Energy Density, Pressure and Flow At Early Times

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Overview

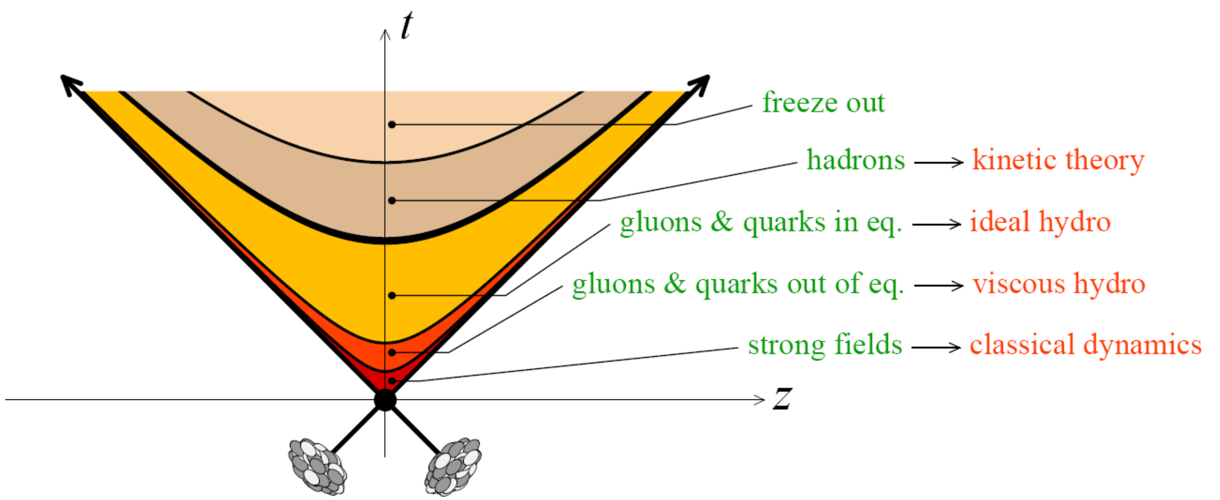
- Classical Gluon Fields and MV Model
- Analytic Solutions for Early Times
- Phenomenology
- Beyond Boost-Invariance

Work in collaboration with

- Guangyao Chen (Texas A&M)
- Joe Kapusta (Minnesota)
- Sener Ozonder (INT/Seattle, Minnesota)

A "Standard Model" of URHICs

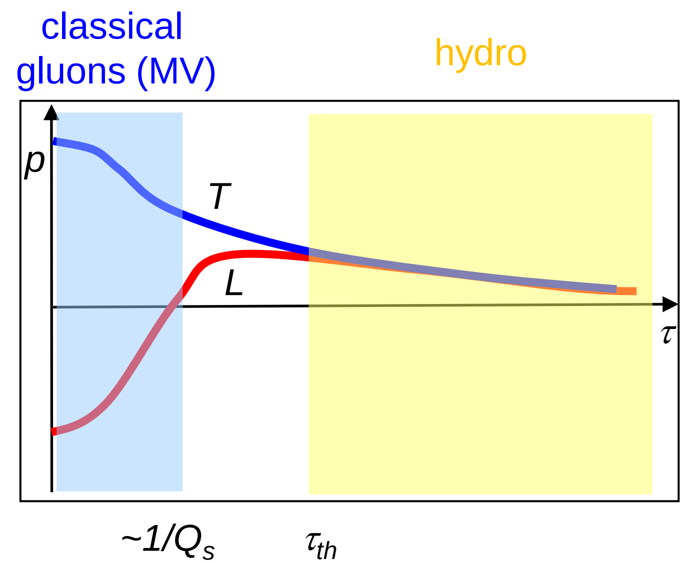
- Bulk evolution: Local thermal equilibrium with small (?) dissipative corrections after time 0.2-1 fm/c.
- Expansion and cooling via viscous hydrodynamics, maybe augmented by transport.



- Pre-equilibrium: strong classical gluon fields (CGC) → rapid (?) thermalization

Motivation

- The simplest classical gluon field model (MV) can be solved analytically for small times (near-field approximation).
- Typical time scale: $1/Q_s$.



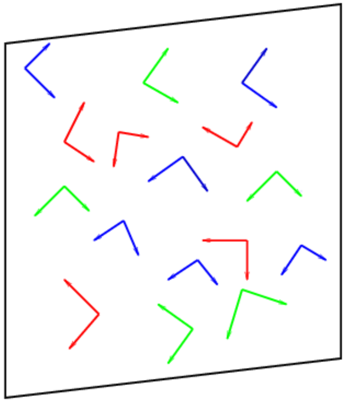
- Notable results to hope fore:
- Analytic expressions for early energy density and pressure.
- Analytic expressions for initial flow [see talk by G. Chen on Saturday]
- "New" phenomena? Global energy-momentum map of the early-time collision!

MV Model: Classical YM Dynamics

- Nuclei/hadrons at asymptotically high energy:
  - Saturated gluon density ~ Q_s^{-2} → scale $Q_s \gg \Lambda_{QCD}$, classical fields.

[L. McLerran, R. Venugopalan]

- Single nucleus: solve Yang-Mills equations $\left[D_\mu, F^{\mu\nu}\right] = J^\nu$ for gluon field $A^\mu(\rho)$.
  - Source = light cone current J (given by SU(3) charge distribution ρ).
  - Calculate observables $O(\rho)$ from the gluon field $A^\mu(\rho)$.
  - ρ from random Gaussian color fluctuations of a color-neutral nucleus.



