



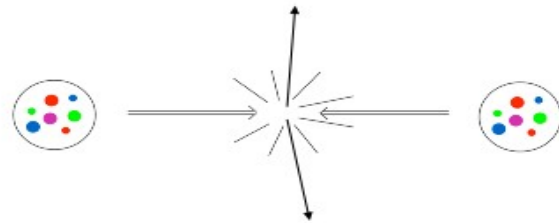
Gluon saturation and production of jets

Krzysztof Kutak

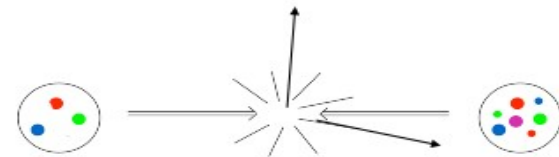


Supported by grant: LIDER/02/35/L-2/10/NCBiR/2011

LHC – as a scanner of gluon



central-central i.e.
not so dense-not so dense



forward-central i.e.
dilute – not so dense

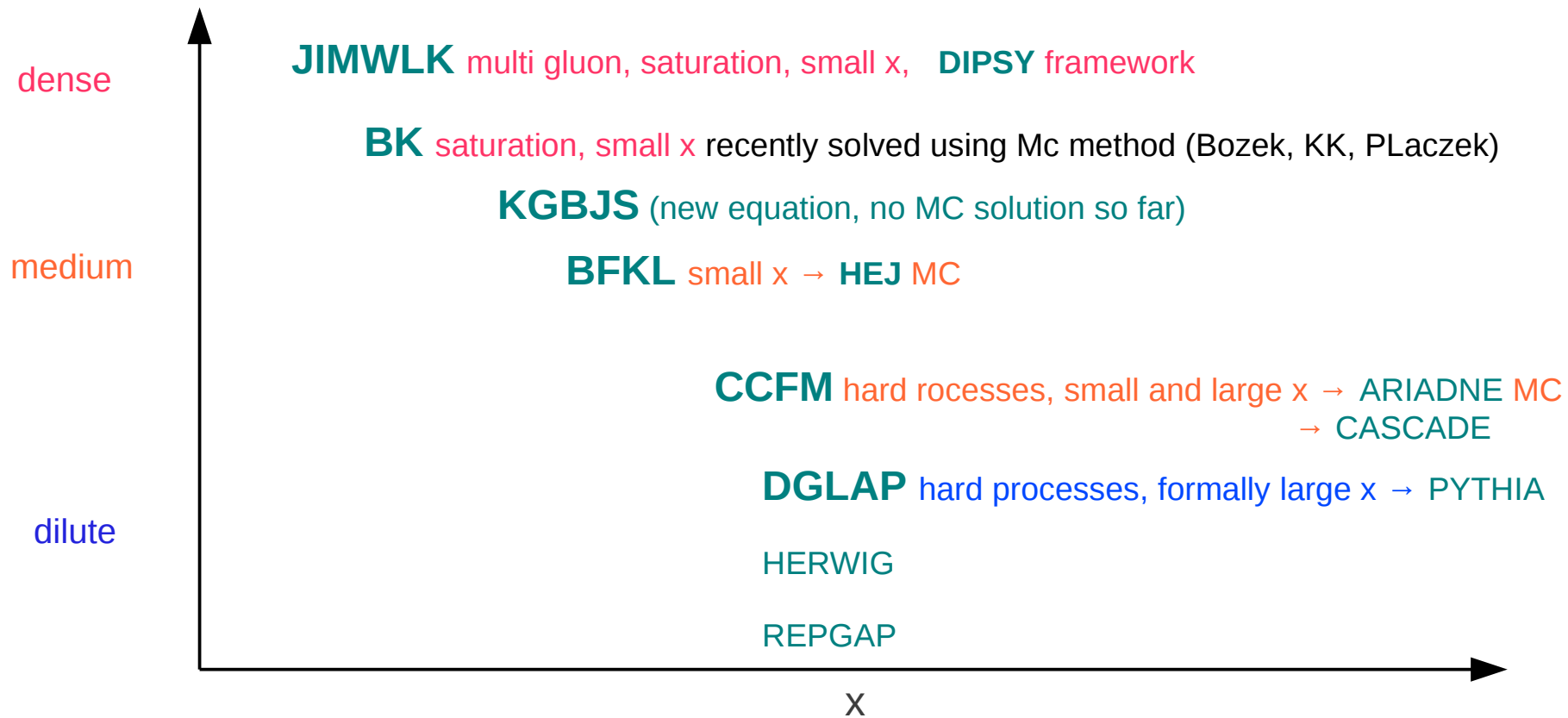


forward-forward i.e.
dilute -dense

$$x_1 = \frac{1}{\sqrt{S}} (p_{t1} e^{y_1} + p_{t2} e^{y_2}) \quad \xrightarrow{y_1 \sim 0, y_2 \gg 0} \quad \sim 1$$

$$x_2 = \frac{1}{\sqrt{S}} (p_{t1} e^{-y_1} + p_{t2} e^{-y_2}) \quad \ll 1$$

The motivation – to understand gluon at low x with the help of exclusive processes



QCD at high energies – high energy factorization

$$\frac{d\sigma}{dy_1 dy_2 d^2p_{1t} d^2p_{2t}} = \sum_{a,b,c,d} \int \frac{d^2k_{1t}}{\pi} \frac{d^2k_{2t}}{\pi} \frac{1}{16\pi^2(x_1 x_2 S)^2} |\overline{\mathcal{M}_{ab \rightarrow cd}}|^2 \delta^2(\vec{k}_{1t} + \vec{k}_{2t} - \vec{p}_{1t} - \vec{p}_{2t})$$

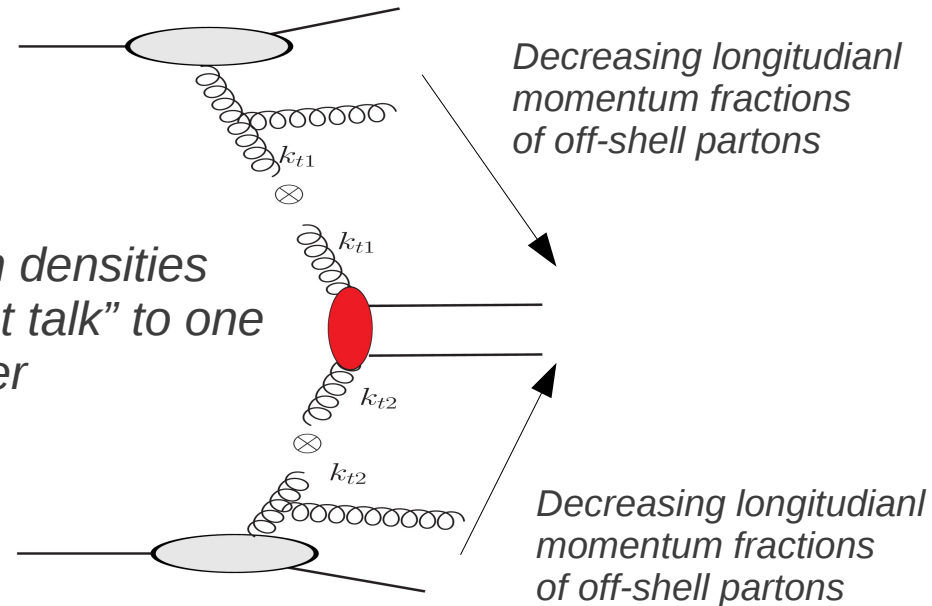
$$\times \mathcal{F}_{a/A}(x_1, k_{1t}^2, \mu^2) \mathcal{F}_{b/B}(x_2, k_{2t}^2, \mu^2) \frac{1}{1 + \delta_{cd}}$$

$$k_1^\mu = x_1 P_1^\mu + \bar{x}_1 P_2^\mu + k_{1t}^\mu \quad k_2^\mu = x_2 P_2^\mu + \bar{x}_2 P_1^\mu + k_{2t}^\mu$$

$$\bar{x}_1 = \frac{k_1^2 + \mathbf{k}^2}{Sx_1} \quad \bar{x}_2 = \frac{k_2^2 + \mathbf{k}^2}{Sx_2}$$

$$|\overline{\mathcal{M}_{ab \rightarrow cd}}|^2 = \frac{2x_1 k_1^{\mu_1} k_1^{\nu_1}}{k_1^2} \frac{2x_2 k_2^{\mu_2} k_2^{\nu_2}}{k_2^2} \mathcal{M}_{ab \rightarrow cd \mu_1 \nu_1} \mathcal{M}_{ab \rightarrow cd \mu_2 \nu_2}^*$$

Parton densities
“do not talk” to one another



Gribov, Levin, Ryskin '81
Ciafaloni, Catani, Hautman '93

Originally derived for quarks in final state.
Lipatov provided general framework.

