Understanding Non-linear hydrodynamic response in HI collisions via Event-Plane correlations

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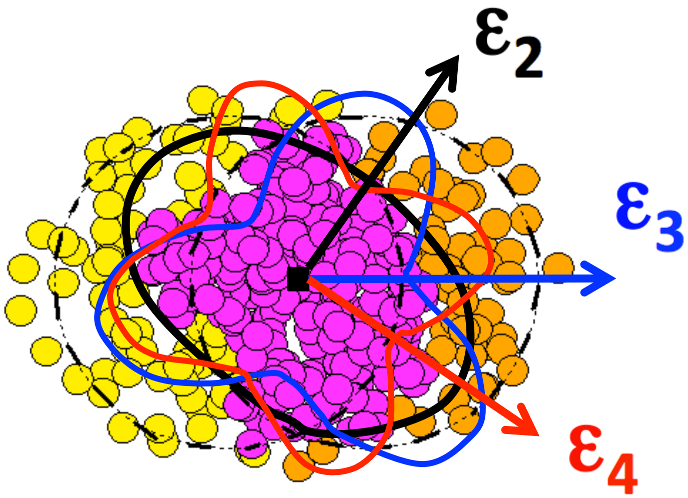
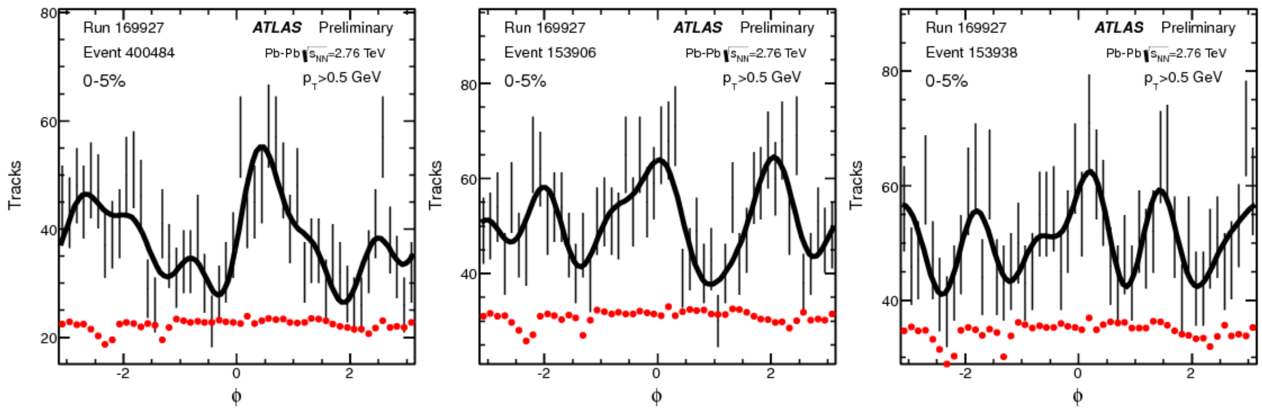
Introduction and motivation



Singles: $\frac{dN}{d\phi} \propto 1 + \sum_n 2\mathrm{v}_n \cos n(\phi - \Phi_n)$

$(\Phi_n - \Phi_m)$ *correlations*



- Understanding the v_n gives understanding of the nature of initial geometry and fluctuations in it.
- Complementary information can be obtained by studying correlations between the phases Φ_n of the v_n.

Origin of the event plane correlations

Representation of flow vector: $\vec{v}_n \equiv (v_n \cos n\Phi_n, v_n \sin n\Phi_n) \equiv v_n e^{-in\Phi_n}$

Hydro response is linear for v_2 and v_3: $v_n \propto \varepsilon_n$ and $\Phi_n \approx \Phi_n^*$ i.e.

$$v_2 e^{-i2\Phi_2} \propto \epsilon_2 e^{-i2\Phi_2^*}, \quad v_3 e^{-i3\Phi_3} \propto \epsilon_3 e^{-i3\Phi_3^*}$$

Non-linear terms possible for higher n

arXiv:1111.6538

Eccentricities of initial geometry

$$v_4 e^{-i4\Phi_4} = \alpha_4 \, \epsilon_4 e^{-i4\Phi_4^*} + \alpha_{2,4} \left(\epsilon_2 e^{-i2\Phi_2^*}\right)^2 + \ldots$$

Hydrodynamic response to eccentricities

$$= \alpha_4 \, \epsilon_4 e^{-i4\Phi_4^*} + \beta_{2,4} \, v_2^2 e^{-i4\Phi_2} + \ldots$$

Similarly correlations can occur between three planes of different orders:

$$v_5 e^{-i5\Phi_5} = \alpha_5 \epsilon_5 e^{-i5\Phi_5^*} + \alpha_{2,3,5} \epsilon_2 e^{-i2\Phi_2^*} \epsilon_3 e^{-i3\Phi_3^*} + \ldots.$$

$$= \alpha_5 \epsilon_5 e^{-i5\Phi_5^*} + \beta_{2,3,5} v_2 v_3 e^{-i(2\Phi_2 + 3\Phi_3)} + \ldots.$$

Quantifying the two-plane correlations

- The correlations are entirely described by the differential distribution:

$$\frac{dN_{events}}{d(k(\Phi_n - \Phi_m))} \quad : \quad k = LCM(m,n)$$

- The multiplication by the *Lowest common multiple, 'k'* removes the *n*/*m*-fold ambiguity in Φ_m/Φ_n.

- The distribution can be expanded as a Fourier series.
  - The Fourier coefficients $V^J_{n,m}$ quantify the strength of the correlation.

$$\frac{dN_{events}}{d(k(\Phi_n - \Phi_m))} = 1 + 2\sum_{j=1}^{\infty} V^j_{n,m} \cos(j \times k(\Phi_n - \Phi_m))$$

$$V^j_{n,m} = \langle \cos(j \times k(\Phi_n - \Phi_m)) \rangle$$

- Observables in general: $\langle \cos(c_1\Phi_1 + 2c_2\Phi_2 ... + lc_l\Phi_l) \rangle$

$$c_1 + 2c_2 ... + lc_l = 0$$

Accounting for detector resolution

True planes : Φ_n

Measured planes: Ψ_n (different than true planes due to finite detector resolution)

Measure correlation between EP, followed by a simple resolution correction.

Desired correlator

Observed correlator

$$\left\langle \cos k\left(\Phi_n - \Phi_m\right)\right\rangle = \frac{\left\langle \cos k\left(\Psi_n - \Psi_m\right)\right\rangle}{\mathrm{Res}\left\{k\Psi_n\right\}\mathrm{Res}\left\{k\Psi_m\right\}}$$

Resolution for individual planes

$$\mathrm{Res}\left\{k\Psi_n\right\} = \sqrt{\left\langle \cos^2\left(k\Psi_n - k\Phi_n\right)\right\rangle}$$

Three plane correlation

$$\left\langle \cos(nc_n\Phi_n + mc_m\Phi_m + lc_l\Phi_l)\right\rangle = \frac{\left\langle \cos(nc_n\Psi_n + mc_m\Psi_m + lc_l\Psi_l)\right\rangle}{\mathrm{Res}\{nc_n\Psi_n\}\mathrm{Res}\{mc_m\Psi_m\}\mathrm{Res}\{lc_l\Psi_l\}}$$

List of correlators measured

Two plane

$$\langle \cos 4(\Phi_2 - \Phi_4) \rangle$$
$$\langle \cos 8(\Phi_2 - \Phi_4) \rangle$$
$$\langle \cos 12(\Phi_2 - \Phi_4) \rangle$$
$$\langle \cos 6(\Phi_2 - \Phi_3) \rangle$$
$$\langle \cos 6(\Phi_2 - \Phi_6) \rangle$$
$$\langle \cos 6(\Phi_3 - \Phi_6) \rangle$$
$$\langle \cos 12(\Phi_3 - \Phi_4) \rangle$$
$$\langle \cos 10(\Phi_2 - \Phi_5) \rangle$$

Three plane

"2-3-5"

$$\langle \cos(2\Phi_2 + 3\Phi_3 - 5\Phi_5) \rangle$$
$$\langle \cos(-8\Phi_2 + 3\Phi_3 + 5\Phi_5) \rangle$$

"2-4-6"

$$\langle \cos(2\Phi_2 + 4\Phi_4 - 6\Phi_6) \rangle$$
$$\langle \cos(-10\Phi_2 + 4\Phi_4 + 6\Phi_6) \rangle$$

"2-3-4"

$$\langle \cos(2\Phi_2 - 6\Phi_3 + 4\Phi_4) \rangle$$
$$\langle \cos(-10\Phi_2 + 6\Phi_3 + 4\Phi_4) \rangle$$

Weighted correlations : Scalar-Product Method

- Scalar product method (weighted correlations) is improvement of EP method that takes into consideration Event-by-Event flow fluctuations

- Each event is weighted by the flow vector magnitude in that event

$$\vec{q}_n = (q_n \cos n\Phi_n, q_n \sin n\Phi_n) \qquad q_n \to \frac{\sum w v_n}{\sum w}, w = \mathrm{E_T} \text{ or } \mathrm{p_T}$$

Ollitrault 1209.2323

- The weighted EP correlations are defined as

$$\left\langle \cos(c_1\Phi_1 + 2c_2\Phi_2 .... + nc_n\Phi_n) \right\rangle \to \left\langle q_1^{c_1} q_2^{c_2} ... q_n^{c_n} \cos(c_1\Phi_1 + ... + nc_n\Phi_n) \right\rangle \Big/ \sqrt{\left\langle q_1^{2c_1} \right\rangle} \sqrt{\left\langle q_2^{2c_2} \right\rangle} ... \sqrt{\left\langle q_n^{2c_n} \right\rangle}$$

