# Υ Suppression in Hot and Cold Nuclear Matter at STAR

or:

# Yield Extraction with Low Statistics

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# Outline

- Current Status
- $Chi^2$ Fitting
- Bias Correction
- Likelihood Fitting
- Composite Signals
- Mismatched line shapes
- The Future

# Current status of Upsilon results

- Our paper was submitted in December
- Now in referee stage
- While responding, we found a critical error
- Now corrected. Back on publication track
- Let's discuss fitting methods
  - Others can learn from my mistake

So without further ado…

# FITTING!

# Example : Yield from a Gaussian pdf

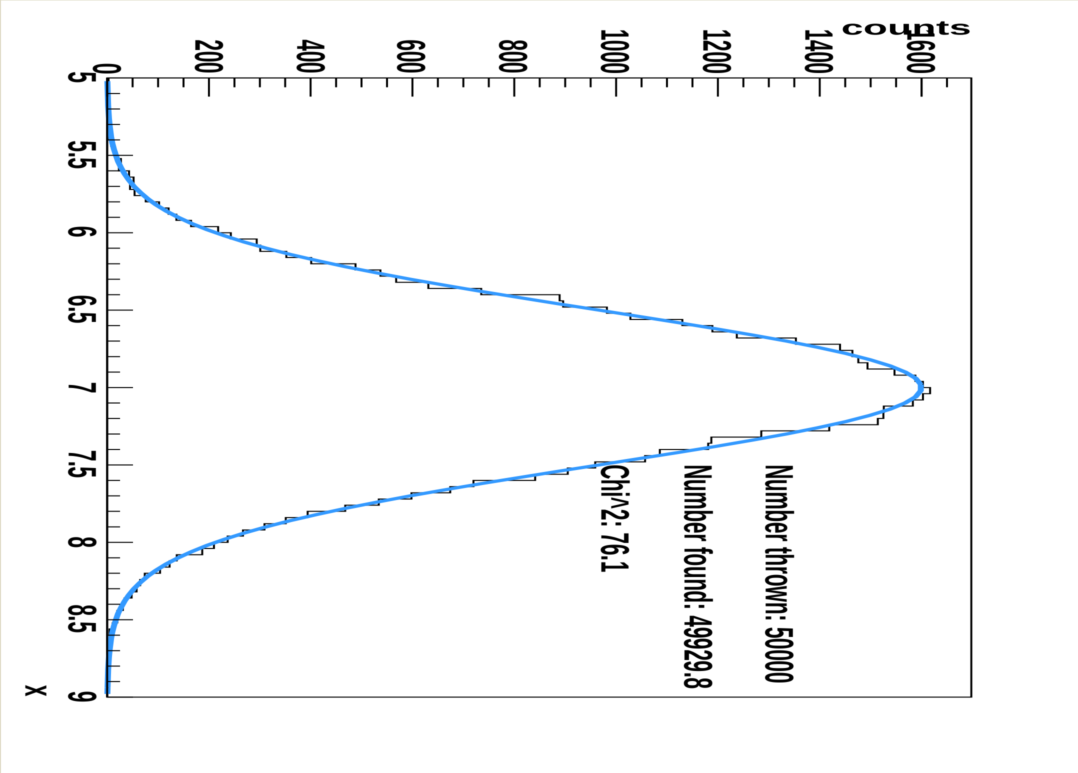
- Suppose we are performing a counting experiment
  - Measuring a particle's mass in the absence of background
    - e.g. time of flight of anti-alpha particle in the STAR detector
  - Distribution is Gaussian, with some mean mass and width given by the detector resolution
    - For simplicity, assume we can make this into a Normal Standard distribution
      - e.g. if the resolution is known, and if the mass of the alpha particle is equal to the mass of the anti-alpha particle.
  - Suppose you want to obtain yield using a fit to the data
    - How would you do it?

# Example : Continued

- One possibility
  - Construct a Gaussian pdf (2 parameters: $\mu$, $\sigma$)
  - Add one parameter as normalization (counts)

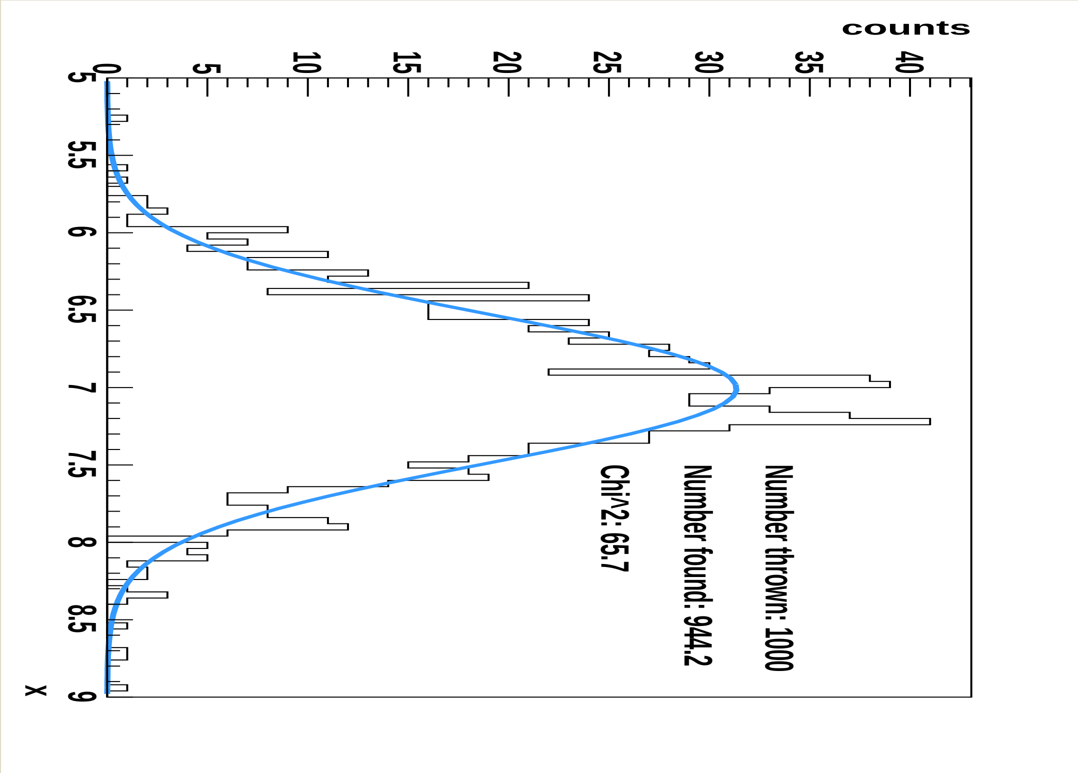
- Is this a good way?

