Thermalization via Hagedorn States

C. Greiner,

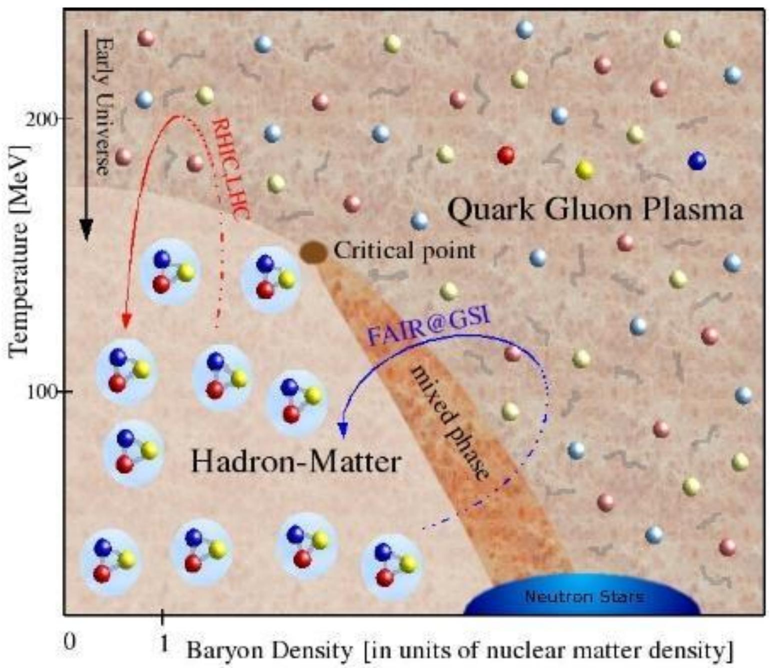
30th winter workshop on nuclear dynamics, Galveston , 2014

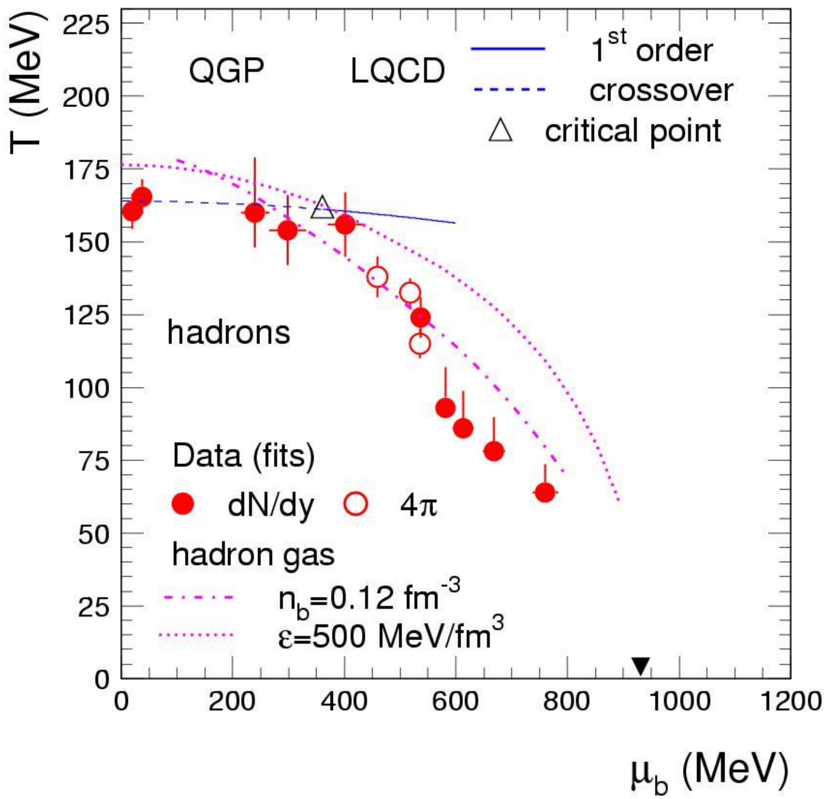
in collaboration with:

M. Beitel, K. Gallmeister and J. Noronha-Hostler

- history of Hagedorn States
- nuclear matter properties including HS
- chemical equilibration at the phase boundary
- why so thermal ? … via phase space 2body decay of HS

M. Beitel, K. Gallmeister, CG, arXiv:1402.1458





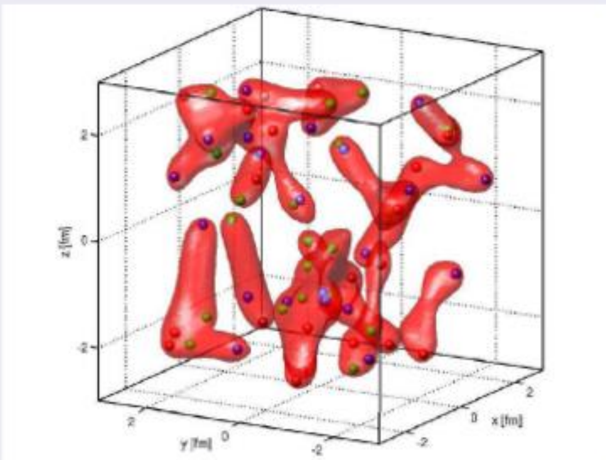
Hadronization

at the phase boundary…?

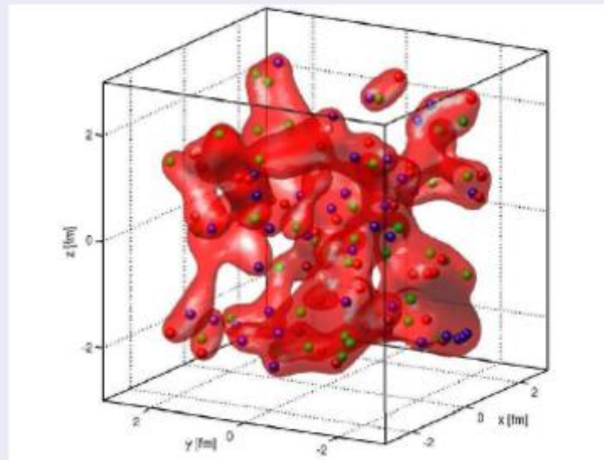
Deconfinement: transition to quark phase

