

# QCD strings and equilibration in high multiplicity pA,pp collisions

Edward Shuryak  
Stony Brook

## **3. Early stages of high multiplicity pA collisions**

Tigran Kalaydzhyan and Edward Shuryak arXiv:1404.1888

## **2. Self-interacting QCD strings and String Balls**

Tigran Kalaydzhyan, Edward Shuryak. Feb 28, 2014.

e-Print: [arXiv:1402.7363](https://arxiv.org/abs/1402.7363)

## **1. New Regimes of Stringy (Holographic) Pomeron and High Multiplicity pp and pA Collisions**

Edward Shuryak, Ismail Zahed Nov 4, 2013. 24 pp.

e-Print: [arXiv:1311.0836](https://arxiv.org/abs/1311.0836)

# why strings?

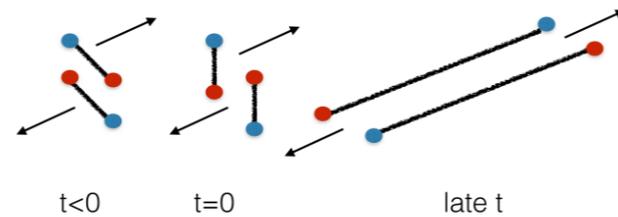
- using pQCD (quarks and gluons) it is very hard to explain the QCD phase transition to confinement
- using strings it is relatively easy: it is due to their exponentially growing density of states => the Hagedorn phenomenon near  $T_H$ :
- condensation of “thermal scalar” mode => deconfined phase
- string balls prepare small size and large entropy of black holes

# outline

- min.bias pp, pA: Pomerons, strings, spaghetti, Lund model
- QCD strings and their interaction
- spaghetti collapse
- (fundamental) string balls
- QCD string balls
- holographic Pomeron and its phases
- stringballs in high multiplicity pp

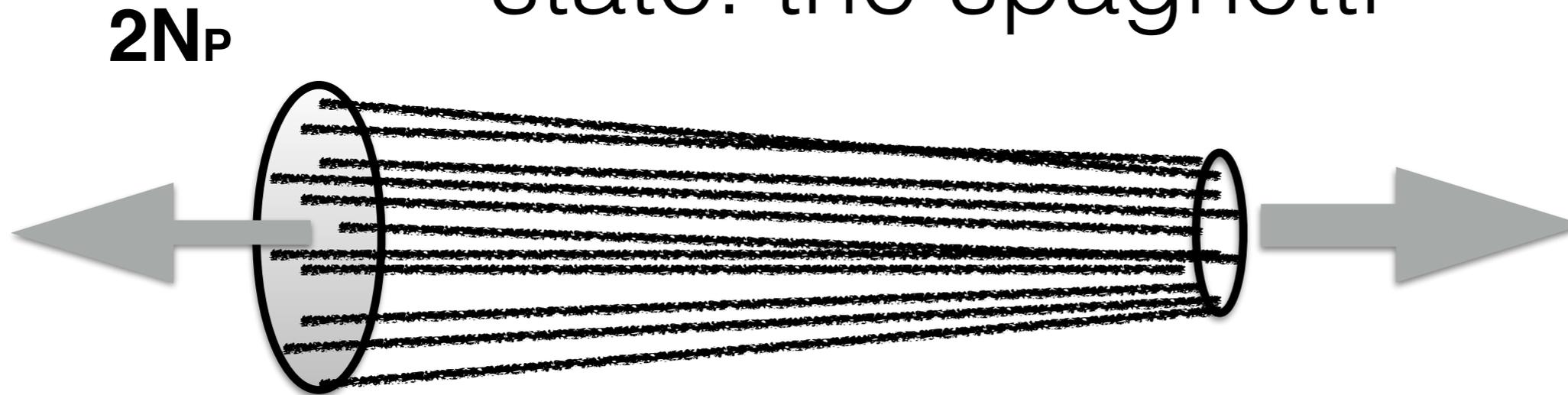
# brief history of QCD strings

- **1960's: Regge phenomenology, Veneziano amplitude. Strings have exponentially growing density of states  $N(E)$**
- **1970's Polyakov, Susskind  $\Rightarrow$  Hagedorn phenomenon near deconfinement**



- **1980's: Lund model (now Pythia, Hijing): string stretching and breaking**
- **1990-now lattice studies. Dual Abrikosov flux tubes. (Very few) papers on string interaction**
- **2013 Zahed et al: holographic Pomeron and its regimes**

the simplest multi-string  
state: the spaghetti



$$N(\text{strings})=2N(\text{Pomerons})$$

in small multiplicity bins strings are broken independently (the Lund model),

but **one should obviously think about their interaction if their number grows**

# intro into pA collisions

## Multiplicity distribution from CMS, pPb

maximal mean number of participants  
is along the Pb diameter, about 16

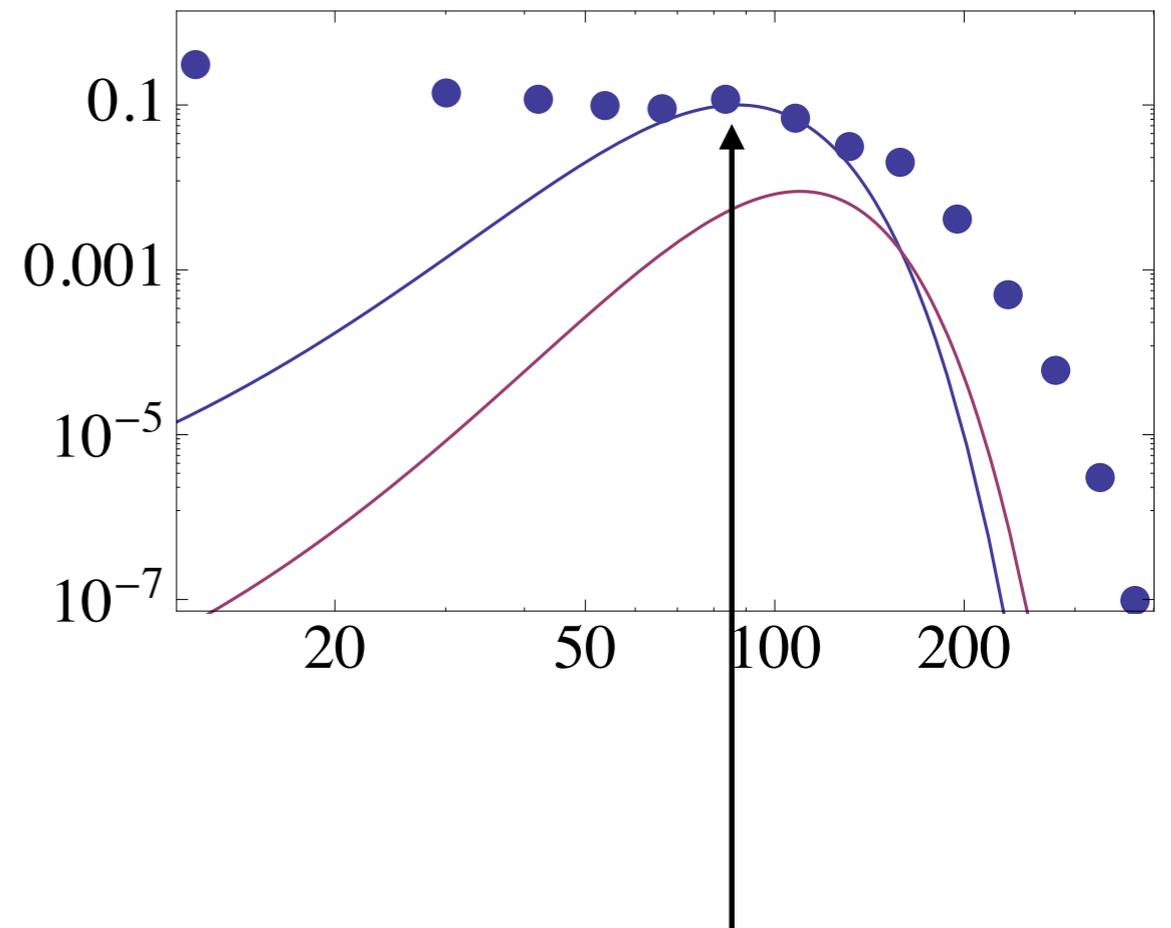
blue line is Poisson with  $\langle N_p \rangle = 16$

red with  $\langle N_p \rangle = 20$

geometry — columns with smaller  $N_p$  -  
explains well the left side (Bozek 2011)

what explains the large tail to the right?

not the “wounded nuclei model” in w  
 $N_{ch}/N_p = \text{const}$  (= independent string  
fragmentation, Lund model)



**are the two sides  
any different?**

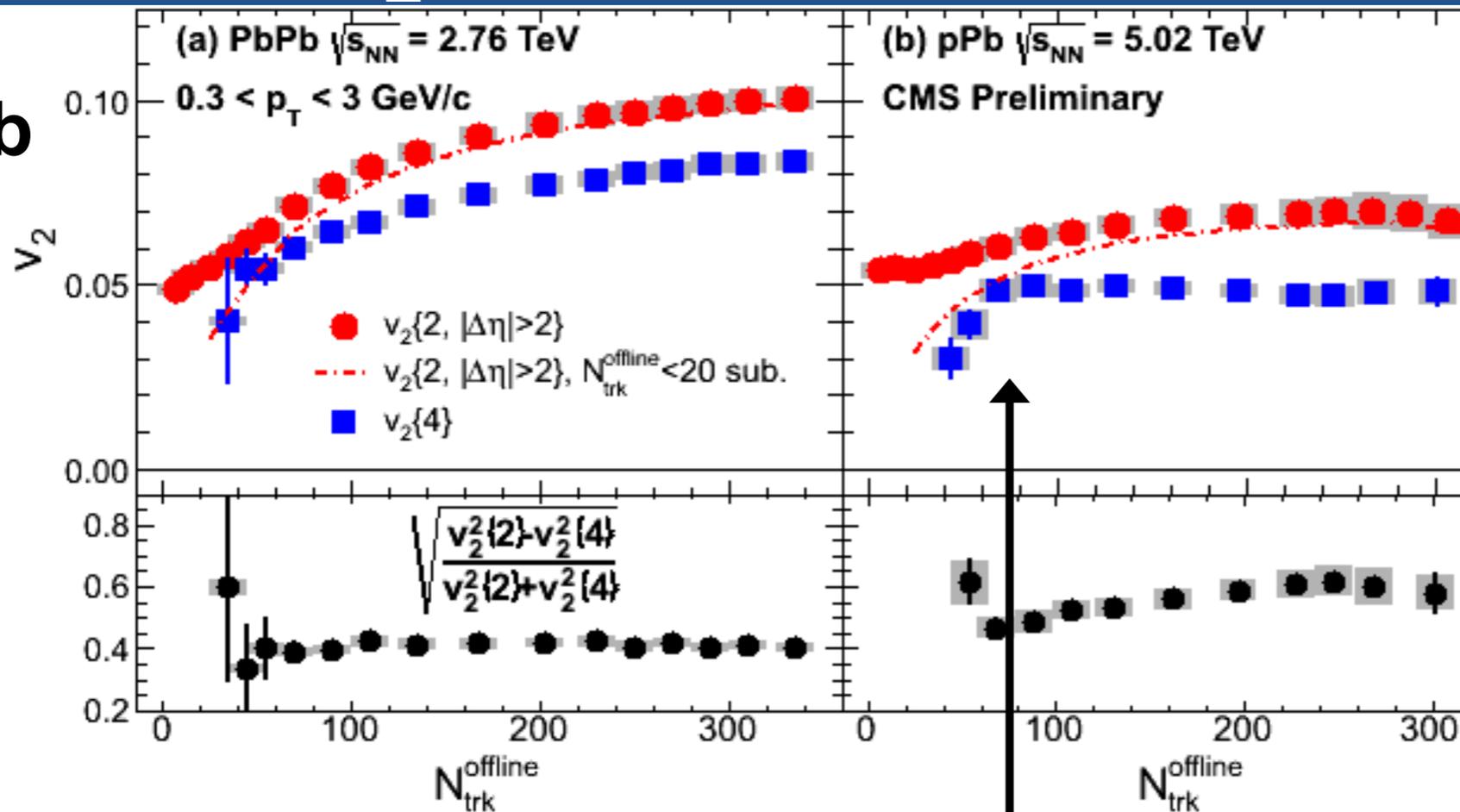
**one needs to explain extra multiplicity,  
and — more importantly — appearance of  
radial, elliptic and triangular flows**