- The formula for applying resolution corrections becomes:

arXiv:1307.0980

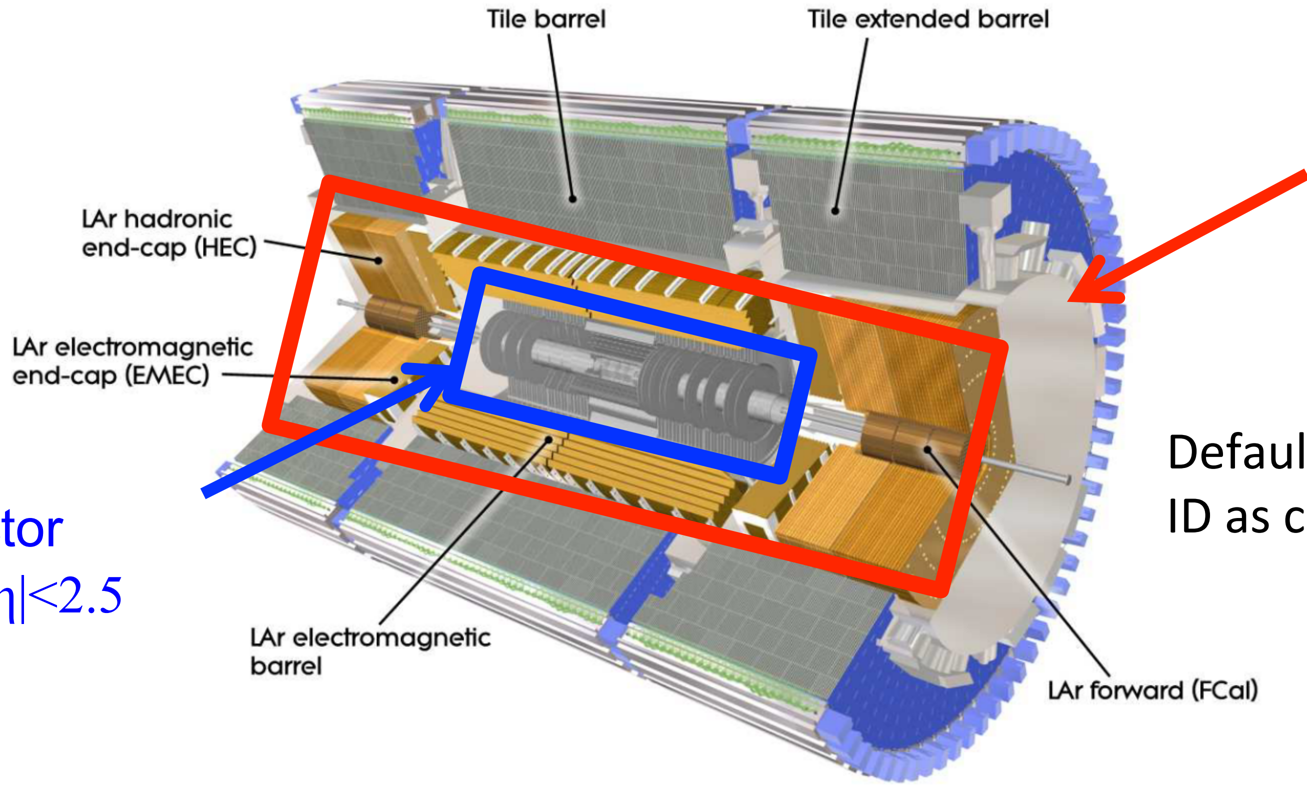
Desired correlator

Observed correlator

$$\left\langle \cos(c_1\Phi_1 + ... + nc_n\Phi_n) \right\rangle_{Weighted} = \frac{\left\langle q_1^{obs,c_1} \times ...... \times q_n^{obs,c_n} \cos(c_1\Psi_1 + ... + nc_n\Psi_n) \right\rangle}{\sqrt{\left\langle (q_1^{obs,P} q_1^{obs,N})^{c_1} \cos\left(c_1(\Psi_1^P - \Psi_1^N)\right) \right\rangle} .... \sqrt{\left\langle (q_n^{obs,P} q_n^{obs,N})^{c_n} \cos\left(nc_n(\Psi_n^P - \Psi_n^N)\right) \right\rangle}}$$

Resolution for individual planes (2SE method)

Choice of detectors



ECal+FCal
$|\eta|<4.9$

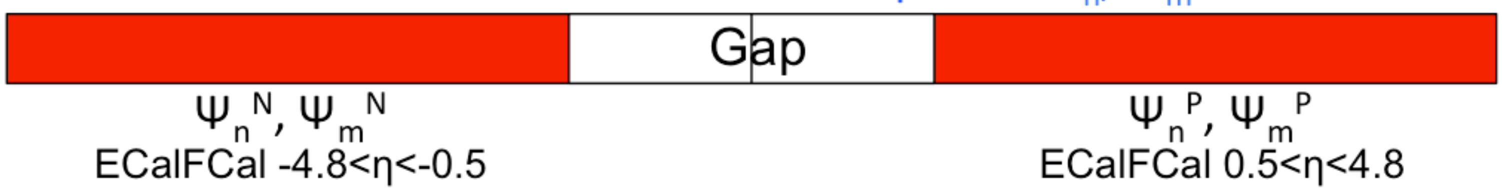
Inner detector
Tracks $|\eta|<2.5$

Default use Calorimeter, ID as cross check

Subevents used for two-plane correlations and their η coverages			
Default	$\mathrm{ECalFCal_P}$ $\eta \in(0.5,4.8)$		$\mathrm{ECalFCal_N}$ $\eta \in(-4.8,-0.5)$
Cross-check	$\mathrm{ID_P}$ $\eta \in(0.5,2.5)$		$\mathrm{ID_N}$ $\eta \in(-2.5,-0.5)$
Subevents used for three-plane correlations and their η coverages			
Default	$\mathrm{ECal_P}$ $\eta \in(0.5,2.7)$	FCal $|\eta| \in(3.3,4.8)$	$\mathrm{ECal_N}$ $\eta \in(-2.7,-0.5)$
Cross-check	$\mathrm{ID_P}$ $\eta \in(1.5,2.5)$	ID $\eta \in(-1.0,1.0)$	$\mathrm{ID_N}$ $\eta \in(-2.5,-1.5)$

Obtaining raw event-plane correlations

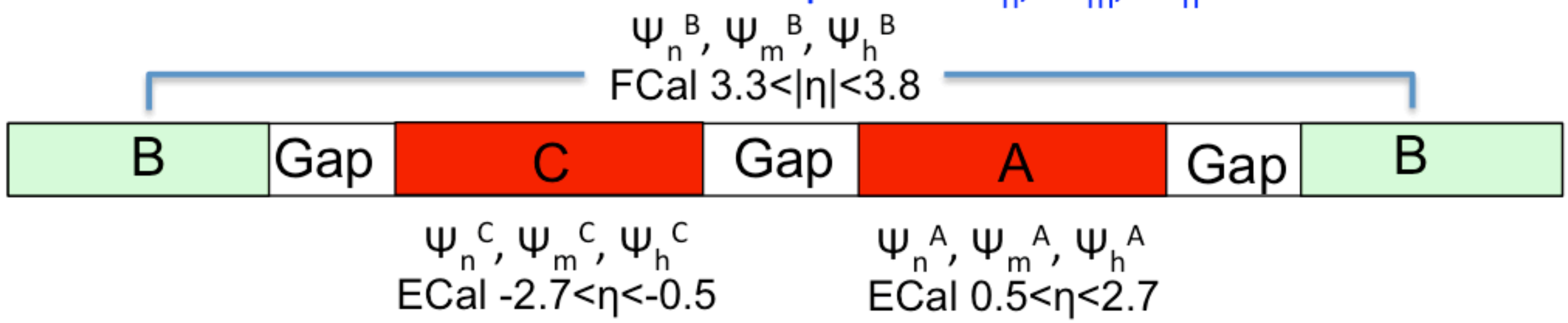
Correlation with two planes Ψ_n, Ψ_m



Each event has two combinations $k(\Psi_n^N-\Psi_m^P)$ and $k(\Psi_n^P-\Psi_m^N)$ with the same resolution

So just combine into one measurement

Correlation with three planes Ψ_n, Ψ_m, Ψ_h

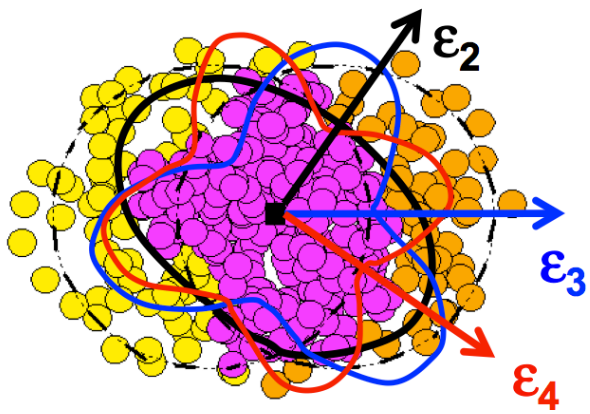


Each event have 3!=6 combinations

A & C are symmetric so only 3 combined measurements: Type 1,2,3

- Gap is required to remove autocorrelation, more important for Res.
- Event-mixing to check acceptance effect (planes taken from different events)