MV Model: Classical YM Dynamics

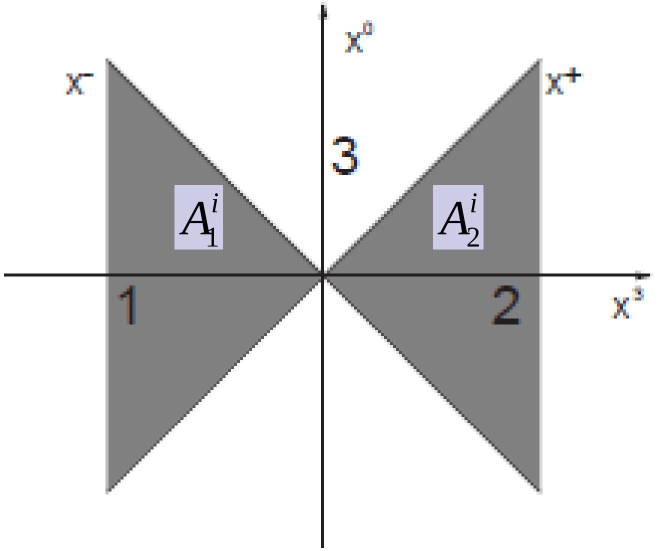
- Two nuclei: intersecting currents J_1, J_2 (given by ρ_1, ρ_2), calculate gluon field $A^\mu(\rho_1, \rho_2)$ from YM.
- Equations of motion

$$\frac{1}{\tau^3}\partial_\tau \tau^3 \partial_\tau A - \left[D^i, \left[D^i, A\right]\right] = 0$$
$$\frac{1}{\tau}\left[D^i, \partial_\tau A_\perp^i\right] - ig\tau\left[A, \partial_\tau A\right] = 0$$
$$\frac{1}{\tau}\partial_\tau \tau \partial_\tau A_\perp^i - ig\tau^2\left[A, \left[D^i, A\right]\right] - \left[D^j, F^{ji}\right] = 0$$

$$A^\pm = \pm x^\pm A(\tau, x_\perp)$$
$$A^i = A_\perp^i(\tau, x_\perp)$$

- Boundary conditions

$$A_\perp^i(\tau = 0, x_\perp) = A_1^i(x_\perp) + A_2^i(x_\perp)$$
$$A(\tau = 0, x_\perp) = -\frac{ig}{2}\left[A_1^i(x_\perp), A_2^i(x_\perp)\right]$$



[A. Kovner, L. McLerran, H. Weigert]

Analytic Solution: Small Time Expansion

- Numerical solutions available [Krasnitz, Venugopalan][Lappi][Schenke et al.]
- Here: analytic solution using small-time expansion for gauge field

$$A(\tau, x_\perp) = \sum_{n=0}^{\infty} \tau^n A_{(n)}(x_\perp)$$

$$A_\perp^i(\tau, x_\perp) = \sum_{n=0}^{\infty} \tau^n A_{\perp(n)}^i(x_\perp)$$

- Recursive solution for gluon field:

$$A_{(n)} = \frac{1}{n(n+2)} \sum_{k+l+m=n-2} \left[D_{(k)}^i, \left[D_{(l)}^i, A_{(m)}\right]\right]$$

$$A_{\perp(n)}^i = \frac{1}{n^2}\left(\sum_{k+l=n-2} \left[D_{(k)}^j, F_{(l)}^{ji}\right] + ig \sum_{k+l+m=n-4} \left[A_{(k)}, \left[D_{(l)}^i, A_{(m)}\right]\right]\right)$$

[RJF, J. Kapusta, Y. Li, 2006]
[Fujii, Fukushima, Hidaka, 2009]

  - 0th order = boundary conditions

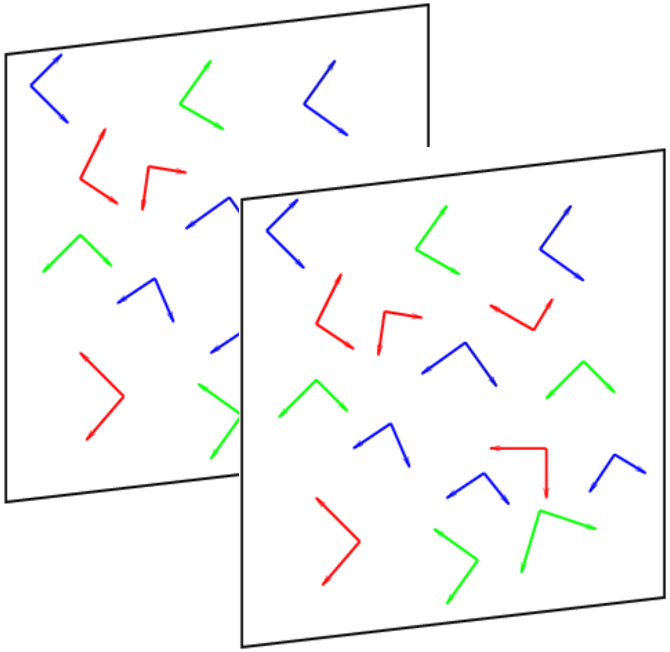
$$A_{\perp(0)}^i(x_\perp) = A_1^i(x_\perp) + A_2^i(x_\perp)$$

$$A_{(0)}(x_\perp) = -\frac{ig}{2}\left[A_1^i(x_\perp), A_2^i(x_\perp)\right]$$

Fields: Before Collision

- Before the collision: color glass = pulse of strictly transverse (color) electric and magnetic fields, mutually orthogonal, with random color orientations, in each nucleus.