# CMS pPb: $v_2$ from 2 and 4-particles

## $v_2$ in pPb and PbPb

PbPb

pPb



$v_2$  smaller in pPb than PbPb

$v_2\{4\}$  drops at low multiplicity

$$v_2\{2\} = \sqrt{\langle v_2 \rangle^2 + \sigma_{v_2}^2}$$

$$v_2\{4\} = \sqrt{\langle v_2 \rangle^2 - \sigma_{v_2}^2}$$

**Which is the real end of hydro!**

4 particle one is a clear sign of collectivity  
 it has clear onset at multiplicity of around 80

in AA fluctuations are too large

# High-multiplicity $pp$ and $pA$ collisions: Hydrodynamics at its edge

Edward Shuryak and Ismail Zahed

We predicted the radial flow in  $pp/pA$  to be **even stronger than in central AA**

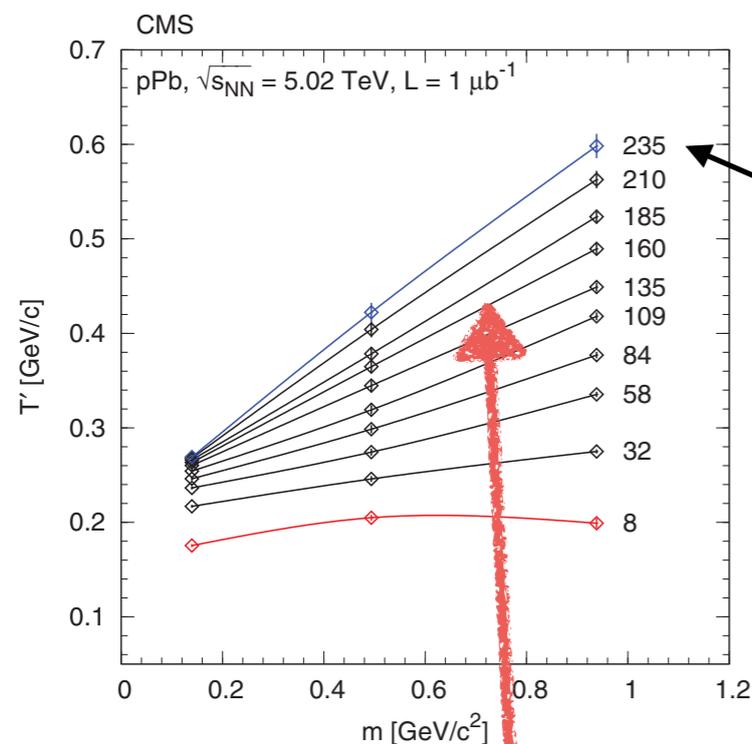


FIG. 8. (Color online) The slopes of the  $m_{\perp}$  distribution  $T'$  (GeV) as a function of the particle mass, from [13]. The numbers on the right

**Not the Mt scaling at large Ntr => not a large Qs but a collective flow:  $p=m v$**

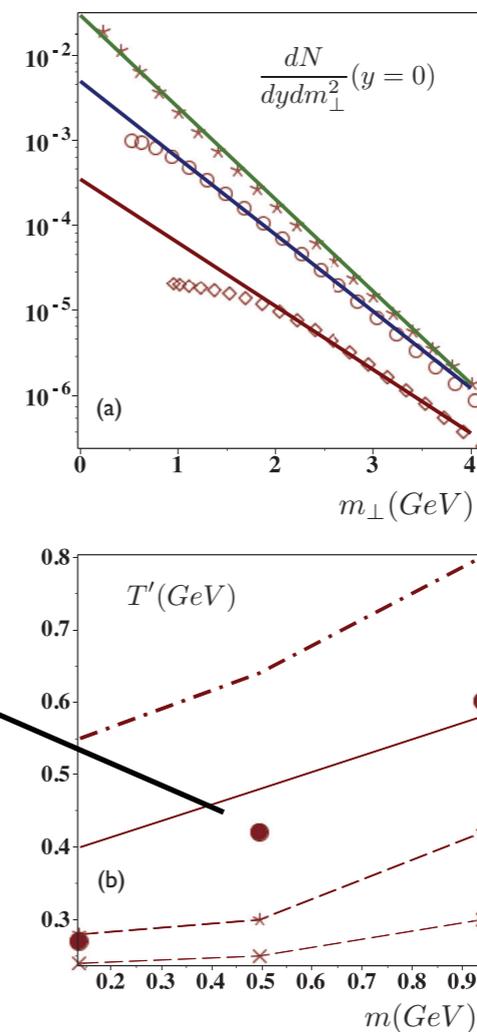


FIG. 9. (Color online) (a) A sample of spectra calculated for  $\pi$ ,  $K$ ,  $p$ , top-to-bottom, versus  $m_{\perp}$  (GeV), together with fitted exponents. (b) Comparison of the experimental slopes  $T'(m)$  versus the particle mass  $m$  (GeV). The solid circles are from the highest multiplicity bin data of Fig. 8, compared to the theoretical predictions. The solid and dash-dotted lines are our calculations for freeze-out temperatures  $T_f = 0.17, 0.12$  GeV, respectively. The asterisk-marked dashed lines are for Epos LHC model, diagonal crosses on the dashed line are for AMTP model.

# 1 flux tube on the lattice

**The dual superconductor:  
Higgs=monopole condensate**

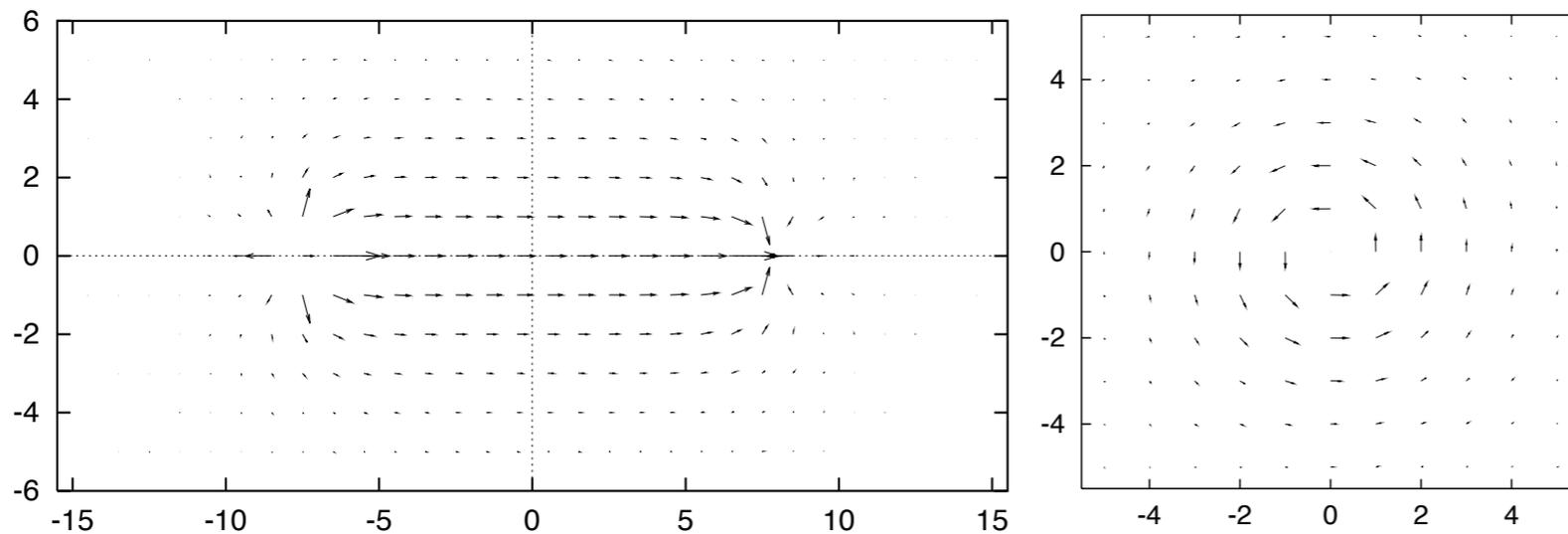
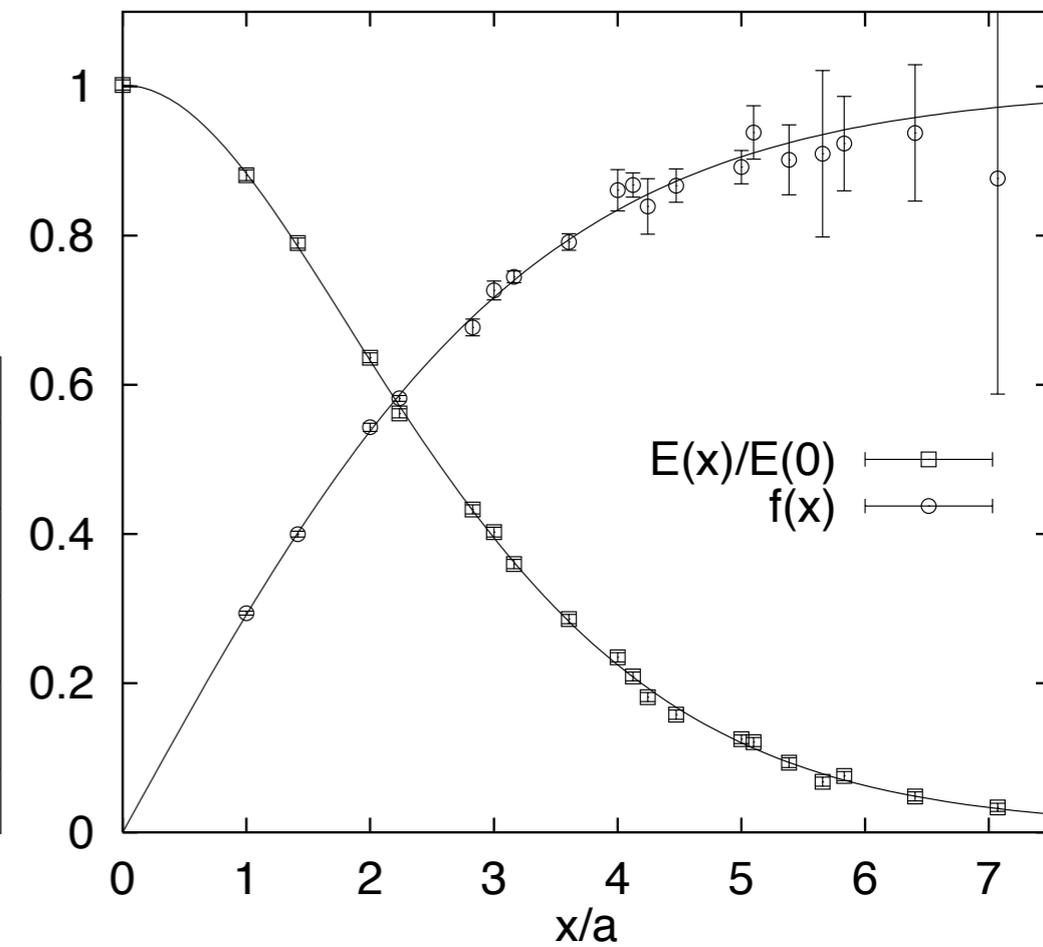
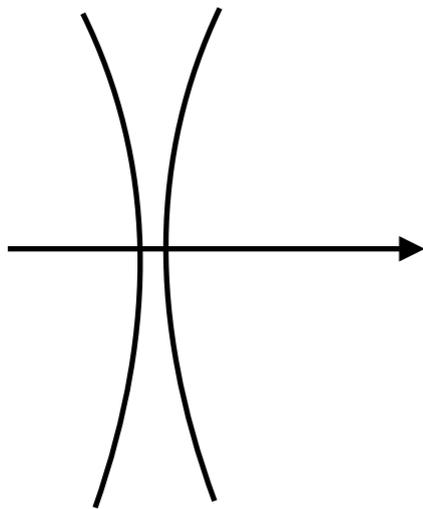


Fig. 7. Electric field  $\mathbf{E}$  and magnetic super current  $\mathbf{k}$  between two static sources.