Expectations from Glauber model

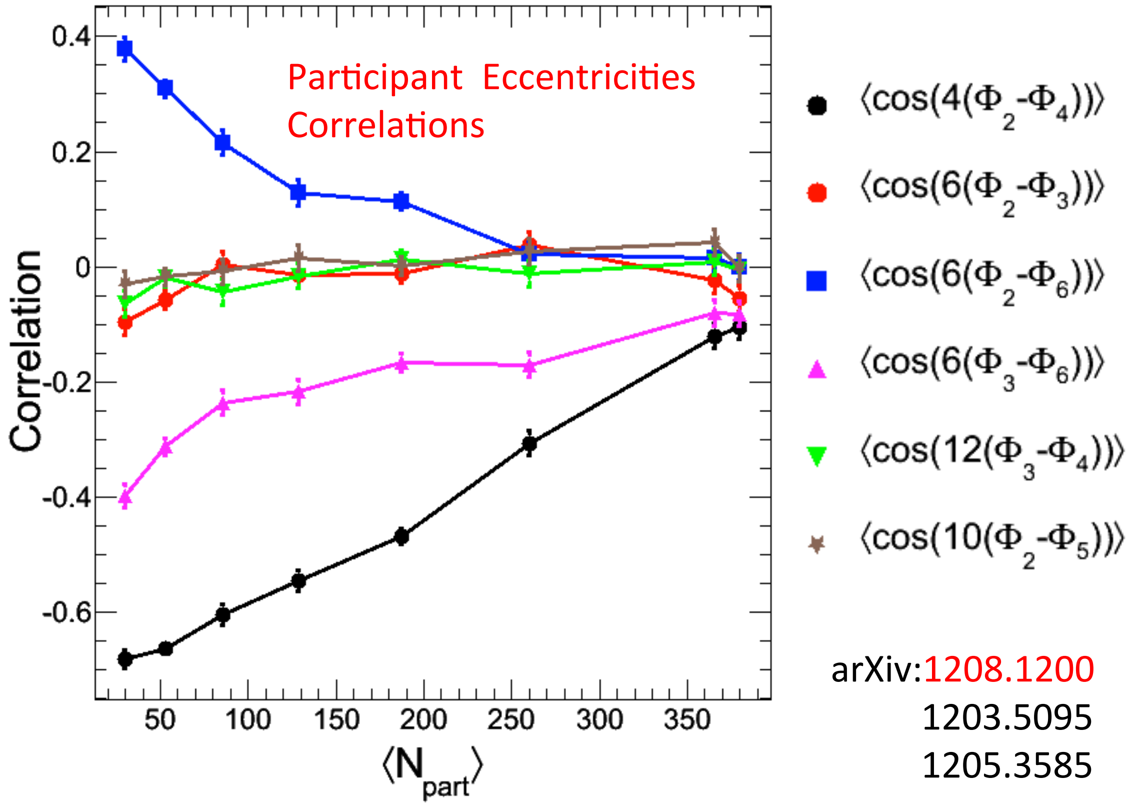


- Plane directions in configuration space

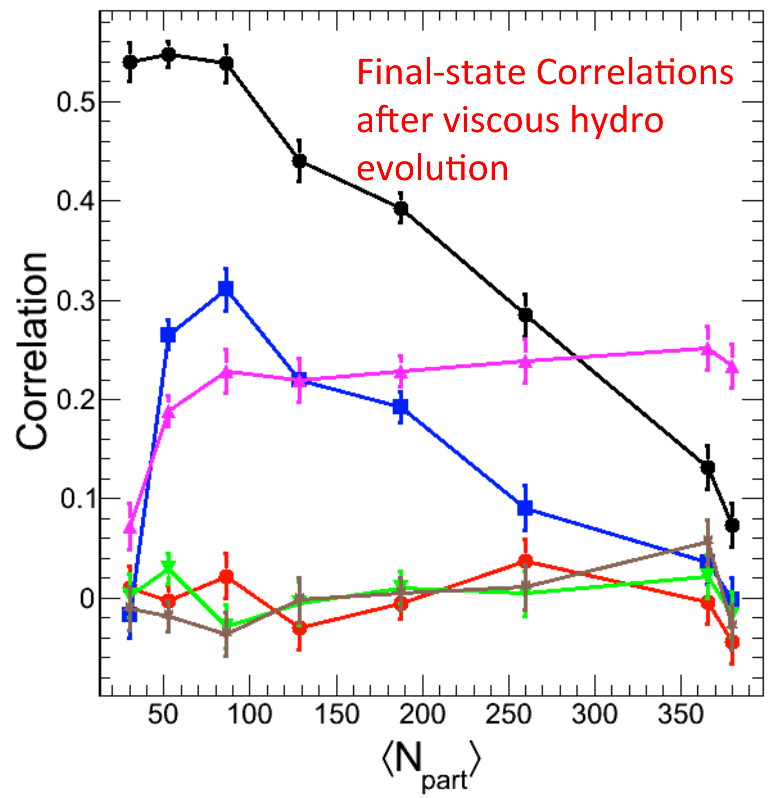
$$\varepsilon_n = \sqrt{\frac{\langle r^n \cos(n\phi)\rangle^2 + \langle r^n \sin(n\phi)\rangle^2}{r^n}}$$

$$\Phi_n = \frac{\mathrm{atan}\left(\langle r^n \sin(n\phi)\rangle, \langle r^n \cos(n\phi)\rangle\right) + \pi}{n}$$

- Expected to be strongly modified by medium evolution in the final state (Qiu and Heinz, arXiv:1208.1200)



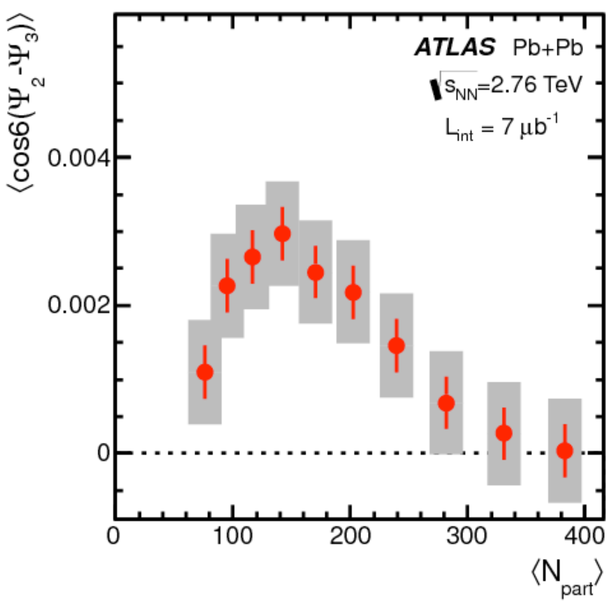
arXiv:1208.1200
1203.5095
1205.3585



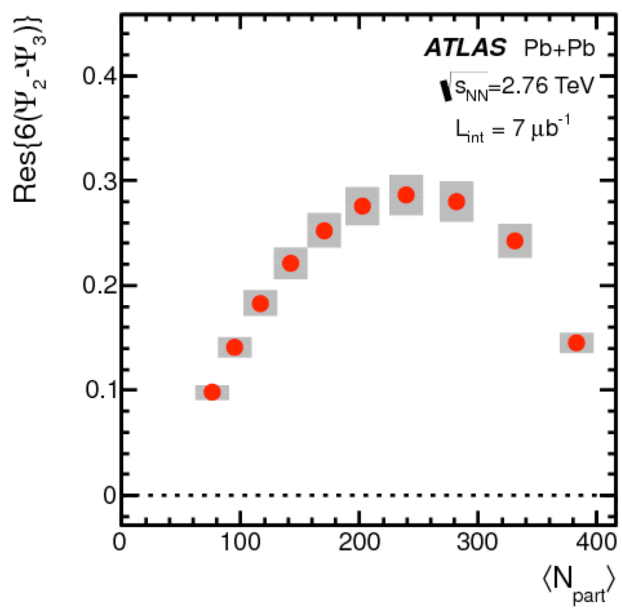
Correlation between Φ_2 and Φ_3

$$\frac{\langle \cos 6(\Psi_2 - \Psi_3) \rangle}{\mathrm{Res}\{6\Psi_2\}\mathrm{Res}\{6\Psi_3\}} = \langle \cos 6(\Phi_2 - \Phi_3) \rangle$$

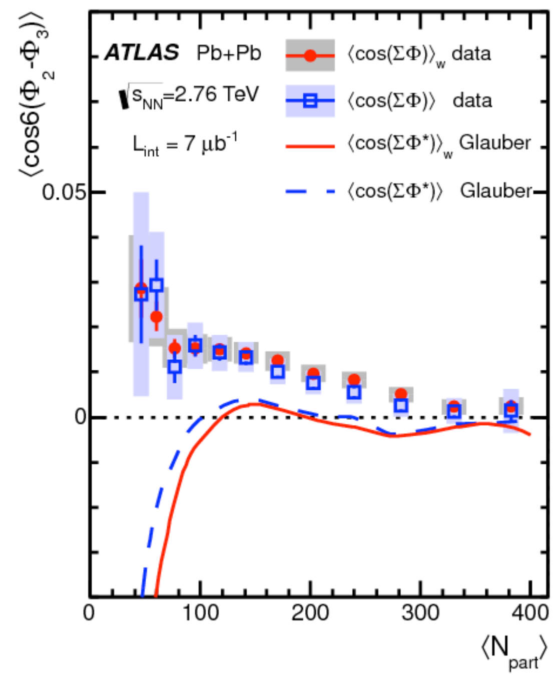
Observed correlation



Combined Res

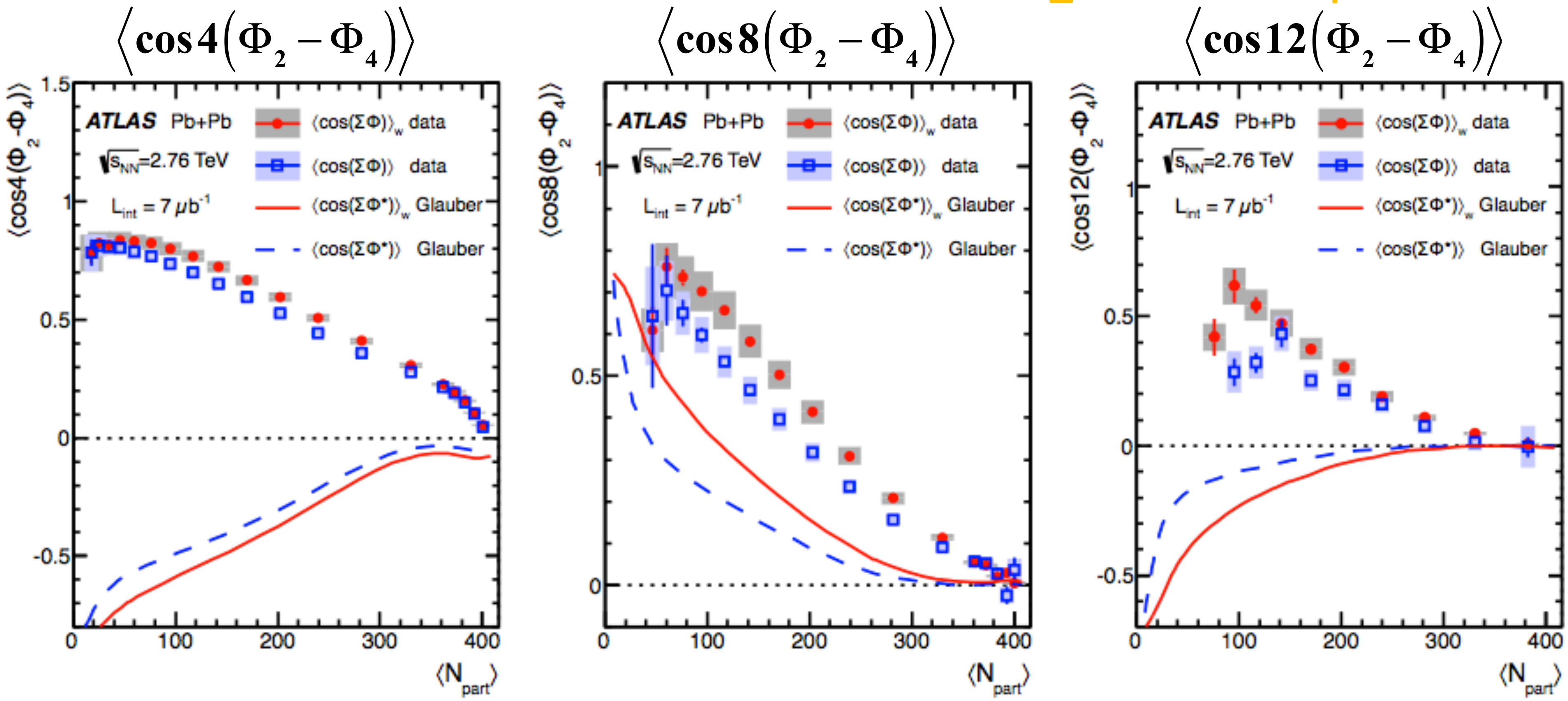


Corrected result



Small observed signal, good resolution→ small corrected signal (<0.02)

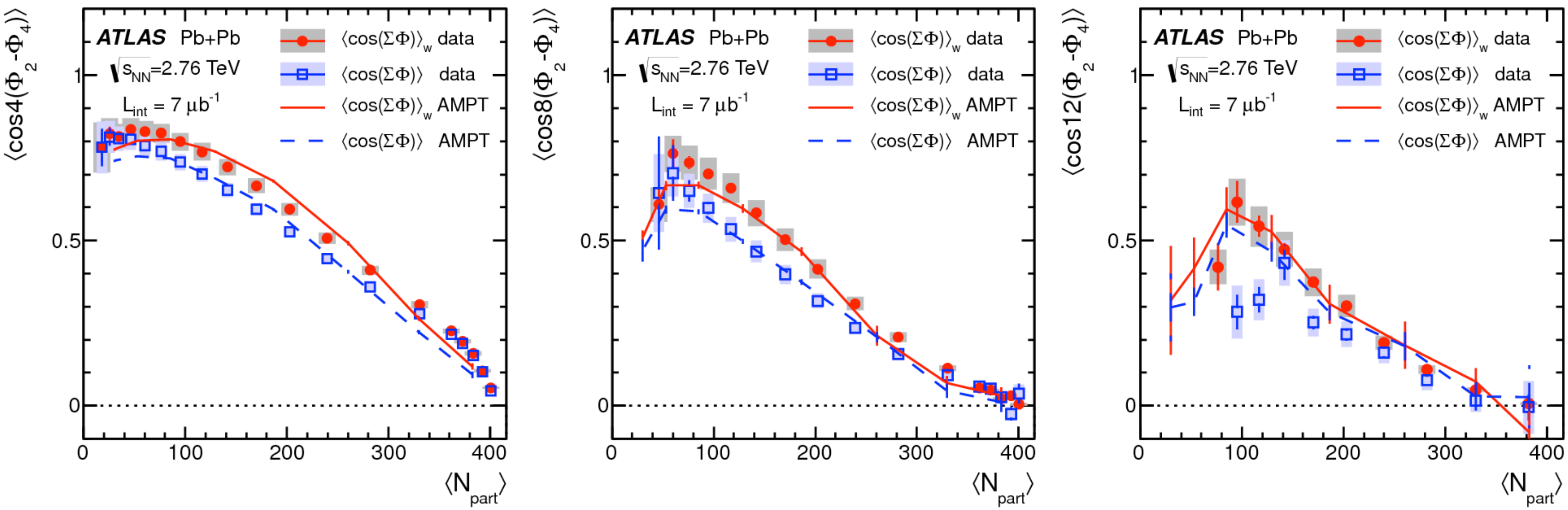
Correlation between Φ_2 and Φ_4



- Coefficients decrease slowly with *j*, imply a sharp Φ_2-Φ_4 correlation.
- Very different from correlations in initial state (Glauber)
- What happens if we include final-state-interactions?

