu)^2\,\tilde{S}(0,\mu)\; e^{\tilde{\tilde{E}}}$

$\equiv C\, e^{\tilde{\tilde{E}}}$

~~$\tilde{E} = \alpha_s(\Gamma_0 \tilde{L}^2 + \gamma_0 \tilde{L})$~~

~~$+ \alpha_s^2(\Gamma_0$~~

$$\tilde{\tilde{E}} \sim \alpha_s(\tilde{L}^2 + \tilde{L}) + \alpha_s^2(\tilde{L}^3 + \tilde{L}^2 + \tilde{L}) + \alpha_s^3(\tilde{L}^4 + \tilde{L}^3 + \tilde{L}^2 + \tilde{L}) + \cdots \qquad \tilde{L} = \ln(e^{\gamma_E}\nu)$$

↓ LL   ↓ NLL   ↓ NNLL   ↓ N^3LL ...

coefficients are simply related to Γ, γ anomalous dims.

compare $$R(\tau) = \underbrace{H_2(Q^2,\mu)\,\tilde{J}(0,\mu)\,\tilde{S}(0,\mu)\, e^{\bar{E}}}_{\text{same expansion}}\; \underbrace{\exp\left[\sum_{n=2}^{\infty}\frac{1}{n!}\bar{E}^{(n)}\partial^n_{\bar{E}'}\right]\frac{1}{\Gamma(1-\bar{E}')}}_{\text{makes up "extra" terms in Laplace transforms.}}$$

same expansion: $\tilde{L} \to L \equiv \ln\frac{1}{\tau}$

makes up "extra" terms in Laplace transforms.

↑

$\tilde{\sigma}(\nu) \longleftrightarrow R(\tau)$ — equal to the order you keep these differential operator terms

— also why ~~guess that~~ we keep Γ, γ to same order in $\bar{E}'$ (Ω) as $\bar{E}$ (K)

— relation maintained up to subleading terms in $N^k LL'$ counting, not quite in $N^k LL$ counting.

From SCET objects, define,

$$E_F(\mu,\mu_F) = K_F(\mu,\mu_F) - \int_{\mu_F}^{\mu} \frac{d\mu'}{\mu'} \frac{d\ln \tilde{F}(0,\mu')}{d\ln\mu'}$$

Then SCET $\sigma \Rightarrow$

(no logs)

$$\sigma(\tau) = \frac{1}{\tau} H_2(Q^2,\mu_H) \left(\frac{\mu_H}{Q}\right)^{\omega_H} e^{K_H} \tilde{J}(0,\mu)^2 \tilde{S}(0,\mu)$$

$$\times e^{2E_J + E_S} \exp\left\{ \sum_{n=2}^{\infty} \frac{1}{n!} \left(2E_J^{(n)}(\mu,\mu_J)\, \partial^n_{2E_J'} + E_S^{(n)}(\mu_S)\, \partial^n_{E_S'} \right) \right\}$$

$$\times \left(\frac{Q}{\mu_J} (e^{\gamma_E} \tau_a)^{\frac{1}{2-a}} \right)^{2E_J'} \left(\frac{Q\tau e^{\gamma_E}}{\mu_S} \right)^{E_S'} \frac{1}{\Gamma\left(\frac{2}{2-a}E_J' + E_S'\right)}$$

where $E_F^{(n)}(\mu,\mu_F) = \dfrac{d^n E_F(\mu,\mu_F)}{d(\ln\mu_F)^n}$

That is, dQCD = SCET → puts some more terms into E_F exponents.

with
$\mu_H = Q$
$\mu_J = Q(e^{\gamma_E}\tau)^{\frac{1}{2-a}}$
$\mu_S = Q e^{\gamma_E}\tau$

$A = \Gamma_{cusp}$
$B_F = \gamma_F - \dfrac{d\ln\tilde{F}(0,\mu_F)}{d\ln\mu_F}$

↓

generalizes to all-orders form with jet/soft scale $\mu_{J,S}$ dependence $\Rightarrow$ better uncertainty estimates, "scale profiles"

III. $\sigma(\tau)$ vs. $R(\tau)$

$$
\begin{array}{ccc}
R(\tau,\mu_F) & \xrightarrow{d/d\tau} & \sigma(\tau,\mu_F) \\
\big\downarrow \mu_F \sim Q\,\tau_a^{1/j_F} & & \big\downarrow \\
R(\tau) & \xrightarrow[d/d\tau]{} & \sigma(\tau)
\end{array}
$$

equal at infinite accuracy

fewer terms at finite $N^kLL^{(\prime)}$ accuracy

[label σ_n]

$$\sigma_R(\tau_a) = \frac{d}{d\tau_a} R(\tau_a, \mu_F(\tau_a))$$

$$= \sigma_n + \delta\sigma_R(\tau_a)$$

ensures relation $\int_0^\tau d\tau' \, \sigma_{N^kLL^{(\prime)}}(\tau') = R^{N^kLL}(\tau)$