G. Martens et al. Phys. Rev. D 70 / 73 (2010)

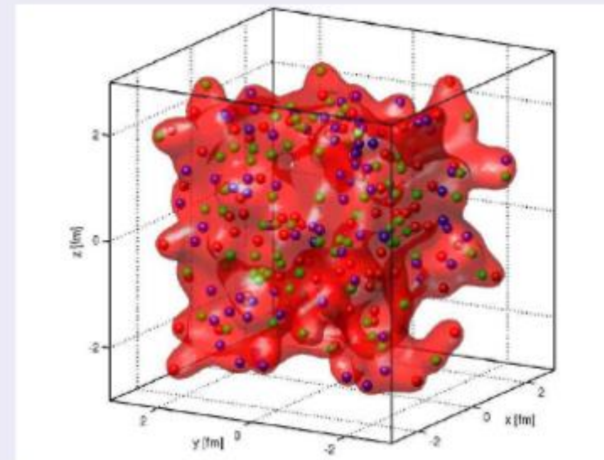
$n = 0.5\mathrm{fm}^{-3}$



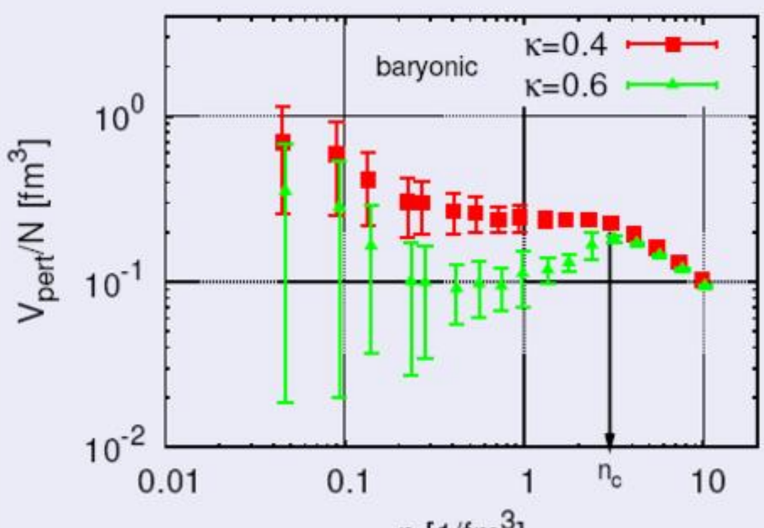
$n = 1.0\mathrm{fm}^{-3}$



$n = 2.0\mathrm{fm}^{-3}$



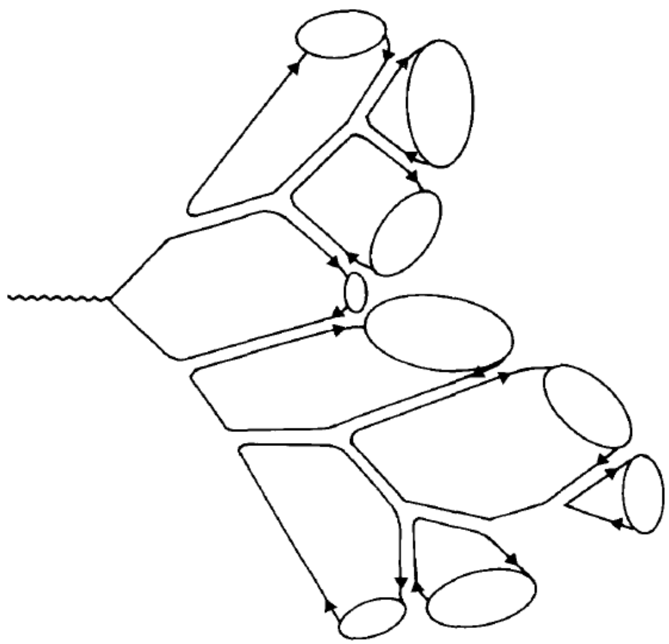
bag volume/particle

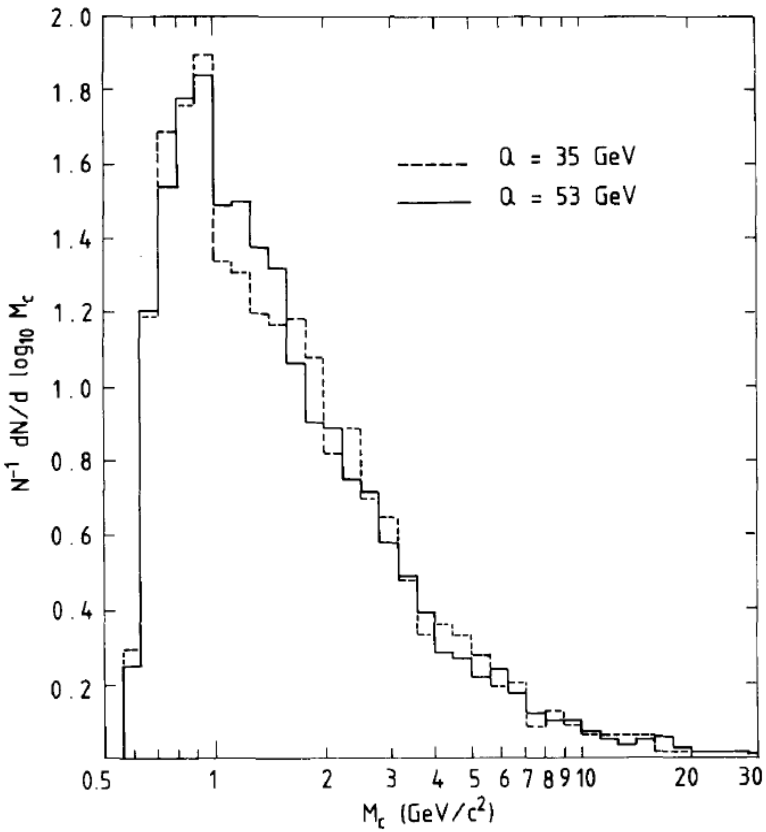


- formation of color neutral clusters at small densities
- particle number/cluster rises
- critical density at maximal overlap ($n \approx 2\mathrm{fm}^{-3}$ or $\varepsilon \approx 1.1\mathrm{GeV}/\mathrm{fm}^3$)
- percolation transition

Color Singlet cluster and their distribution

B.R. Webber, Nucl. Phys. B 238 (1984)





- The blobs (right) represent colour singlet clusters as basis for hadronization
- Distribution of colour singlet cluster mass (left) in e+-e- annihilation at c.m. energies of Q=35 GeV and Q=53 GeV
- this colour singlet clusters might be identified as Hagedorn States

History

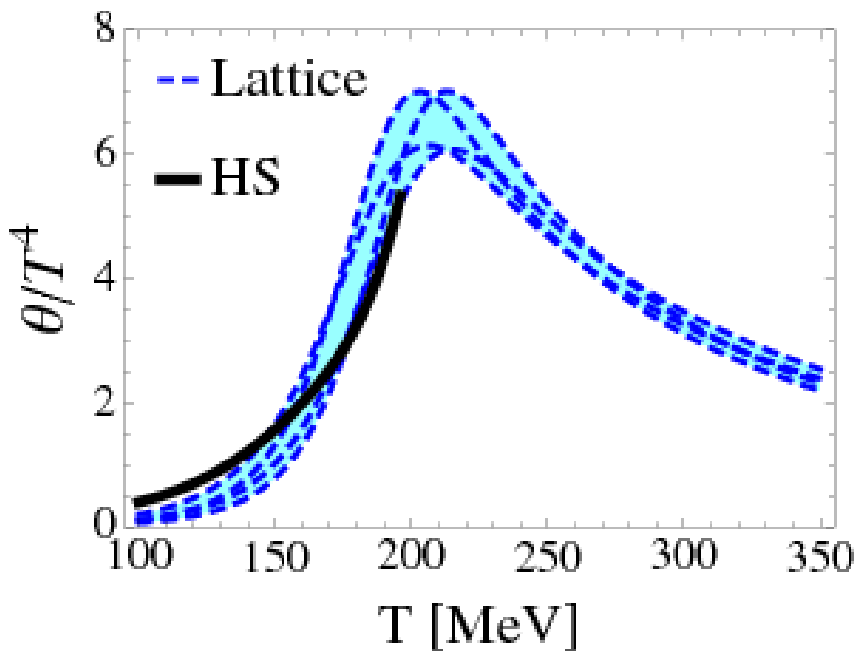
- 1965 R. Hagedorn postulated the "Statistical Bootstrap Model" before QCD
- fireballs and their constituents are the same
- nesting fireballs into each other leads to self-consistency condition (bootstrap equation)
- solution is exponentially rising common known as Hagedorn spectrum
- slope of Hagedorn Spectrum determined by Hagedorn temperature