Correlation between Φ_2 and Φ_4

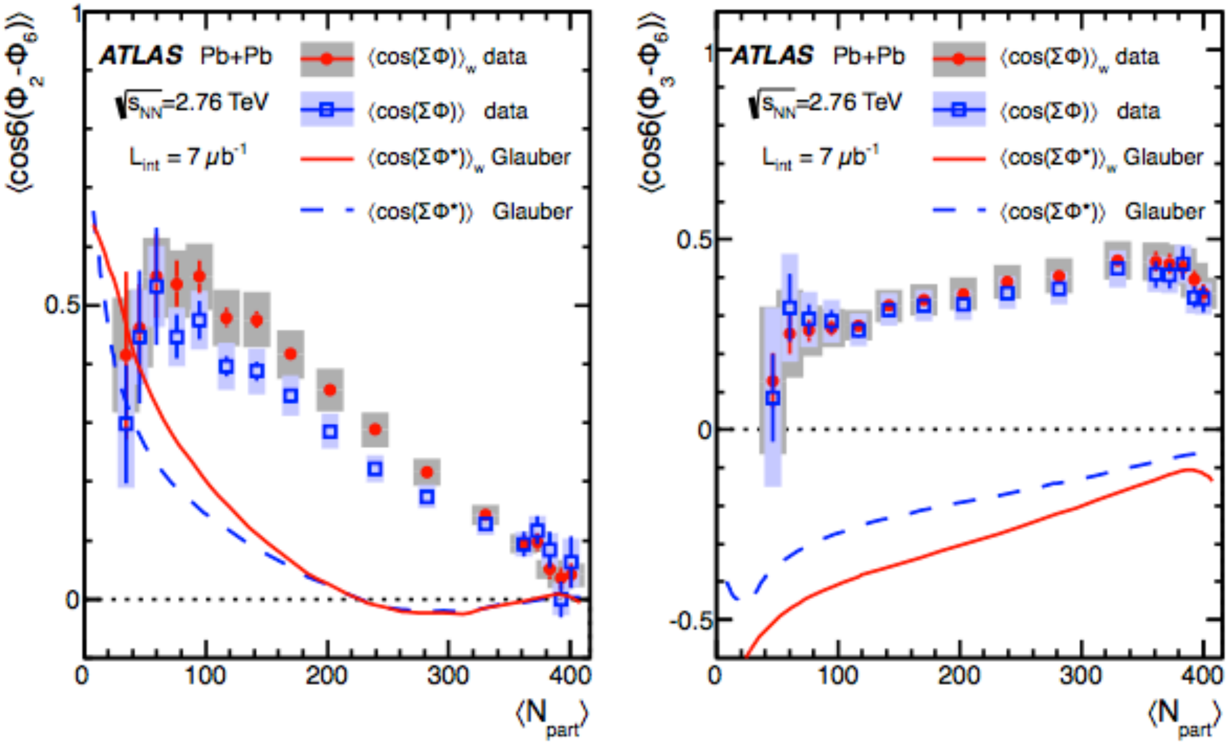
$\left\langle \cos 4\left(\Phi_2 - \Phi_4\right)\right\rangle$ $\left\langle \cos 8\left(\Phi_2 - \Phi_4\right)\right\rangle$ $\left\langle \cos 12\left(\Phi_2 - \Phi_4\right)\right\rangle$



- Correlations beautifully reproduced in AMPT model
  - AMPT results from **arXiv:1307.0980** (Bhalerao et. al.)
  - Model tuned to reproduce v_n also reproduces EP correlations

Correlation of Φ_2 or Φ_3 with Φ_6

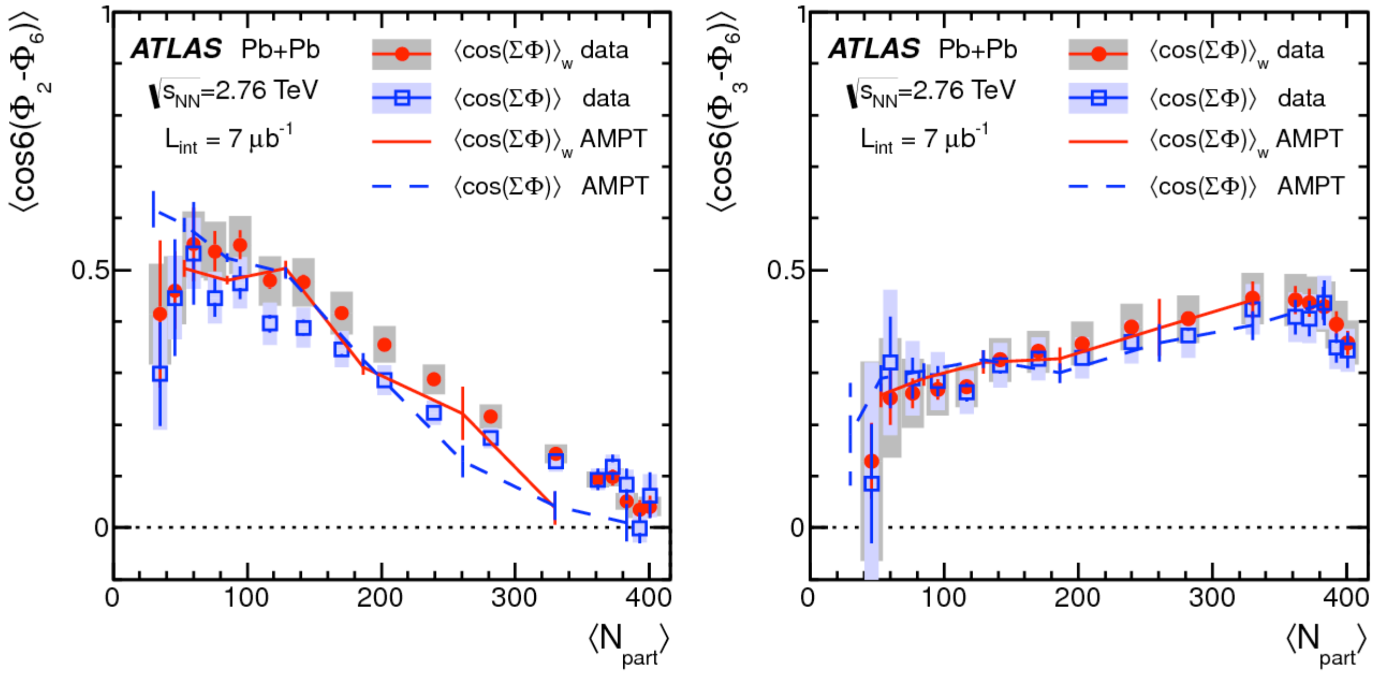
$\langle \cos 6(\Phi_2 - \Phi_6) \rangle$ $\langle \cos 6(\Phi_3 - \Phi_6) \rangle$



- Φ_2 and Φ_3 weakly correlated, but both strongly correlated with Φ_6.
- They show opposite centrality dependence
  - Φ_2-Φ_6 correlation may due to average geometry..
  - But Φ_3-Φ_6 correlation?
  - v_6 dominated by non-linear contribution: v_2^3, v_3^2 ?

Correlation of Φ_2 or Φ_3 with Φ_6

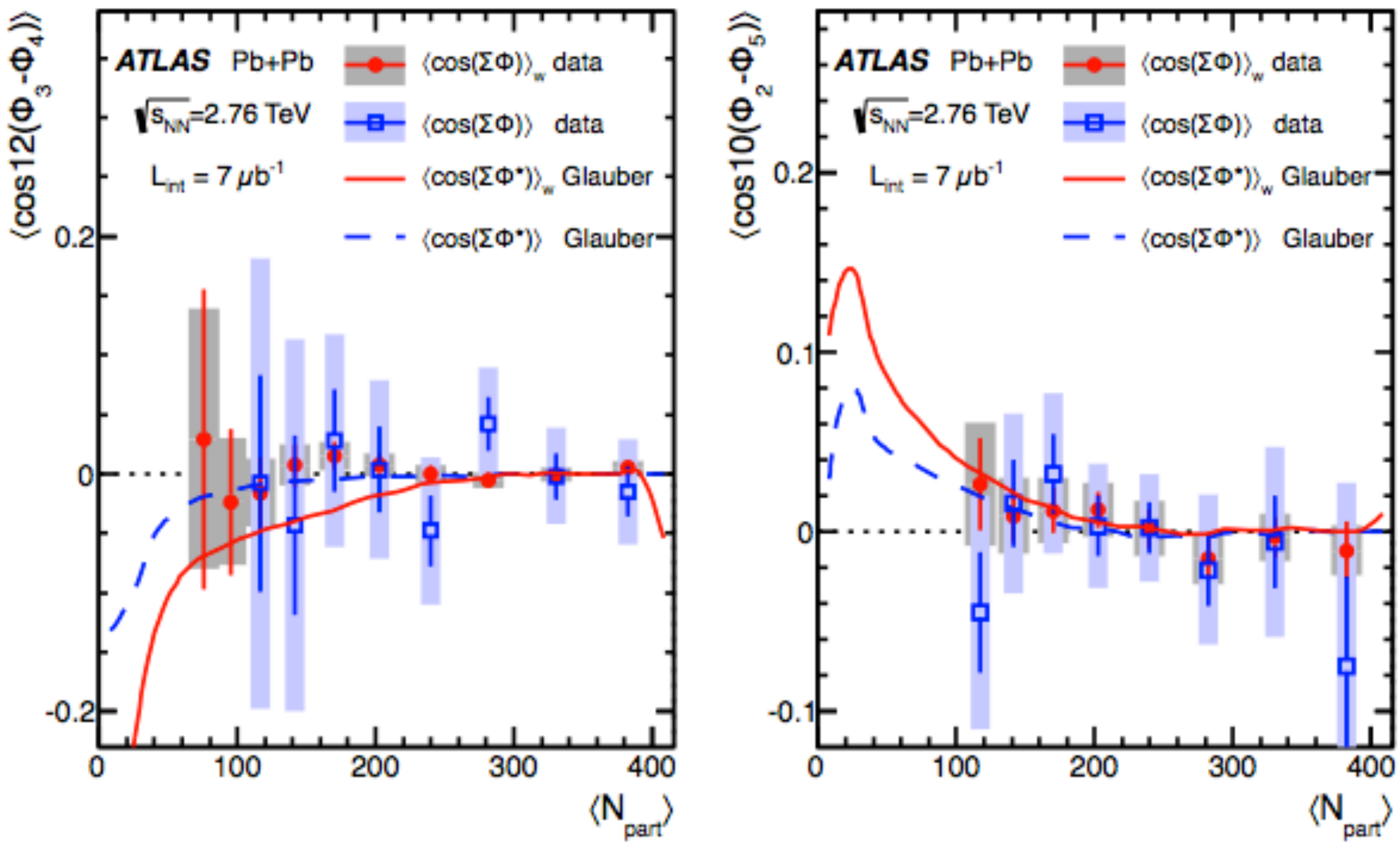
$\left\langle \cos 6\left(\Phi_2 - \Phi_6\right)\right\rangle$ $\left\langle \cos 6\left(\Phi_3 - \Phi_6\right)\right\rangle$