- integrates to correct total σ.
- performs better in large $\ln\frac{1}{\tau}$ region (peak)
- instabilities in far tail $\tau \sim 1$ [AFHMS]

at NLL order $\sigma_n(\tau) = \frac{1}{\tau} \frac{e^{K_{NLL} + \gamma_E \eta_{NLL}}}{\Gamma(-\eta_{NLL})}$

$$= -\frac{\eta_{NLL}}{\tau} \frac{e^{K_{NLL}+\gamma_E \eta_{NLL}}}{\Gamma(1-\eta_{NLL})}$$

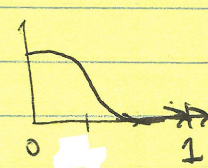
$$= \frac{\alpha_s \Gamma_0 \ln\frac{1}{\tau}}{\tau}(1 + \cdots)$$

missing γ_0 contained in $\delta\sigma_R$.

i.e. do not truncate σ_n according to usual N^kLL tables. σ_R OK. σ_n OK at N^kLL' order.

we provide $\sigma_R(\tau_a, \mu_H, \mu_J, \mu_S)$

— interpolate between peak & far tail $\delta\sigma_R(\tau_a)\, F^{interp}(\tau_a)$



[switch to plots]

Supplemental Formulas (side boards)

Laplace transform $\quad \tilde{F}(\nu) \equiv \mathcal{L}\{F\}(\nu) = \int_0^\infty d\tau \, e^{-\nu\tau} F(\tau)$

$$\tilde{L} \equiv \ln(\nu e^{\gamma_E}) \qquad L \equiv \ln\frac{1}{\tau}$$

$$\mathcal{L}(1) = \frac{1}{\nu}(1)$$
$$\mathcal{L}(L) = \frac{1}{\nu}(\tilde{L})$$
$$\mathcal{L}(L^2) = \frac{1}{\nu}\left(\tilde{L}^2 + \frac{\pi^2}{6}\right)$$
$$\mathcal{L}(L^3) = \frac{1}{\nu}\left(\tilde{L}^3 + \frac{\pi^2}{2}\tilde{L} + 2\zeta_3\right)$$
$$\mathcal{L}(L^4) = \frac{1}{\nu}\left(\tilde{L}^4 + \pi^2\tilde{L}^2 + 8\zeta_3\tilde{L} + \frac{3\pi^4}{20}\right)$$
$$\vdots$$

$$\mathcal{L}^{-1}\{\tilde{F}\}(\tau) = \int_{\gamma-i\infty}^{\gamma+i\infty} \frac{d\nu}{2\pi i} e^{\nu\tau} \tilde{F}(\nu)$$

$$\mathcal{L}^{-1}\left\{\frac{1}{\nu}\right\} = 1$$
$$\mathcal{L}^{-1}\left\{\frac{1}{\nu}\tilde{L}\right\} = L$$
$$\mathcal{L}^{-1}\left\{\frac{1}{\nu}(\tilde{L}^2)\right\} = L^2 - \frac{\pi^2}{6}$$
$$\mathcal{L}^{-1}\left\{\frac{1}{\nu}(\tilde{L}^3)\right\} = L^3 - \frac{\pi^2}{2}L - 2\zeta_3$$
$$\mathcal{L}^{-1}\left\{\frac{1}{\nu}(\tilde{L}^4)\right\} = L^4 - \pi^2 L^2 - 8\zeta_3 L + \frac{\pi^4}{60}$$
$$\vdots$$

SCET

$$\sigma(\tau_a) = H_2(Q^2,\mu)\int dt_a^n \, dt_a^{\bar{n}} \, dk_s \, \delta\!\left(\tau_a - \frac{t_a^n + t_a^{\bar{n}}}{Q^{2-a}} - \frac{k_s}{Q}\right) J_n^a(t_a^n,\mu) \, J_{\bar{n}}(t_a^{\bar{n}},\mu) \, S_2^a(k_s,\mu)$$

$$\tilde{\sigma}(\nu_a) = H_2(Q^2,\mu)\,\tilde{J}_n^a\!\left(\frac{\nu_a}{Q^{2-a}},\mu\right)\tilde{J}_{\bar{n}}^a\!\left(\frac{\nu_a}{Q^{2-a}},\mu\right)\tilde{S}_2^a\!\left(\frac{\nu_a}{Q},\mu\right)$$

$$\mu\frac{d}{d\mu}\tilde{F}(\nu,\mu) = \tilde{\gamma}_F(\nu,\mu)\tilde{F}(\nu,\mu) \qquad \text{for } F = J, S$$

$$\tilde{\gamma}_F(\nu,\mu) = -\kappa_F \Gamma_{cusp}[\alpha_s(\mu)] \ln\!\left(\left(\frac{\mu}{Q}\right)^{j_F} \nu e^{\gamma_E}\right) + \gamma_F[\alpha_s(\mu)]$$