# Simple Extraction



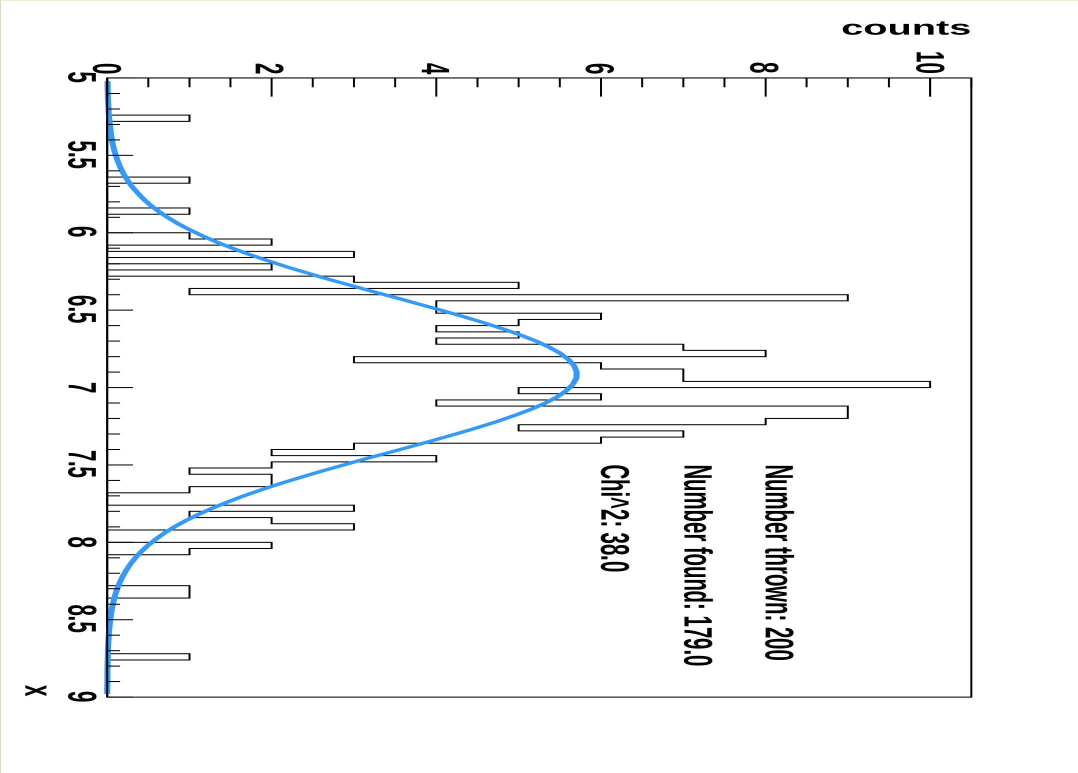

- Signal only; no background
- Signal is a simple Gaussian
- Throw a known number, extract result from chi$^2$ fit
  - 0.1% error

# Simple Extraction (lower statistics)




- Only 2% of previous sample
- 5.6% error

# Simple Extraction (even lower statistics)




- 20% of previous sample
- 10.5% error

# Least squares with binned data

- Least squares: minimize $\chi^2$ with

$$\chi^2(\vec{\theta}) = \sum_{i=1}^{N} \frac{\left(y_i - \lambda_i(\vec{\theta})\right)^2}{\lambda_i(\vec{\theta})}$$

  - assumes Poisson statistics, i.e.: mean = variance
  - here, $\lambda$ is PDF which we assume (hope?) is parent distribution.
- Modified least-squares method: minimize $\chi^2$ with

$$\chi^2(\vec{\theta}) = \sum_{i=1}^{N} \frac{\left(y_i - \lambda_i(\vec{\theta})\right)^2}{y_i}$$

  - Easier computationally: use data, instead of assumed PDF
  - Can lead to problems if a bin contains no entries

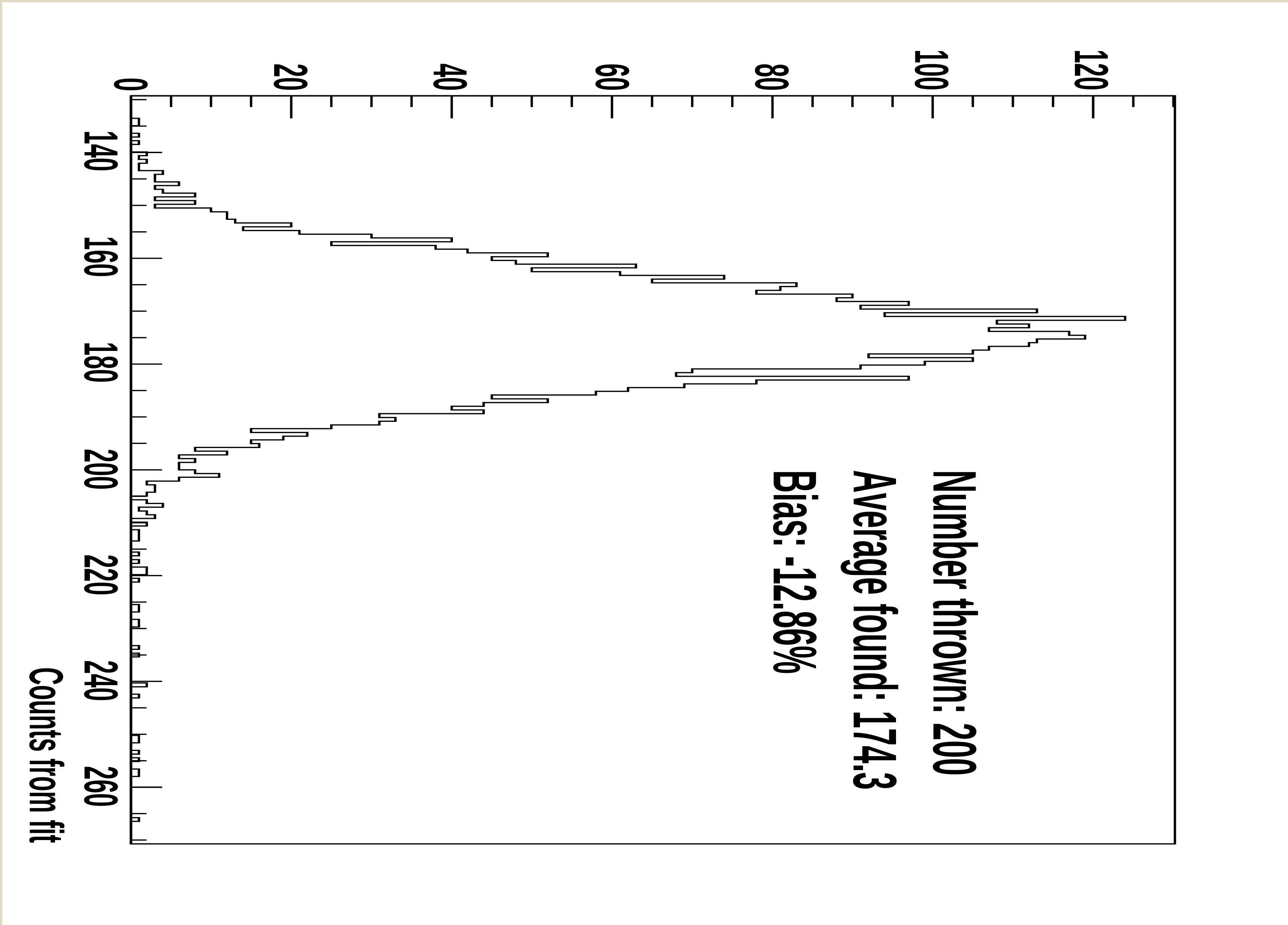
# Adjustable normalization: careful!

- If we add an arbitrary normalization:

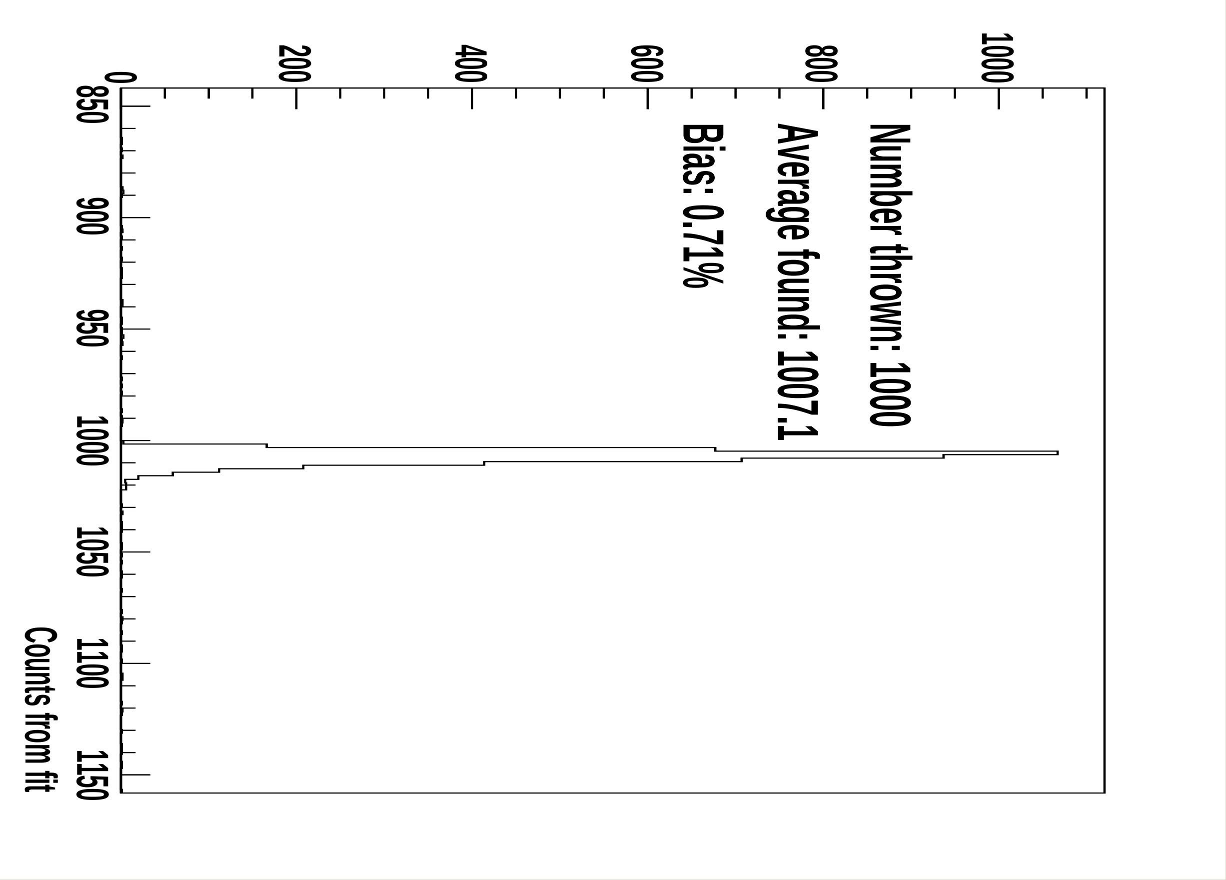
$$\lambda_i(\vec{\theta},\nu)=\nu\int_{x_i^{\min}}^{x_i^{\max}} f(x;\vec{\theta})\,dx=\nu p_i(\vec{\theta})$$

  - Here, $\nu$ **is the arbitrary normalization constant.**
    - Suitably normalized, it can be used to estimate the yield.
  - Minimizing $\chi^2$ leads to an estimator of the normalization:
    - For least squares: $\nu_{LS}=\nu+\frac{\chi^2}{2}$
    - For modified least squares: $\nu_{MLS}=\nu-\chi^2$
  - In our example of 1,000 events: N =944.2 +/- 30.9
    - What method do you think ROOT is using by default?