- Final state interactions reproduce the correlations

Φ_3 vs Φ_4 and Φ_2 vs Φ_5

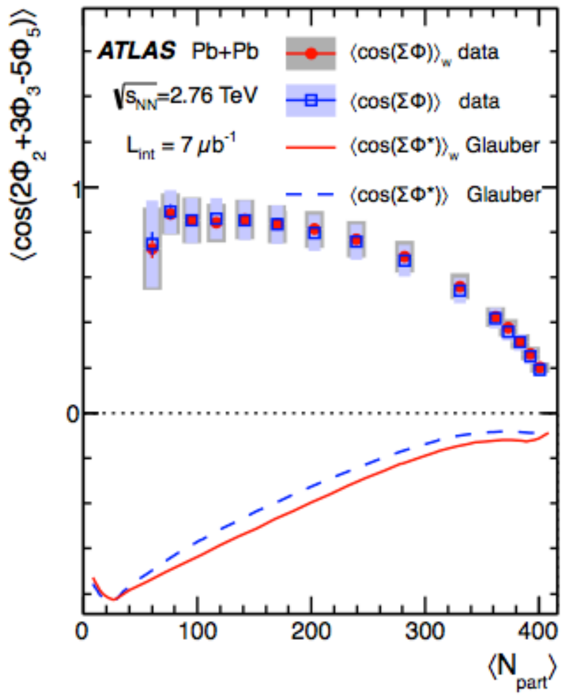
$\langle \cos 12(\Phi_3 - \Phi_4) \rangle$ $\langle \cos 10(\Phi_2 - \Phi_5) \rangle$

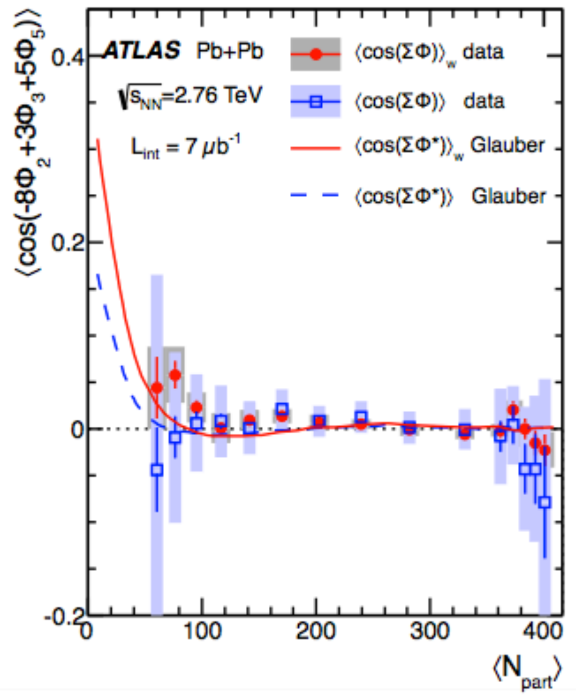


correlations are weak (< few %)

Three-plane : "2-3-5" correlation

$$\left\langle \cos\left(2\Phi_2 + 3\Phi_3 - 5\Phi_5\right)\right\rangle \qquad \left\langle \cos\left(-8\Phi_2 + 3\Phi_3 + 5\Phi_5\right)\right\rangle$$





$$v_5 e^{i5\Phi_5} = \alpha_5 \varepsilon_5 e^{i5\Phi_5} + \beta_{2,3,5} v_2 e^{i2\Phi_2} v_3 e^{i3\Phi_3}$$

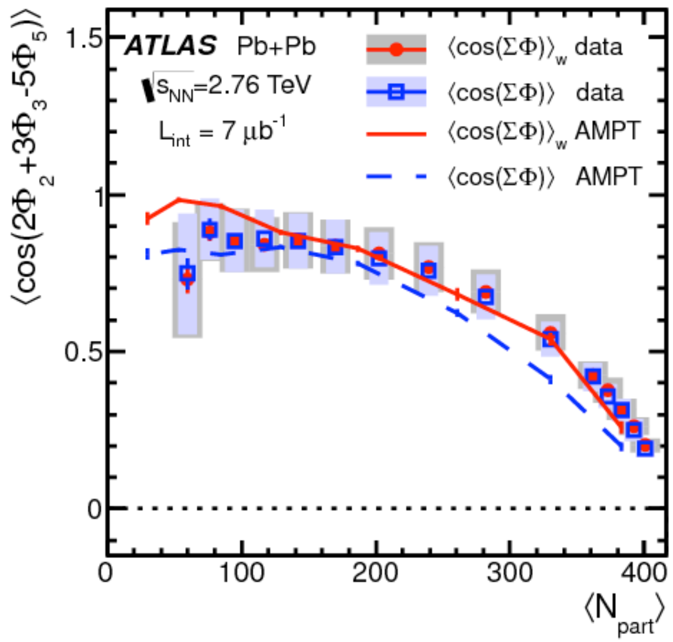
$(2\Phi_2 + 3\Phi_3 - 5\Phi_5) = 3(\Phi_3 - \Phi_2) - 5(\Phi_5 - \Phi_2)$

$(-8\Phi_2 + 3\Phi_3 + 5\Phi_5) = 3(\Phi_3 - \Phi_2) + 5(\Phi_5 - \Phi_2)$

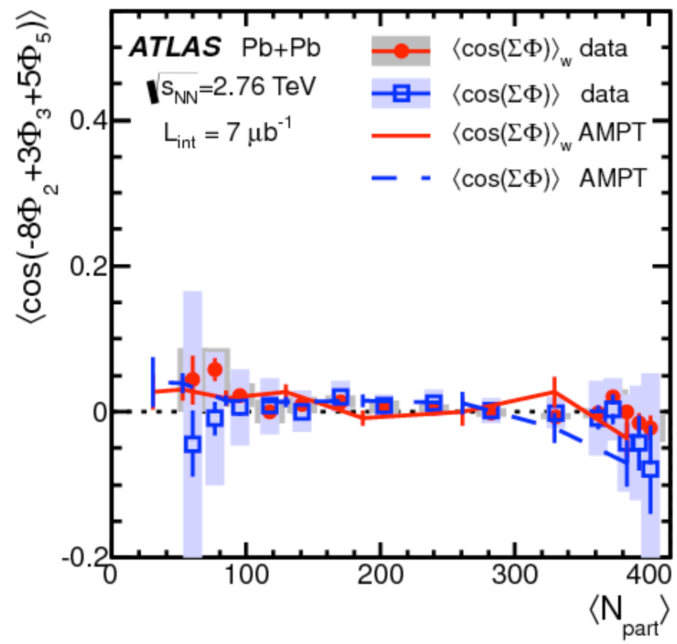
- Φ_5 and Φ_3 are individually weakly correlated with Φ_2
- But $(2\Phi_2 + 3\Phi_3 - 5\Phi_5)$ correlation is non-zero
- Glauber geometry does not match the correlation

Three-plane : "2-3-5" correlation

$\left\langle \cos\left(2\Phi_2 + 3\Phi_3 - 5\Phi_5\right)\right\rangle$



$\left\langle \cos\left(-8\Phi_2 + 3\Phi_3 + 5\Phi_5\right)\right\rangle$



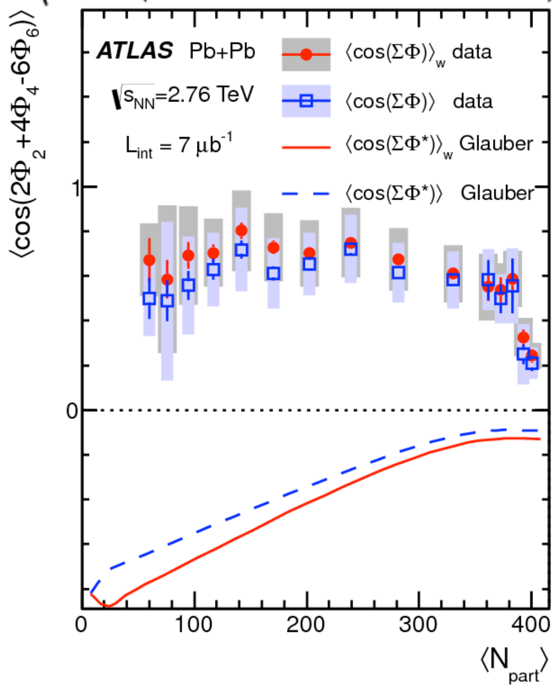
$(2\Phi_2 + 3\Phi_3 - 5\Phi_5) = 3(\Phi_3 - \Phi_2) - 5(\Phi_5 - \Phi_2)$

$(-8\Phi_2 + 3\Phi_3 + 5\Phi_5) = 3(\Phi_3 - \Phi_2) + 5(\Phi_5 - \Phi_2)$

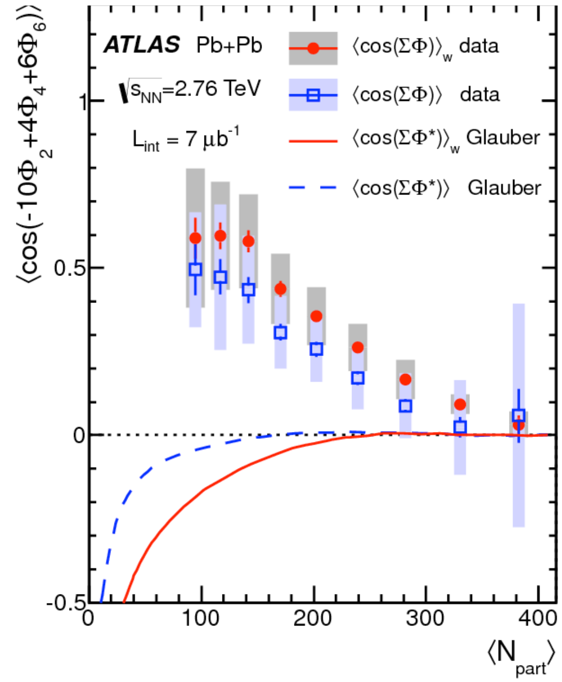
- Φ_5 and Φ_3 are individually weakly correlated with Φ_2
- But $(2\Phi_2 + 3\Phi_3 - 5\Phi_5)$ correlation is non-zero
- Glauber geometry does not match the correlation