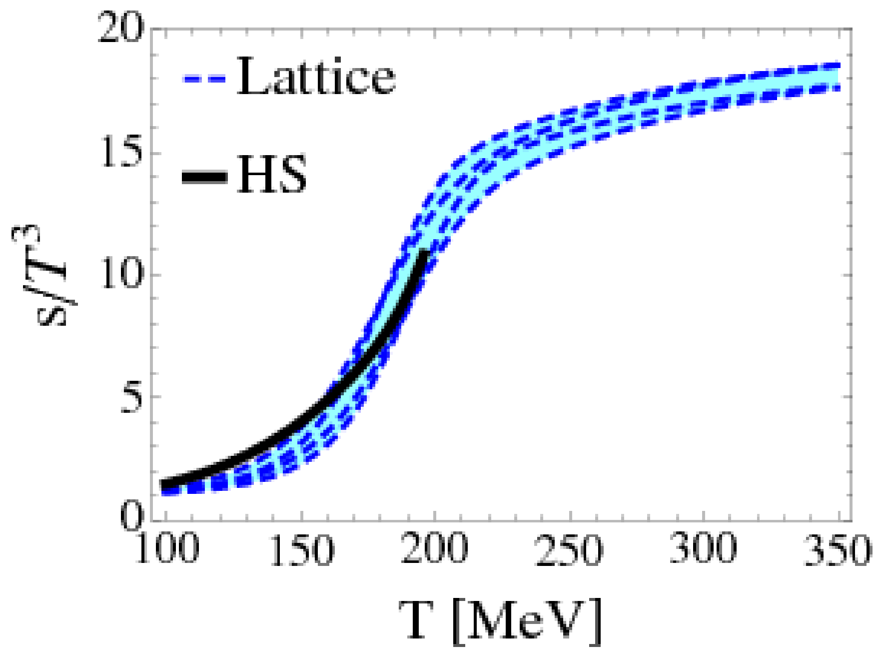
Hadron Resonance Gas with Hagedorn States and comparison to lattice QCD close to T_{critical}

J. Noronha-Hostler, J. Noronha, CG, PRL 103 (2009), PRC 86 (2012)

- **Hagedorn spectrum**: $\rho_{HS} \sim m^{-a} \exp[m/T_H]$
- **RBC collaboration**:

$$\longrightarrow \qquad \rho = \int_{M_0}^{M} \frac{A}{[m^2+m_r^2]^{\frac{5}{4}}} e^{\frac{m}{T_H}} dm$$

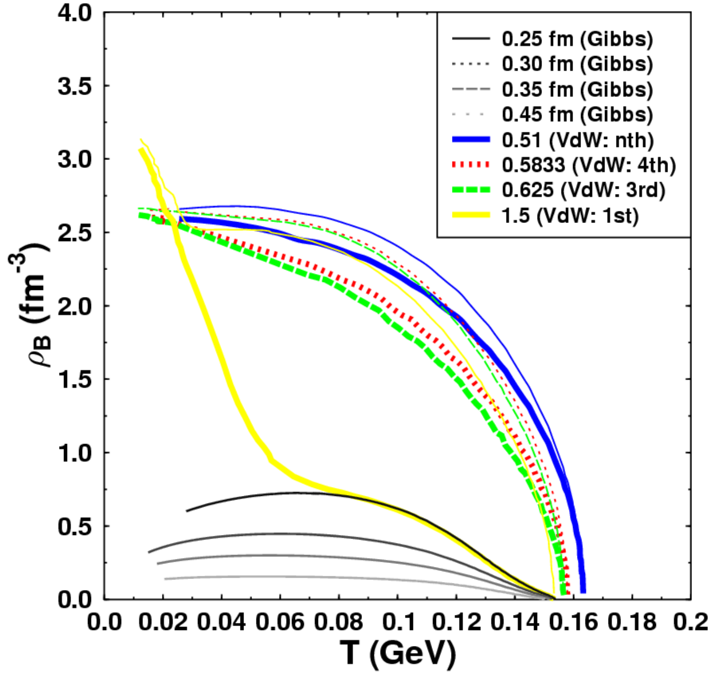




The order and shape of QGP phase transition

I.Zakout, CG and J. Schaffner-Bielich, NPA 781 (2007) 150,

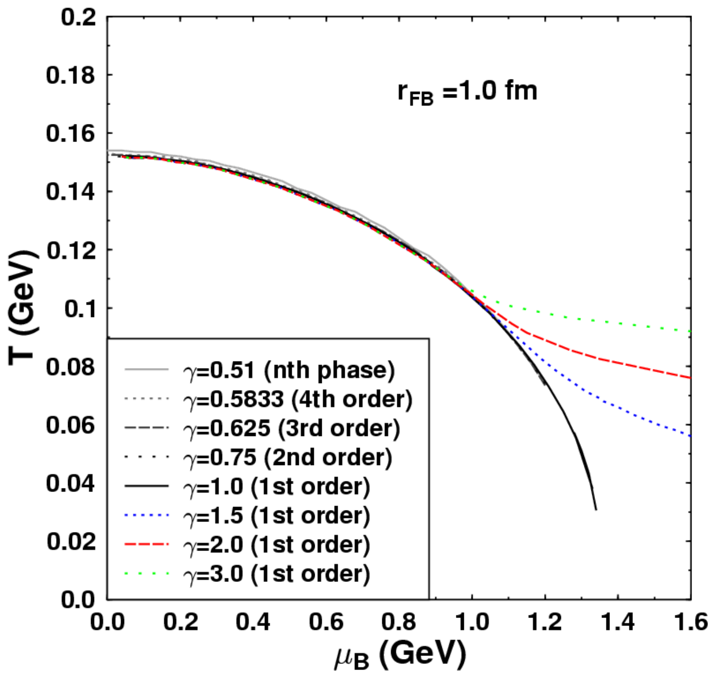
PRC78 (2008)

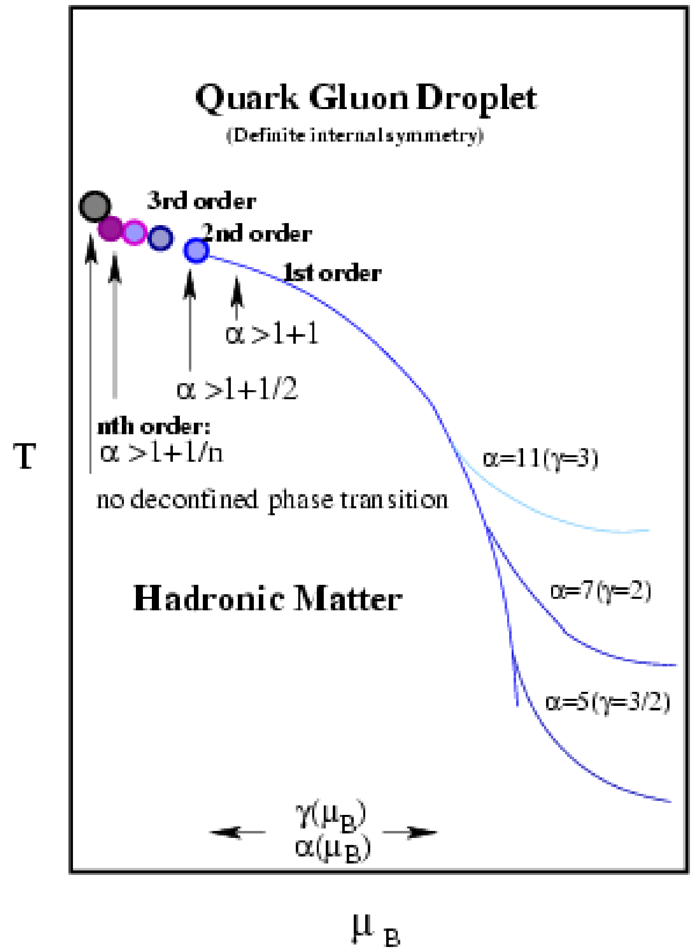


$$\Big\} \gamma = \frac{\alpha+1}{4}$$

density of states:

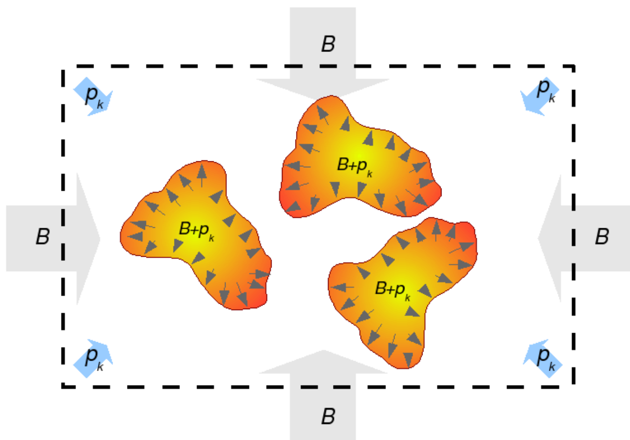
$$\boxed{\rho(m,v) \sim c\, m^{-(\alpha+2)}\, e^{\frac{m}{T_{H[B]}}}\, \delta(m-4Bv)}$$





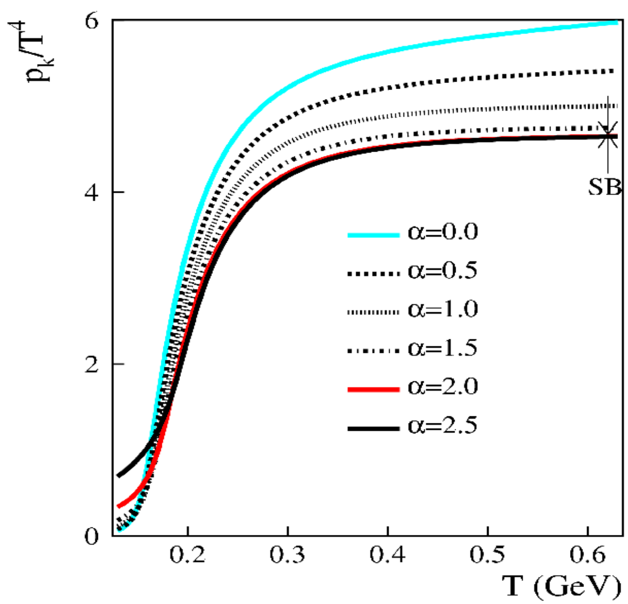
$$\alpha(\mu_B)$$

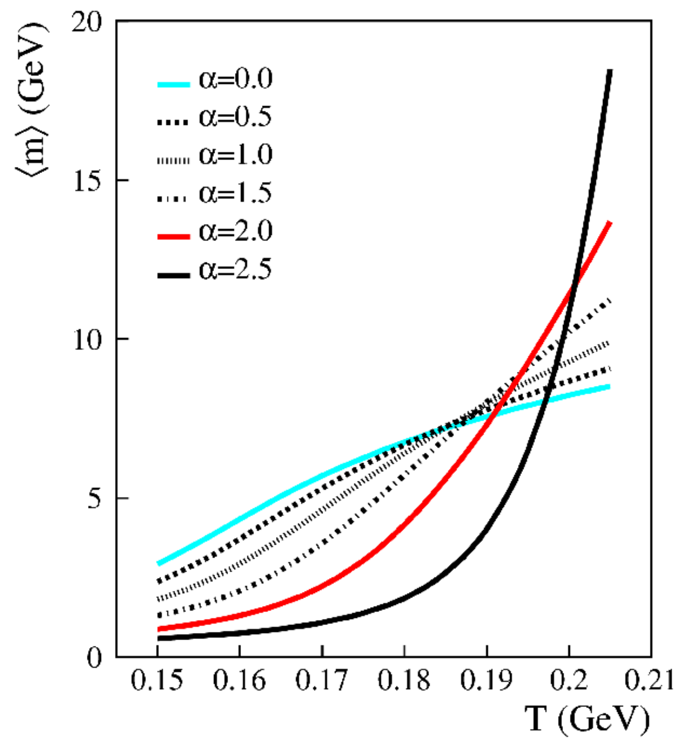
Crossover transition in bag-like models

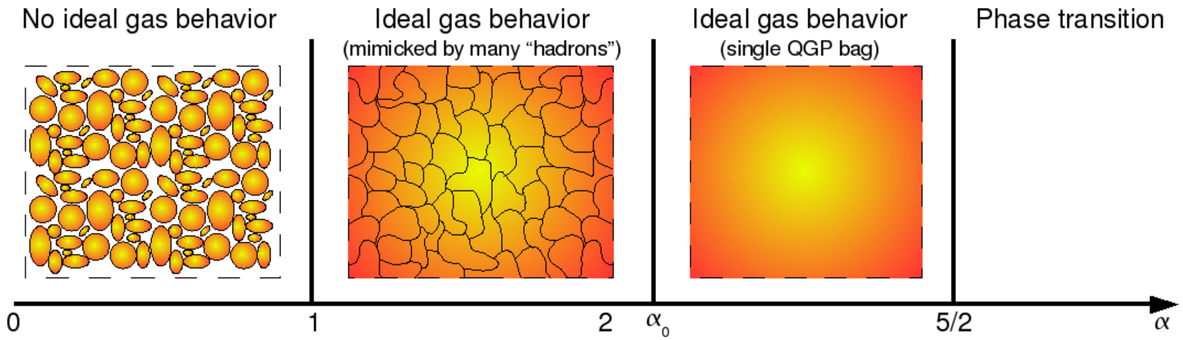


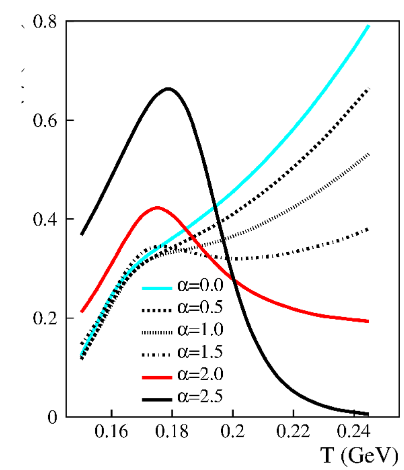
L. Ferroni and V. Koch, PRC79 (2009) 034905

density of states: $\boxed{\rho(m) \sim c\, m^{-(\alpha)}\, e^{\frac{m}{T_H[B]}}}$