# …and this isn't a fluctuation

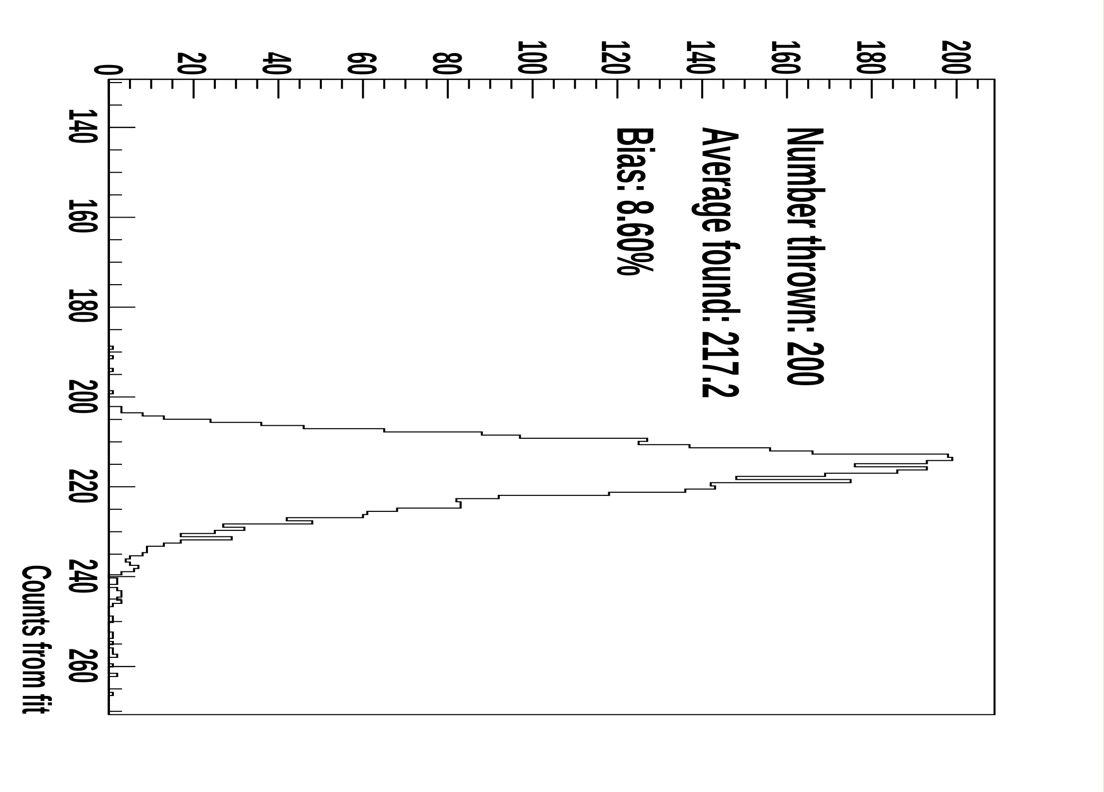


# But it can be corrected (somewhat)



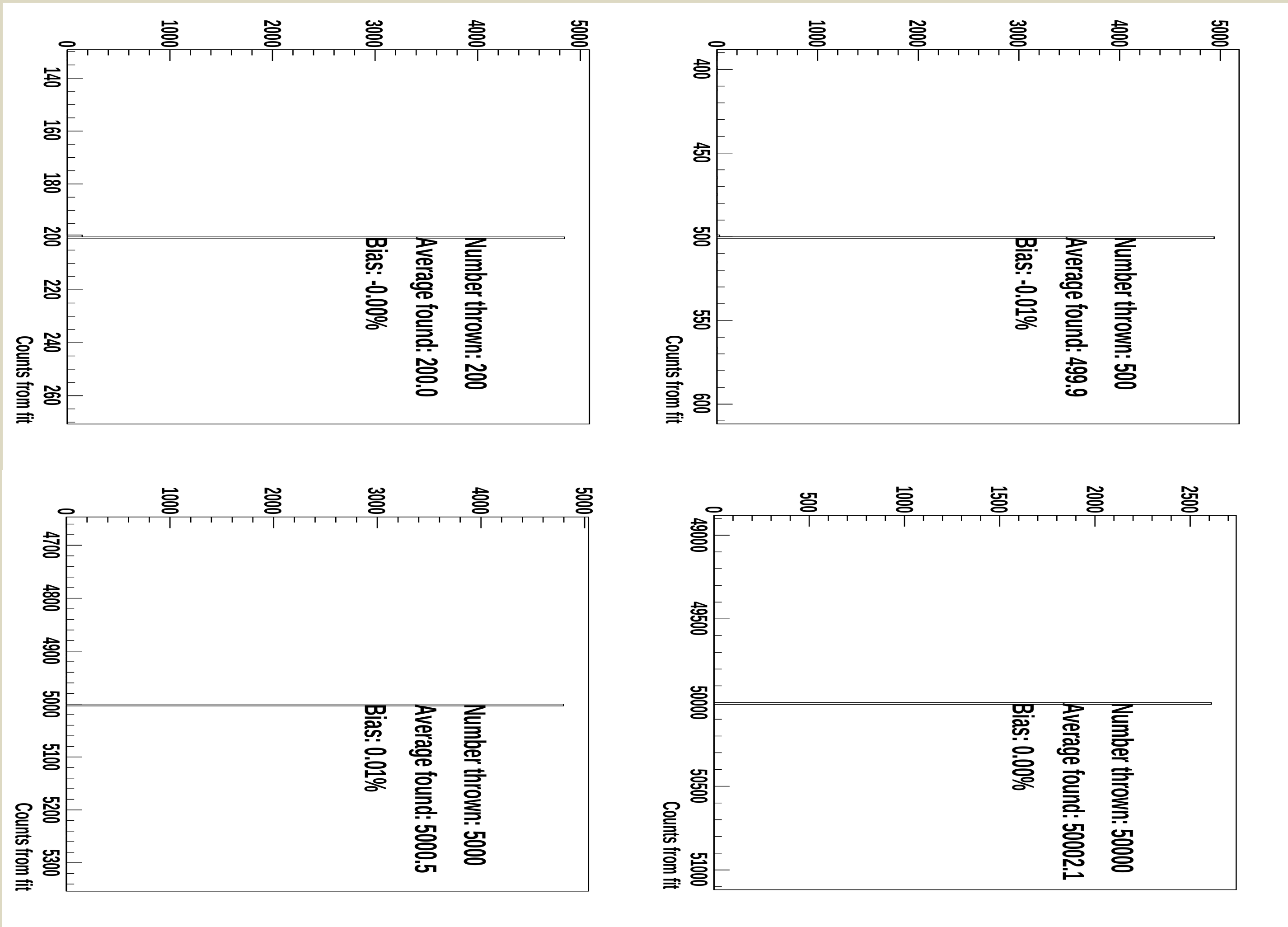

- 1000 thrown per event
- Yield corrected by adding $chi^2$
- 0.71% bias vs -5.8%

# Until you go back to low statistics



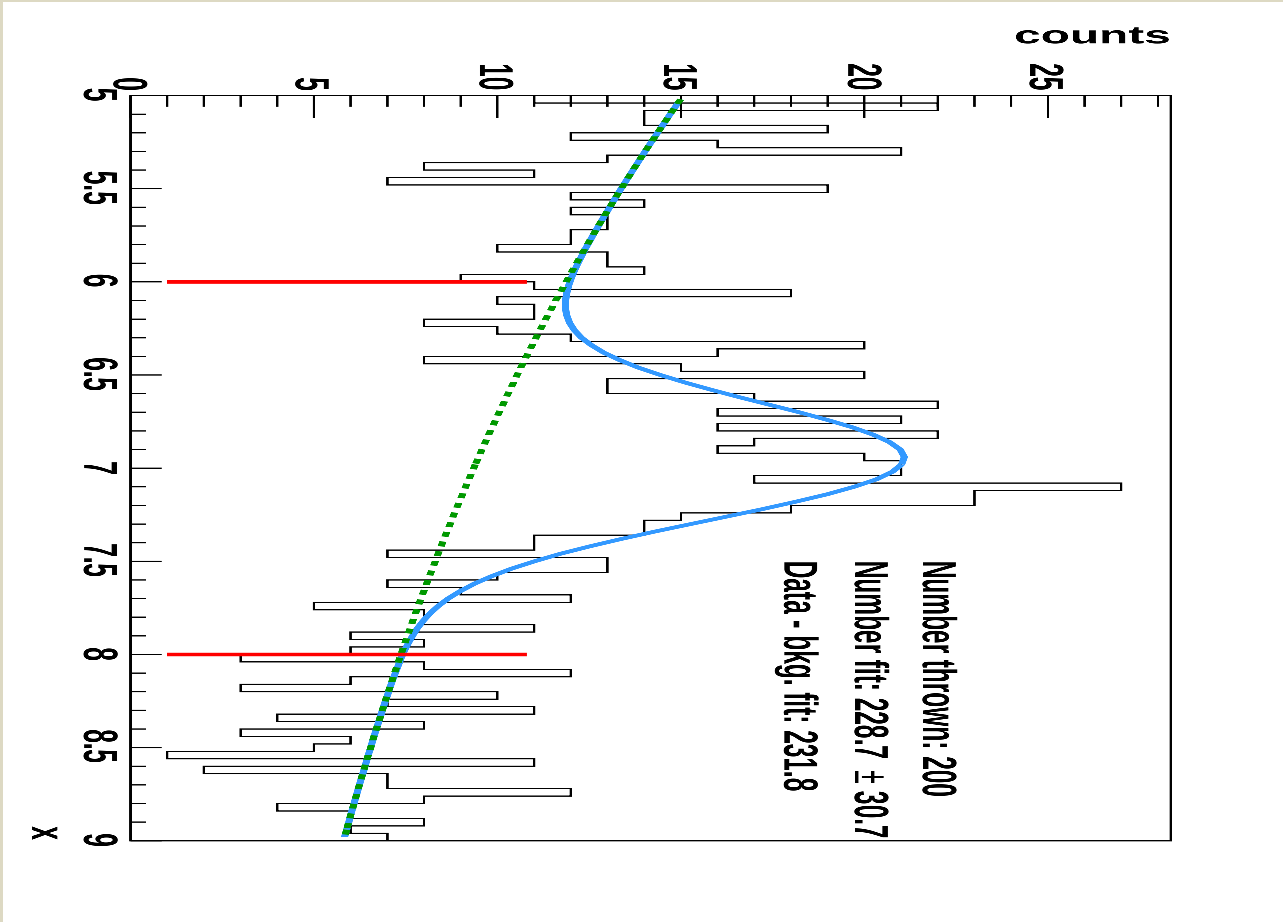

- 200 thrown per event
- Yield corrected by adding chi$^2$
- 8.6% bias vs -12.7% → Not a huge improvement
  - Empty-bin effect