**Interaction  
strongly grows  
near  $T_c$**



## 2 flux tubes on the lattice

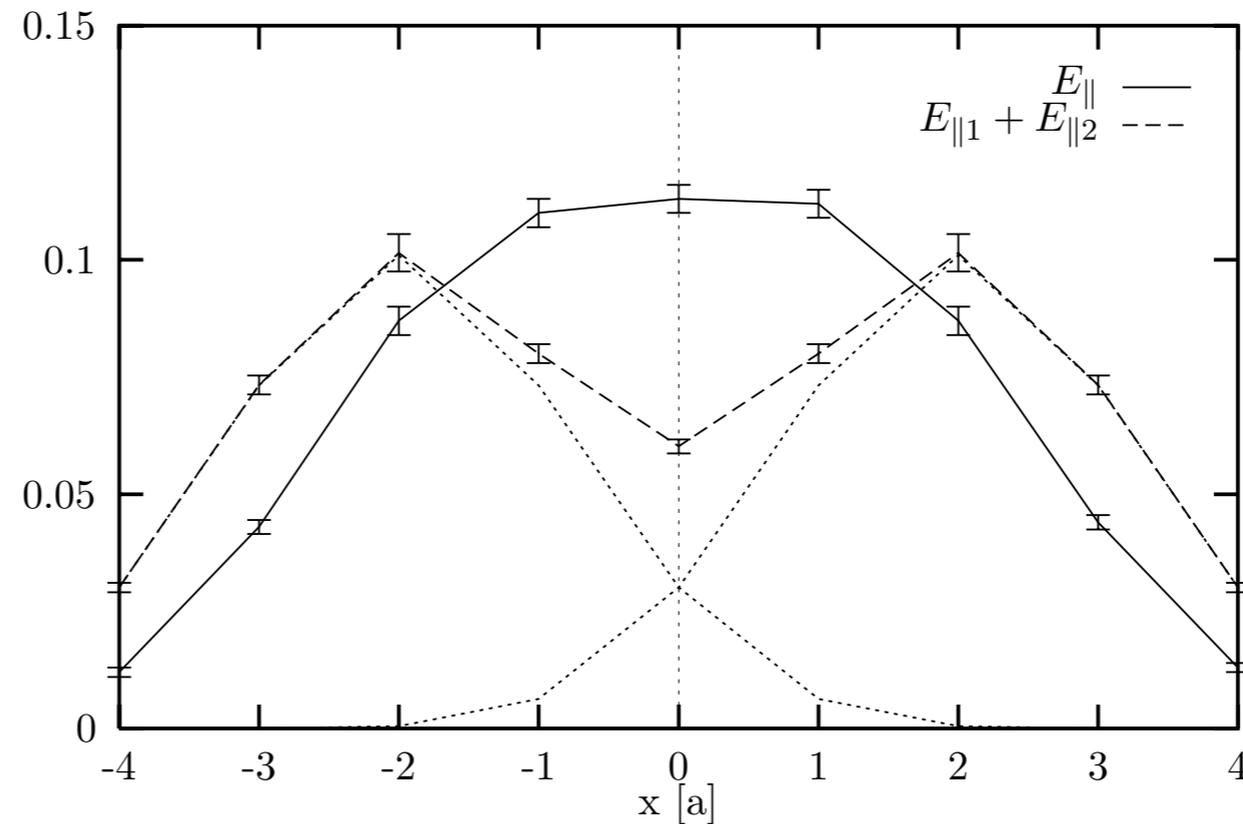


Figure 12: Longitudinal electric field profile of two interacting flux tubes in the symmetry plane ( $E_{\parallel}$ , solid line). The length of flux tubes is  $d = 22a$ , the transverse distance of equal charges is  $4a$ . For comparison, the dotted lines show the results for single flux tubes at  $x = -2a$  and  $x = +2a$ , and the dashed line corresponds to the superposition  $E_{\parallel 1} + E_{\parallel 2}$  of these two non-interacting flux tubes.

**M. Zach, M. Faber and P. Skala, Nucl. Phys.  
B 529, 505 (1998) [hep-lat/9709017].**

# string interaction via sigma meson exchange

our fit uses  
the sigma mass  
600 MeV

$$\frac{\langle \sigma(r_{\perp})W \rangle}{\langle W \rangle \langle \sigma \rangle} = 1 - CK_0(m_{\sigma} \tilde{r}_{\perp})$$

$$\tilde{r}_{\perp} = \sqrt{r_{\perp}^2 + s_{string}^2}$$

T. Iritani, G. Cossu and S. Hashimoto, arXiv:1311.0218

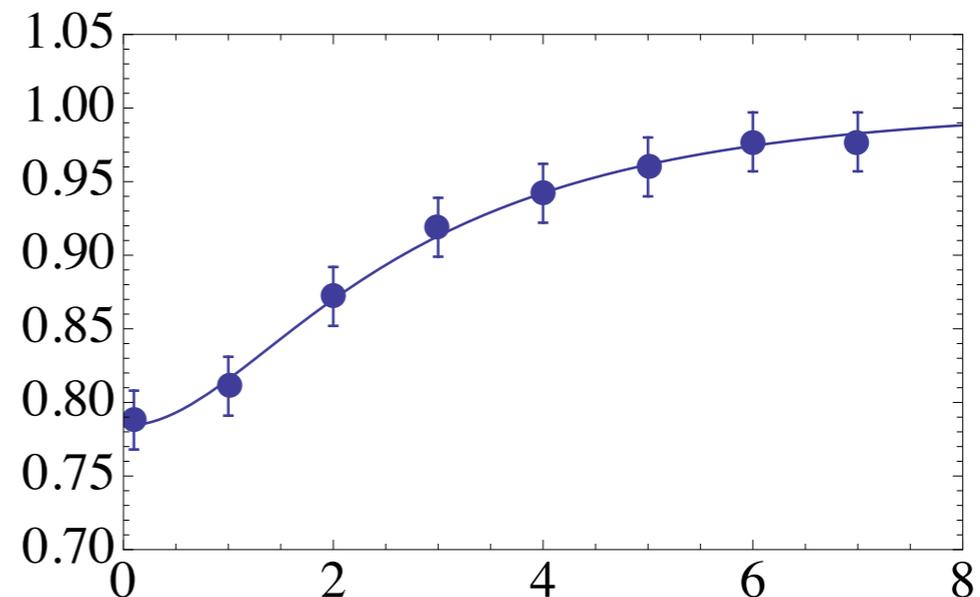


FIG. 2. (Color online). Points are lattice data from [12], the curve is expression (8) with  $C = 0.26$ ,  $s_{string} = 0.176$  fm.

**So the sigma cloud around a string is there!**

# 2d spaghetti collapse

Basically strings can be viewed as a 2-d gas of particles with unit mass and forces between them are given by the derivative of the energy (8) , and so

$$\ddot{\vec{r}}_i = \vec{f}_{ij} = \frac{\vec{r}_{ij}}{\tilde{r}_{ij}} (g_N \sigma_T) m_\sigma 2K_1(m_\sigma \tilde{r}_{ij}) \quad (19)$$

$$E_{tot} = \sum_i \frac{v_i^2}{2} - 2g_N \sigma_T \sum_{i>j} K_0(m_\sigma r_{ij})$$

with  $\vec{r}_{ij} = \vec{r}_i - \vec{r}_j$  and “regularized”  $\tilde{r}$  (9).