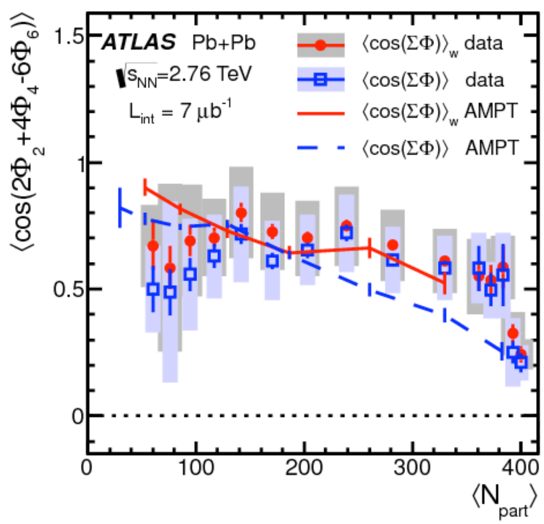
Three-plane : “2-4-6” correlation

$$\left\langle \cos\left(2\Phi_2 + 4\Phi_4 - 6\Phi_6\right)\right\rangle$$



$$\left\langle \cos\left(-10\Phi_2 + 4\Phi_4 + 6\Phi_6\right)\right\rangle$$

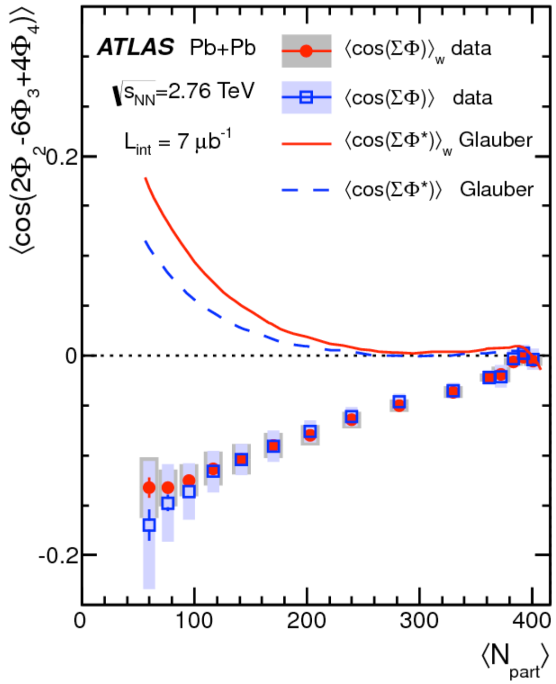




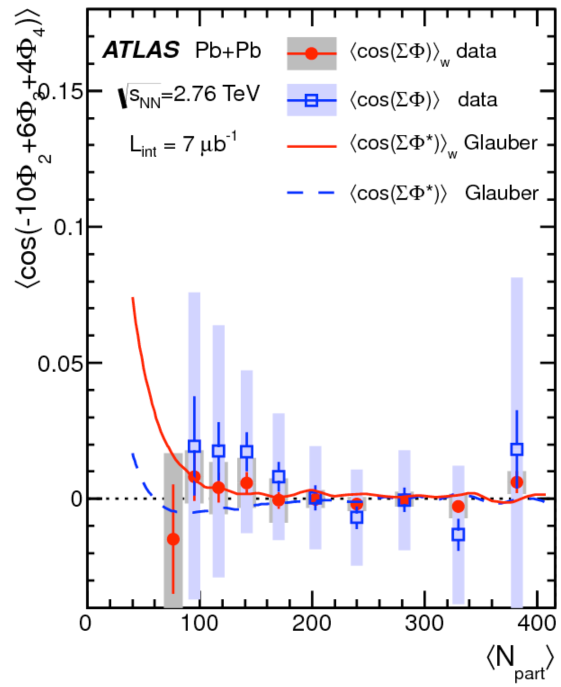
AMPT

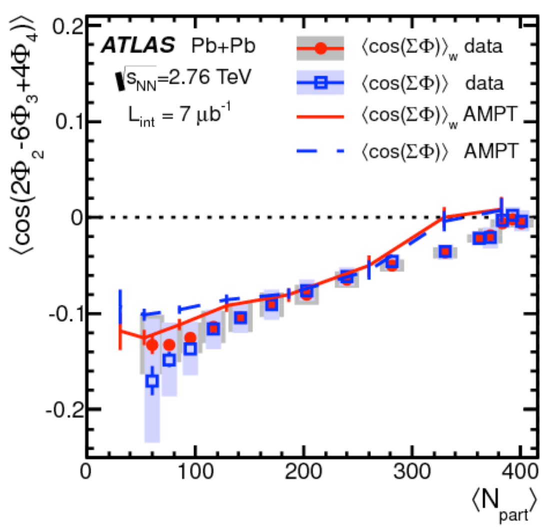
Three-plane : "2-3-4" correlation

$$\left\langle \cos\left(2\Phi_2 - 6\Phi_3 + 4\Phi_4\right)\right\rangle$$



$$\left\langle \cos\left(-10\Phi_2 + 6\Phi_3 + 4\Phi_4\right)\right\rangle$$





AMPT

Alternative parameterization of initial geometry

- Typically initial geometry in Heavy-Ion collisions is quantified by the eccentricities ε_n:

$$\varepsilon_n e^{in\Phi_n} \equiv -\frac{\left\langle r^n e^{in\phi_r} \right\rangle}{\langle r^n \rangle}$$

- In a recent paper (arXiv:1206.1905) Teaney and Yan have pointed out that it might be better to quantify the initial geometry by cumulants c_n

- The cumulants are related to the eccentricities by:

$$c_2 e^{i2\Phi_2} \equiv -\frac{\langle z^2 \rangle}{\langle r^2 \rangle}, \qquad z = re^{i\phi}$$

$$c_3 e^{i3\Phi_3} \equiv -\frac{\langle z^3 \rangle}{\langle r^3 \rangle},$$

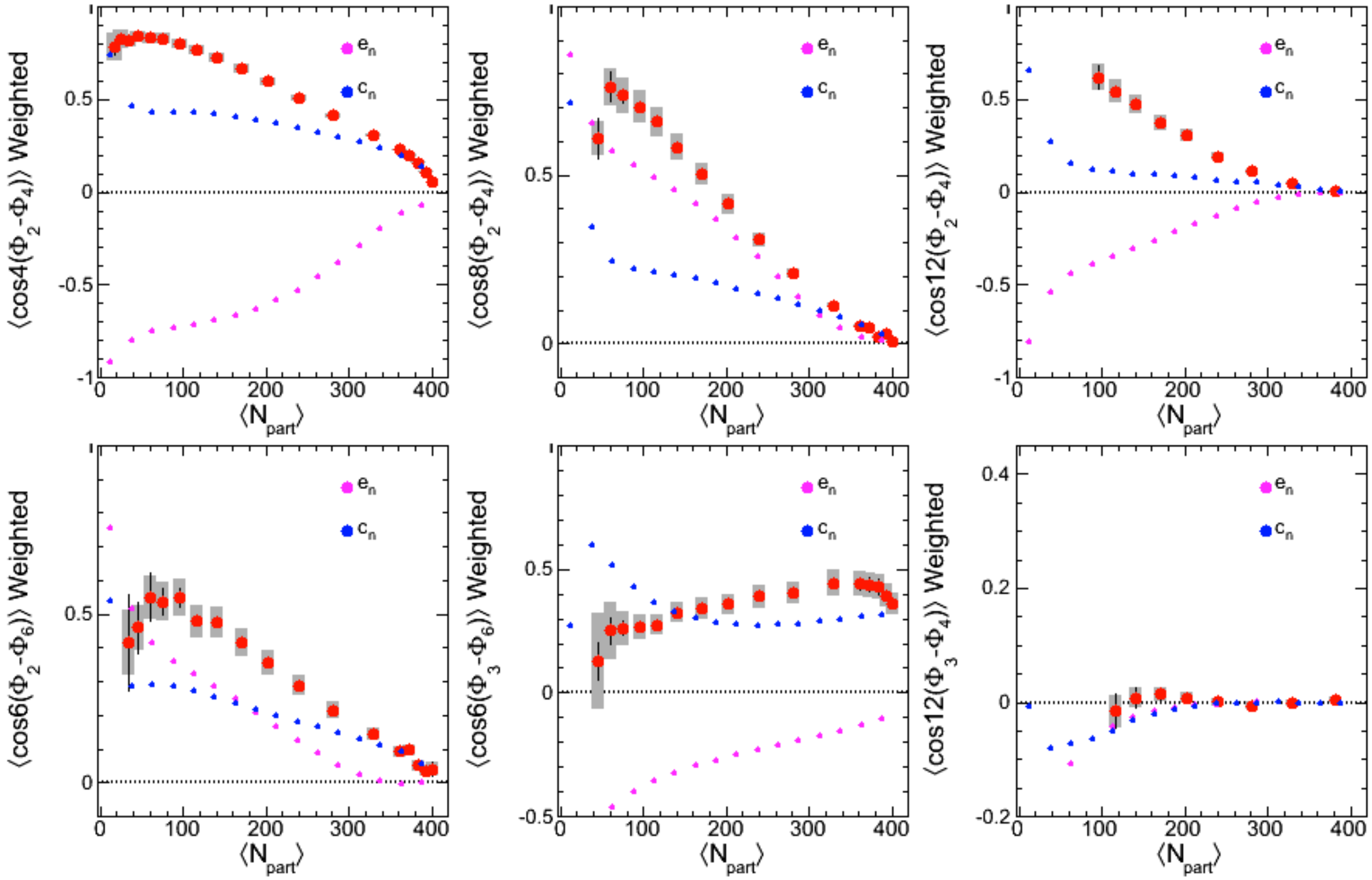
$$c_4 e^{i4\Phi_4} \equiv -\frac{1}{\langle r^4 \rangle}\left[\langle z^4 \rangle - 3\langle z^2 \rangle^2\right],$$

$$c_5 e^{i5\Phi_5} \equiv -\frac{1}{\langle r^5 \rangle}\left[\langle z^5 \rangle - 10\langle z^2 \rangle\langle z^3 \rangle\right],$$

$$c_6 e^{i6\Phi_6} = -\frac{1}{\langle r^6 \rangle}\left[\langle z^6 \rangle - 15\langle z^4 \rangle\langle z^2 \rangle - 10\langle z^3 \rangle^2 + 30\langle z^2 \rangle^3\right]$$

- Is this parameterization better?