$$F_1^{i+} = \delta(x^-) A_1^i$$



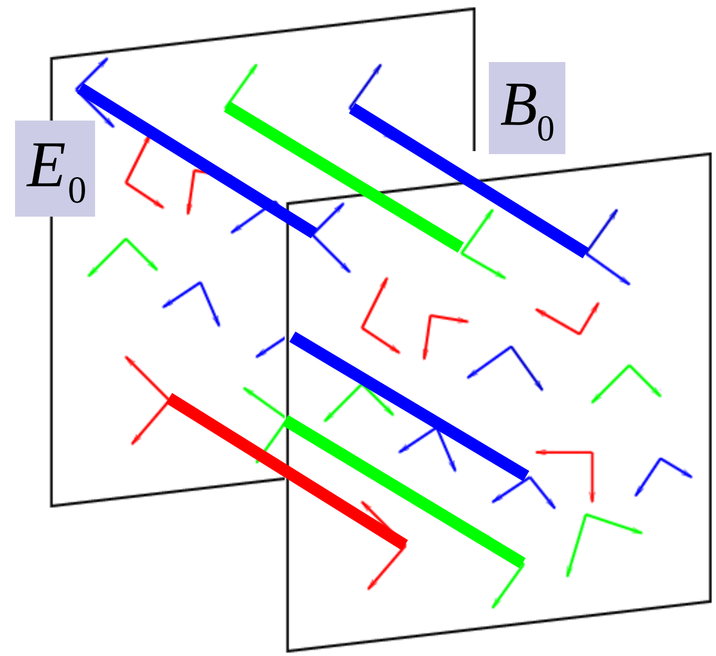
$$F_2^{i-} = \delta(x^+) A_2^i$$

Fields: At Collision

- Before the collision: color glass = pulse of strictly transverse (color) electric and magnetic fields, mutually orthogonal, with random color orientations, in each nucleus.
- Immediately after overlap (forward light cone, $\tau \to 0$): strong *longitudinal* electric & magnetic fields E_0, B_0. Non-abelian effect.

$$F^{+-}_{(0)} = ig\left[A^i_1, A^i_2\right] \quad \leftarrow \quad E_0$$

$$F^{21}_{(0)} = ig\varepsilon^{ij}\left[A^i_1, A^j_2\right] \quad \leftarrow \quad B_0$$



[L. McLerran, T. Lappi, 2006]
[RJF, J.I. Kapusta, Y. Li, 2006]

Fields: Into the Forward Light Cone

- Once the longitudinal fields E_0, B_0 are seeded, the next step in the time evolution can be understood in terms of the QCD versions of Ampere's, Faraday's and Gauss' Law.
- Transverse fields with rapidity-even and -odd contributions

$$E^i = -\frac{\tau}{2}\left(\sinh\eta\left[D^i, E_0\right] + \cosh\eta\, \varepsilon^{ij}\left[D^j, B_0\right]\right)$$

$$B^i = \frac{\tau}{2}\left(\cosh\eta\, \varepsilon^{ij}\left[D^j, E_0\right] - \sinh\eta\left[D^i, B_0\right]\right)$$

[G. Chen, RJF, PLB 723 (2013)]

- Longitudinal fields up to $O(\tau^2)$

$$E^3 = E_0 + \frac{\tau^2}{4} D^i D^i E_0$$

$$B^3 = B_0 + \frac{\tau^2}{4} D^i D^i B_0$$

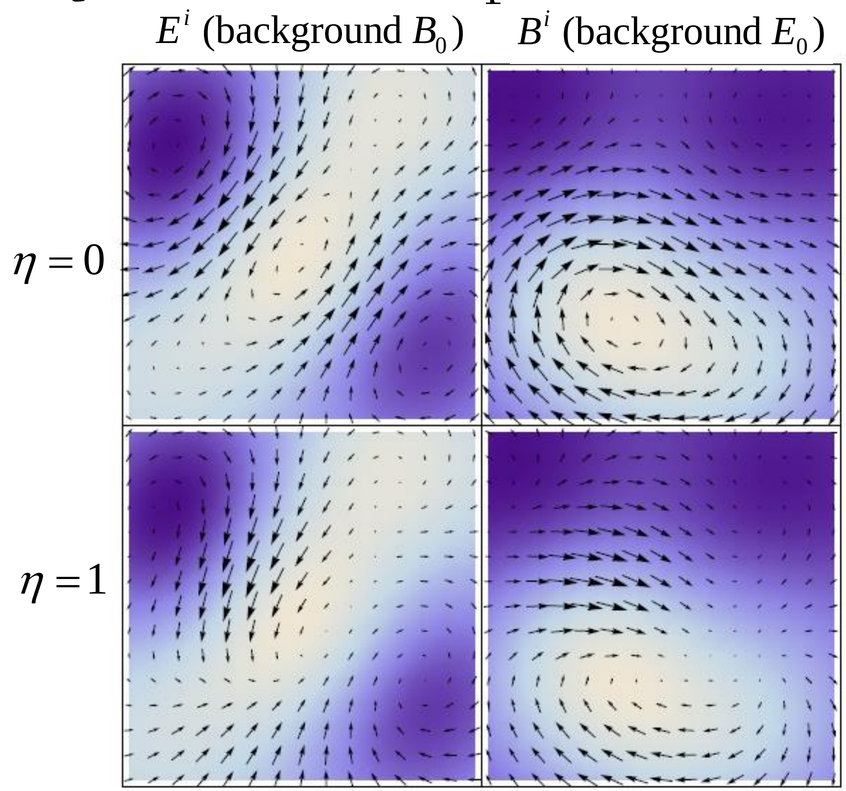


Figure 2: Transverse electric fields (left panels) and magnetic fields (right panels) at $\eta = 0$ (upper panels) and $\eta = 1$ (lower panels) in an abelian example for a random distribution of fields A_1^i, A_2^i. The initial longitudinal fields B_0 (left panels) and E_0 (right panels) are indicated through the density of the background (lighter color = larger values). At $\eta = 0$ the fields are divergence-free and clearly following Ampére's and Faraday's Laws, respectively.

Analytic Solution: Small Time Expansion

- Convergence for weak field limit: recover analytic solution for all times.

$$A_{(n)} = \frac{2}{n!!^2\,(n+2)} \left(-k_\perp\right)^{n/2} A_{(0)} \qquad A^{\mathrm{LO}}\left(\tau,\mathbf{k}_\perp\right) = \frac{2A_{(0)}\left(\mathbf{k}_\perp\right)}{k_\perp \tau} J_1\left(k_\perp \tau\right)$$

$$A^i_{\perp(n)} = \frac{1}{n!!^2} \left(-k_\perp\right)^{n/2} A^i_{\perp(0)} \qquad A^{i\,\mathrm{LO}}_\perp\left(\tau,\mathbf{k}_\perp\right) = A^i_{\perp(0)}\left(\mathbf{k}_\perp\right) J_0\left(k_\perp \tau\right)$$

- Convergence for strong fields: convergence radius ~ $1/Q_s$ for averaged quantities like energy density.