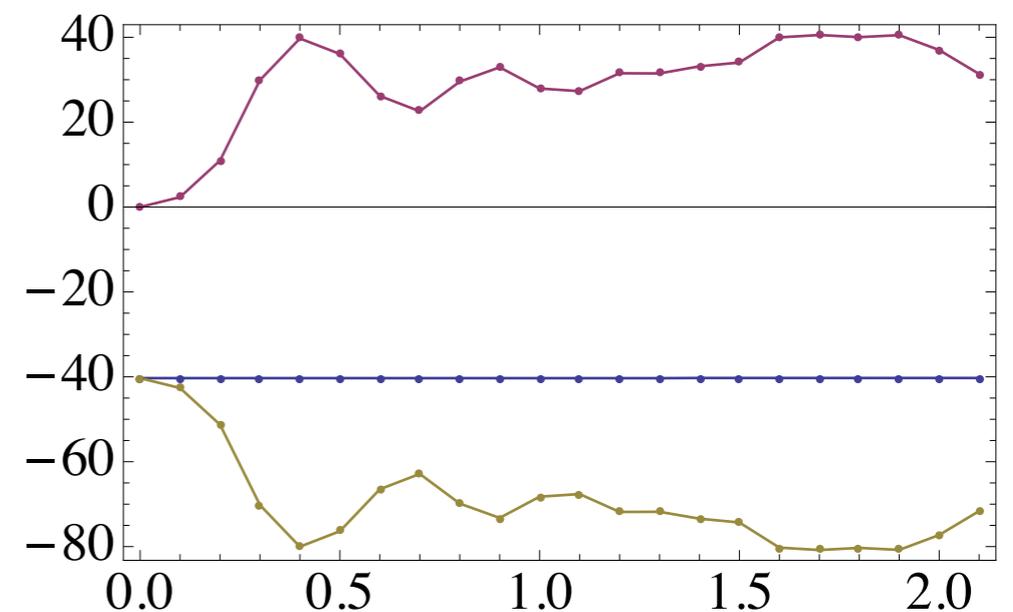
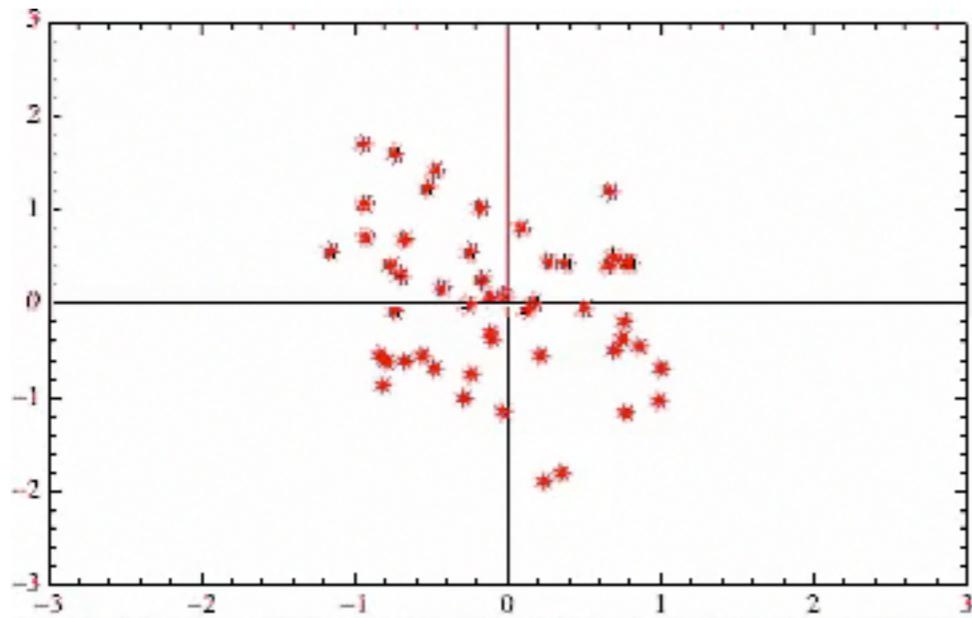
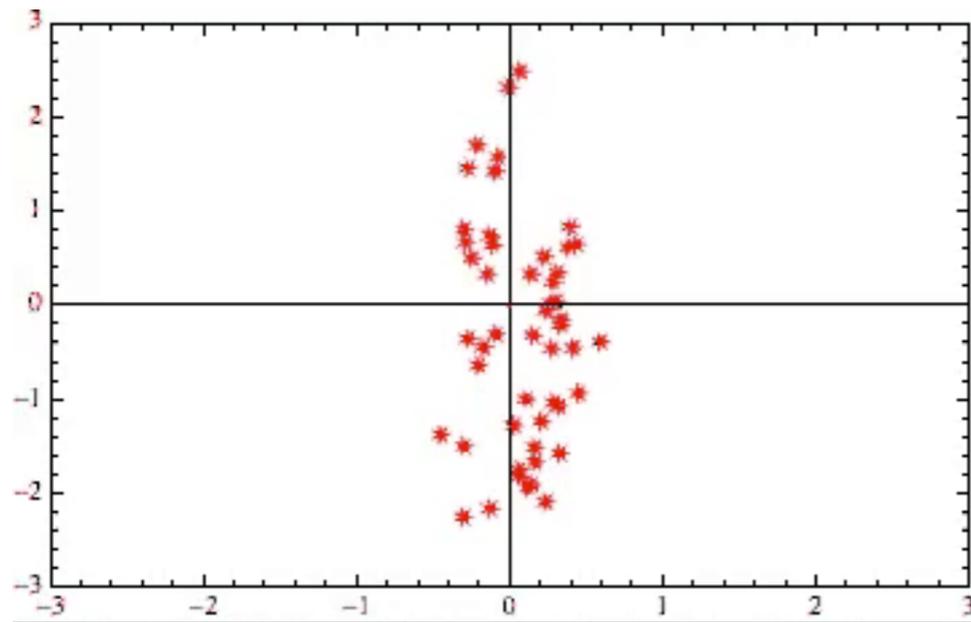
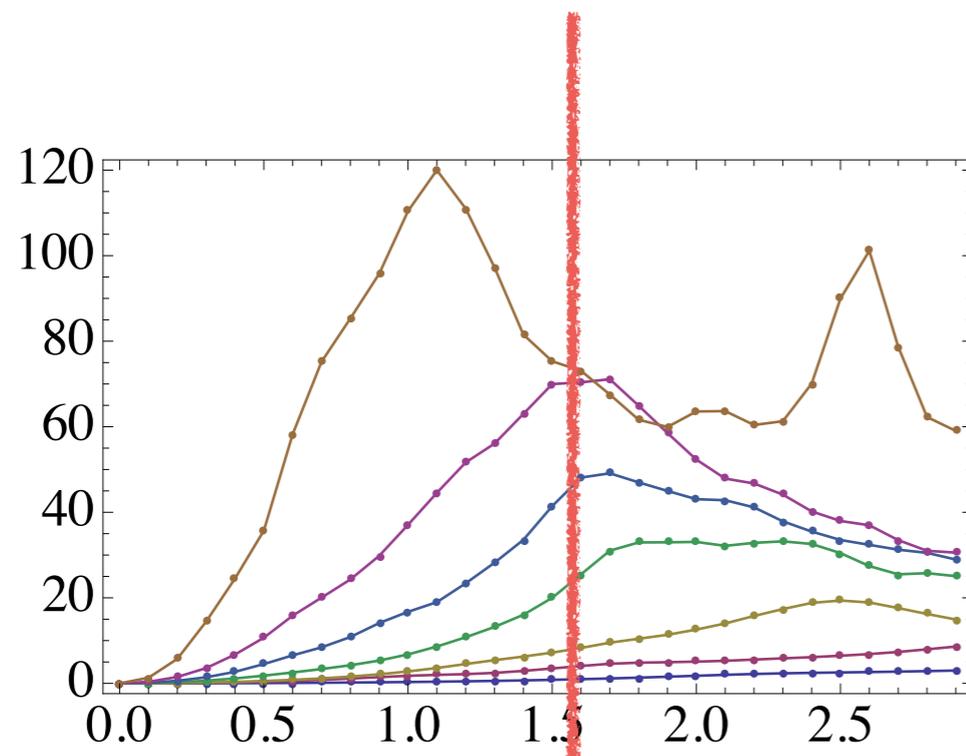


FIG. 4. (Color online) The (dimensionless) kinetic and potential energy of the system (upper and lower curves) for the same example as shown in Fig. 6, as a function of time  $t$ (fm). The horizontal line with dots is their sum, namely  $E_{tot}$ , which is conserved.



peripheral AA  
 contraction in x first  
 (and only: limited  
 time scale)



$g_N\sigma_T = 0.01, 0.02, 0.03, 0.05, 0.08, 0.10, 0.20.$

string stretching - about 1fm/c  
 1/4 period of yo-yo - another 0.5  
 so too small coupling does not work

# collective sigma field

before and after collapse

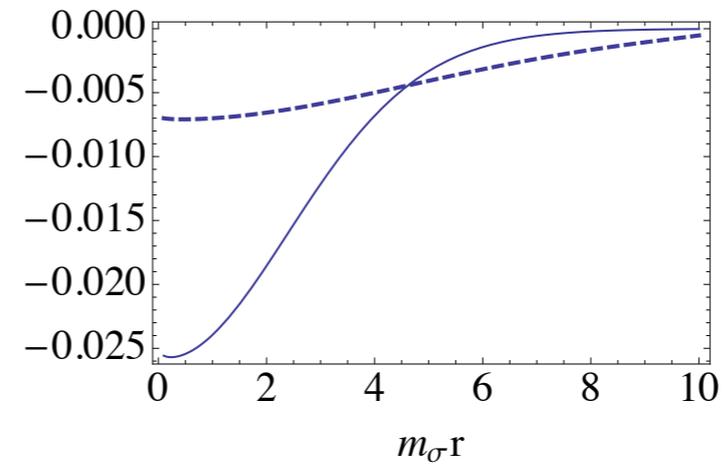
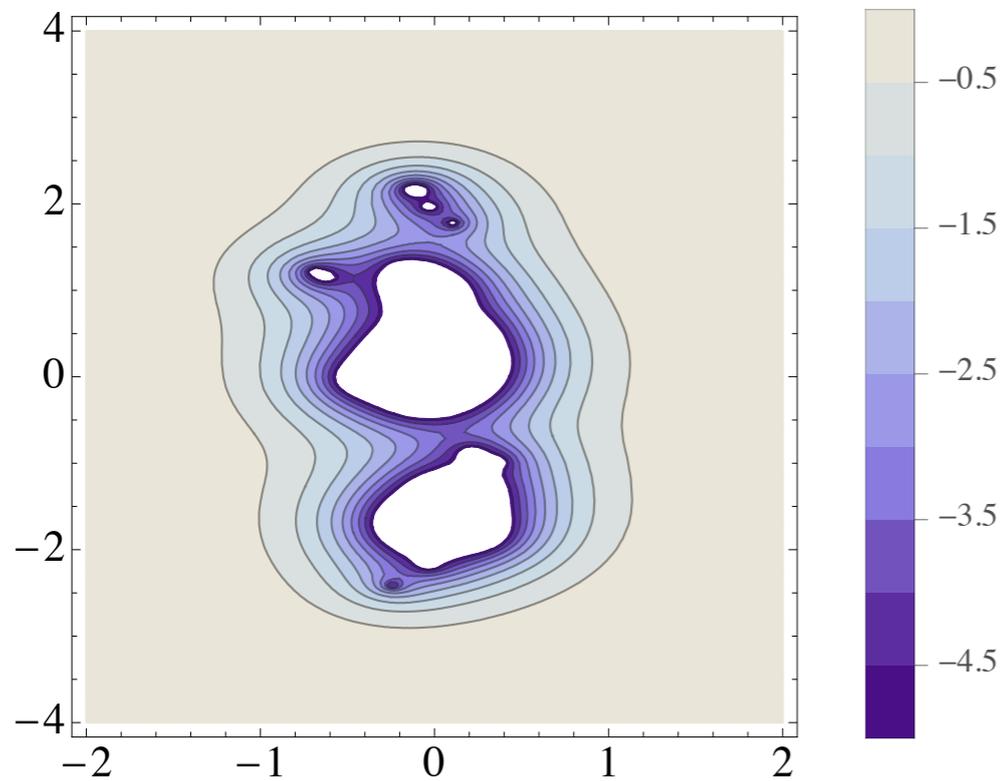


FIG. 4: The mean field (normalized as explained in the text) versus the transverse radius in units of inverse  $m_\sigma$ . The dashed and solid curves correspond to the source radii  $R = 1.5$  and  $0.7$  fm, respectively.

FIG. 10: Instantaneous collective potential in units  $2g_N\sigma_T$  for an AA configuration with  $b = 11$  fm,  $g_N\sigma_T = 0.2$ ,  $N_s = 50$  at the moment of time  $\tau = 1$  fm/ $c$ . White regions correspond to the chirally restored phase.

Field gradient at the edge  
leads to quark pair production:  
QCD analog of Hawking radiation

# SELF-INTERACTING STRINGS

The lightest  $JPC = 0^{++}$   $\sigma$  meson, or  $f_0(500)$  in the PDG13 listings. Its mass  $m_\sigma = 0.4 - 0.55$  GeV is comparable to its width  $\Gamma_\sigma = 0.4 - 0.7$  GeV:

**Sigma mass is expected to decrease near  $T_c$  as it meets its chiral partner, the pion**

It is basically responsible for binding nuclei

$$V_{NN}(r) = \frac{g_{\sigma NN}^2}{4\pi} \frac{\exp(-m_\sigma r)}{r}$$

$$g_N^{max} = \frac{g_{\sigma NN}^2}{4\pi m_N^2} \approx \frac{357}{4\pi} \frac{m_\sigma^2}{m_N^4} \approx 13 \text{ GeV}^{-2}$$

For non-nuclear physicists it may be worth reminding at this point that in  $NN$  case it is nearly completely cancelled by the repulsive vector  $\omega$  exchanges, coupled to the nucleon baryon number. We also remind that this sigma term can be found in phenomenological potentials such as Paris and Bonn ones, or the so-called Walecka model of nuclear forces. More recent treatment uses a more accurate “correlated  $\pi\pi$ ” exchange to account for it.

For non-string theorists it may be worth reminding that the fundamental strings and D-branes have also certain charges and repulsive vector forces, canceling attractive ones and making them “BPS-protected”. Our QCD string is not like that, it is just a bosonic string without charges, there are no traces of supersymmetry or BPS protection.