Correlations In initial geometry



See also
arXiv:1312.3689
Teaney & Yan

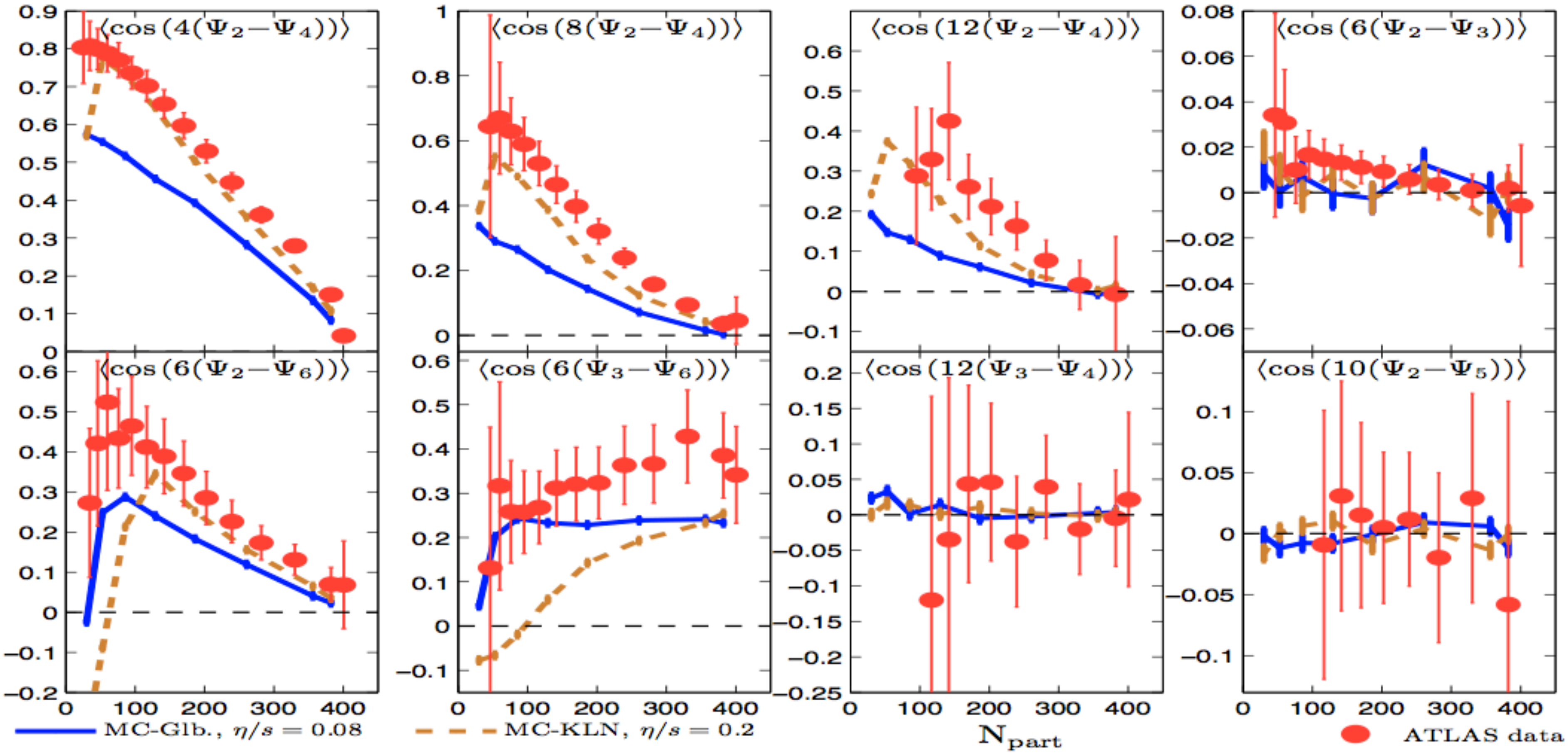
Compare correlation between cumulants to the ATLAS EP correlations

1. Do much better job than the correlations between the ε_n
2. Indicative that when we define initial geometry in terms of ε_n, we have to take into consideration a large degree on non-linear response in generation of the v_n

$$v_n e^{i\Phi_n} \propto \varepsilon_n e^{i\tilde{\Phi}_n} + \text{significant non-linear contribution from } \varepsilon_m (m < n)$$

$$v_n e^{i\Phi_n} \propto c_n e^{i\tilde{\Phi}_n} + \text{small non-linear contribution from } c_m (m < n)$$

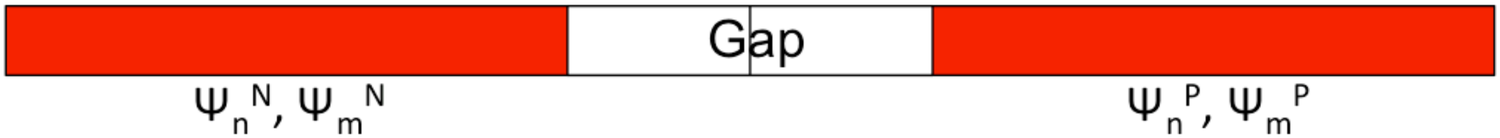
Can also constrain η/s, initial geometry