Energy Momentum Tensor

- Initial ($\tau = 0$) structure of the energy-momentum tensor from purely londitudinal fields

$$T^{\mu\nu}_{\mathrm{f}(0)} = \begin{pmatrix} \varepsilon_0 & & & \\ & \varepsilon_0 & & \\ & & \varepsilon_0 & \\ & & & -\varepsilon_0 \end{pmatrix}$$

$$\varepsilon_0 = \frac{1}{2}\left(E_0^2 + B_0^2\right)$$

Transverse pressure
$P_T = \varepsilon_0$

Longitudinal pressure
$P_L = -\varepsilon_0$

Energy Momentum Tensor

- Flow emerges from pressure at order τ^1:

$\vec{S} = \vec{E} \times \vec{B}$

$$T_f^{\mu\nu} = \begin{pmatrix} \varepsilon_0 + O(\tau^2) & \alpha^1 \cosh\eta + \beta^1 \sinh\eta & \alpha^2 \cosh\eta + \beta^2 \sinh\eta & O(\tau^2) \\ \alpha^1 \cosh\eta + \beta^1 \sinh\eta & \varepsilon_0 + O(\tau^2) & O(\tau^2) & \alpha^1 \sinh\eta + \beta^1 \cosh\eta \\ \alpha^2 \cosh\eta + \beta^2 \sinh\eta & O(\tau^2) & \varepsilon_0 + O(\tau^2) & \alpha^2 \sinh\eta + \beta^2 \cosh\eta \\ O(\tau^2) & \alpha^1 \sinh\eta + \beta^1 \cosh\eta & \alpha^2 \sinh\eta + \beta^2 \cosh\eta & -\varepsilon_0 + O(\tau^2) \end{pmatrix}$$

- Transverse Poynting vector gives transverse flow.

[RJF, J.I. Kapusta, Y. Li, (2006)]
[G. Chen, RJF, PLB 723 (2013)]

$$S^i_{\text{even}} = \frac{\tau}{2} \cosh\eta \left(E_0 \left[D^i, E_0 \right] + B_0 \left[D^i, B_0 \right] \right) = \alpha^i \cosh\eta$$

$$S^i_{\text{odd}} = \frac{\tau}{2} \sinh\eta \, \varepsilon^{ij} \left(E_0 \left[D^j, B_0 \right] - B_0 \left[D^j, E_0 \right] \right) = \beta^i \sinh\eta$$

$$\alpha^i = -\frac{\tau}{2} \nabla^i \varepsilon_0$$

Like hydrodynamic flow, determined by gradient of transverse pressure $P_T = \varepsilon_0$; even in rapidity.

$$\beta^i = \frac{\tau}{2} \varepsilon^{ij} \left(\left[D^j, B_0 \right] E_0 - \left[D^j, E_0 \right] B_0 \right)$$

Non-hydro like; odd in rapidity ??

[see talk by G. Chen on Saturday]

Energy Momentum Tensor

- Corrections to energy density and pressure at second order in time

- Example: energy density and pressure

Depletion/increase of energy density due to transverse flow

$$T^{00} = \varepsilon_0 - \frac{\tau^2}{8}\left[2\nabla^i\alpha^i + \sinh 2\eta\, \nabla^i\beta^i + (2-\cosh 2\eta)\delta\right] + O(\tau^4)$$

New term

$$T^{ii} = \varepsilon_0 - \frac{\tau^2}{8}\left[2\nabla^i\alpha^i + 2\delta + (-1)^i\omega\right] + O(\tau^4)$$

$$T^{33} = -\varepsilon_0 + \frac{\tau^2}{8}\left[2\nabla^i\alpha^i - \sinh 2\eta\, \nabla^i\beta^i + (2+\cosh 2\eta)\delta\right] + O(\tau^4)$$

- New terms:

$$\delta = \left(D^i E_0\right)\left(D^i E_0\right) + \left(D^i B_0\right)\left(D^i B_0\right)$$

$$\omega = \left(D^1 E_0\right)^2 - \left(D^2 E_0\right)^2 + \left(D^1 B_0\right)^2 - \left(D^2 B_0\right)^2$$

Modelling Color Charges

- So far color charge densities ρ_1, ρ_2 fixed.
- MV: Gaussian distribution around color-neutral average

$$\left\langle \rho_i^a(x) \right\rangle = 0$$

$$\left\langle \rho_i^a(x_1)\rho_j^b(x_2) \right\rangle = \frac{g^2}{N_c^2-1}\delta_{ij}\delta^{ab}\lambda_i(x_1^\mp)\delta(x_1^\mp - x_2^\mp)\delta^2(\mathbf{x}_{1T}-\mathbf{x}_{2T}) \qquad \mu_i = \int dx^\mp \lambda_i(x^\mp)$$

- Sample distribution to obtain event-by-event observables.
- Next: analytic calculation of expectation values (as function of average color charge densities μ_1, μ_2).

- Transverse flow comes from gradients in nuclear profiles.
- Original MV model $\mu = const.$
- Here: relaxed condition, μ constant on length scales $1/Q_s$, allow variations on larger length scales $1/m$ where $m \ll Q_s$.

$$\mu^2(\mathbf{x}_T) >> m^{-1}\left|\nabla^i \mu^2(\mathbf{x}_T)\right| >> m^{-2}\left|\nabla^i\nabla^j \mu^2(\mathbf{x}_T)\right|$$

[G. Chen, RJF, PLB 723 (2013)]
[G. Chen et al., in preparation]

Calculating Expectation Values

- Example:

$$\langle \delta \rangle = \left\langle \left(D^i E_0\right)\left(D^i E_0\right) + \left(D^i B_0\right)\left(D^i B_0\right) \right\rangle$$

$$= [color\#] \left\langle \left(D^i A_1^k\right)\left(D^i A_1^m\right) \right\rangle \left\langle A_2^l A_2^n \right\rangle + [color\#] (ig)^2 \left\langle A_1^i A_1^k A_1^i A_1^m \right\rangle \left\langle A_2^l A_2^n \right\rangle + \ldots$$

- Need to evaluate "higher twist" expectation values.

$$\left\langle \left(D^i A_1^k\right)\left(D^i A_1^m\right) \right\rangle, \left\langle A_1^k \left(D^2 A_1^m\right) \right\rangle, \ldots$$

[Fujii, Fukushima, Hidaka, PRC 79 (2009)]
[G. Chen et al., in preparation]

- Number of terms increases rapidly with order in time (~ 600 for τ^4).