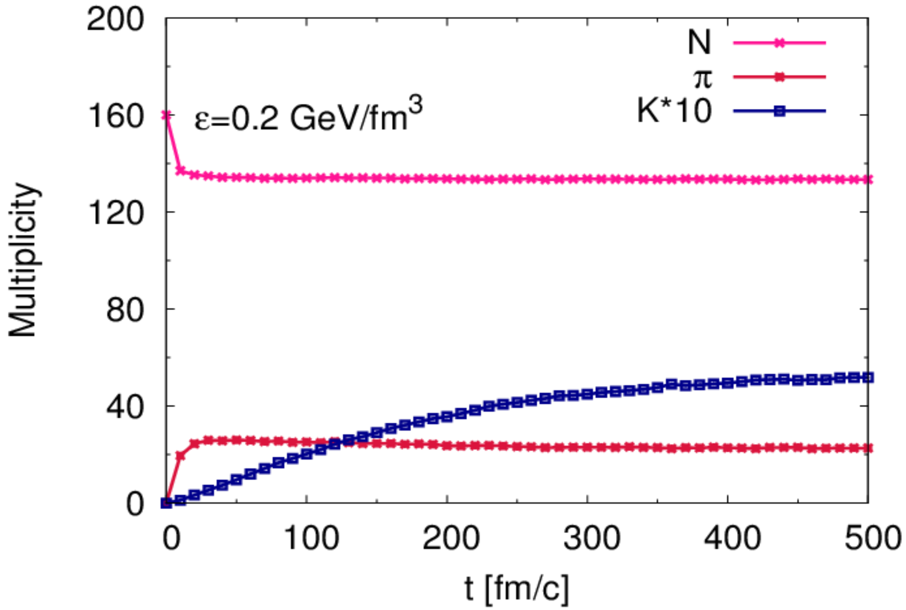


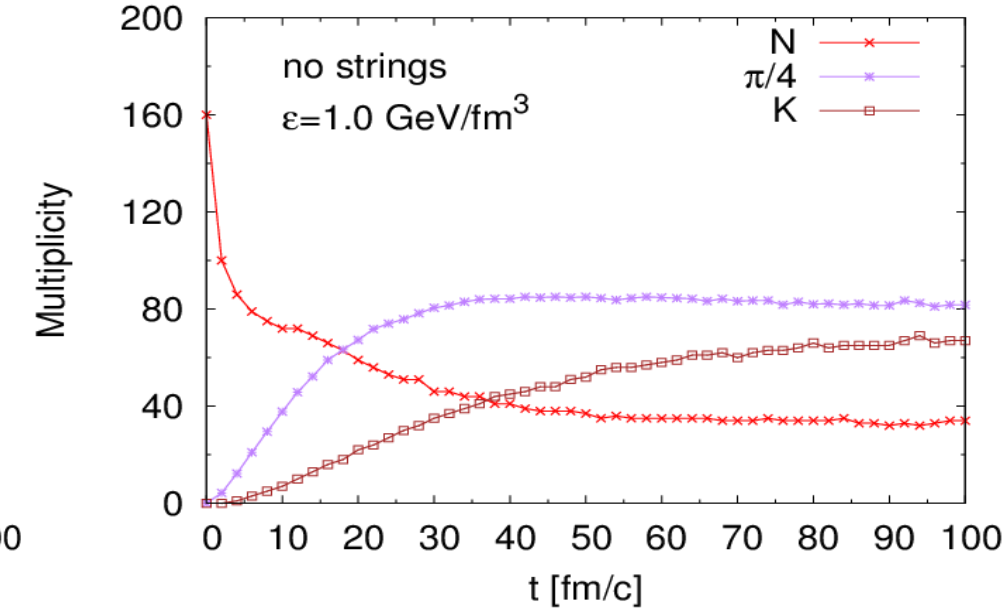






Problem: Chem. Equil. in UrQMD (box) too long





- UrQMD: microscopic transport model for p-p, p-N, A-A collisions for SIS up to LHC energies
- detailed balance violated by strings and some hadronic decay !

Application of Hagedorn states

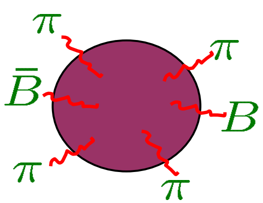
- at SPS energies chem. equil. time is 1-3 fm/c

$$\mathrm{n}_1 \pi + n_2 K \leftrightarrow Y + p$$

- at RHIC energies chem. equil. time is 10 fm/c

  with same approach

- fast chem. equil. mechanism through Hagedorn states



$$\big(n_1 \pi + n_2 K + n_3 K \leftrightarrow\big) \;\; HS \leftrightarrow B + B + X$$

- dyn. evolution through set of coupled rate equations
  leads to 5 fm/c for BB pairs

J. Noronha-Hostler et al. PRL 100 (2008)
J. Noronha-Hostler et al. J. Phys. G 37 (2010)
J. Noronha-Hostler et al. Phys. Rev. C 81 (2010)

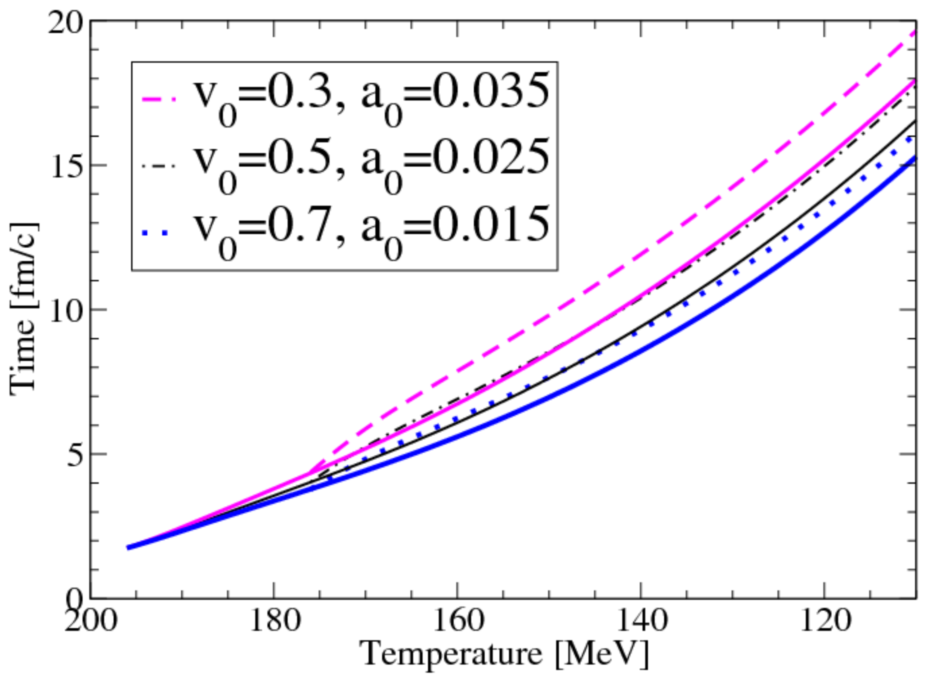
Rate Equations

$$n\pi \leftrightarrow HS \leftrightarrow n'\pi + B\bar{B}$$

J. Noronha-Hostler, CG, I. Shovkovy, PRL 100:252301, 2008

$$\begin{aligned}
\dot{N}_i &= \Gamma_{i,\pi}\left[N_i^{eq}\sum_n B_{i,n}\left(\frac{N_\pi}{N_\pi^{eq}}\right)^n - N_i\right] \\
&+ \Gamma_{i,X\bar{X}}\left[N_i^{eq}\left(\frac{N_\pi}{N_\pi^{eq}}\right)^{\langle n_{i,x}\rangle}\left(\frac{N_{X\bar{X}}}{N_{X\bar{X}}^{eq}}\right)^2 - N_i\right] \\
\dot{N}_\pi &= \sum_i \Gamma_{i,\pi}\left[N_i\langle n_i\rangle - N_i^{eq}\sum_n B_{i,n} n\left(\frac{N_\pi}{N_\pi^{eq}}\right)^n\right] \\
&+ \sum_i \Gamma_{i,X\bar{X}}\langle n_{i,x}\rangle\left[N_i - N_i^{eq}\left(\frac{N_\pi}{N_\pi^{eq}}\right)^{\langle n_{i,x}\rangle}\left(\frac{N_{X\bar{X}}}{N_{X\bar{X}}^{eq}}\right)^2\right] \\
\dot{N}_{X\bar{X}} &= \sum_i \Gamma_{i,X\bar{X}}\left[N_i - N_i^{eq}\left(\frac{N_\pi}{N_\pi^{eq}}\right)^{\langle n_{i,x}\rangle}\left(\frac{N_{X\bar{X}}}{N_{X\bar{X}}^{eq}}\right)^2\right]
\end{aligned}$$