Recently new approach consistent with Lipatov's action allowed for formulation and numerical calculation of **any tree level amplitude with off-shell gluons in initial state**

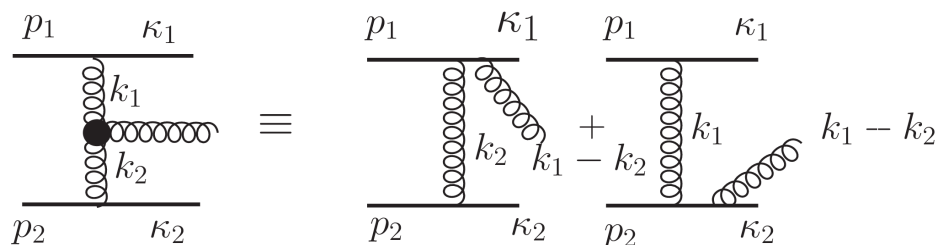
Van Hameren, Kotko, KK '12

Generalized to p-A

Dominguez, Huan, Marquet, Xiao '10

The BFKL evolution

Balitsky, Fadin, Kuraev, Lipatov '77

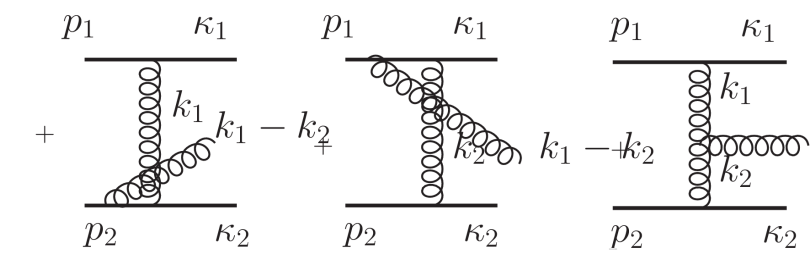


$$1 \gg \alpha_1 \gg \alpha_2$$

$$1 \gg |\beta_1| \gg |\beta_2|$$

$$k_1 = \alpha_1 p_1 + \beta_1 p_2 + k_{1t}$$

$$k_2 = \alpha_2 p_1 + \beta_2 p_2 + k_{2t}$$



$$iA_{2 \rightarrow 3}^{\rho} = (-2ig_2 p_1^{\mu}) t_{mj}^a \left(\frac{-i}{k_1^2} \right) f_{abc} g_s \Gamma_{\mu\nu}^{\rho}(k_1, k_2) \left(\frac{-i}{k_2^2} \right) (-2ig_s p_2^{\nu}) t_{nl}^b$$

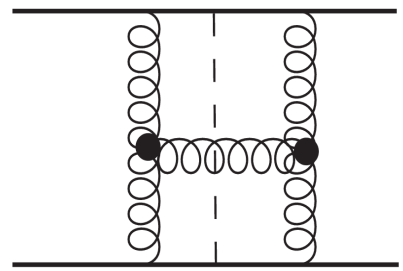
• Known also for SM YM

• Studied also in context of AdS/CFT

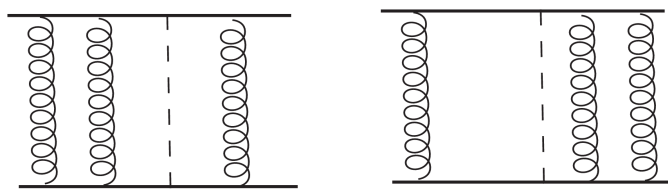
• Known up to NLO

• No saturation

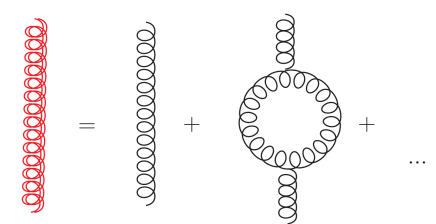
• No applicable to final states: "evolution without observer"



$$J^{\mu} = -ig\bar{u}(p_1 + q)\gamma^{\mu}u(p_1) \approx -2ig_s p_1^{\mu}$$

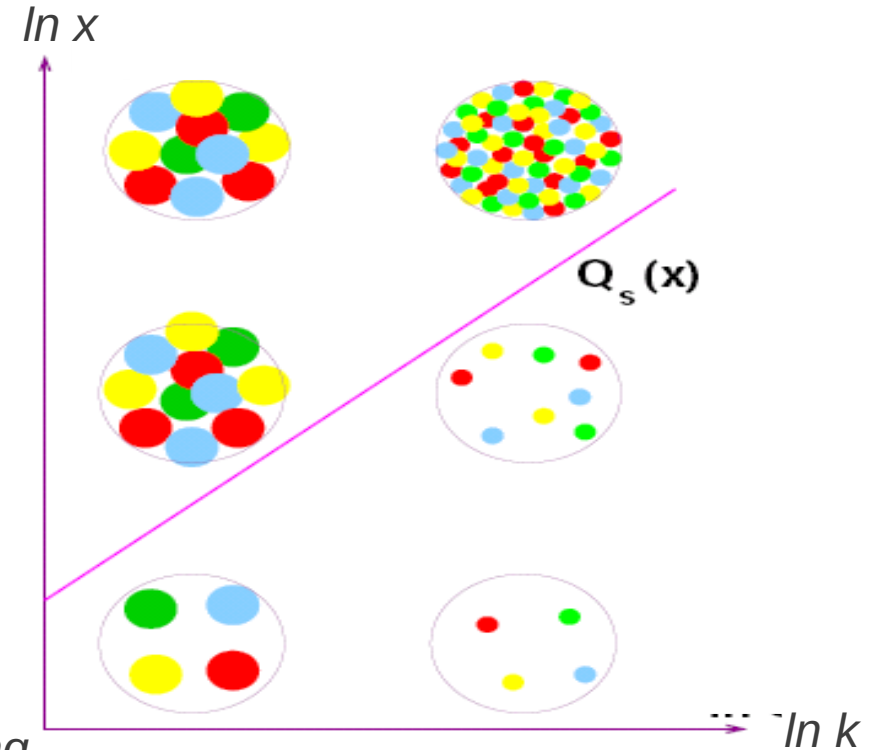


reggeized gluon



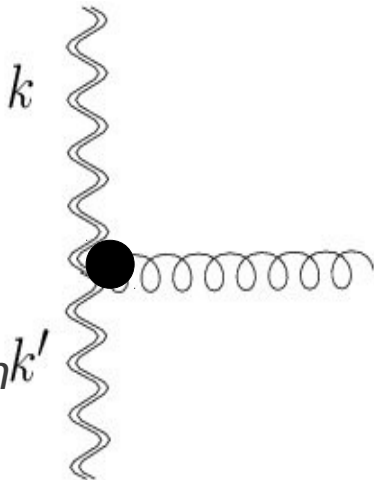
High energy factorization and saturation

Saturation – state where number of gluons stops growing due to high occupation number. way to fulfill unitarity requirements in high energy limit of QCD. More generally saturation is an example of **percolation** which has chance to happen since partons have size $1/k_t$ and hadron has finite size. Cross sections (e.g. F_2) change their behavior from power like to **logarithmic like**.



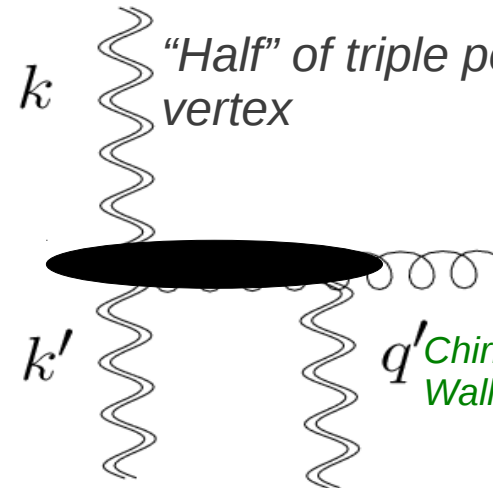
On microscopic level it means that gluon apart splitting recombine

splitting



recombination

Nonlinear evolution equations
BK, JIMWLK
CGC framework
DIPSY

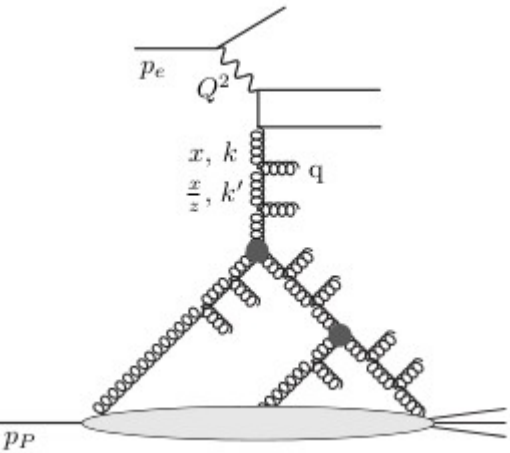
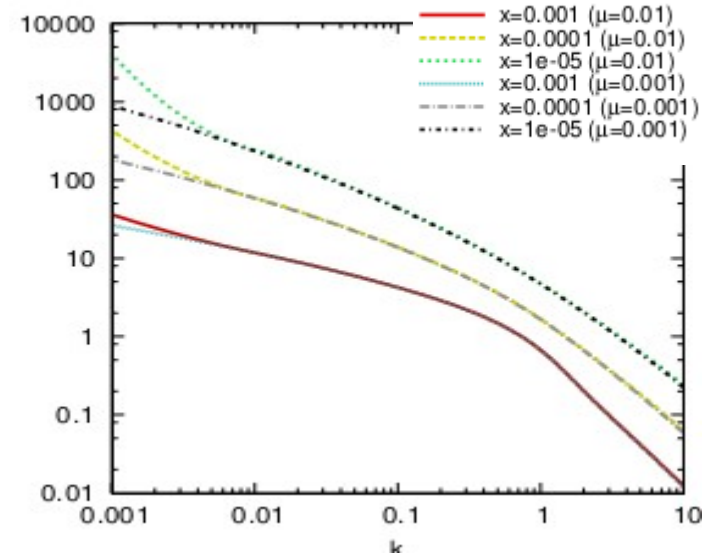
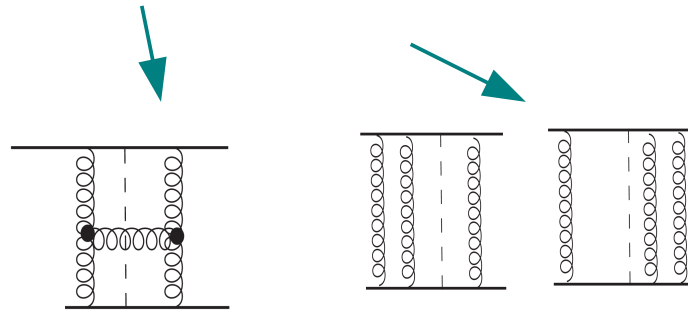