$$\tilde{F}(\nu,\mu) = \tilde{F}(\nu,\mu_0)\, e^{K_F(\mu,\mu_0)}\left(\left(\frac{\mu_0}{Q}\right)^{j_F} e^{\gamma_E}\nu\right)^{\omega_F(\mu,\mu_0)}$$

$$K_F(\mu,\mu_0) = -j_F\kappa_F\int_{\mu_0}^{\mu}\frac{d\mu'}{\mu'}\Gamma_{cusp}[\alpha_s(\mu')]\ln\frac{\mu'}{\mu_0} + \int_{\mu_0}^{\mu}\frac{d\mu'}{\mu'}\gamma_F[\alpha_s(\mu')]$$

$$\omega_F(\mu,\mu_0) = -\kappa_F\int_{\mu_0}^{\mu}\frac{d\mu'}{\mu'}\Gamma_{cusp}[\alpha_s(\mu')]$$

~~$\sigma(\tau_a) =$~~

$$\tilde{\sigma}(\nu_a) = H_2(Q^2,\mu_H)\,\tilde{J}(L_J(\mu_J),\mu_J)\,\tilde{S}(L_S(\mu_S),\mu_S)\, e^{K} \left(\frac{\mu_H}{Q}\right)^{\omega_H} \left(\frac{\mu_J(\nu e^{\gamma_E})}{Q}\right)^{\frac{2(2-a)\omega_J}{2-a}} \left(\frac{\mu_S \nu e^{\gamma_E}}{Q}\right)^{\omega_S}$$

$$L_F(\mu_F) \equiv \ln\left(\left(\frac{\mu}{Q}\right)^{j_F} e^{\gamma_E}\nu\right)$$

(Becher, Neubert; Schwartz; Lee, Hornig, Ovanesyan) ⋮

$$\sigma_{(n)}(\tau_a) = H_2(Q^2,\mu_H)\,\frac{e^{K}}{\tau}\left(\frac{\mu_H}{Q}\right)^{\omega_H}\left(\frac{\mu_J}{Q\tau^{\frac{1}{2-a}}}\right)^{\frac{2(2-a)\omega_J}{2-a}}\left(\frac{\mu_S}{Q\tau}\right)^{\omega_S}$$

$$\times\ \tilde{J}^2\left(\partial_\Omega + \ln\left[\left(\frac{\mu_J}{Q}\right)^{2-a}\frac{1}{\tau}\right],\mu_J\right)\tilde{S}\left(\partial_\Omega + \ln\frac{\mu_S}{Q\tau},\mu_S\right)\frac{e^{\gamma_E\Omega}}{\Gamma(-\Omega)}$$

$$K \equiv K_H(\mu,\mu_H) + 2K_J(\mu,\mu_J) + K_S(\mu,\mu_S)$$

$$\Omega \equiv 2\omega_J(\mu,\mu_J) + \omega_S(\mu,\mu_S)$$

usual rules:

	Γ_{cusp}	γ_F	$\beta[\alpha_s]$	$H,\tilde{J},\tilde{S}$		$H,\tilde{J},\tilde{S}$
LL	α_s	1	α_s	1	LL	1
NLL	α_s^2	α_s	α_s^2	1	NLL′	α_s
NNLL	α_s^3	α_s^2	α_s^3	α_s	NNLL′	α_s^2
N^3LL	α_s^4	α_s^3	α_s^4	α_s^2	N^3LL′	α_s^3

NLL angularities "dQCD" Berger, Sterman hep-ph/0307394 + priv. comm.

SCET Hornig, Lee, Ovanesyan 0901.3780

SCET $\quad \sigma(\tau) = \dfrac{1}{\tau}\dfrac{e^{K+\gamma_E\Omega}}{\Gamma(-\Omega)}$

dQCD $\quad \sigma(\tau) = \dfrac{e^{-E(1/\tau)}}{\Gamma(-E'(1/\tau))}$

meanwhile

$$E(\ln \tfrac{1}{\tau}) = 4 \int_{Q(e^{\gamma_E}\tau)^{1/2-a}}^{Q} \frac{d\mu'}{\mu'} A[\alpha_s] \ln \frac{\mu'}{Q}$$

$$- \frac{4}{1-a} \int_{Qe^{\gamma_E}\tau}^{Q(e^{\gamma_E}\tau)^{\frac{1}{1-a}}} \frac{d\mu'}{\mu'} A[\alpha_s] \ln \frac{\mu'}{Q(e^{\gamma_E}\tau)}$$

$$+ 2 \int_{Q(e^{\gamma_E}\tau)^{\frac{1}{1-a}}}^{Q} \frac{d\mu'}{\mu'} B_J[\alpha_s] + \int_{Qe^{\gamma_E}\tau}^{Q} \frac{d\mu'}{\mu'} B_S[\alpha_s]$$