Averaged Density and Flow

- Energy density ~ product of nuclear gluon distributions ~ product of color source densities

$$\varepsilon_0 = \frac{g^6 N_c \left(N_c^2 - 1\right)}{8\pi} \mu_1 \mu_2 \ln^2 \frac{Q^2}{m^2}$$

[T. Lappi, 2006]
[RJF, Kapusta, Li, 2006]
[Fujii, Fukushima, Hidaka, 2009]

- "Hydro" flow:

$$\alpha^i = -\tau \frac{g^6 N_c \left(N_c^2 - 1\right)}{64\pi^2} \nabla^i \left(\mu_1 \mu_2\right) \ln^2 \frac{Q^2}{m^2}$$

- "Odd" flow term:

[G. Chen, RJF, PLB 723 (2013)]
[G. Chen et al., in preparation]

$$\beta^i = -\tau \frac{g^6 N_c \left(N_c^2 - 1\right)}{64\pi^2} \left(\mu_2 \nabla^i \mu_1 - \mu_1 \nabla^i \mu_2\right) \ln^2 \frac{Q^2}{m^2}$$

  - With $E_0 = ig\left[A_1^i, A_2^i\right]$, $B_0 = ig\,\varepsilon^{ij}\left[A_1^i, A_2^j\right]$ we have $\left\langle E_0 \nabla^i B_0 \right\rangle = -\left\langle B_0 \nabla^i E_0 \right\rangle$

Higher Orders in Time

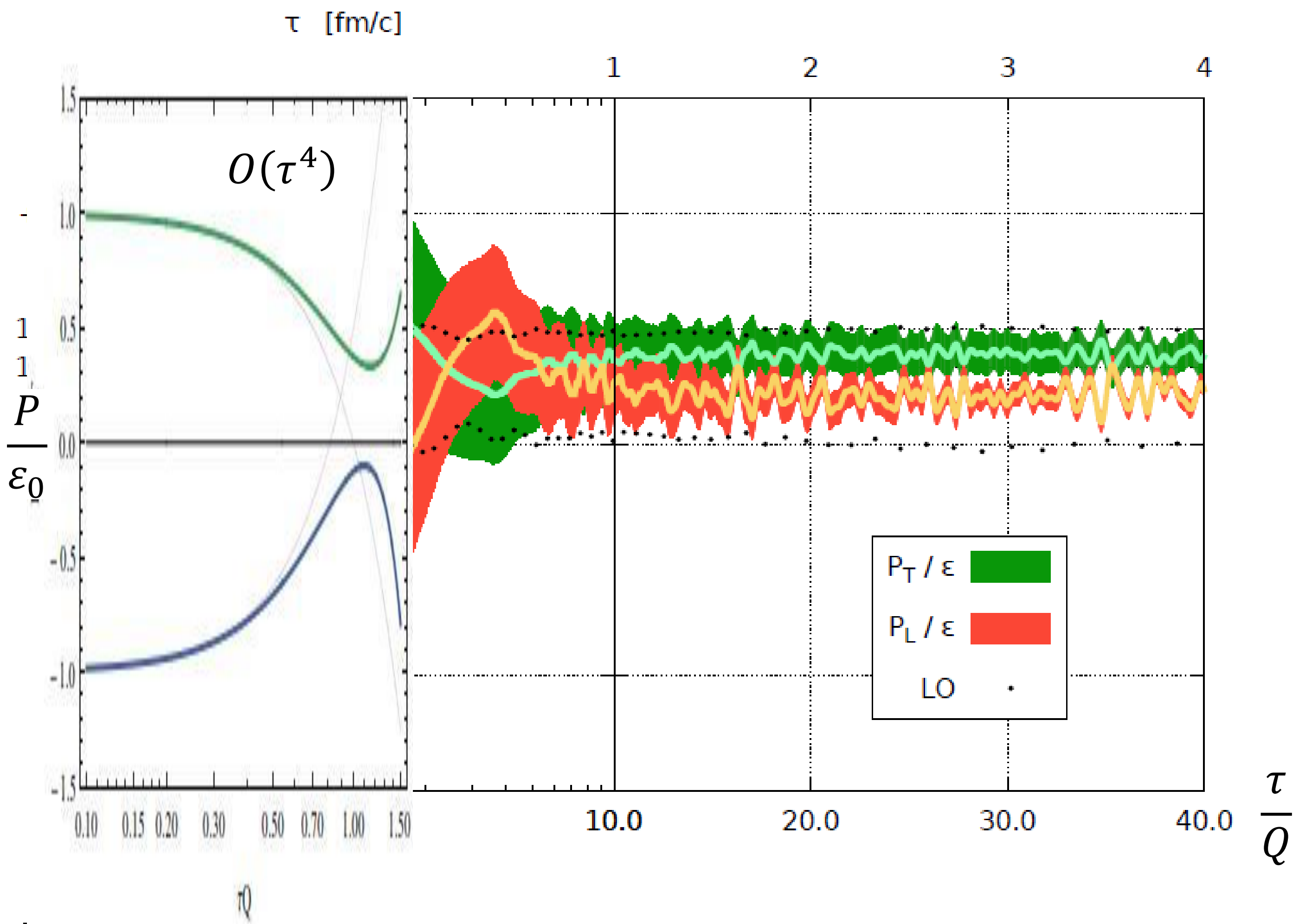
- Generally: powers of τ go with factors of Q or factors of transverse gradients:
  - $\sim\tau\mu^{-1}\nabla^i\mu$
  - $\sim\tau Q$
- Example for order τ^2 term: $\frac{\delta}{\varepsilon_0} = 4Q^2 \ln^{-1}\frac{Q^2}{m^2} + \left(\frac{\Delta\mu_1}{\mu_1} + \frac{\Delta\mu_2}{\mu_2}\right) + \ldots$
- Rather simple pocket formulas if gradients and logs are neglected:

$$\varepsilon = \varepsilon_0 - \frac{1}{2}(Q\tau)^2 + O(Q\tau)^4$$

$$\frac{P_L}{P_T} = -\frac{1-\frac{3}{2}(Q\tau)^2}{1-(Q\tau)^2} + O(Q\tau)^4$$

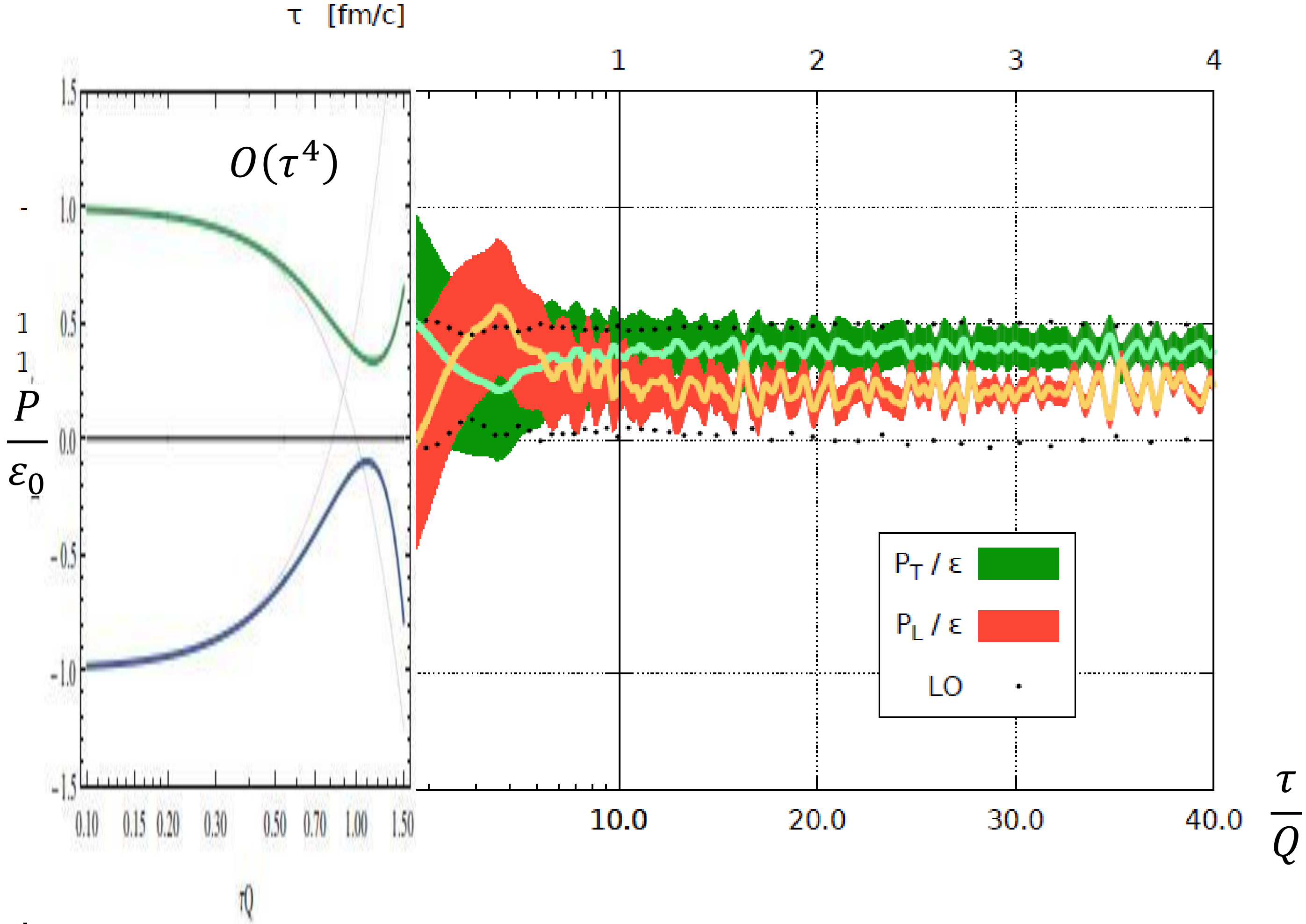
Comparison with Numerical Solutions

- CGC in a box [F. Gelis, T. Epelbaum, arxiv:1307.2214]



Comparison with Numerical Solutions

- CGC in a box [F. Gelis, T. Epelbaum, arxiv:1307.2214]
- 4[th] order in time: preliminary



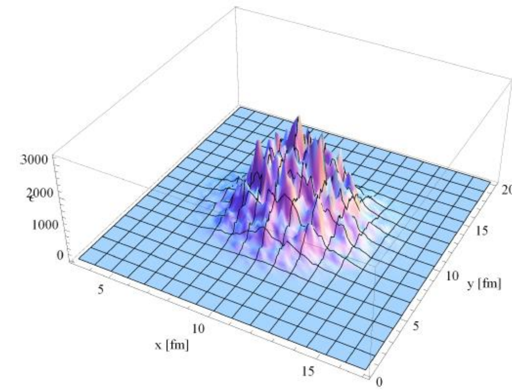
Comparison with Numerical Solutions

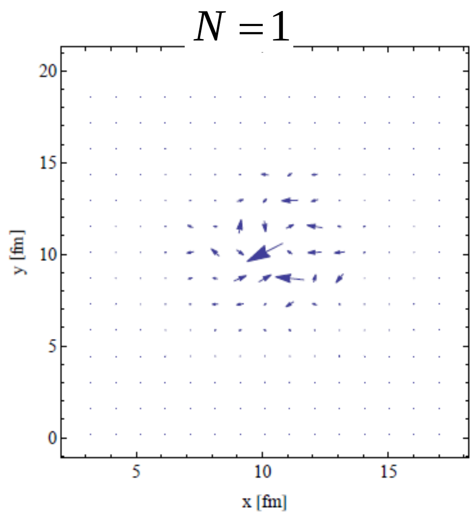
- Caveat: only a qualitative comparison at this point.
- Qualitative features for pressure and energy density up to 1/Q recovered.
- Classical theory sufficient up to 1/Q.