Bartels, Wusthoff '95
Bartels, KK, 06

Chirilli, Szymanowski,
Wallon '10

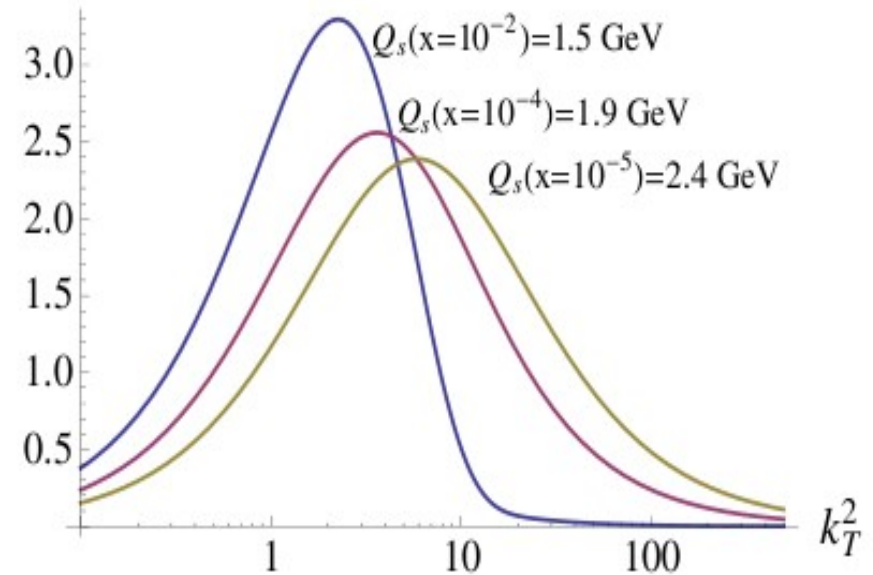
The BFKL and BK evolutions - solutions

$$\mathcal{F}(x, k^2) = \mathcal{F}_0(x, k^2) + \bar{\alpha}_s \int_{x/x_0}^1 \frac{dz}{z} \int_0^\infty \frac{dl^2}{l^2} \left[\frac{l^2 \mathcal{F}(x/z, l^2) - k^2 \mathcal{F}(x/z, k^2)}{|k^2 - l^2|} + \frac{k^2 \mathcal{F}(x/z, k^2)}{\sqrt{(4l^4 + k^4)}} \right]$$



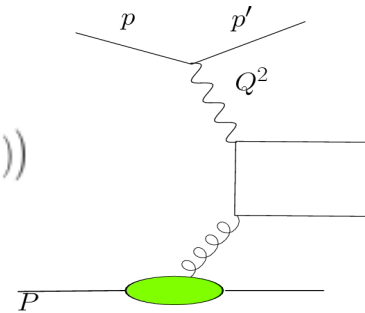
$$\mathcal{F}(x, k^2) = \mathcal{F}_0(x, k^2) + \bar{\alpha}_s \int_{x/x_0}^1 \frac{dz}{z} \int_0^\infty \frac{dl^2}{l^2} \left[\frac{l^2 \mathcal{F}(x/z, l^2) - k^2 \mathcal{F}(x/z, k^2)}{|k^2 - l^2|} + \frac{k^2 \mathcal{F}(x/z, k^2)}{\sqrt{(4l^4 + k^4)}} \right]$$

$$- \frac{2\alpha_s^2 \pi}{N_c R^2} \int_{x/x_0}^1 \frac{dz}{z} \left\{ \left[\int_{k^2}^\infty \frac{dl^2}{l^2} \mathcal{F}(x/z, l^2) \right]^2 + \mathcal{F}(x/z, k^2) \int_{k^2}^\infty \frac{dl^2}{l^2} \ln \left(\frac{l^2}{k^2} \right) \mathcal{F}(x/z, l^2) \right\}$$



BFKL applied to DIS - some recent results

$$F_2(x, Q^2) = \frac{Q^2}{4\pi^2} \alpha_s \sum_q e_q^2 \int d^2k \mathcal{F}(x, k^2) (S_L(k^2, Q^2, m_q^2) + S_T(k^2, Q^2, m_q^2))$$

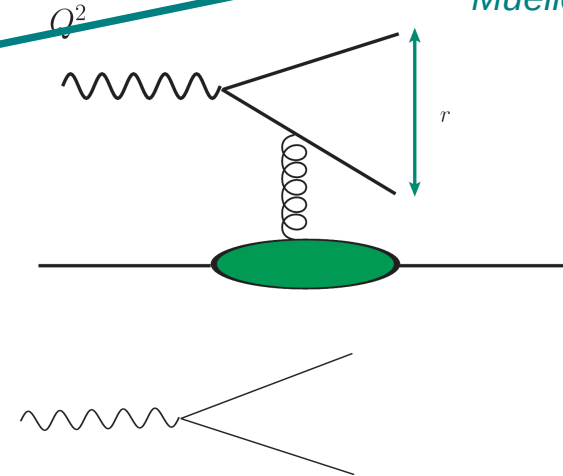


In the dipole formalism

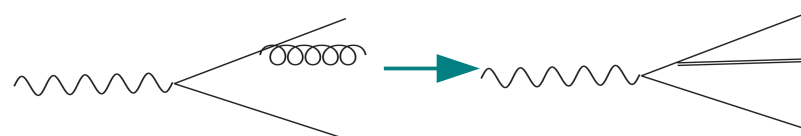
$$F_2(x, Q^2) = \frac{Q^2}{4\pi^2 \alpha_{em}} \int d^2b \int_0^1 dz \int d^2r (|\psi_L(z, r)|^2 + |\psi_T(z, r)|^2) N(x, r, b)$$

Mueller, Patel '95

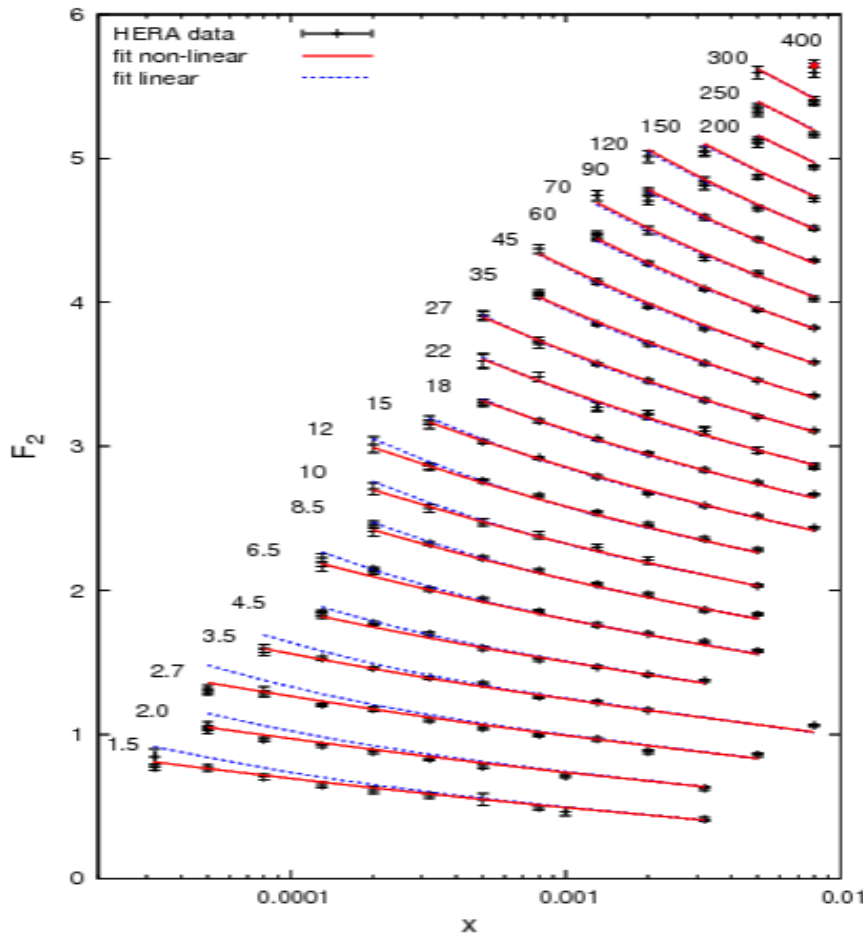
Forward scattering amplitude



Large Nc



$$\mathcal{F}(x, k^2) = \frac{N_c}{\alpha_s (2\pi)^3} \int d^2b \int d^2r e^{ik \cdot r} \nabla_r^2 N(r, b, x)$$



Sapeta, KK '12

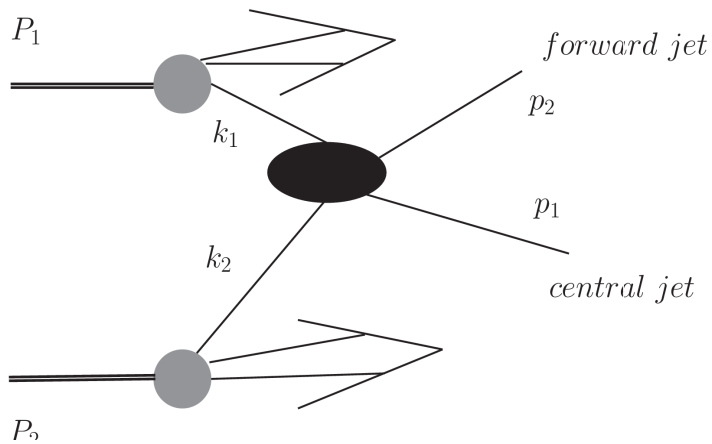
BK equation with corrections of higher order

High energy prescription and forward-central di-jets

Deak, Jung, Hautmann Kutak
JHEP 0909:121,2009

$$\frac{d\sigma}{dy_1 dy_2 dp_{1t} dp_{2t} d\Delta\phi} = \sum_{a,c,d} \frac{p_{t1} p_{t2}}{8\pi^2 (x_1 x_2 S)^2} |\mathcal{M}_{ag \rightarrow cd}|^2 x_1 f_{a/A}(x_1, \mu^2) \mathcal{F}_{g/B}(x_2, k^2) \frac{1}{1 + \delta_{cd}}$$

$$S = 2P_1 \cdot P_2$$



- Resummation of logs of x and logs of hard scale
- Knowing well parton densities at large x one can get information about low x physics
- Framework goes recently under name "k_t framework"