Expanding fireball

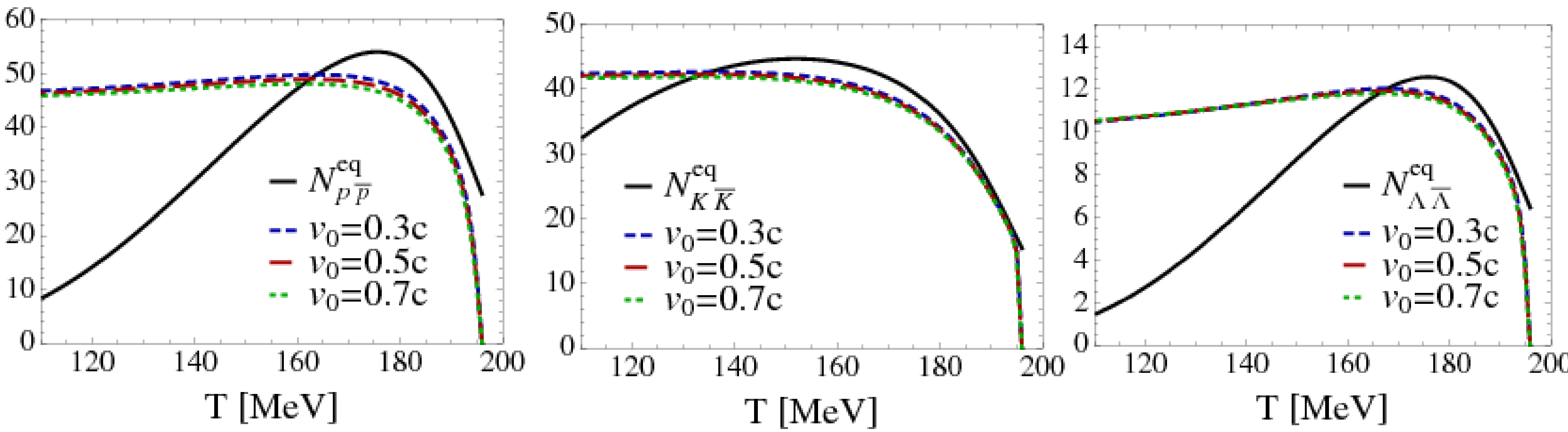


$$V(t) = \pi\, ct \left(r_0 + v_0(t - t_0) + \frac{1}{2} a_0 (t - t_0)^2 \right)^2$$

$$\text{const.} = s(T)V(t) \sim \frac{S_\pi}{N_\pi} \int \frac{dN_\pi}{dy} dy$$

Varying parameters has only small effect!

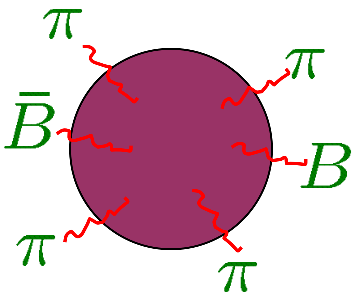
$$\Delta t = 2 - 3\ \frac{fm}{c}$$

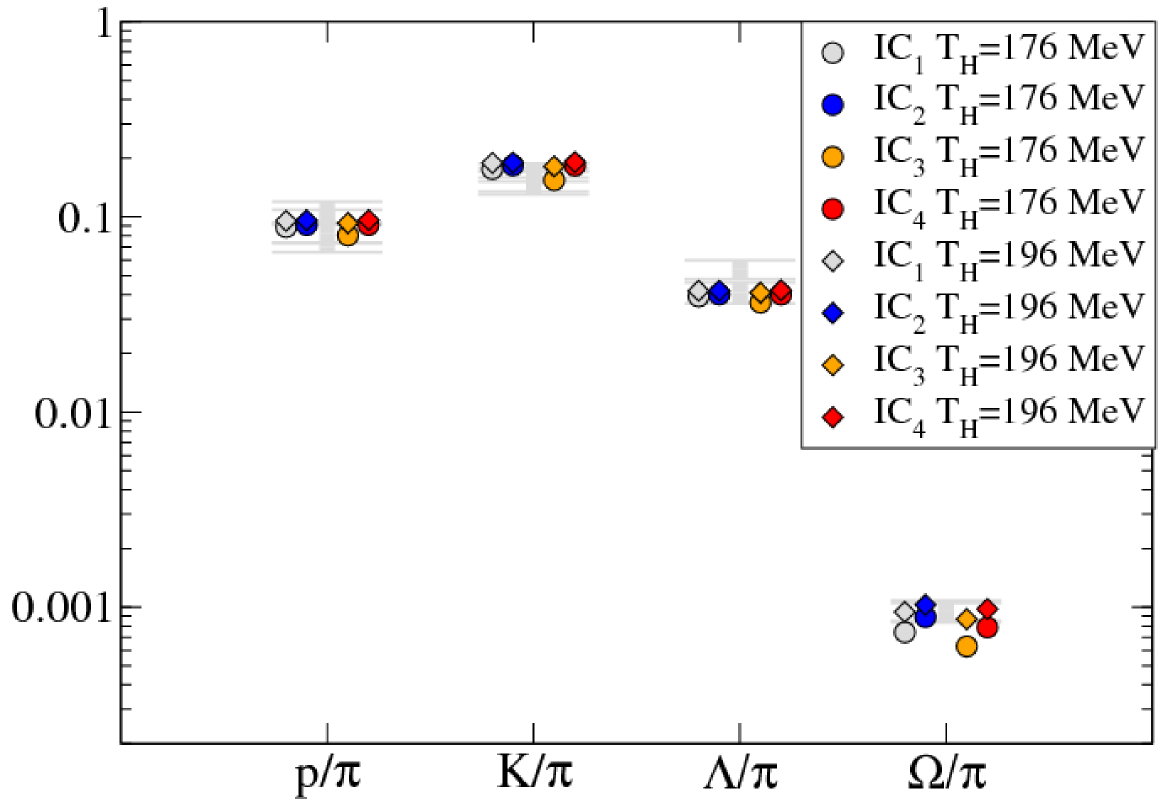


intermediate summary

Potential Hagedorn States close to critical temperature:

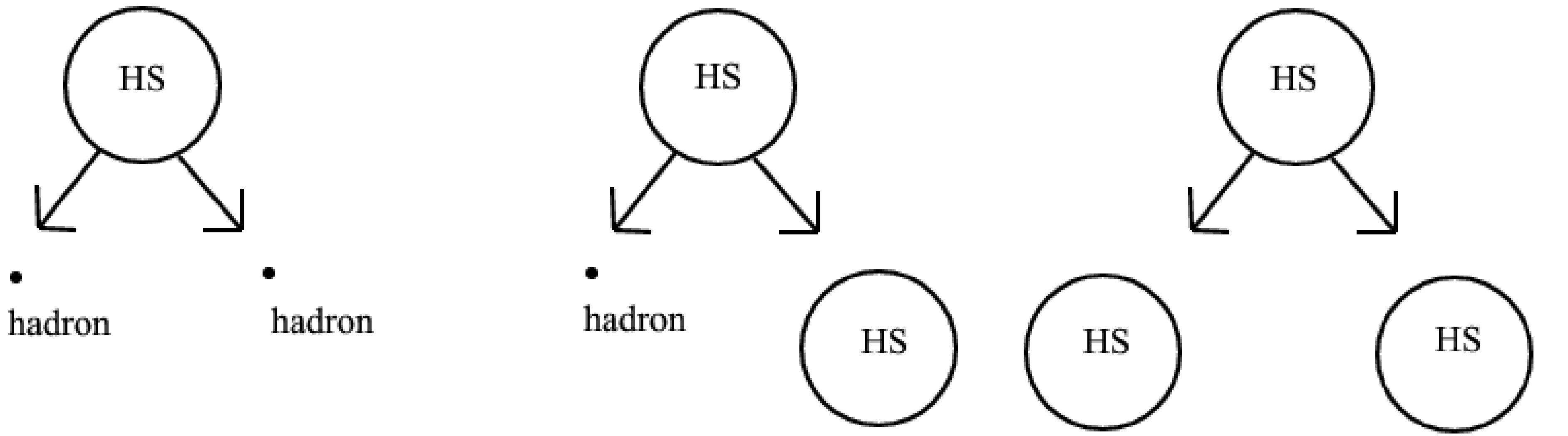
- **can explain fast chemical equilibration by HS regeneration**
- **roughly:** $\Delta\epsilon_{HS} \approx 0.3 - 0.5\, GeV/fm^3$
- **roughly:** $\Gamma_{HS}^{tot} \approx 0.5 - 1\, GeV$
- **smaller shear viscosity of QCD matter at** T_c
- **Future: embedding into UrQMD**