# But really, just use Likelihood



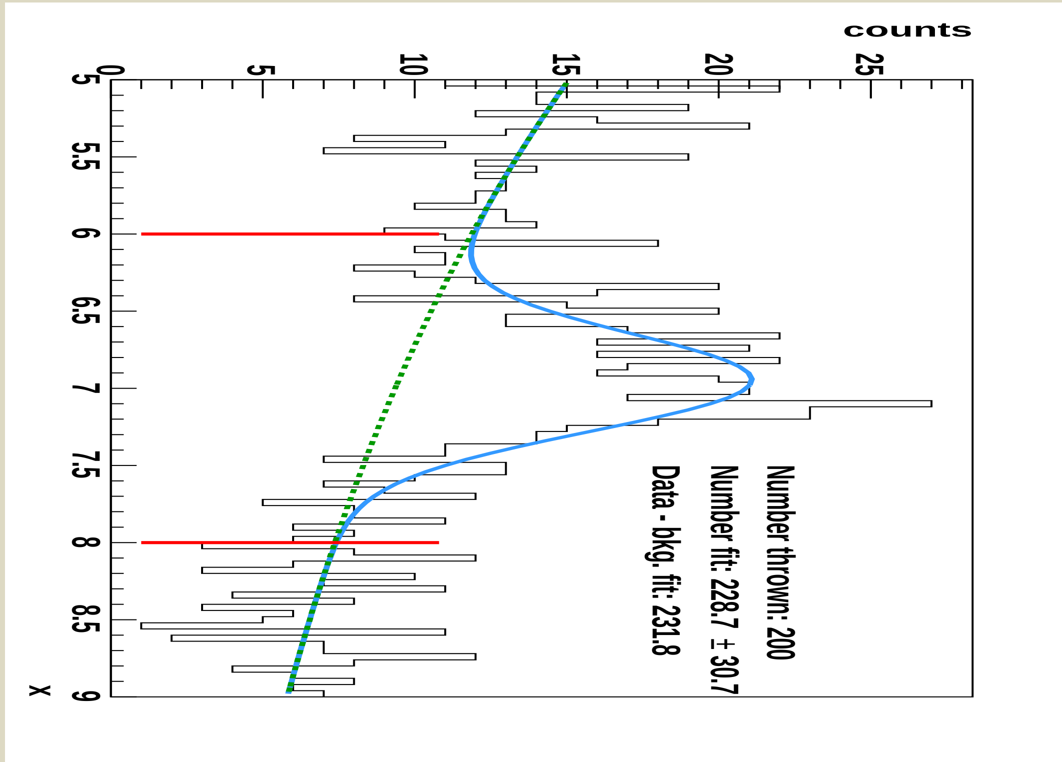
# BUT THE WORLD ISN'T THAT SIMPLE

# Extracting yields from composite signals



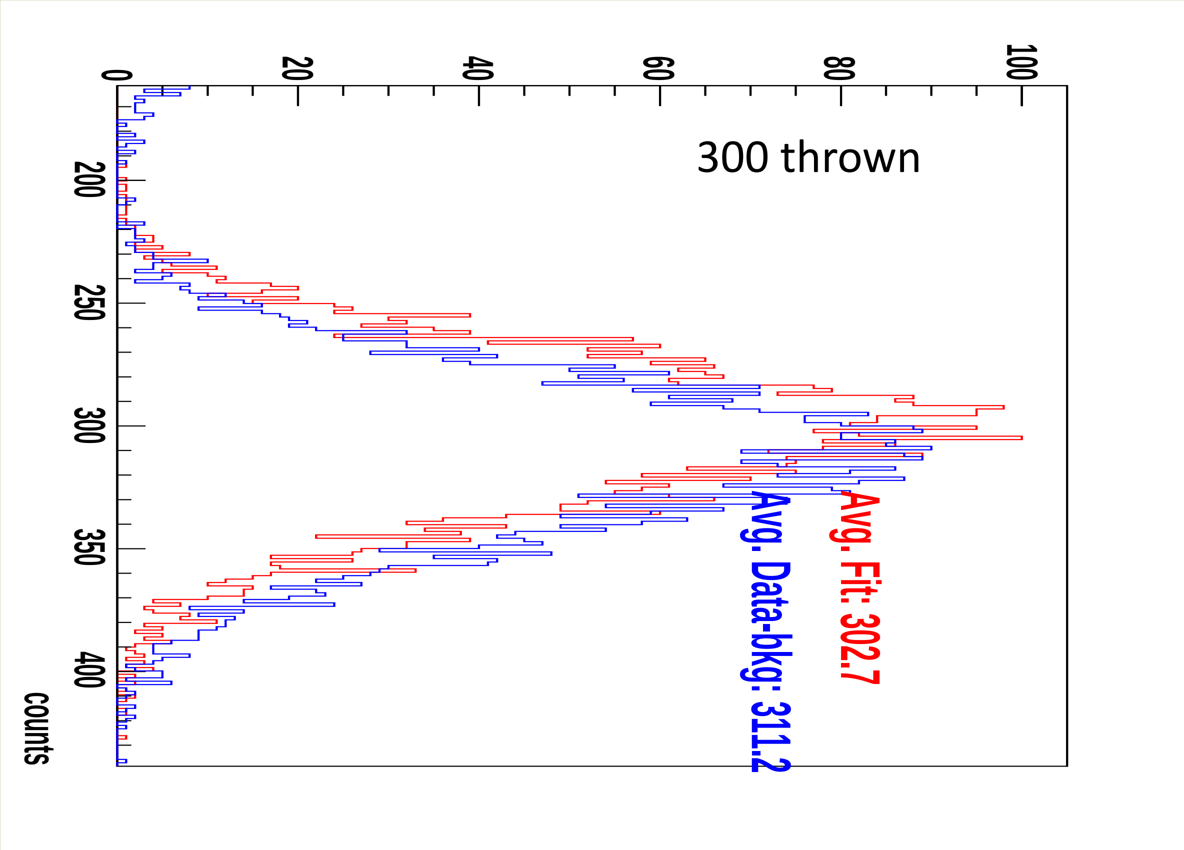

- Signal is a Gaussian
- Background is an exponential

# Extracting yields from composite signals



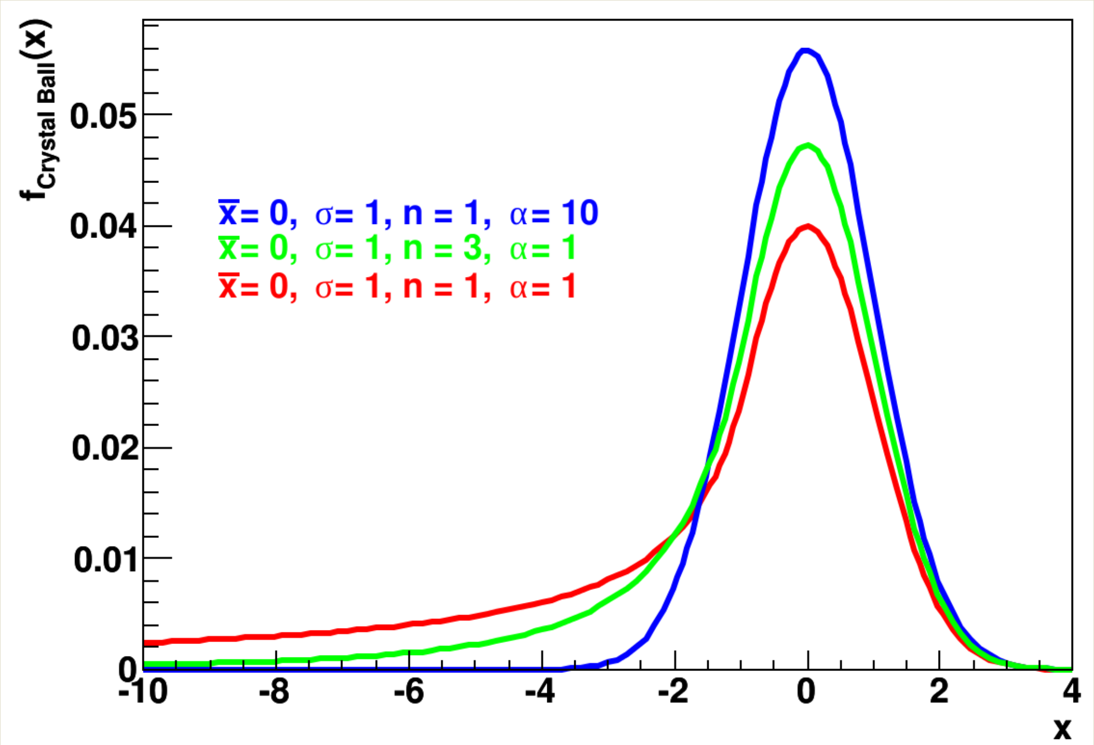

- 1) Extraction via fit parameters
- 2) Integrate data and subtract background (green dashed curve)

# Extracting yields from composite signals




- Extraction from fit outperforms subtraction
- But we know the lineshape here….