# fundamental string balls

A string ball can be naively generated by a “random walk” process, of  $M/M_s$  steps, where  $M_s \sim 1/\sqrt{\alpha'}$  is the typical mass of a straight string segment. If so, the string entropy scales as the number of segments

$$S_{ball} \sim M/M_s \quad (1)$$

$$\frac{R_{ball,r.w.}}{l_s} \sim \sqrt{M}$$

The Schwarzschild radius of a black hole in  $d$  spatial dimensions is

$$R_{BH} \sim (M)^{\frac{1}{(d-2)}} \quad (2)$$

and the Bekenstein entropy

$$S_{BH} \sim Area \sim M^{\frac{d-1}{d-2}} \quad (3)$$

Can be matched for one  $M$  only  $\Rightarrow$  critical string ball  
its Hawking  $T$  is the Hagedorn  $T_H$

Damour and Veneziano

entropy of a self-interacting string ball of radius  $R$  and mass  $M$ ,

$$S(M, R) \sim M \left(1 - \frac{1}{R^2}\right) \left(1 - \frac{R^2}{M^2}\right) \left(1 + \frac{g^2 M}{R^{d-2}}\right) \quad (5)$$

where all numerical constants are for brevity suppressed and all dimensional quantities are in string units given

**even for a very small  $g$ , the importance of the last term depends not on  $g$  but on  $g^2 M$ . So, very massive balls can be influenced by a very weak gravity (what, indeed, happens with planets and stars)**

# Our lattice model for string balls

$$Z \sim \int dL \exp \left[ \frac{L}{a} \ln(2d-1) - \frac{\sigma_T L}{T} \right], \quad (18)$$

and hence the Hagedorn divergence happens at

$$T_H = \frac{\sigma_T a}{\ln(2d-1)}. \quad (19)$$

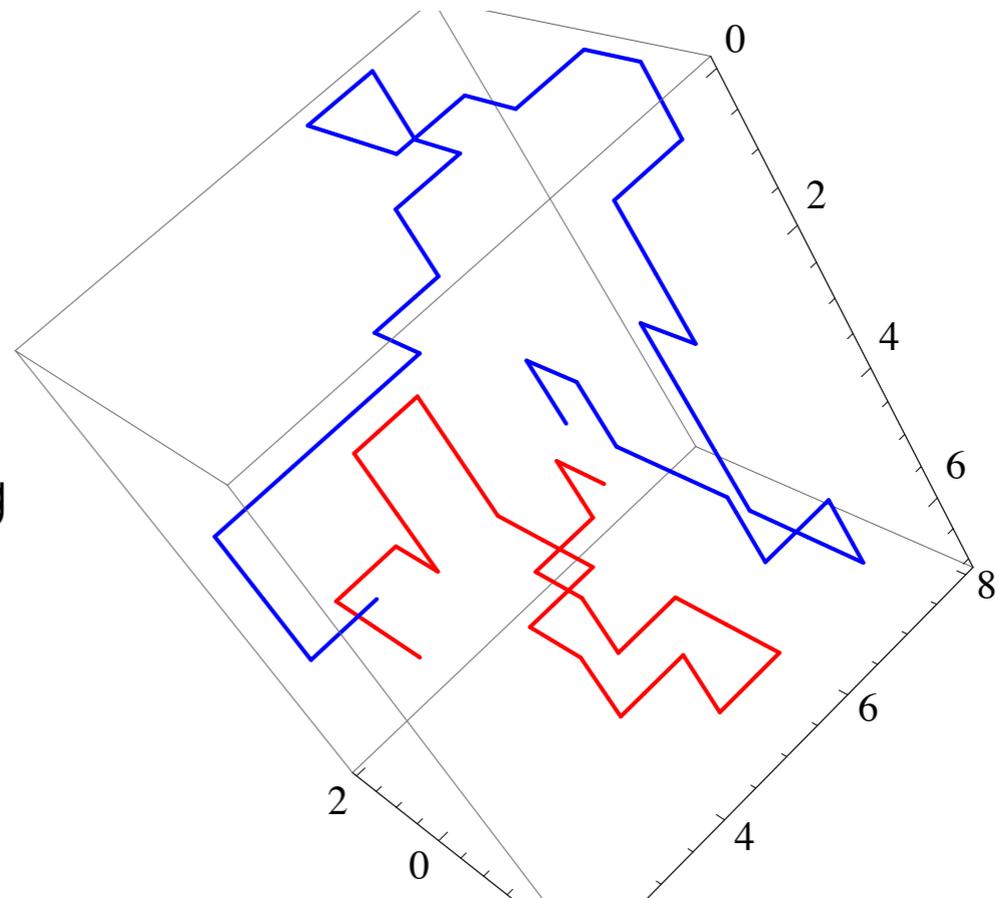
Setting  $T_H = 0.30 \text{ GeV}$ , according to the lattice data mentioned above and the string tension, we fix the 3-dimensional spacing to be

$$a_3 = 2.73 \text{ GeV}^{-1} \approx 0.54 \text{ fm}. \quad (20)$$

$$E_{\text{plaquette}} = 4\sigma_T a \approx 1.9 \text{ GeV}, \quad (21)$$

is amusingly in the ballpark of the lowest glueball masses of QCD. (For completeness: the lowest “meson” is one link or mass  $0.5 \text{ GeV}$ , and the lowest “baryon” is three links –  $1.5 \text{ GeV}$  of string energy – plus that of the “baryon junction”.)

Example of non-interacting strings



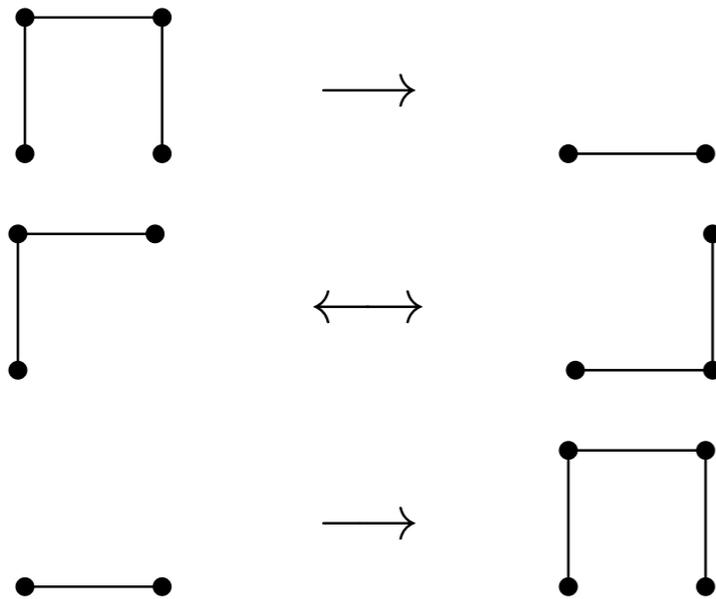
The most compact (volume-filling or Hamiltonian) string wrapping visits each site of the lattice. If the string is closed, then the number of occupied links is the same as the number of occupied sites. Since in  $d = 3$  each site is shared among 8 neighboring cubes, there is effectively only one occupied link per unit cube, and this wrapping produces the maximal energy density,

$$\frac{\epsilon_{max}}{T_c^4} = \frac{\sigma_T a}{a^3 T_c^4} \approx 4.4 \quad (22)$$

(we normalized it to a power of  $T_c$ , the highest temperature of the hadronic phase). It is instructive to compare it to the energy density of the gluonic plasma, for which we use the free Stefan-Boltzmann value

$$\frac{\epsilon_{gluons}}{T^4} = (N_c^2 - 1) \frac{\pi^2}{15} \approx 5.26 \quad (23)$$

# Self-interacting string balls



Metropolis algorithm, updates,  $T(x)$  instead of a box  
Yukawa self-interaction

we observe a new regime: the **entropy-rich self-balanced string balls** separated by 2 phase transitions