J. Noronha-Hostler, M. Beitel, CG, I.Shovkovy, PRC 81(2010)

Hagedorn state decay modes



M. Beitel, K. Gallmeister, CG, arXiv:1402.1458

Bootstrap equation and Hagedorn state total decay width

(S. Frautschi PRD 3, C. Hamer et al. PRD 4)

$$\tau_{\vec{C}}(m) = \frac{R^3}{3\pi m} \sum_{\vec{C}_1, \vec{C}_2} \iint dm_1 dm_2 \, \tau_{\vec{C}_1}(m_1)\, m_1$$

$$\times\, \tau_{\vec{C}_2}(m_2)\, m_2 \; p_{cm}(m, m_1, m_2)\, \delta^{(3)}\left(\vec{C} - \vec{C}_1 - \vec{C}_2\right)$$

- Bootstrap equation with four-momentum and strict charge conservation **(B,S,Q)**

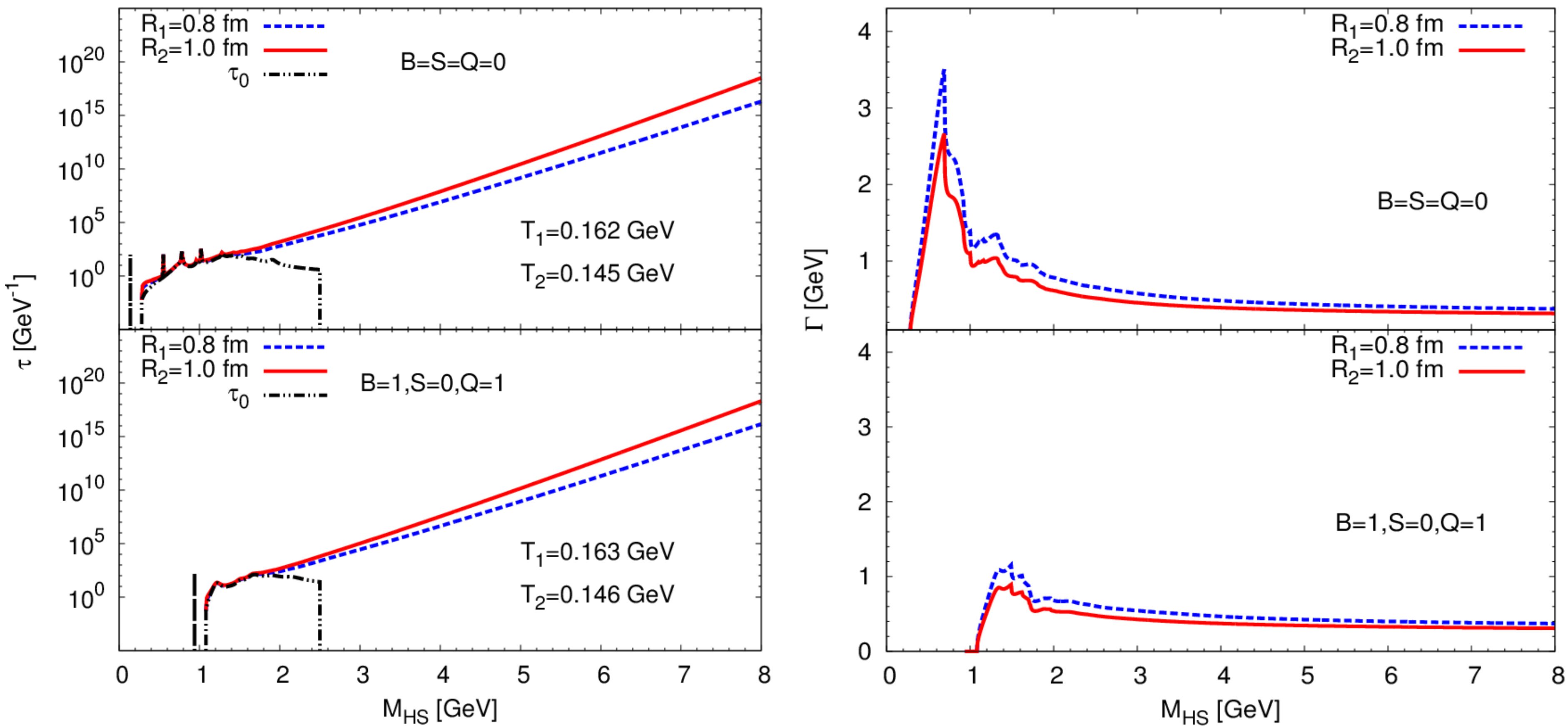
$$\Gamma_{\vec{C}}(m) = \frac{\sigma}{2\pi^2 \tau_{\vec{C}}(m)} \sum_{\vec{C}_1, \vec{C}_2} \iint dm_1 dm_2 \tau_{\vec{C}_1}(m_1)\, \tau_{\vec{C}_2}(m_2)$$

$$\times\, p_{cm}(m, m_1, m_2)^2\, \delta^{(3)}\left(\vec{C} - \vec{C}_1 - \vec{C}_2\right)$$

- Total decay width of Hagedorn state by application of the principle of detailed balance

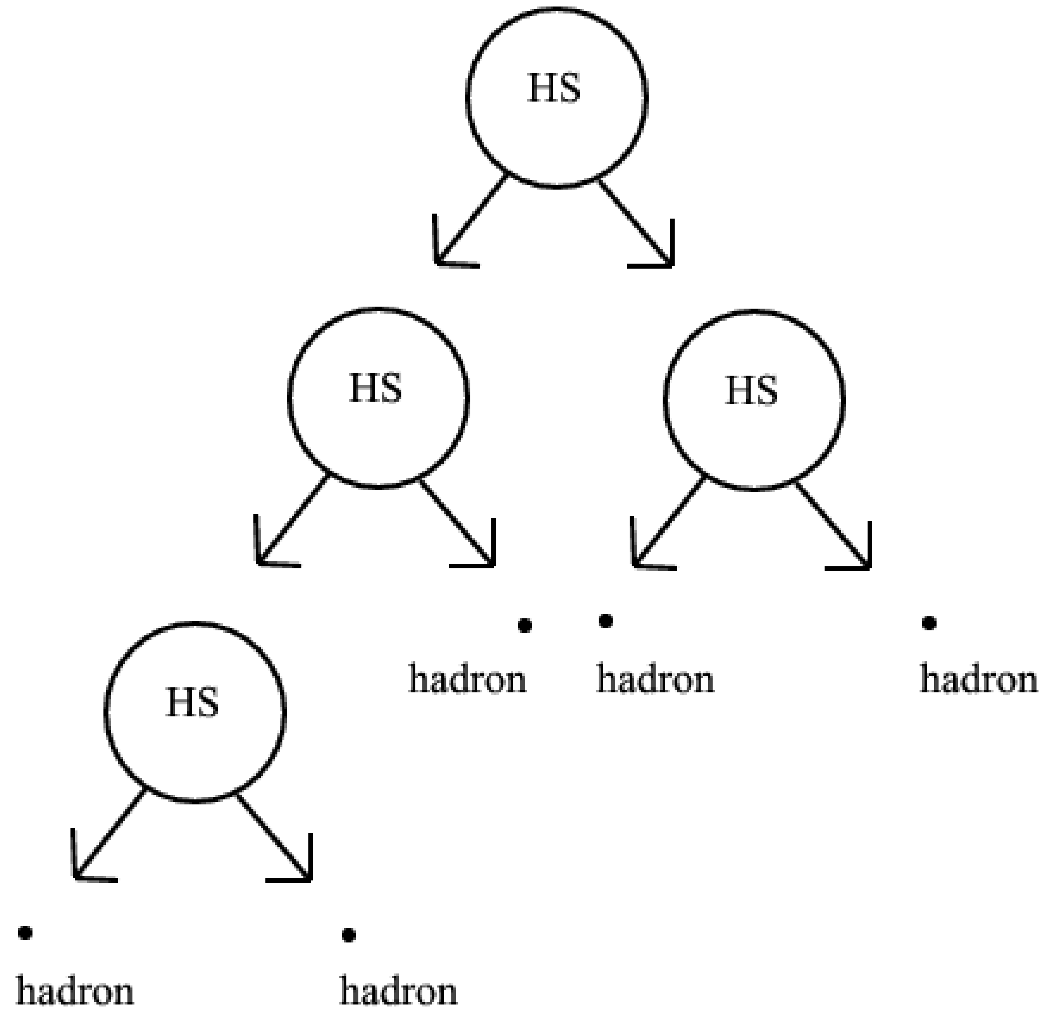
Hagedorn spectra and Hagedorn state decay widths