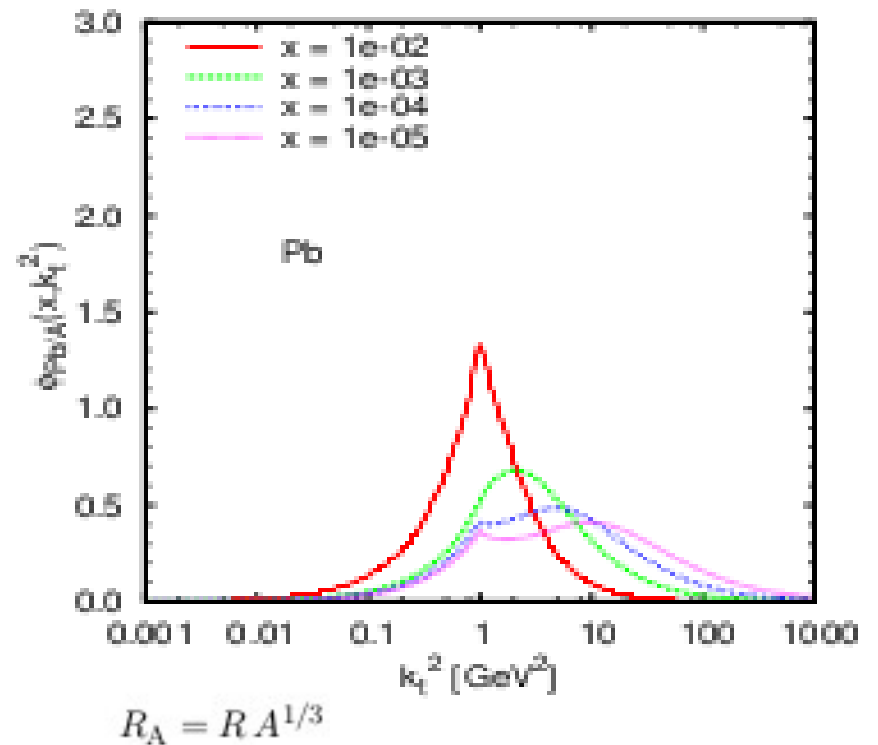
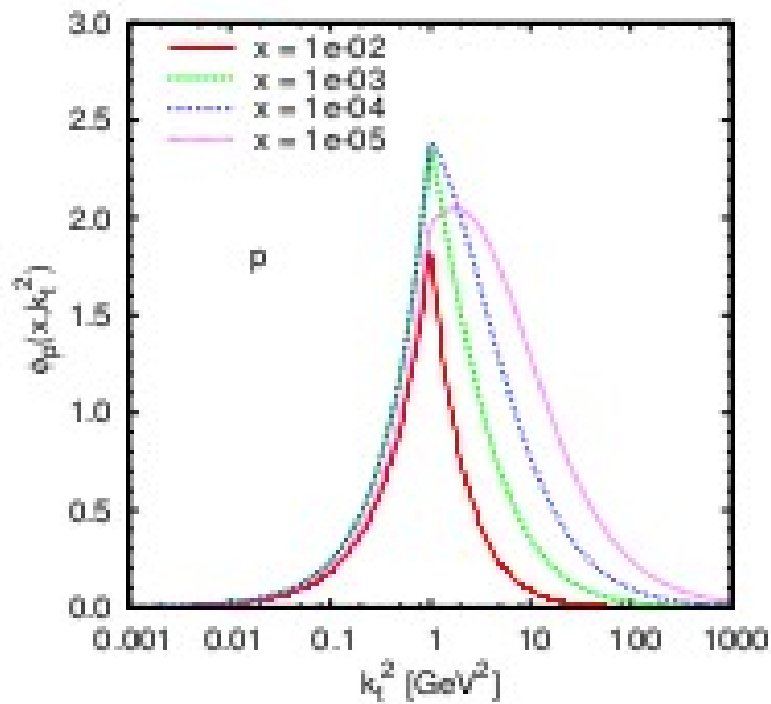
$$x_1 = \frac{1}{\sqrt{S}} (p_{t1} e^{y_1} + p_{t2} e^{y_2}) \quad \xrightarrow{y_1 \sim 0, y_2 \gg 0} \sim 1$$

$$x_2 = \frac{1}{\sqrt{S}} (p_{t1} e^{-y_1} + p_{t2} e^{-y_2}) \quad \ll 1$$

$$k_1^\mu = x_1 P_1^\mu$$

$$k_2^\mu = x_2 P_2^\mu + k_t^\mu$$

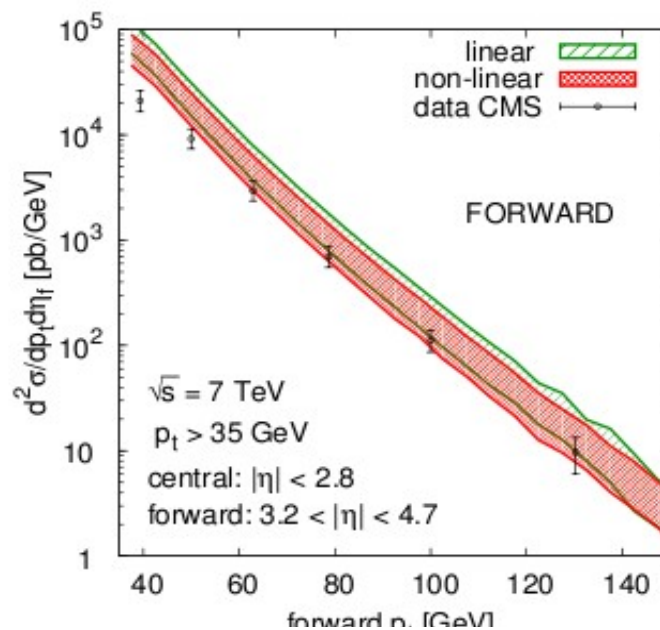
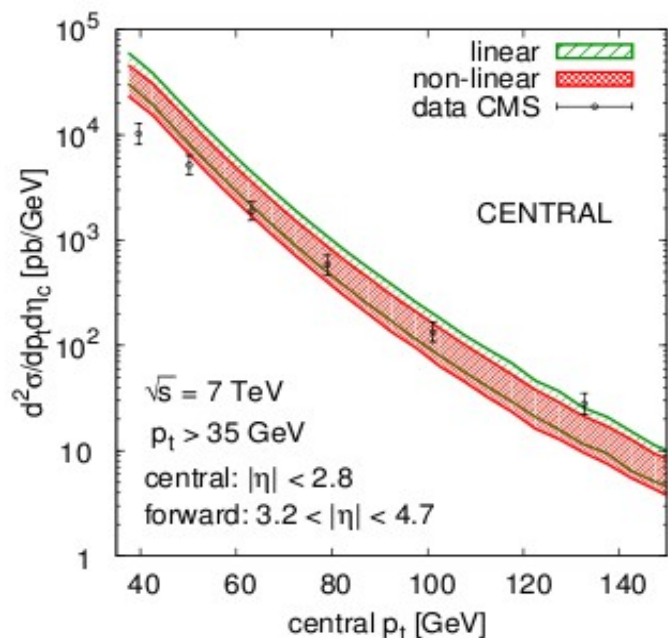
Glue in p vs. glue in Pb



Nonlinear equation for unintegrated gluon density.
 Related to BK via Fourier transform
 Includes corrections of higher order Kwiecincki, KK 2002
 Fitted to latest HERA data Sapeta, KK 2011