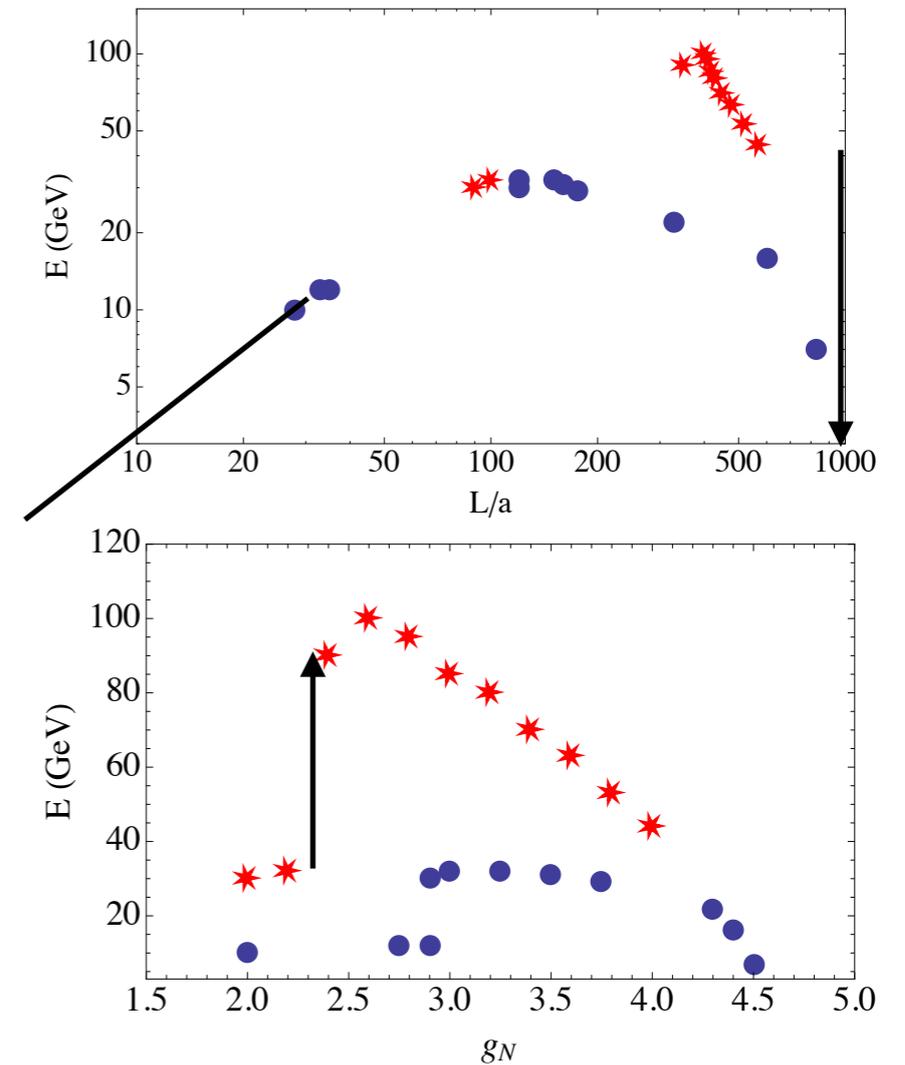


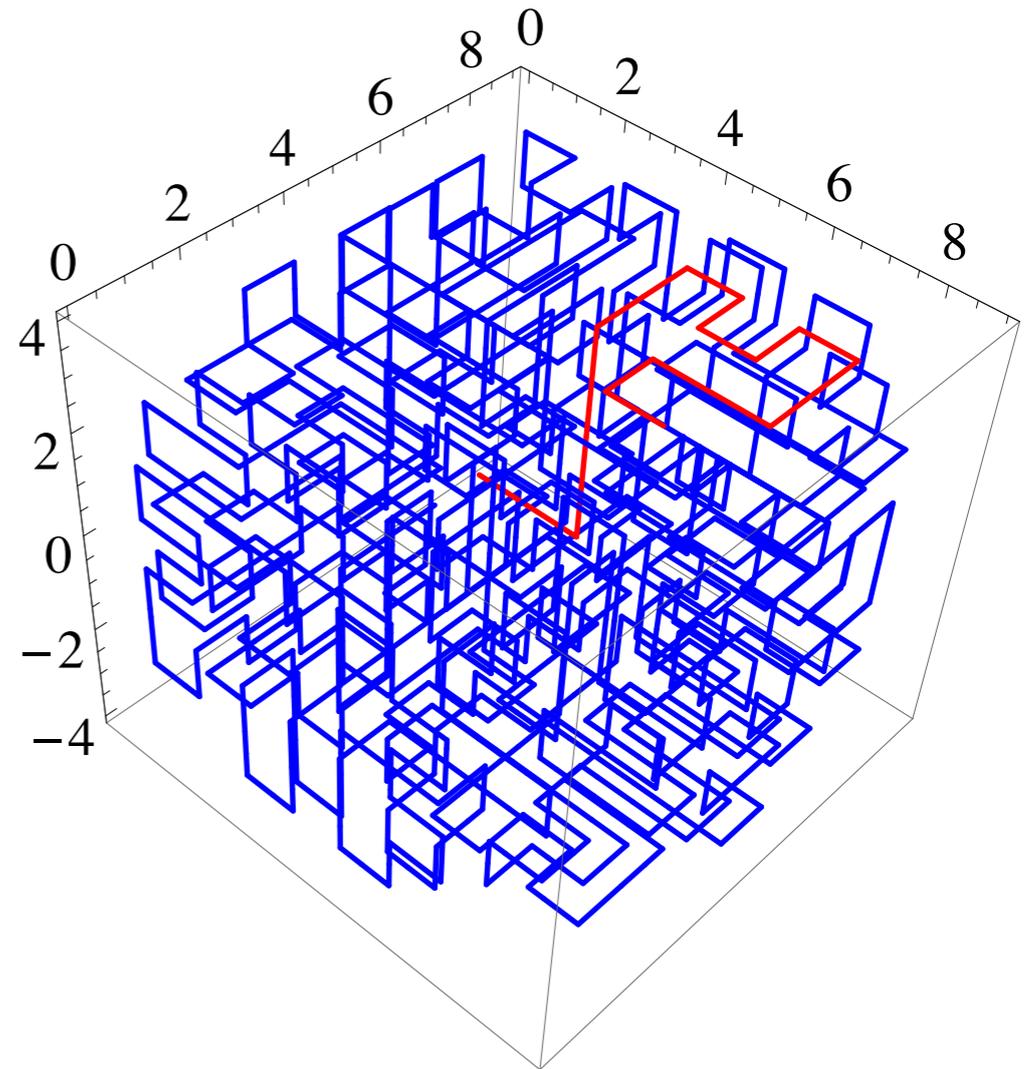
FIG. 7: Upper plot: The energy of the cluster  $E$  (GeV) versus the length of the string  $L/a$ . Lower plot: The energy of the cluster  $E$  (GeV) versus the “Newton coupling”  $g_N$  ( $\text{GeV}^{-2}$ ). Points show the results of the simulations in setting  $T_0 = 1 \text{ GeV}$  and size of the ball  $s_T = 1.5a, 2a$ , for circles and stars, respectively.

# extreme example

in spite of a very large string length

$L/a \sim 700$ , the total energy is only  $E \approx 17\text{GeV}$ ,

as a result of the balancing between  
the string tension and self- interaction.



$$\frac{\hat{q}}{s} \approx \text{const} \quad ?$$

## Jet quenching during the mixed phase

It has however been pointed out long ago [24] that large experimental values of  $v_2$  are difficult to explain by any simple model of quenching, in particular, they were in a strong contradiction with the simplest assumption (30). One possible solution to this puzzle has been suggested few years ago in Ref. [6]: the  $v_2$  data can be reproduced, if  $\hat{q}$  is significantly enhanced in the mixed phase. More

Here we want to point out that a natural explanation for the enhanced  $\hat{q}$  in the mixed phase can be provided by the strings. As far as we know, the “kicks” induced by the color electric field inside the QCD strings has been ignored in all jet quenching phenomenology: only the fields of “charges” (quarks and gluons in QGP, hadrons alternatively) were included. in the spherical Debye approxima-

$$\hat{q} = \frac{d\langle p_{\perp}^2 \rangle}{dl}, \quad \langle p_{\perp}^2 \rangle \approx (gEr_s)^2 ;$$

$$E(x) = \frac{\Phi_e}{2\pi r_s^2} K_0(x/r_s)$$

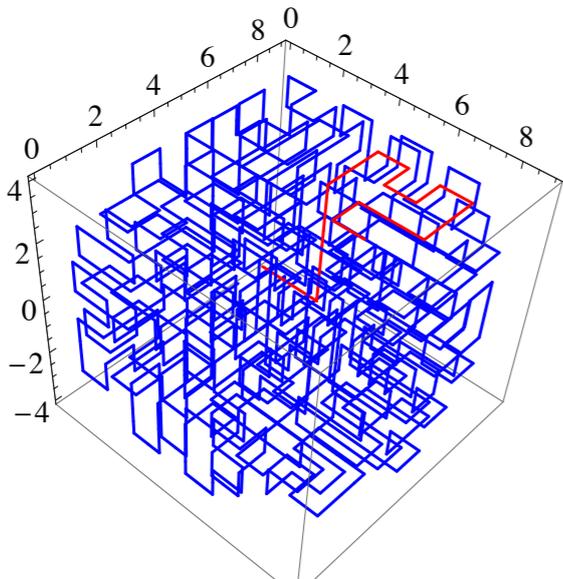
the string radius  $r_s = 1/(1.3\text{GeV}) = 0.15 \text{ fm}$ .

$$\hat{q} \approx \frac{16}{3} \alpha_s \sigma_T \frac{\bar{L} r_s}{\text{fm}^3} .$$

string length inside

1 fm<sup>3</sup>

$$\hat{q}_{min} = 0.028, \quad \hat{q}_{max} = 0.10 \left( \frac{\text{GeV}^2}{\text{fm}} \right) .$$



across the mixed phase, to be compared with values by the Jet coll. at  $T_c$   $\hat{q}_{min} = 0.025, \quad \hat{q}_{max} = 0.15 \left( \frac{\text{GeV}^2}{\text{fm}} \right)$

**But in high entropy self-supporting balls it can be up to one order of magnitude larger!**

# Two historic views on the Pomeron

$$\frac{d\sigma}{dt} \sim s^{\alpha(t)}$$

*Prehistoric: Regge, Pomeranchuk, Gribov*

$$\alpha(t) = \alpha(0) + \alpha' t + \dots$$

intercept + string scale

1960's: Veneziano  
dual resonance  
amplitude =>  
appearance of **strings**

1970's QCD  
Gribov, Lipatov =>  
gluon ladders =>  
**BFKL**

So, do we have **two different Pomerons**,  
**soft (strongly coupled) and hard (weakly coupled)?**

# Holographic Pomeron based on AdS/QCD

- the answer to the question is No:
- **Only one Pomeron** because the gauge description on the boundary is **dual to** string description in the bulk. Weak and strong coupling are its limits
- Concrete model of this type has been worked out by Zahed et al (Stoffers, Basar, Kharzeev): I will call it **Z+**
- **Our main statement:** as a function of  $b$  there are three distinct regimes: **subcritical**, near-critical and **supercritical!**

# the “tube”

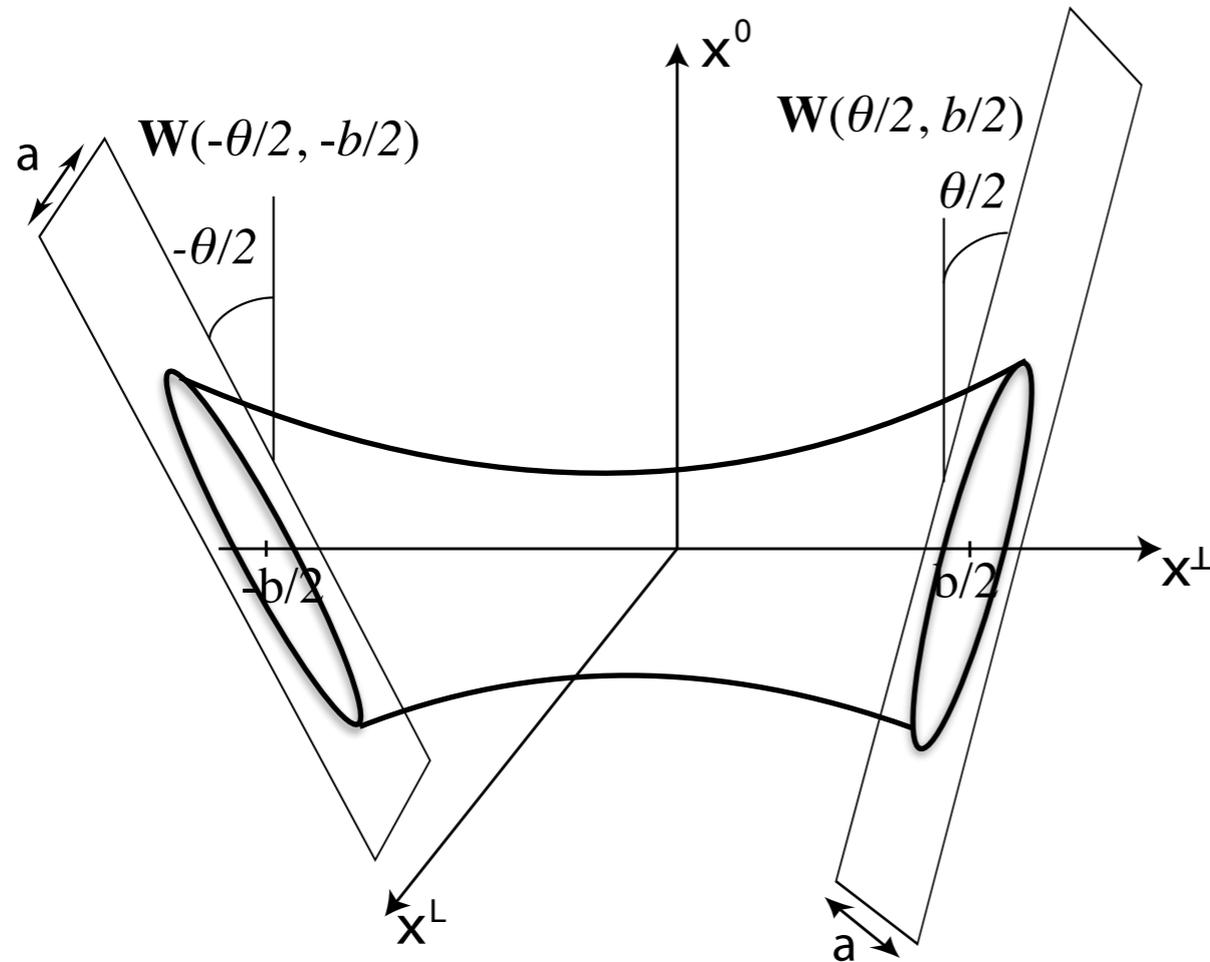


FIG. 1: Dipole-dipole scattering configuration in Euclidean space. The dipoles have size  $a$  and are  $b$  apart. The dipoles are tilted by  $\pm\theta/2$  (Euclidean rapidity) in the longitudinal  $x_0x_L$  plane.