# The Crystal Ball function

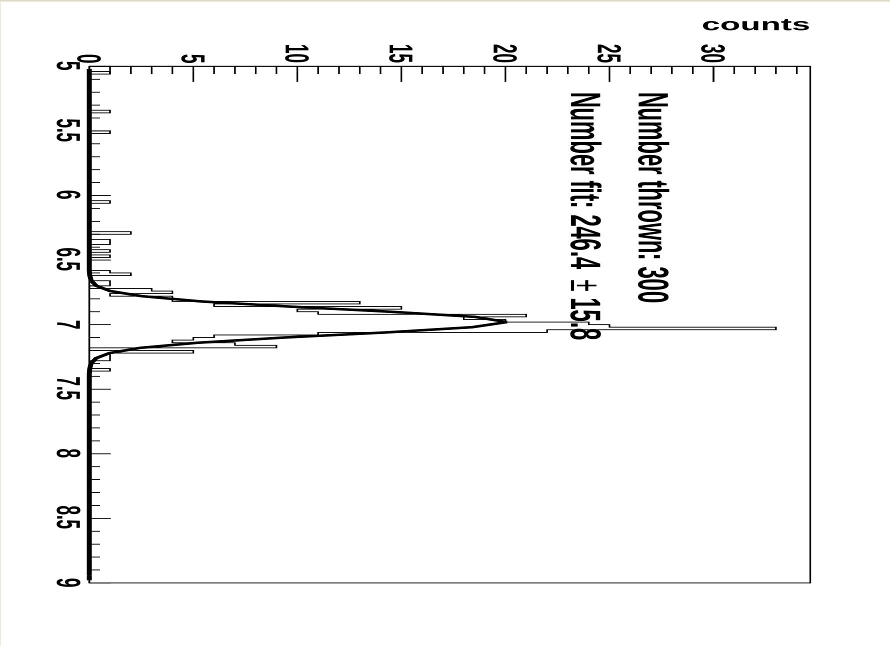



$$f(x;\alpha,n,\bar{x},\sigma) = N \cdot \begin{cases} \exp(-\frac{(x-\bar{x})^2}{2\sigma^2}), & \text{for } \frac{x-\bar{x}}{\sigma} > -\alpha \\ A \cdot (B - \frac{x-\bar{x}}{\sigma})^{-n}, & \text{for } \frac{x-\bar{x}}{\sigma} \leqslant -\alpha \end{cases}$$

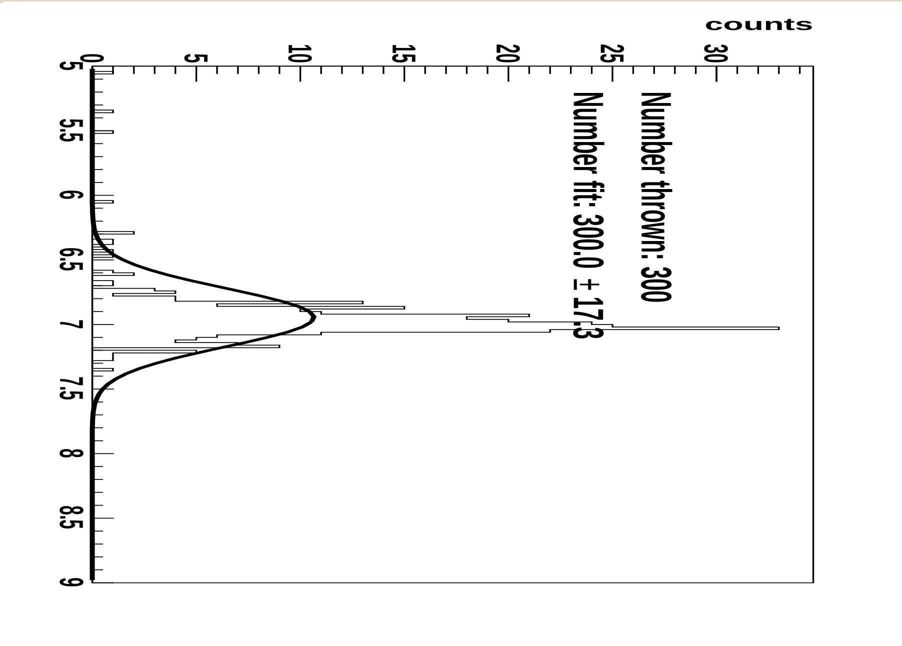
- Gaussian paired with power law
- Models “lossy” functions

# $Chi^2$ vs Likelihood with incorrect line shapes

$Chi^2$



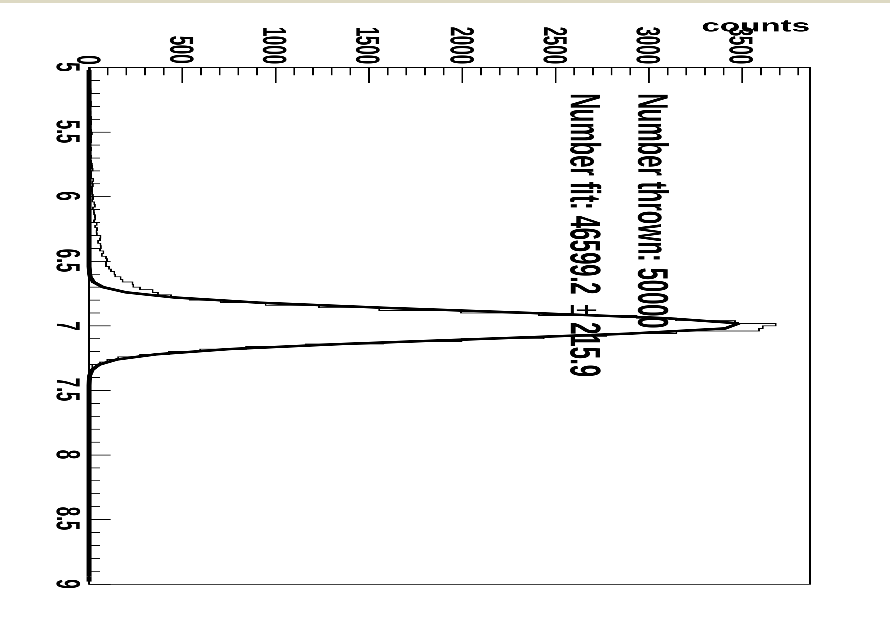

Likelihood



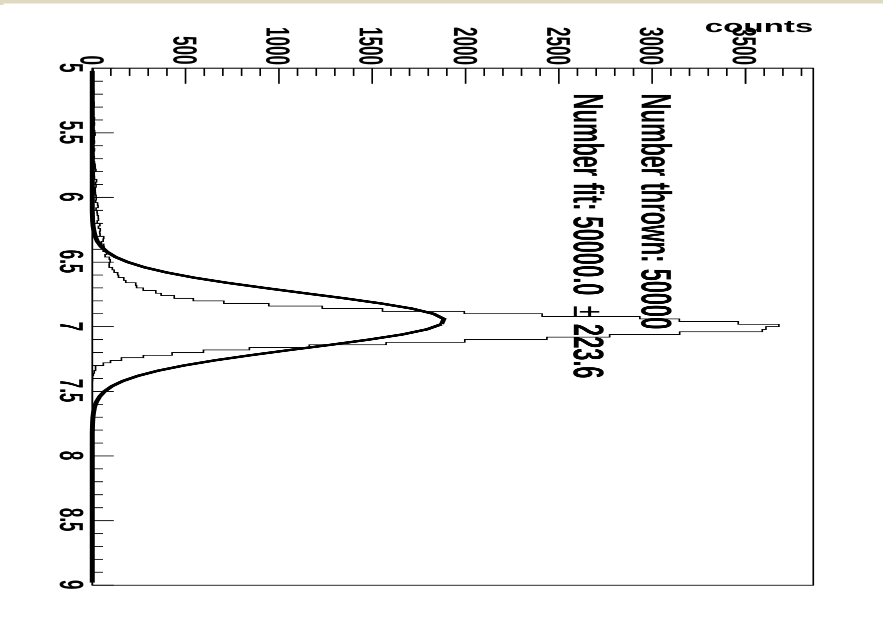

- Threw 300 events with a Crystal Ball function and fit it to a Gaussian
- $Chi^2$ gets the shape right (somewhat)
- Likelihood gets the yield right