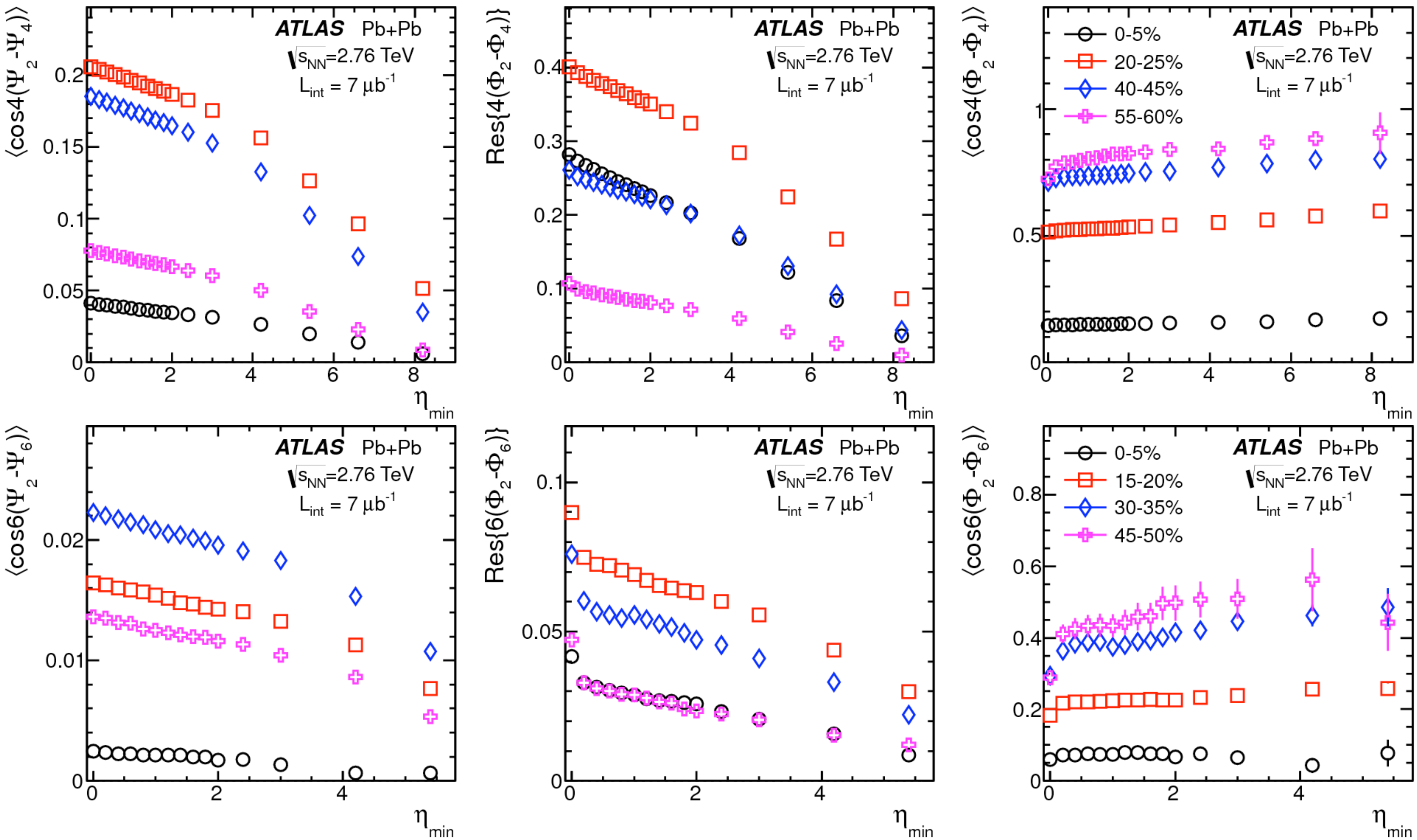


arXiv:1208.1200 : Qui & Heinz

Dependence on η gap : EP method

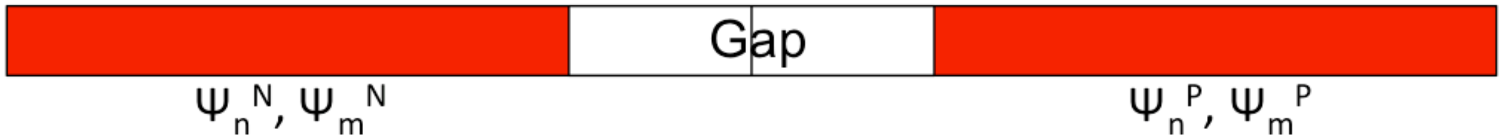
Correlation with two planes Ψ_n, Ψ_m

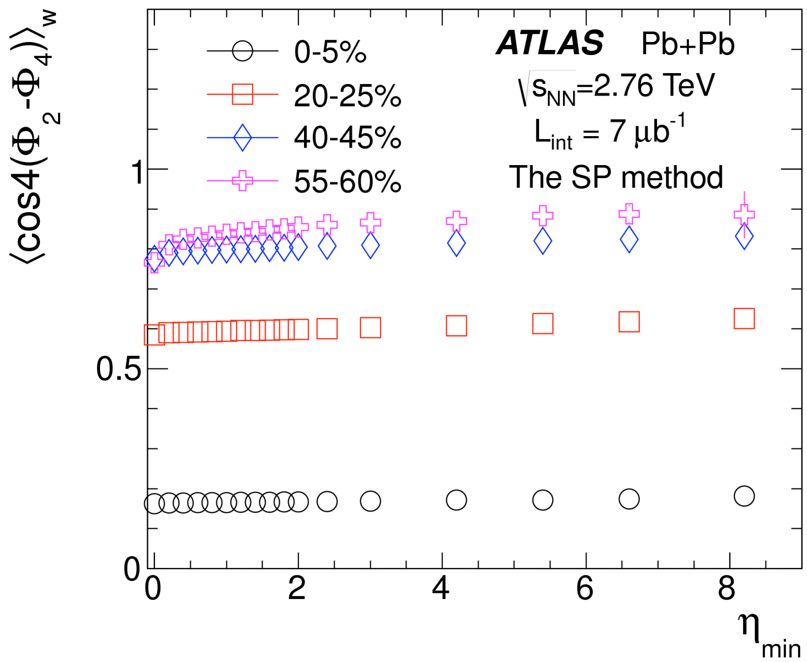


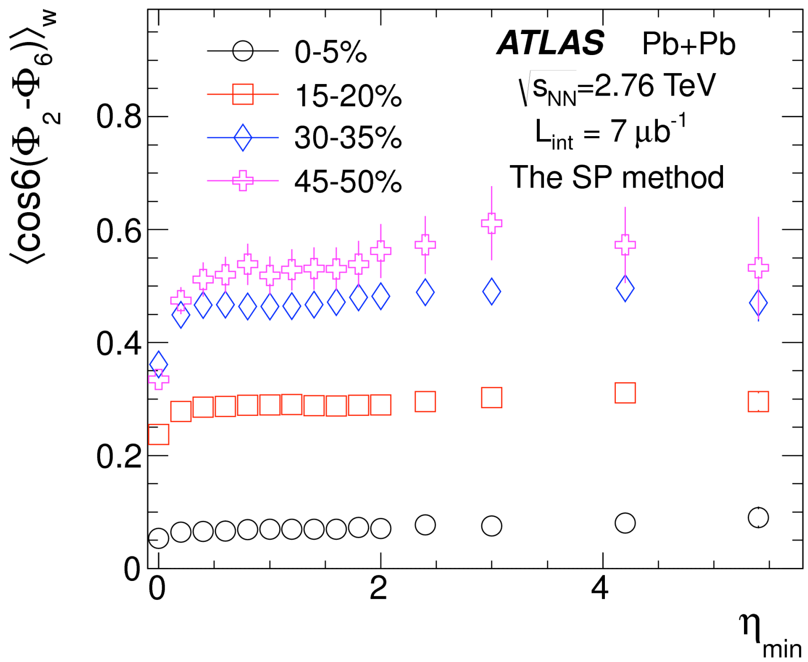


Dependence on η gap : SP method

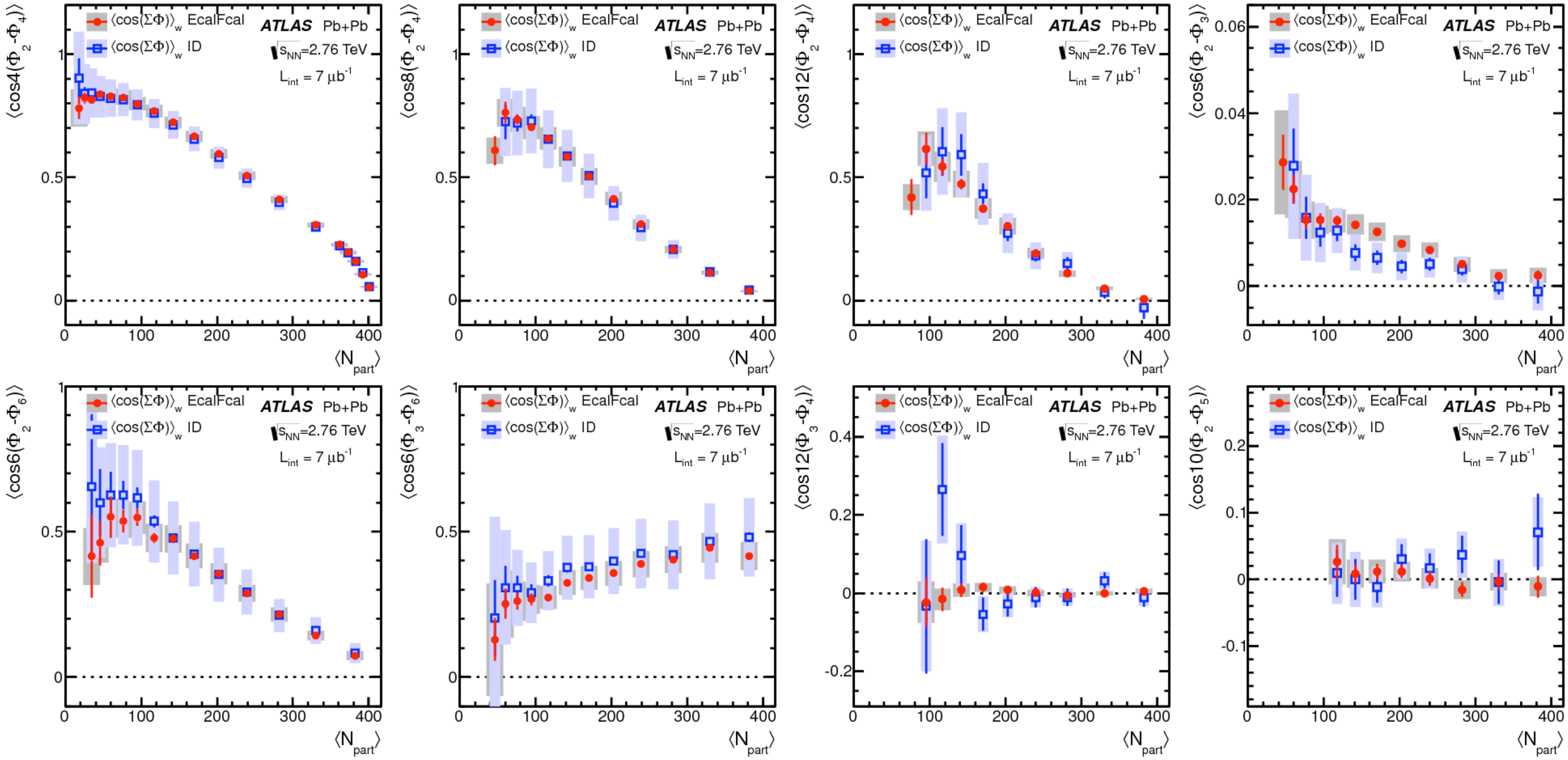
Correlation with two planes Ψ_n, Ψ_m



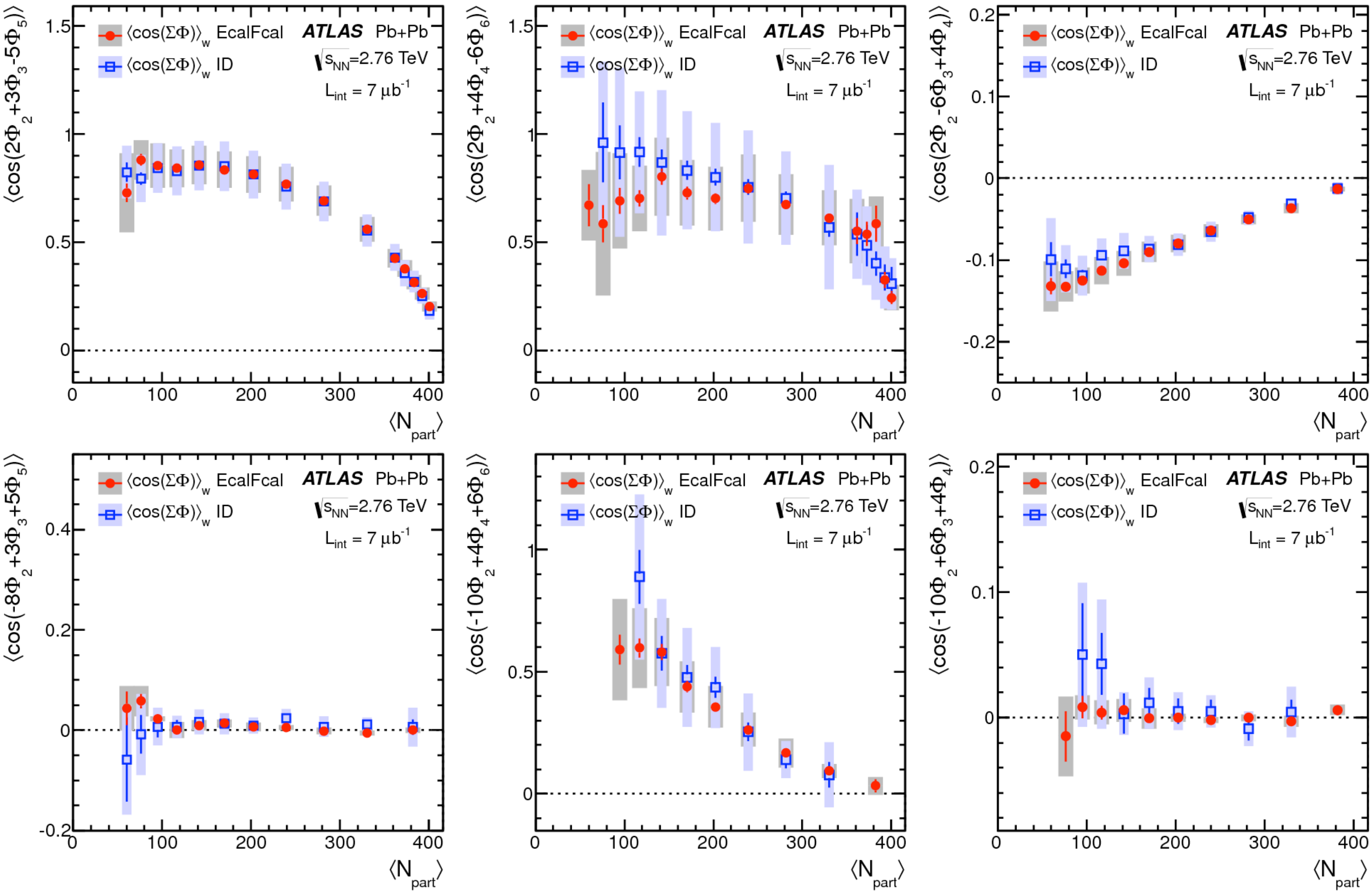




Two-plane correlations : ID



Three-plane correlations : ID



Event plane correlations: Summary

- ATLAS has measured correlations between two and three event planes
  - Significant correlations are observed for

$$\langle\cos(4(\Phi_2-\Phi_4))\rangle, \langle\cos(8(\Phi_2-\Phi_4))\rangle, \langle\cos(12(\Phi_2-\Phi_4))\rangle, \langle\cos(6(\Phi_2-\Phi_6))\rangle, \langle\cos(6(\Phi_3-\Phi_6))\rangle$$

$$\langle\cos(2\Phi_2+3\Phi_3-5\Phi5)\rangle, \langle\cos(2\Phi_2+4\Phi_4-6\Phi_6)\rangle \text{ and } \langle\cos(-10\Phi_2+4\Phi_4+6\Phi6)\rangle$$

  - Correlation is very small but nonzero for $\langle\cos(6(\Phi_2-\Phi_3))\rangle$
  - Correlation is negative for $\langle\cos(2\Phi_2-6\Phi_3+4\Phi_4)\rangle$

- Completely new flow observable

- Most non-zero correlations very different than Glauber model ε_n correlations.
- Indicate that these are generated dynamically via hydrodynamic evolution.

Qiu and Heinz, arXiv:1208.1200
Teaney and Yan, arXiv:1206.1905

- This measurement provides new constraints for models.
  - Further constraints on η/s, initial geometry

- Indicate that cumulants might be better parameterization of initial geometry