- **If cut horizontally, it describes production of a pair of open strings**
- **If cut vertically, it describes an exchange by a closed string**
- **string fluctuations are included mode-by-mode**

$$\frac{1}{-2is} \mathcal{T}(s, t) \approx \frac{\pi^2 g_s^2 a^2}{2} \sum_{k=1}^{k_{\max}} \sum_{n=0}^{\infty} \frac{(-1)^k}{k} \left( \frac{k\pi}{\ln s} \right)^{D_{\perp}/2-1} \times d(n) s^{-2n/k + D_{\perp}/12k + \alpha' t/2k}, \quad (70)$$

**$k=1$  in SU(3),  $n$  is excitation**

The previous literature focuses on what we call the “cold” regime of the string

$$\mathbf{b} \gg \beta \gg \tilde{\beta}_H \quad (17)$$

where the former inequality follows from large collision energy (14) and the latter implies that the string is nearly straight, with small effective excitations (small effective  $T$ ). The meaning of

We will now review the Pomeron results in this setting. The amplitude of the elastic dipole-dipole scattering reads [2–4]

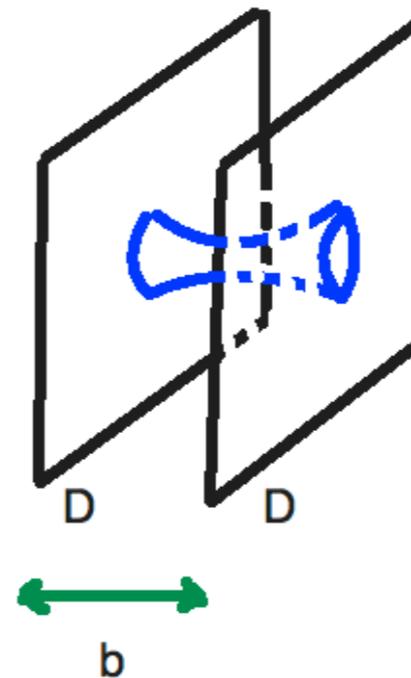
$$\frac{1}{-2is} \mathcal{T}(s, t; k) \approx g_s^2 \int d^2 \mathbf{b} e^{iq \cdot \mathbf{b}} \mathbf{K}_T(\beta, \mathbf{b}; k) \quad (15)$$

$$\mathbf{K}_T(\beta, \mathbf{b}; 1) = \left( \frac{\beta}{4\pi^2 \mathbf{b}} \right)^{D_\perp/2} \times e^{-\sigma \beta \mathbf{b} (1 - (\tilde{\beta}_H/\beta)^2/2)} \times \sum_{n=0.. \infty} d(n) \exp(-2\chi n)$$

**Linear Regge trajectories,  
daughters shifted by 2 down**

$$\beta = \frac{1}{T} = \frac{2\pi \mathbf{b}}{\chi}$$

$$\chi = \log(s)$$



As we mentioned, the expression (18) has been derived in [4] from the semiclassical approach to a Polyakov string, but (to leading order in  $1/\lambda$ ) it can alternatively be derived from a diffusion equation

$$(\partial_\chi + \mathbf{D}_k (\mathbf{M}_0^2 - \nabla_{\mathbf{b}}^2)) \mathbf{K}_T = 0 \quad (20)$$

where the rapidity  $\chi$  interval is the time and the diffusion happens in the (curved) transverse space with the diffusion constant  $\mathbf{D}_k = \alpha'/2k = l_s^2/k$ . This diffusion (20) is nothing else but the Gribov diffusion of the Pomeron, leading on average to an impact parameter  $\langle \mathbf{b}^2 \rangle = \mathbf{D}_k \chi$  for close Pomeron strings. If the “mother dipoles”

**connection to  
Gribov diffusion**

# fluctuations of flat membrane and a tube are different

- The ( $T=0$ ) potential  $\langle W \rangle$  is linear at large  $b$  and Coulombic (pQCD) at small  $b$  : yet **transition is smooth**
- **The tube has a periodic variable  $\Rightarrow$  quantization formally the same as the (Matsubara) thermal formalism  $\Rightarrow$  thus we expect **thermal-like behavior with a nontrivial transition to pQCD regime****

# New Regimes of Stringy (Holographic) Pomeron

1. A “cold” regime, with low string excitations;
2. A “near-critical” or “HPS regime”, in which strings indefinitely increase their energy and entropy, but not their free energy / pressure;
3. An “explosive regime”, in which the string occupies large portion of space and generates sufficient pressure for hydrodynamical explosion.

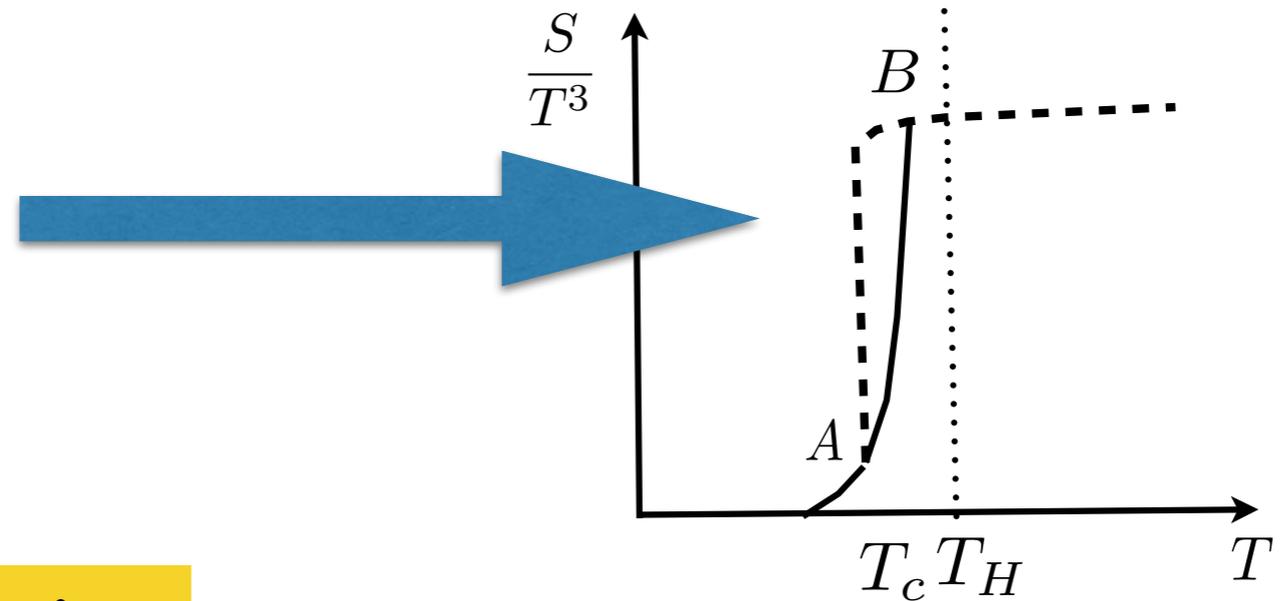


FIG. 3: (Color on-line) Schematic temperature dependence of the entropy density. The dashed line represents equilibrium gluodynamics with a first order transition at  $T = T_c$ . The solid line between points A and B represents the expected behavior of a single string approaching its Hagedorn temperature  $T_H$ .

## Hagedorn-Polyakov-Susskind regime

stringy excitations grows with an excitation energy *exponentially*. To see this, imagine a  $d$ -dimensional lattice with spacing  $a$  and draw all possible strings of length  $L/a$  making all possible turns (except going backward) at each site, that is

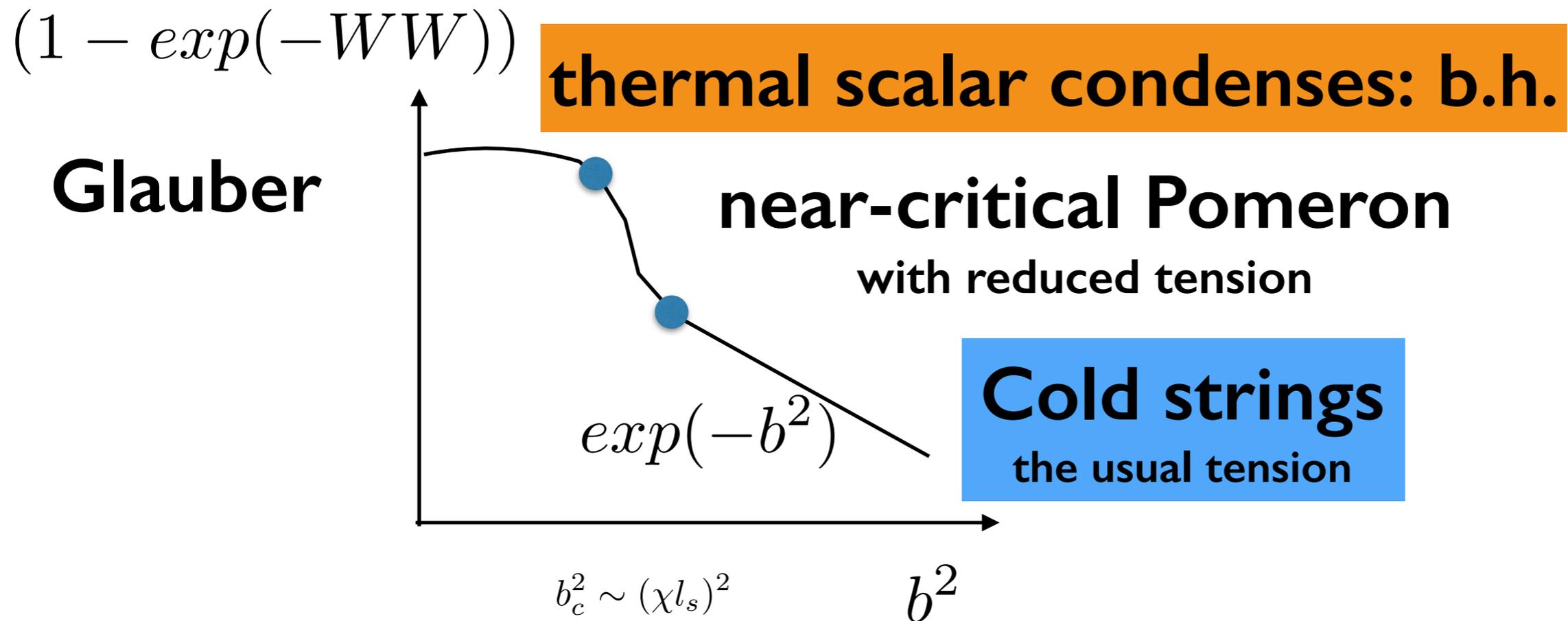
$$N(E) \approx (2d - 1)^{L/a} = e^{E(L)/T_H} \quad (5)$$

where in the last term we changed length into energy using the string tension  $E(L) = \sigma_T L$  and defined

$$T_H = \frac{\sigma_T a}{\ln(2d - 1)} \quad (6)$$

**As  $T \Rightarrow T_H$  the entropy and energy grow, but not free energy (pressure) as  $F = E - TS$  and two terms cancel**