# Still holds at high statistics

Chi$^2$

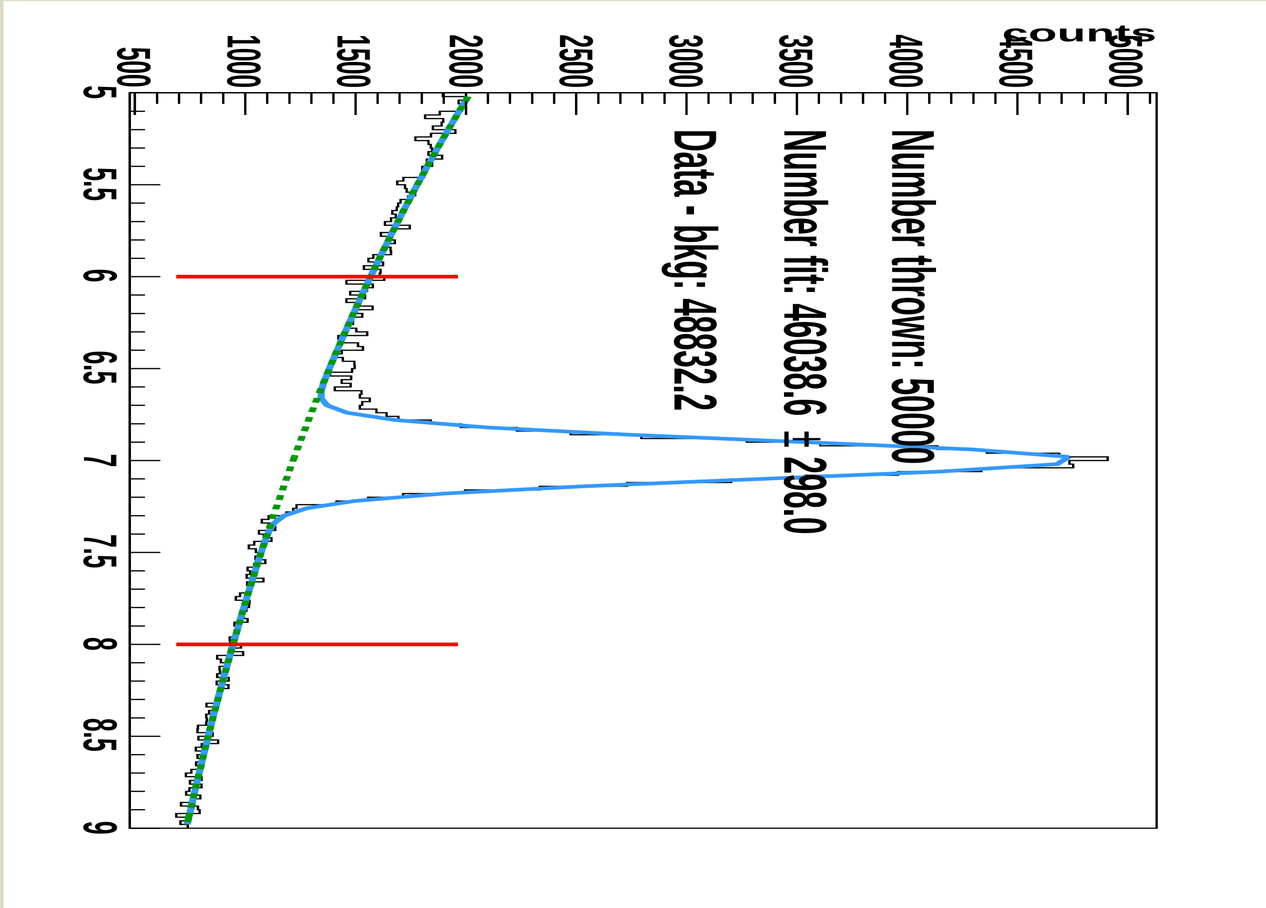



Likelihood



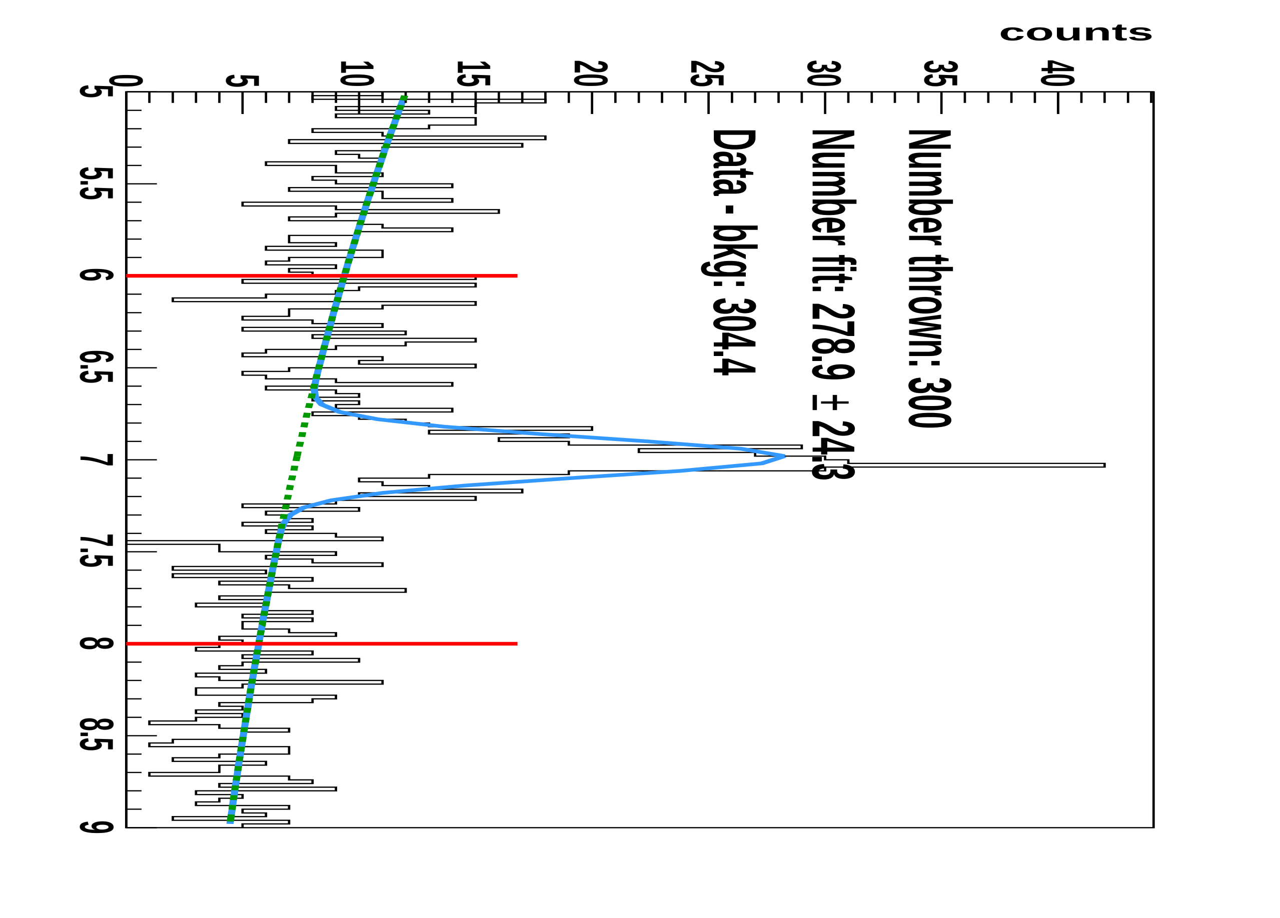

- Threw 50,000 events with a Crystal Ball function and fit it to a Gaussian

# Composite signals with incorrect line shapes



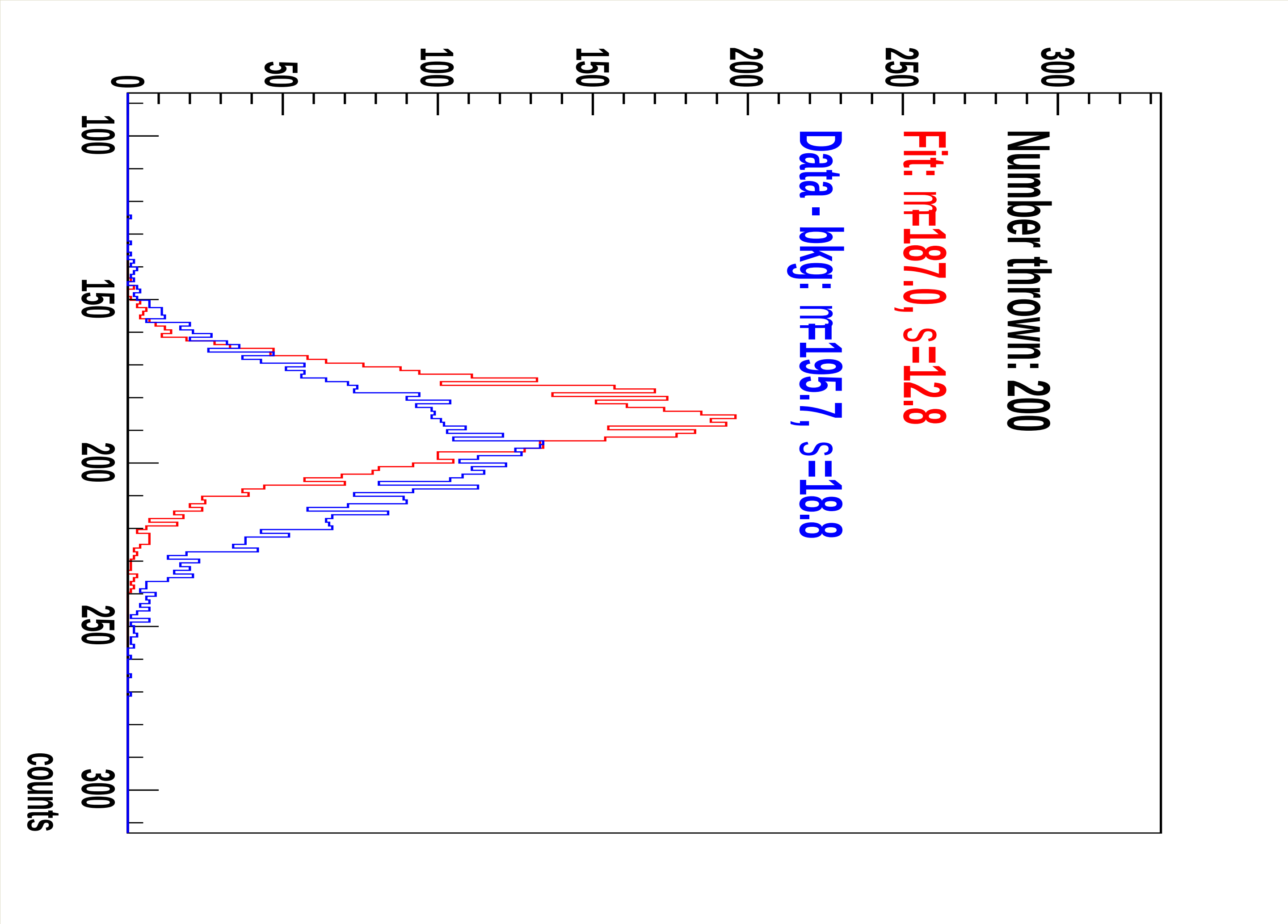

- Signal is Crystal Ball with exponential bkg
- Fit to Gaussian with exponential bkg

# Composite signals with incorrect line shapes

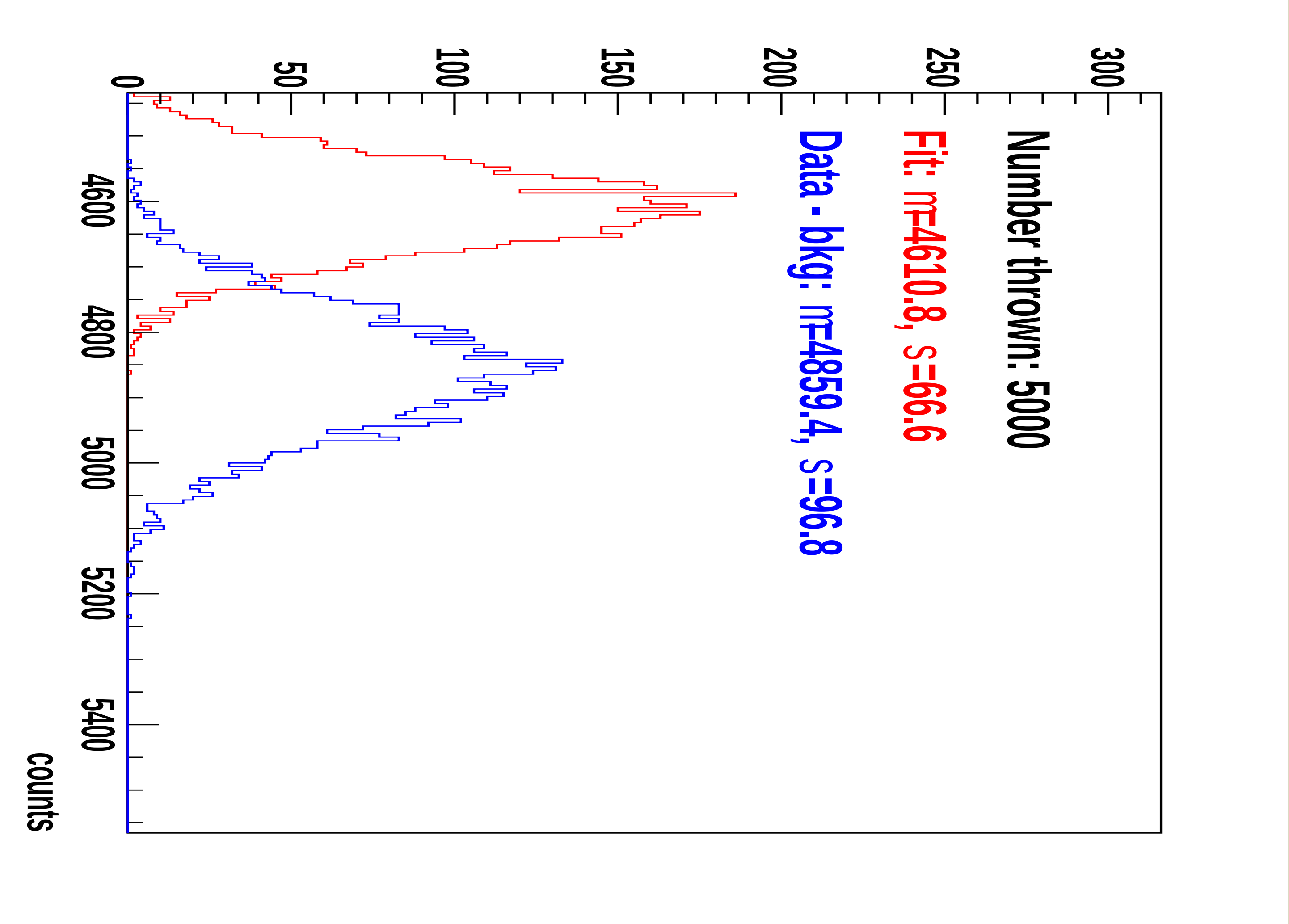



- Signal is Crystal Ball with exponential bkg
- Fit to Gaussian with exponential bkg

# Results of many “incorrect” fits



# Results of many “incorrect” fits



# </talk>

- STAR Upsilon results are in the publication process
- $Chi^2$ fitting is biased at low statistics
  - Use likelihood.
- Be wary of line shape assumptions
  - Integrating data help remove biases
- Keep learning
- Keep sharing