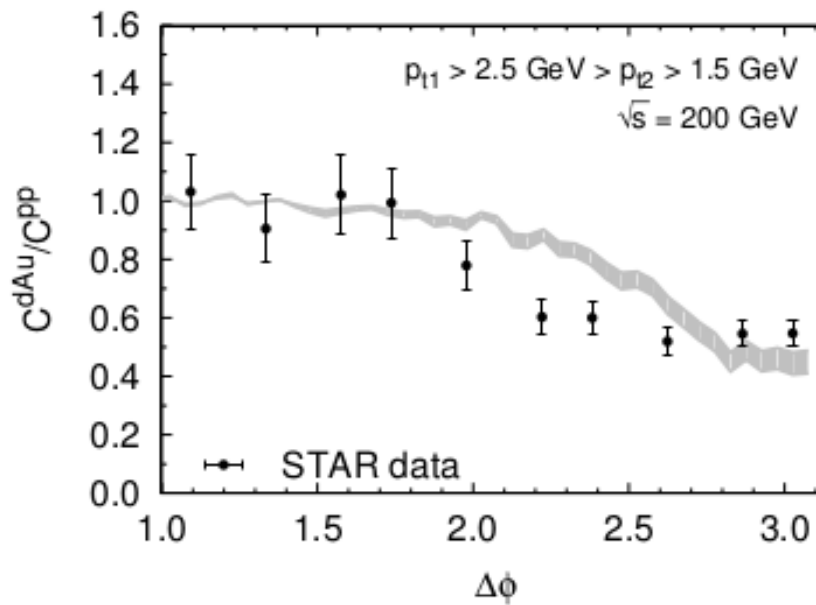
Di-jets p_t spectra

S.Sapeta. KK ,12



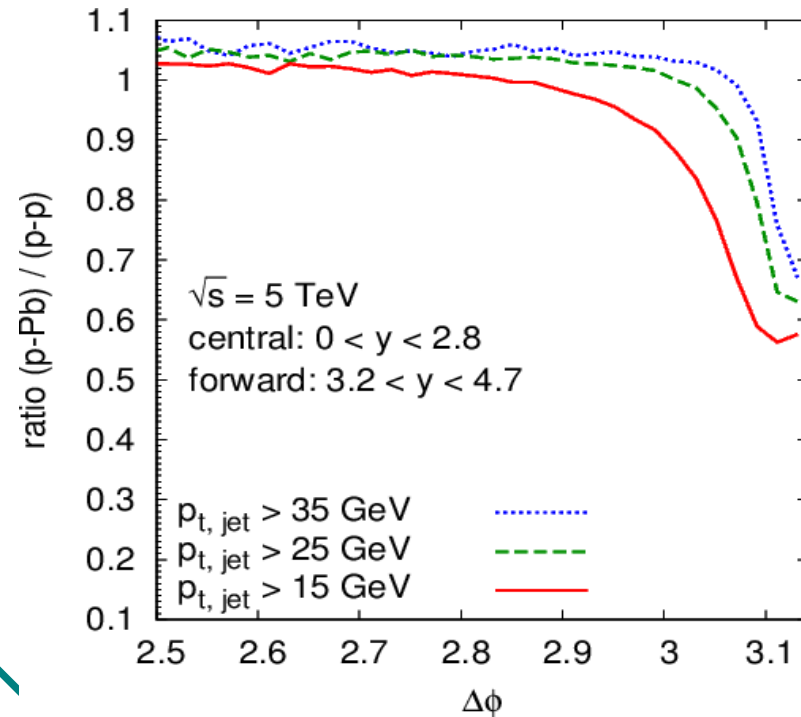
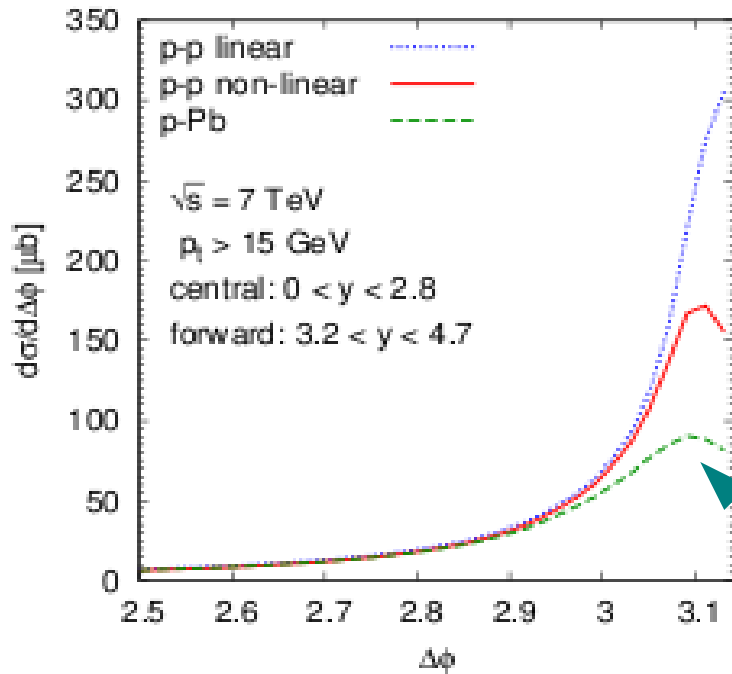
Reasonable agreement.
No usage of partonshower
BK + higher order corrections

At RHIC



Prediction for signatures of saturation in p-p and p-Pb

S.Sapeta. KK '12

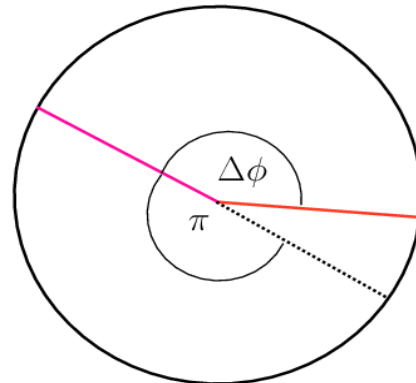


Observable suggested to study BFKL effects

Sabio-Vera, Schwensen '06

Studied also context of RHIC

Albacete, Marquet '10



Reflects $\sim k^2$

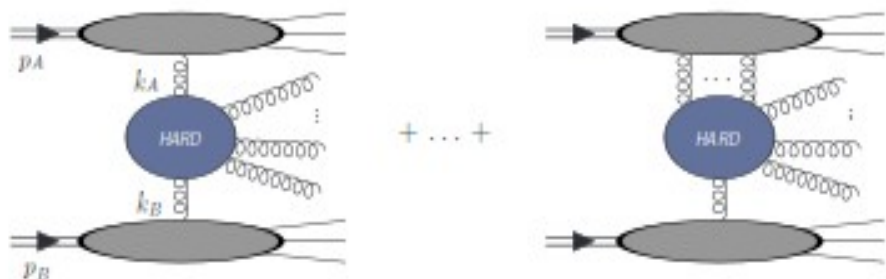
behavior of gluon at small k

but possible corrections from

generalized high energy factorization

HEF applied to three jets

Van Hameren, Kotko, KK ,13



$$k_A^\mu = x_A p_A^\mu + k_{T A}^\mu, \quad k_B^\mu = x_B p_B^\mu + k_{T B}^\mu \sim x_B p_B, \quad x_A \ll x_B$$

p-p and p-Pb collisions

CM energy 5 TeV and 7 TeV

$$p_{T1} > p_{T2} > p_{T3} > p_{T \text{ cut}}$$

anti- k_T clustering with $R = 0.5$

collinear PDF \Rightarrow CTEQ10 NLO set, scale choice $\mu = a(E_1 + E_2 + E_3)$, where the variation of a gives the (large) theoretical uncertainty

calculations are made and cross-checked using LxJet and OSCARS

$$d\sigma_{AB \rightarrow X} = \int \frac{d^2 k_{T A}}{\pi} \int \frac{dx_A}{x_A} \int dx_B \sum_b \mathcal{F}_{g^*/A}(x_A, k_{T A}) f_{b/B}(x_B) d\hat{\sigma}_{g^*b \rightarrow X}(x_A, x_B, k_{T A})$$

$$x_A = \sum_i \frac{|\vec{p}_{T i}}{\sqrt{S}} e^{\eta_i}, \quad x_B = \sum_i \frac{|\vec{p}_{T i}}{\sqrt{S}} e^{-\eta_i}$$

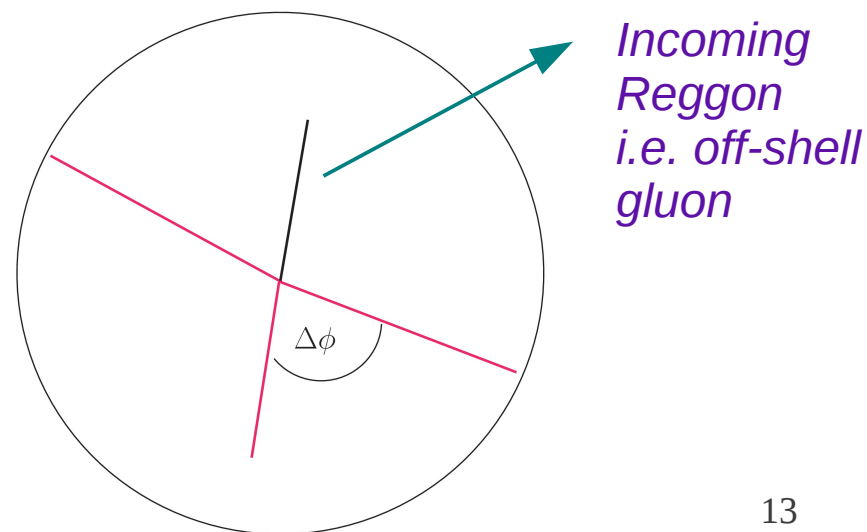
$$\eta_{f0} \leq \eta_i \leq \eta_{f1}$$

$$|\eta_j| \leq \eta_c$$

central

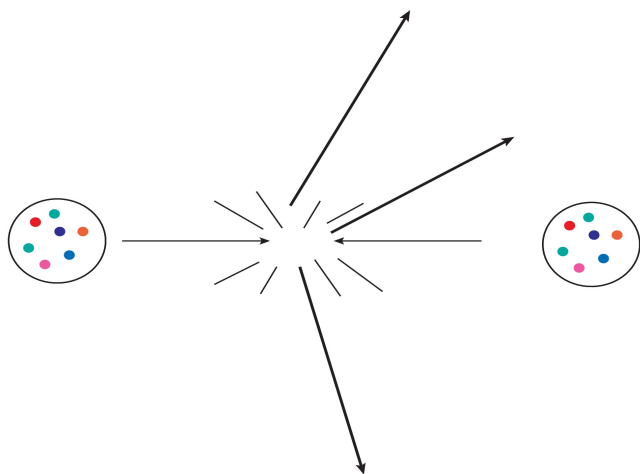
forward

$$|\vec{p}_{T i}| > p_{T \text{ cut}}, \quad i = 1, 2, 3.$$



¹ <http://annapurna.ifj.edu.pl/~pkotko/LxJet.html>

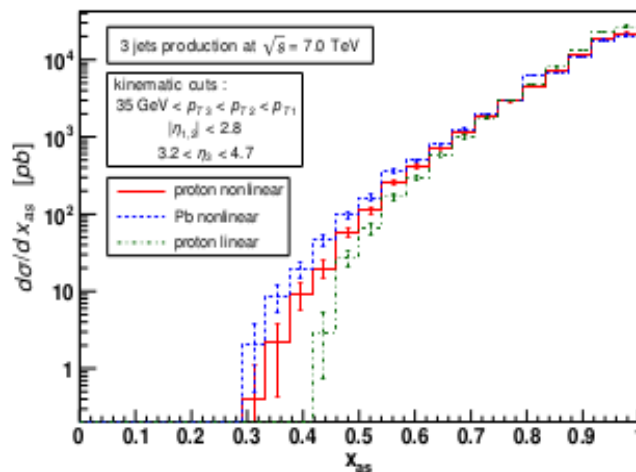
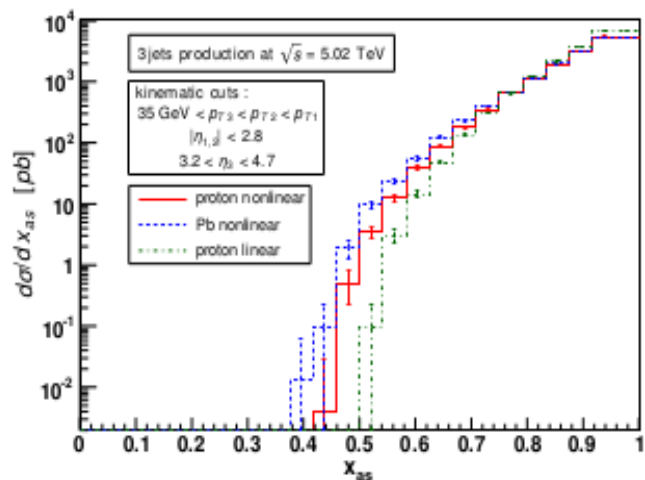
Central-central-forward configuration



two leading jets are in the central region with $|\eta_{1,2}| < 2.8$
 the softest jet is in the forward region $3.2 < \eta_3 < 4.7$