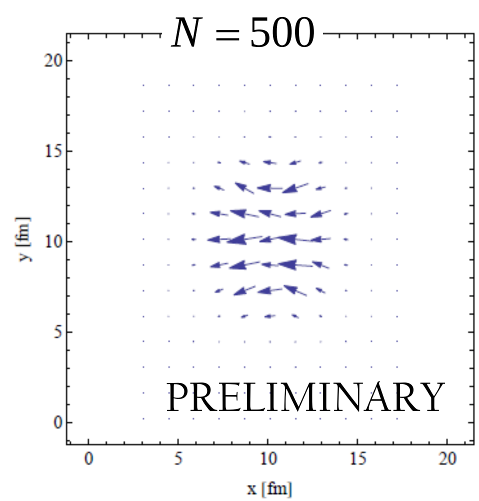
Can We Do It Event-By-Event?

- Yes, MC sampling of time-expansion coefficients.
- Example: “odd” vector β^i in Au+Au (b=4 fm).

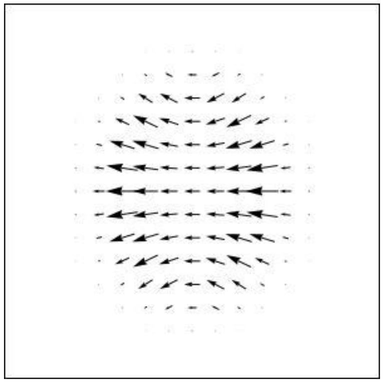
Energy density N=1







Analytic expectation value



- Averaging over events: recover analytic result.

Classical QCD Beyond Boost Invariance

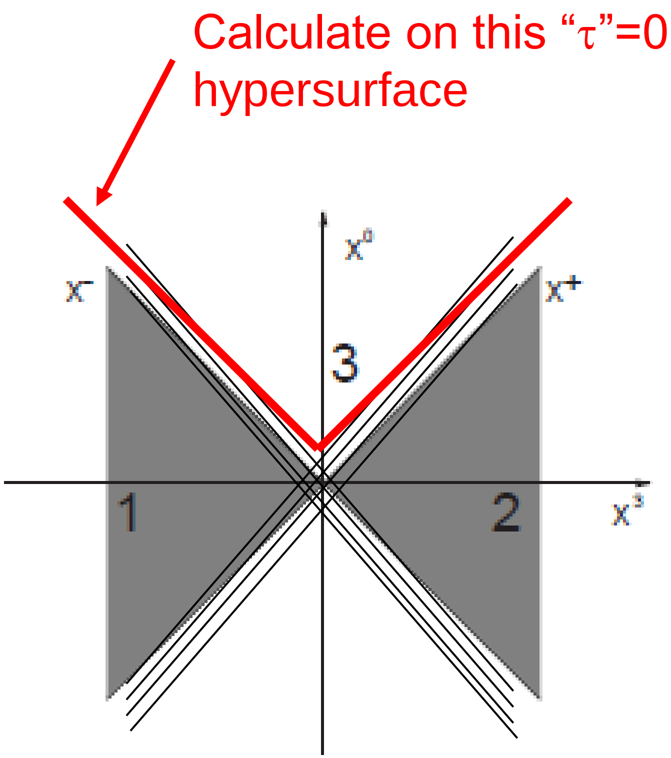
- Real nuclei are slightly off the light cone.
- Classical gluon distributions calculated by Lam and Mahlon.

[C.S. Lam, G. Mahlon, PRD 62 (2000)]

- Nuclear collisions off the light cone?

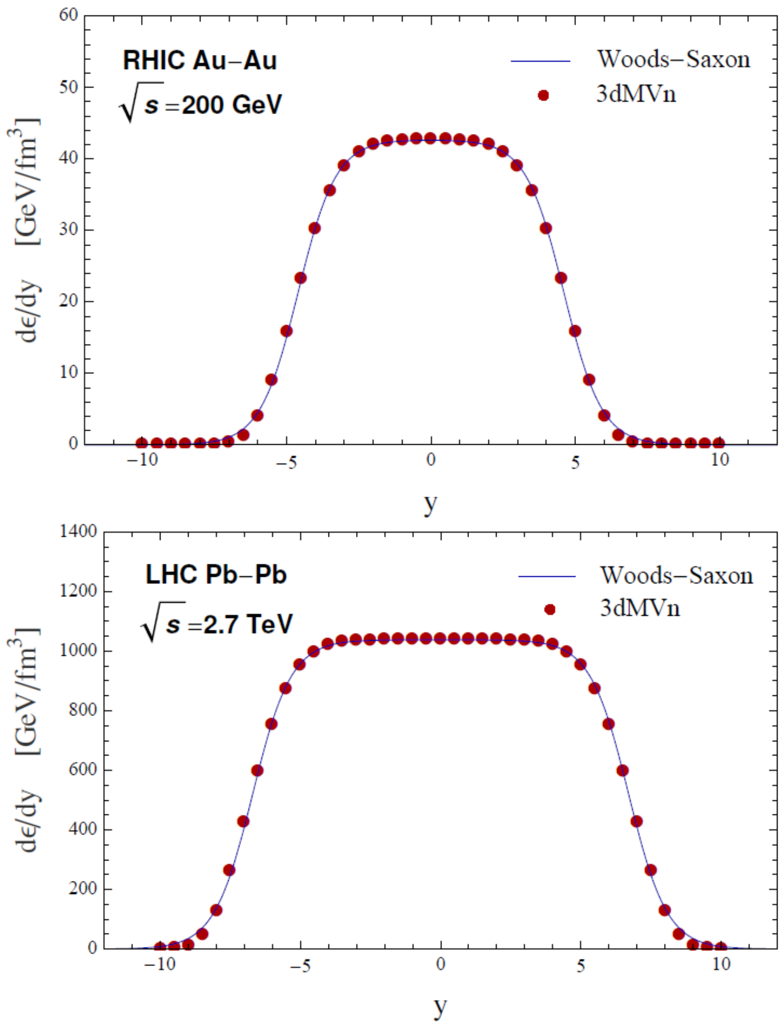
- Here: two simplifying assumptions when passing time $R_A/\gamma \ll$ internal time $1/Q_s$:
  - $d\varepsilon/d\tau = 0 \Rightarrow$ put a light-cone like hypersurface on which to "measure" the energy density just after nuclear overlap
  - New (transverse) field components Lorentz suppressed $\rightarrow$ only count longitudinal fields in initial energy density.