# Can new regimes be seen in the elastic amplitude?



**After integration over  $b$  and dipole sizes,  
can one still be able to see such shape in  $A(t)$  ?**

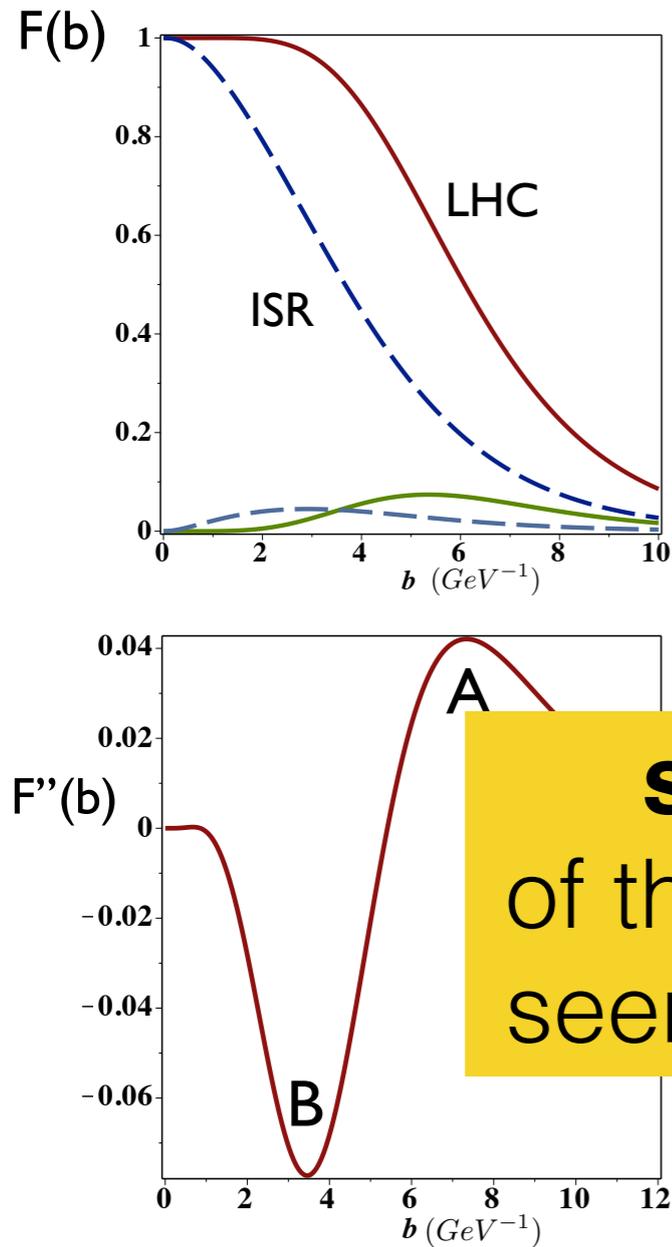


FIG. 4: (Color online) The upper figure shows the imaginary (upper) and real (down) parts of the profile function  $F(s, b)$  versus  $\mathbf{b}(\text{GeV}^{-1})$  for  $\sqrt{s} = 7 \text{ TeV}$  (solid) and  $\sqrt{s} = 63 \text{ GeV}$  (dashed). The lower plot shows the second derivative over  $b$  for  $\sqrt{s} = 7 \text{ TeV}$ . Two maxima correspond to the same points  $A, B$  as in the sketch in Fig. 1.

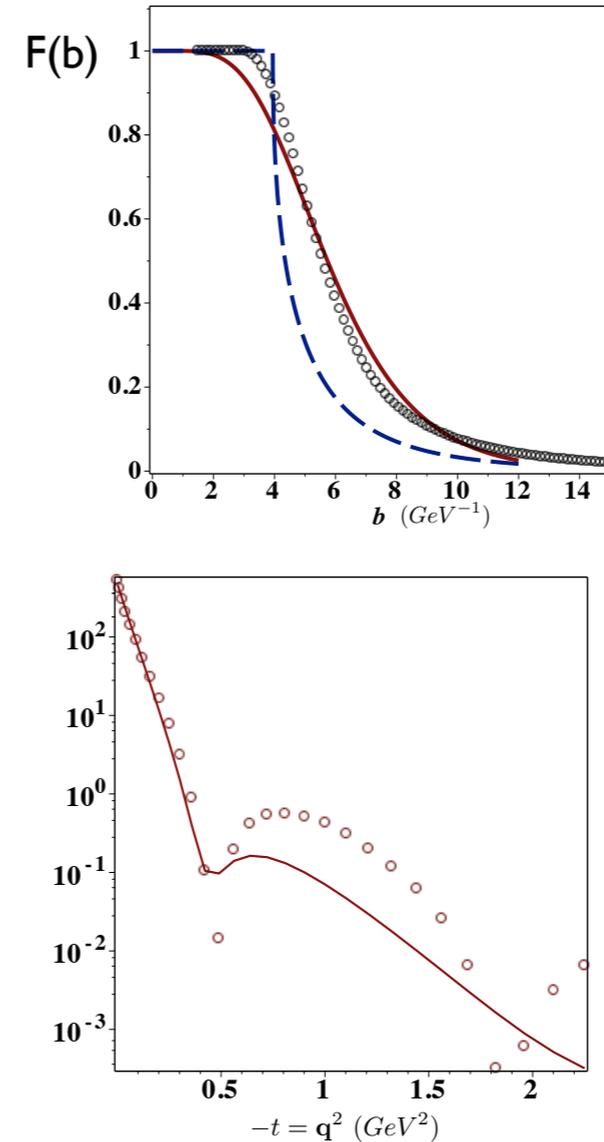


FIG. 12: (Color on-line) The profile function  $F(\mathbf{b})$  versus the impact parameter  $\mathbf{b}$  is shown in the upper plot for LHC  $\sqrt{s} = 7 \text{ TeV}$  energy. The solid line is the same curve as in Fig.4 corresponding to the BSW data parametrization. The dashed line is the shape corresponding to the approximation (59) for fixed sizes of the dipoles  $u_1 = u_2$ , while the circles correspond to the profile with the fluctuating dipoles. The lower plot shows the corresponding absolute value squared of its Bessel transform as a function of momentum transfer.

# summary

- in “central” pA and partly in peripheral AA “spaghetti” state **implodes/explodes like supernova**
- in **the mixed phase** (near  $T_c$ ) **entropy-rich self-bound string balls** can exist: QCD analog of holographic string balls
- (QGP and deconfinement => black holes)
- stringy Pomeron has Euclidean tube derivation => thus thermal description => thus analogy to phase transition and 3 regimes,
- which seems to be **seen in elastic scattering profile**
- **these regimes should show in inelastic events as well: to be done next**

# the near-critical Pomeron

*One can re-sum  
“Luscher terms”  
in the potential*

$$V(R) = \sqrt{\sigma_T^2 R^2 - const}$$

The re-summed result follows the paper of Arvis [38], which obtained the potential induced by the Nambu-Goto string. The result obtains the square root (which will play an important role in what follows)

$$\mathbf{K}_T(\beta, \mathbf{b}; 1) \approx \left( \frac{\beta}{4\pi^2 \mathbf{b}} \right)^{D_\perp/2} e^{-\sigma\beta\mathbf{b} (1 - \tilde{\beta}_H^2/\beta^2)^{1/2}}$$

but the results get clearly  
inapplicable as root argument  
changes sign

**Compare to “cold” one,  
in which it is a small  
correction**

$$\mathbf{K}_T(\beta, \mathbf{b}; 1) \approx \left( \frac{\beta}{4\pi^2 \mathbf{b}} \right)^{D_\perp/2} e^{-\sigma\beta\mathbf{b} (1 - \tilde{\beta}_H^2/2\beta^2)}$$

**Can one figure out  
what to do when  $T > T_H$   
and argument of the sqrt  $< 0$ ?**

very strong dependence on  
the value of the sigma mass

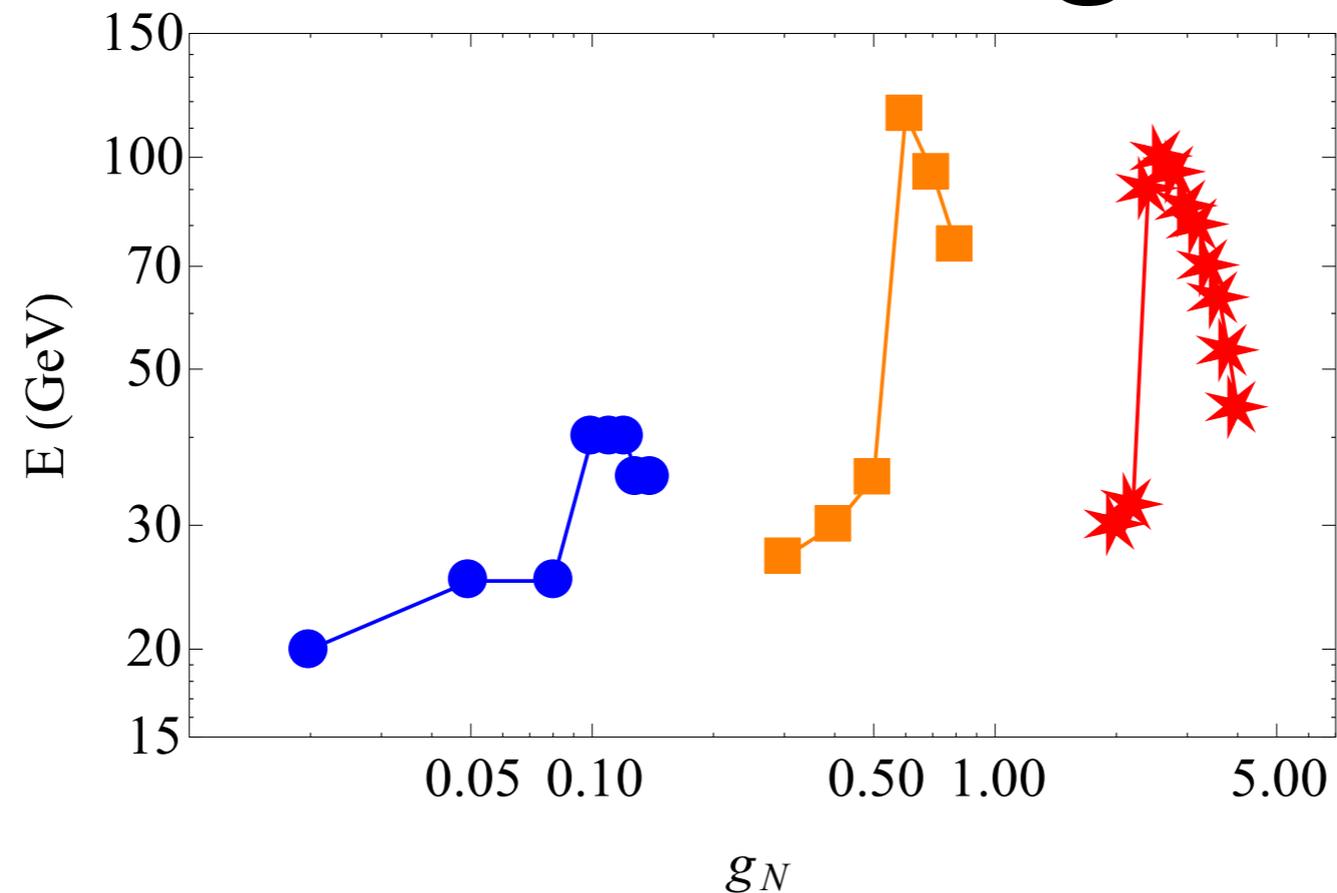


FIG. 9: (Color online) Blue circles, yellow squares and red stars show the dependence of the string ball energy  $E$ (GeV) on the coupling  $g_N$  (GeV<sup>-2</sup>) for  $m_\sigma = 0.1, 0.3, 0.6$  GeV, respectively. These simulations are performed in the setting  $T_0 = 1$  GeV,  $s_T = 2a$ .