$p_{T\text{cut}} = 35 \text{ GeV}$

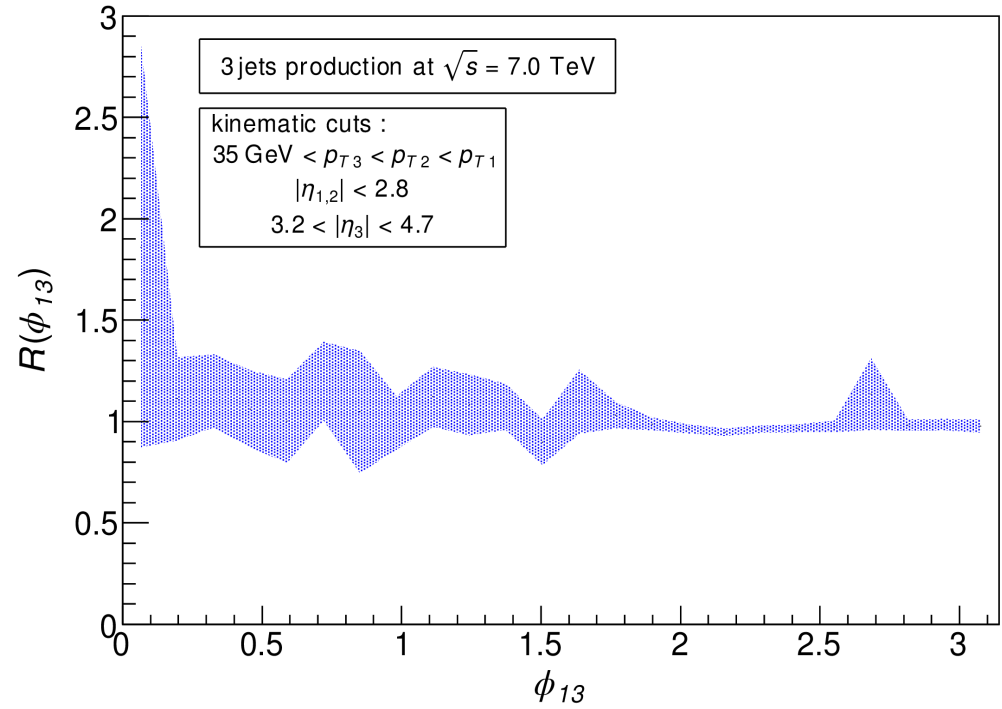
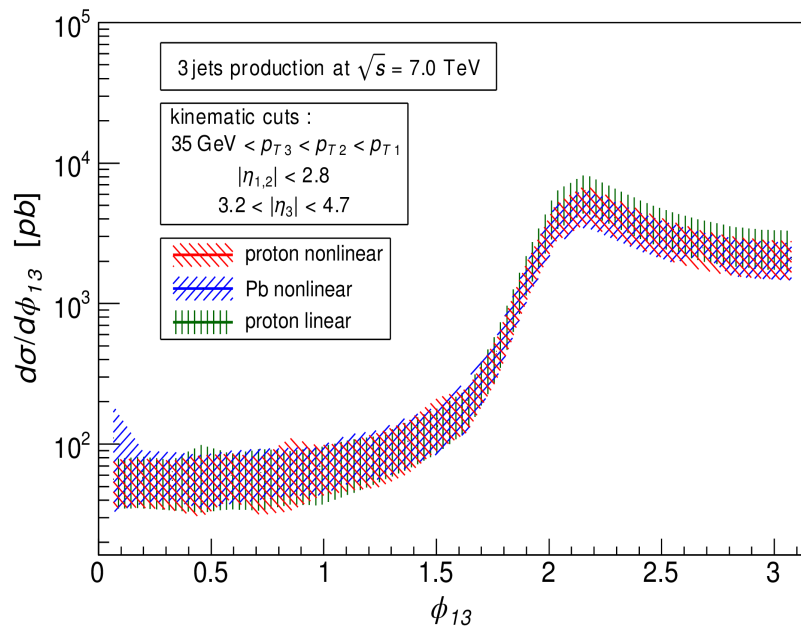
we may restrict additional cuts on the central jets, e.g. $|\vec{p}_{T1} + \vec{p}_{T2}| < D_{\text{cut}}$



$$x_{\text{as}} = \frac{|x_A - x_B|}{x_A + x_B}$$

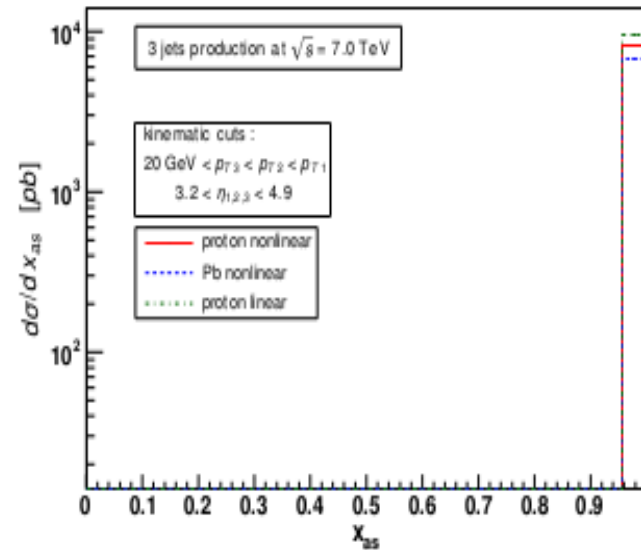
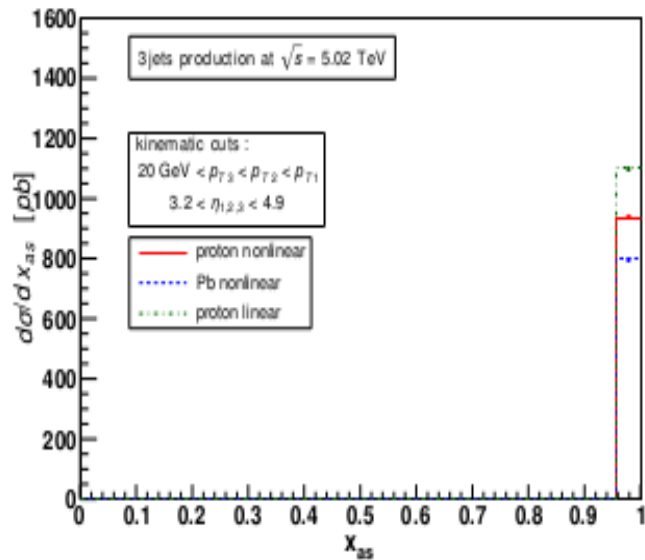
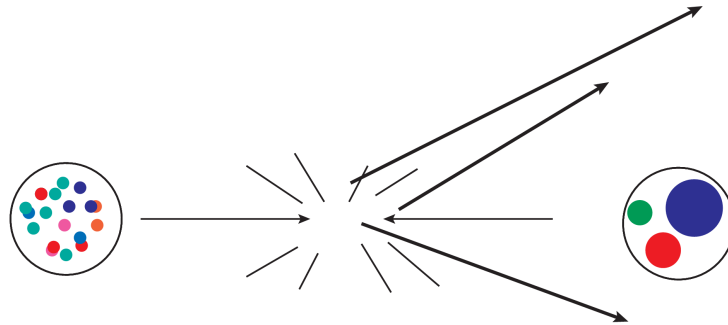
Many symmetric events

Central-central-forward configuration



No noticeable saturation effects

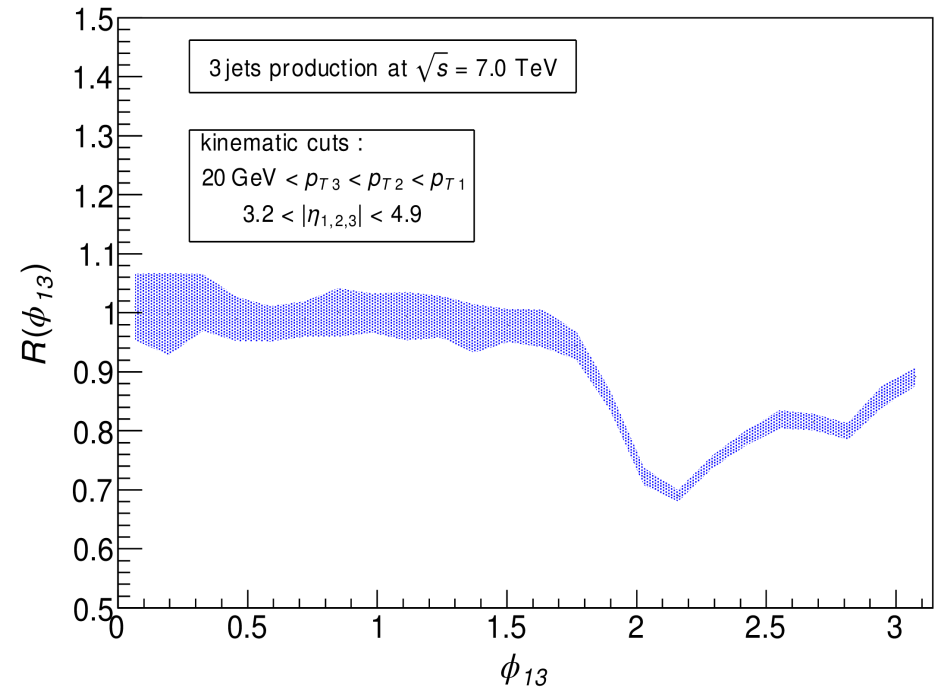
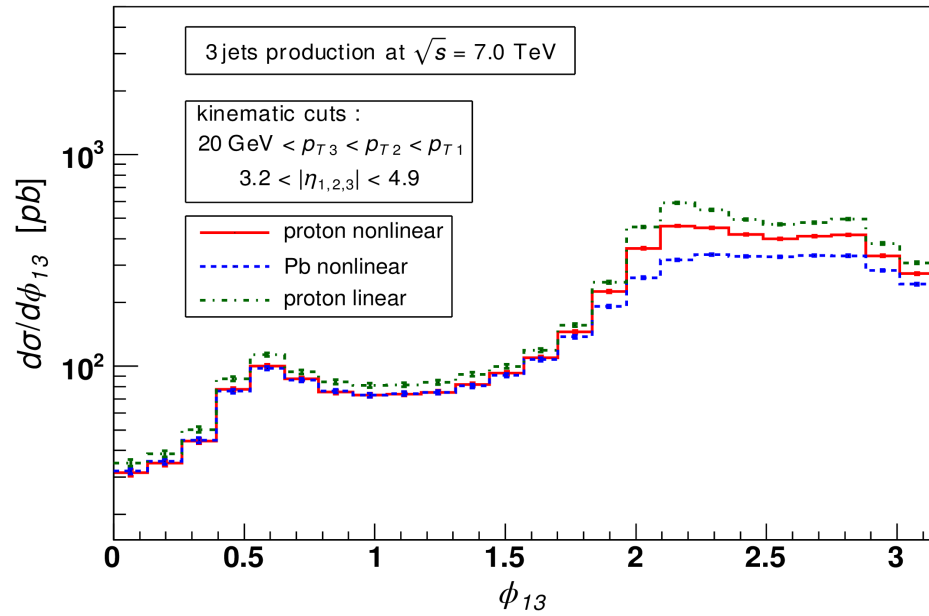
Forward-forward-forward configuration