[S. Ozonder, RJF, PRC 89 (2014)]



Classical QCD Beyond Boost Invariance

- Strategy: use Lam-Mahlon gluon distributions in the "old" light cone formula for ε_0.
- Obtain rapidity profiles with plateau around the center (approximate boost-invariance), rapid fall-off towards beam rapidity.
- Well fitted by Woods-Saxon (see [S. Ozonder, RJF, PRC 89 (2014)] for parameters).



Summary

- Fields and energy momentum tensor for $\tau < 1/Q_s$ can be calculated analytically in the appropriate limit of QCD.
- Simple predictions for homogeneous nuclei.
- Transverse energy flow shows interesting and unique features: directed flow, A+B asymmetries, etc.
- First results for energy density at finite sqrt(s).

Transverse Flow: Visualization

- Transverse Poynting vector for randomly seeded A_1, A_2 fields (abelian case).
- $\eta = 0$: "Hydro-like" flow from large to small energy density
- $\eta \neq 0$: Quenching/amplification of flow due to the underlying field structure.

$(\text{background} = \varepsilon_0)$

$\eta = 0$
(no odd flow)

$\eta = 1$

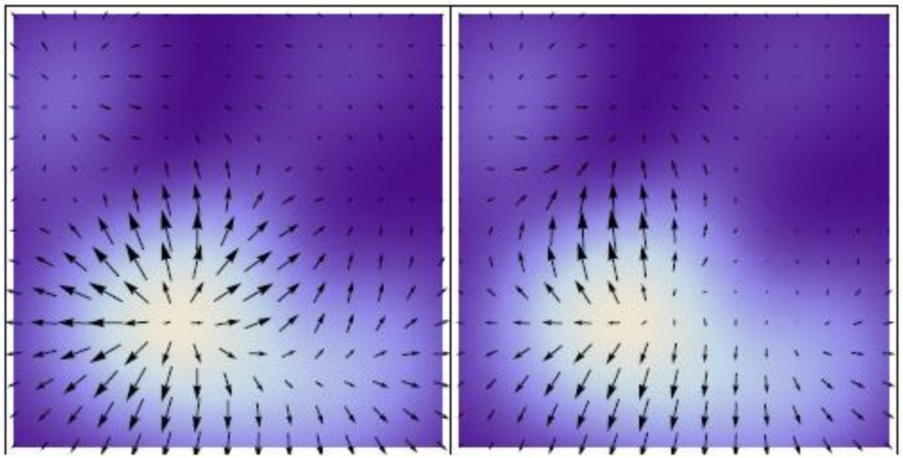


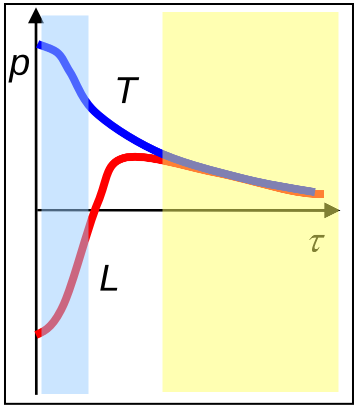
Figure 3: Example for transverse flow of energy for $\eta = 0$ (left panel) and $\eta = 1$ (right panel) in the abelian example for the same random distribution of fields A_1^i, A_2^i as in Fig. 2. The initial energy density T^{00} is shown through the density of the background (lighter color = larger values). At $\eta = 0$ the flow follows the gradient in the energy density in a hydro-like way while away from mid-rapidity energy flow gets quenched in some directions and amplified in others.

Matching to Hydrodynamics

- No equilibration here; see other talks at this workshop.
- Pragmatic solution: extrapolate from both sides ($r(\tau)$ = interpolating fct.)

$$T^{\mu\nu} = T_{\mathrm{f}}^{\mu\nu} r(\tau) + T_{\mathrm{pl}}^{\mu\nu}\left(1 - r(\tau)\right)$$

- Here: fast equilibration assumption: $r(\tau) = \Theta(\tau_0 - \tau)$



- Matching: enforce $\partial_\mu T^{\mu\nu} = 0$
(and other conservation laws).
- Analytic solution possible for matching to ideal hydro.
  - 4 equations + EOS to determine 5 fields in ideal hydro. Up to second order in time:

$$\vec{v}_\perp = \frac{1}{\cosh\eta}\frac{\vec{\alpha}}{\epsilon_0 - \frac{\tau_{th}^2}{8}(-2\triangle\epsilon_0 + \delta) + p},$$

$$v_L = \tanh\eta,$$

$$e + p = \left(\epsilon_0 - \frac{\tau_{th}^2}{8}(-2\triangle\epsilon_0 + \delta) + p\right)\left(1 - \frac{\vec{\alpha}^2}{(\epsilon_0 - \frac{\tau_{th}^2}{8}(-2\triangle\epsilon_0 + \delta) + p)^2}\right)$$

  - Odd flow β drops out: we are missing angular momentum!