$$x_{as} = \frac{|x_A - x_B|}{x_A + x_B}$$

1
totally asymmetric

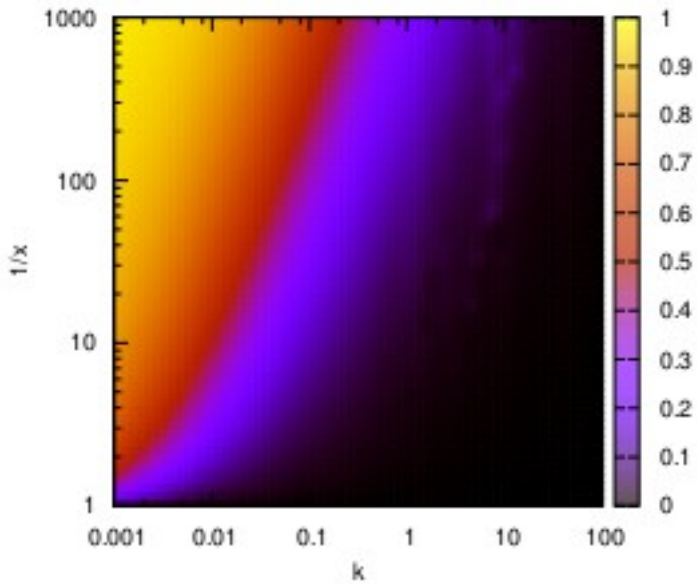
Forward-forward-forward configuration



Saturation effects show up

Saturation scale in KGBJS

BK

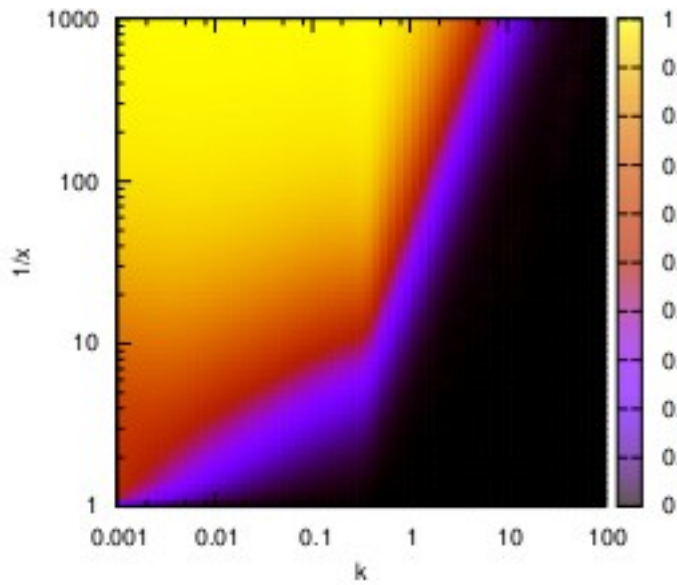


Relative differences between linear and nonlinear

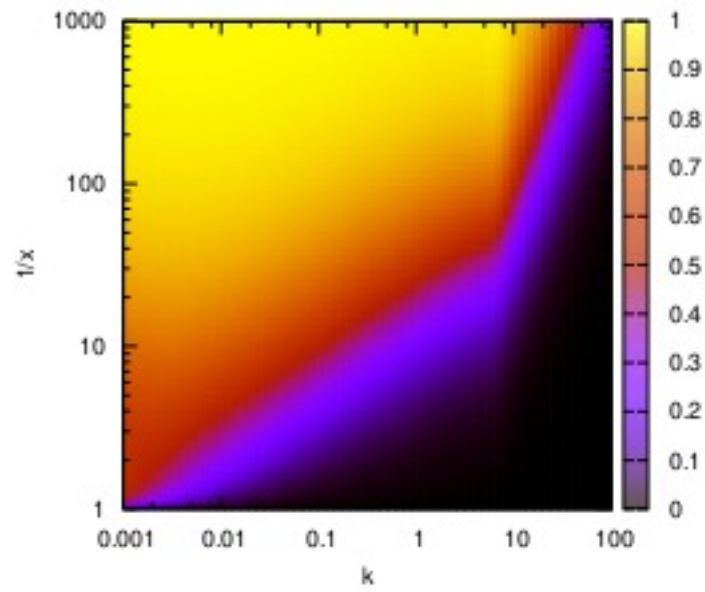
K.K, A. Stasto Eur.Phys.J.C41:343-351,2005

$$\beta(x, k, p) = \frac{|\mathcal{E}_{CCFM}(x, k, p) - \mathcal{E}_{KGBJS}(x, k, p)|}{\mathcal{E}_{CCFM}(x, k, p)}$$

KGBJS



p=1



p=10

Toton, KK '13

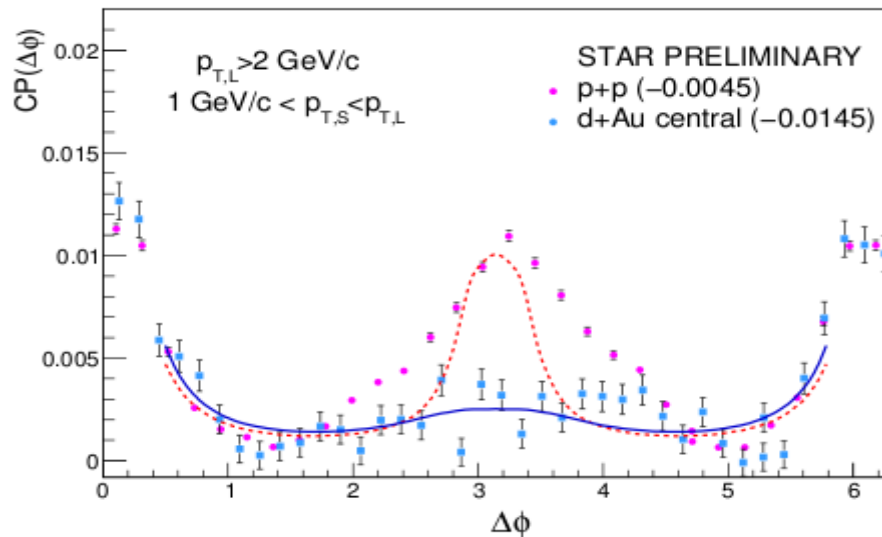
Conclusions and outlook

- *LHC gives opportunity to test parton densities both when the parton density is probed at low x and at low, medium and large kt .*
- *The results for jets give some theoretical hints for saturation*
- *More jet studies on the way i.e. forward forward configuration of di-jets*
- *New evolution equations combining saturation and coherence to be used in the future*

Back up

Saturation and production of forward dijets in d Au

Features: studies allow for direct studies of saturation effects of the correlation function.
d-Au no smearing due to collective flow as in A A



$$CP(\Delta\phi) = \frac{N_{pair}(\Delta\phi)}{N_{trig}}$$

$$N_{pair}(\Delta\phi) = \int_{y_i, |p_{i\perp}|} \frac{dN^{pA \rightarrow h_1 h_2 X}}{d^3 p_1 d^3 p_2}$$

$$N_{trig} = \int_{y, p_{\perp}} \frac{dN^{pA \rightarrow h X}}{d^3 p}$$

•Marquet, Albacete

$$\frac{dN^{qA \rightarrow qgX}}{d^3 k d^3 q} = \frac{\alpha_s C_F}{4\pi^2} \delta(xP^+ - k^+ - q^+) F(\tilde{x}_A, \Delta) \sum_{\lambda\alpha\beta} \left| I_{\alpha\beta}^{\lambda}(z, k_{\perp} - \Delta; \tilde{x}_A) - \psi_{\alpha\beta}^{\lambda}(z, k_{\perp} - z\Delta) \right|^2$$

Generalizes kt factorization

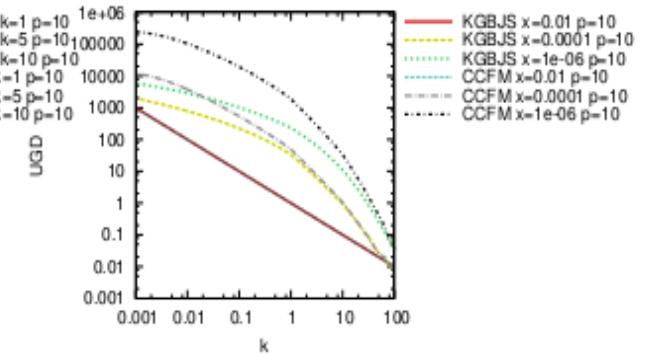
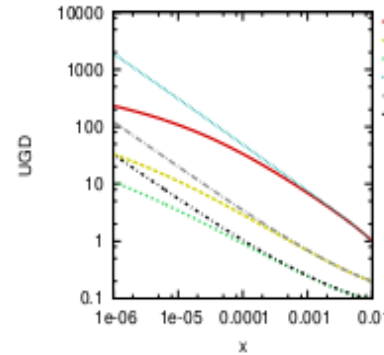
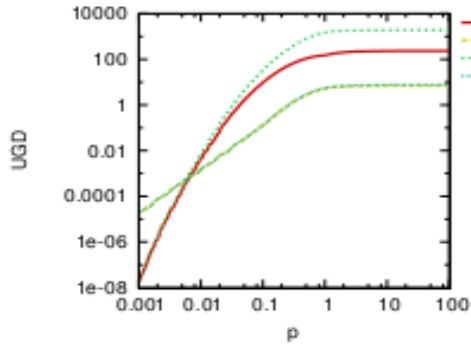
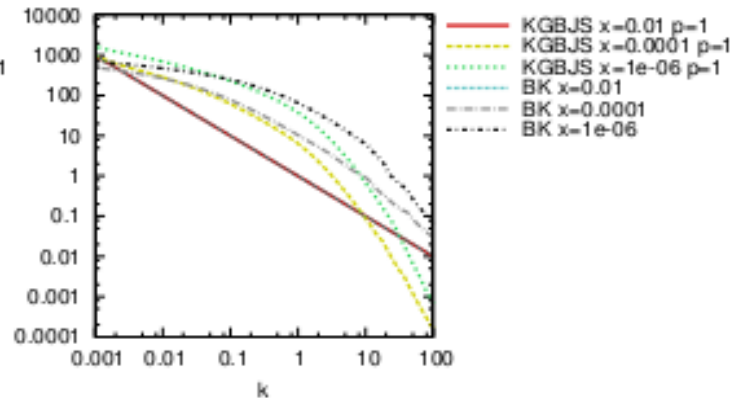
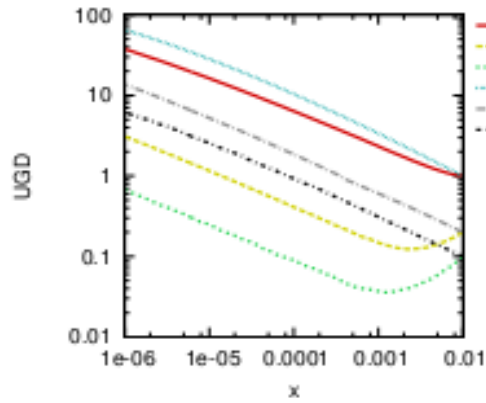
•Tuchin

$$\varphi(x, q^2) = \frac{1}{2\pi^2} \frac{S_{\perp} C_F}{\alpha_s} (1 - e^{-Q_s^2/q^2}) (1 - x)^4$$

KLN approach. Model for gluon density. Gluon does not vanish at small kt.

The KGBJS equation – nonlinear ext. of CCFM

Toton, KK '13



$$\begin{aligned} \phi(x, k) &= \tilde{\phi}_0(x, k) \\ &+ \bar{\alpha}_s \int_{x/x_0}^1 \frac{dz}{z} \int \frac{d^2\mathbf{q}}{\pi q^2} \theta(q^2 - \mu^2) \Delta_R(z, k, \mu) \\ &\left(\phi\left(\frac{x}{z}, k'^2\right) - \frac{q^2}{\pi R^2} \delta(q^2 - k^2) \phi^2\left(\frac{x}{z}, q^2\right) \right) \end{aligned}$$

$$\Delta_R(z, k, \mu) = \exp\left(-\bar{\alpha}_s \log \frac{1}{z} \log \frac{k^2}{\mu^2}\right)$$

$$\begin{aligned} \mathcal{E}(x, k^2, p) &= \mathcal{E}_0(x, k^2, p) \\ &+ \bar{\alpha}_s \int_{x/x_0}^1 \frac{dz}{z} \int \frac{d^2\bar{\mathbf{q}}}{\pi \bar{q}^2} \theta(p - z\bar{q}) \Delta_{ns}(z, k, \bar{q}) \\ &\left(\mathcal{E}\left(\frac{x}{z}, k'^2, \bar{q}\right) - \frac{\bar{q}^2}{\pi R^2} \delta(\bar{q}^2 - k^2) \mathcal{E}^2\left(\frac{x}{z}, \bar{q}^2, \bar{q}\right) \right) \end{aligned}$$

$$\Delta_{ns} = \exp\left(-\bar{\alpha}_s \ln \frac{1}{z} \ln \frac{k^2}{zq^2}\right) \quad \text{for } k^2 > zq^2$$

MPI and unitarity

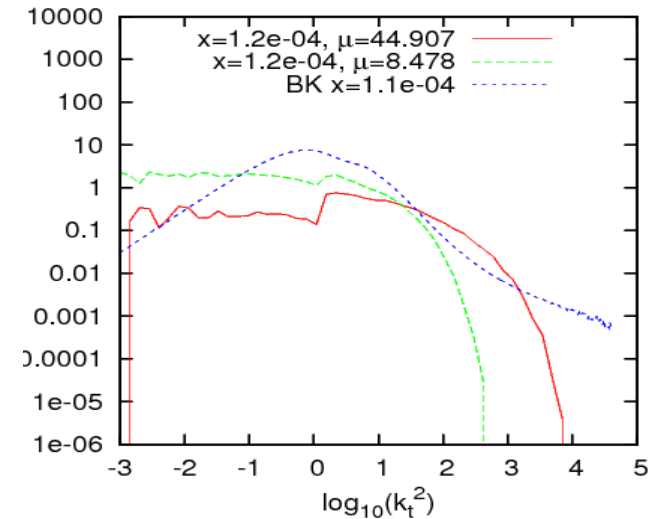
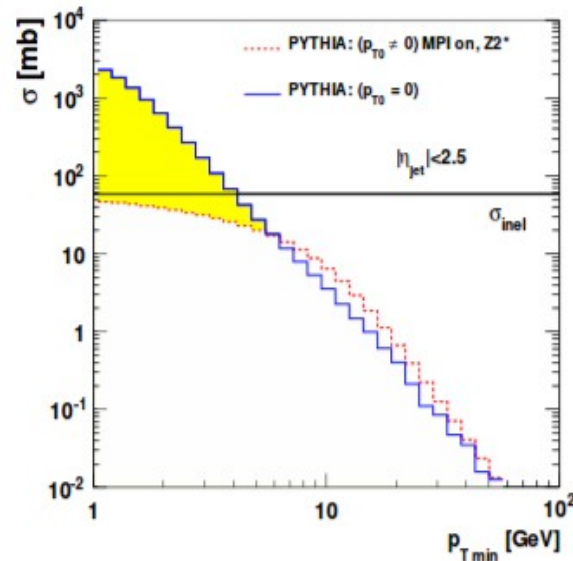
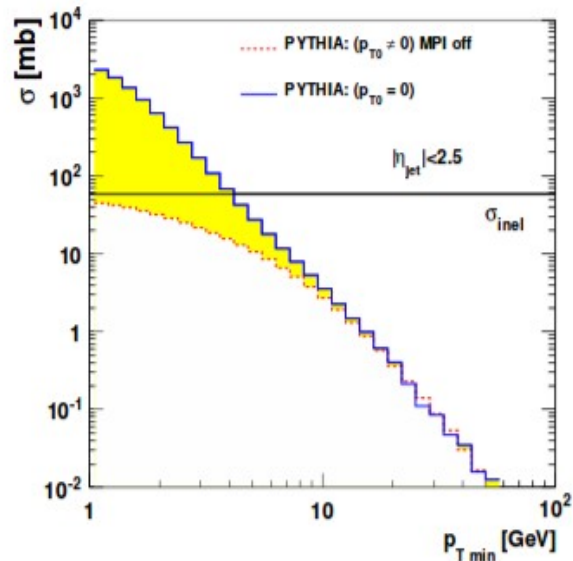
Typical x at LHC $x=10^{-4}$ → theoretically relevant for onset of low x effects

Estimations and MC calculations show that the MPI effects are enhanced at low x.

n quarks $F \sim (x_1 x_2 \dots x_n)^{(-1-\lambda_q)} \sim x^{-2n(1-\lambda_q)}, \quad x_1 \sim x_2 \dots \sim x_n$

single quark $F \sim x^{2(-1-\lambda_q)}$

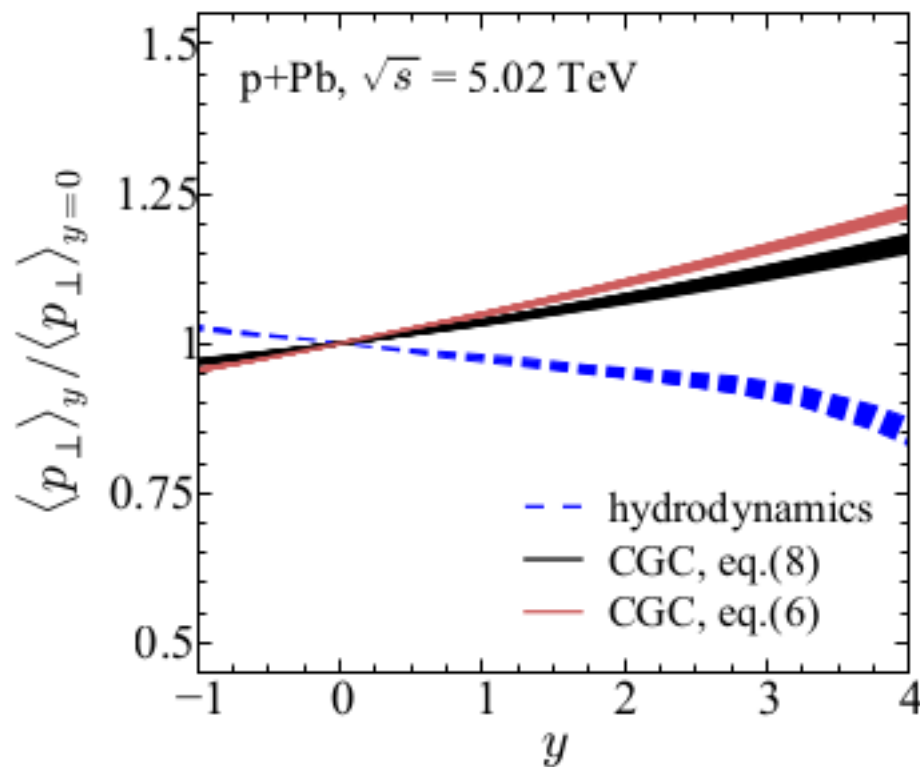
Diehl, Ostermeier, Schafer '12



Various approaches show that suppression of gluon density at low p_T is relevant for jet observables and for unitarization [Grebenyuk, Hautmann, Jung '12](#). This rizes a question of interplay of MPI and saturation.

CGC vs. hydrodynamics

Bozek, Bzdak, Skokov '13,



Collectivity in hydro.
Generation of saturation scale
In CGC