Hagedorn spectra | **Hagedorn total decay widths**



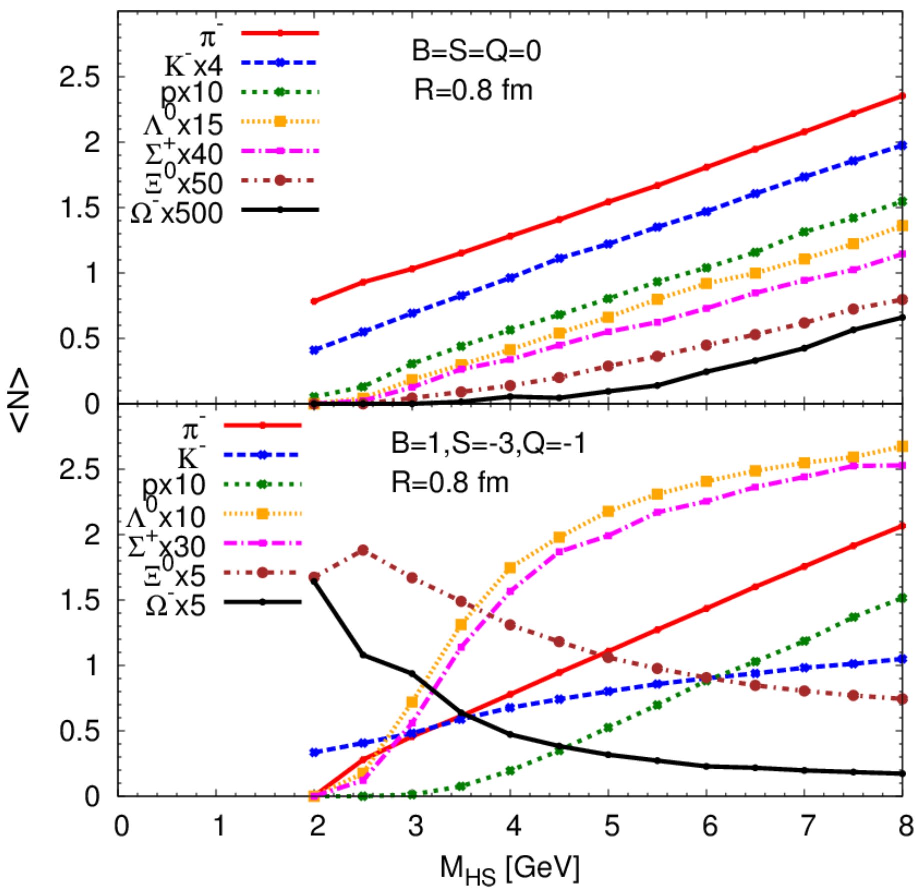
- $\color{red}{\text{large}}$ Hagedorn state → $\color{red}{\text{small}}$ Hagedorn temperature
- Hagedorn temperature $\color{red}{\text{slope parameter}}$, rather independent of charges
- Hagedorn state decay width $\color{red}{\text{constant}}$ in infinite mass limit
- peak in total decay width dependent on Hagedorn state charges, due to combinatorics

One possible Hagedorn state decay chain

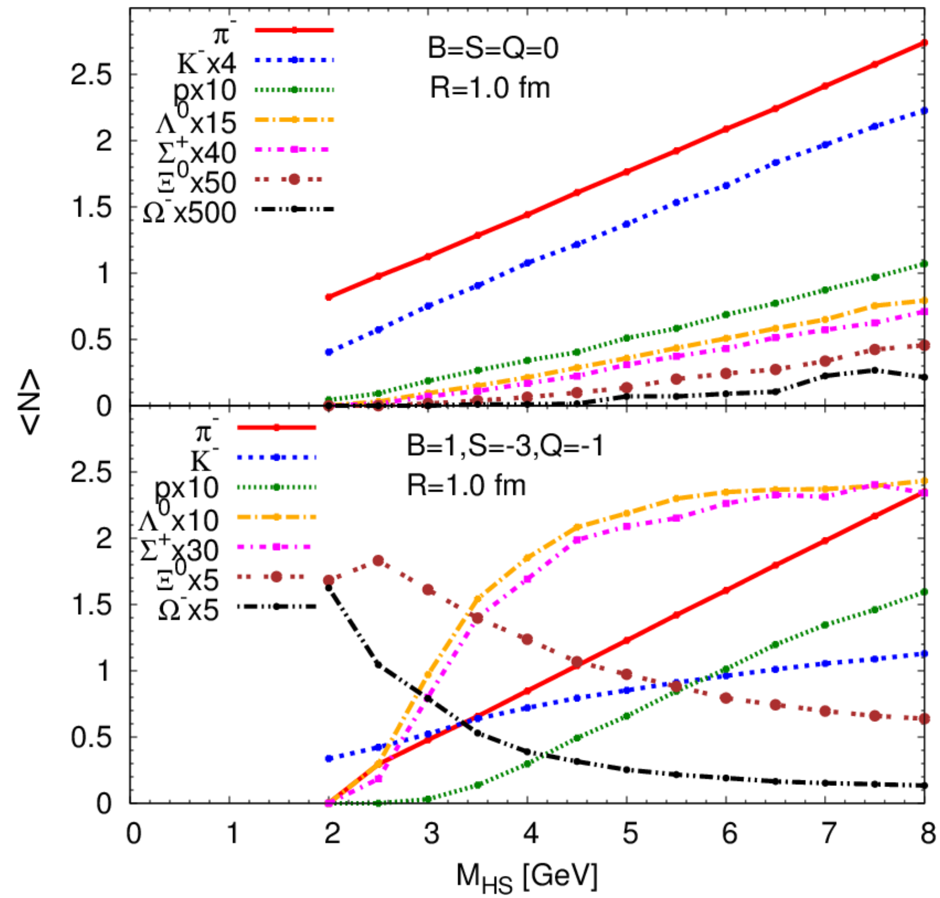


Hadronic multiplicities after Hagedorn state cascading (incl. feeddown)

R=0.8 fm

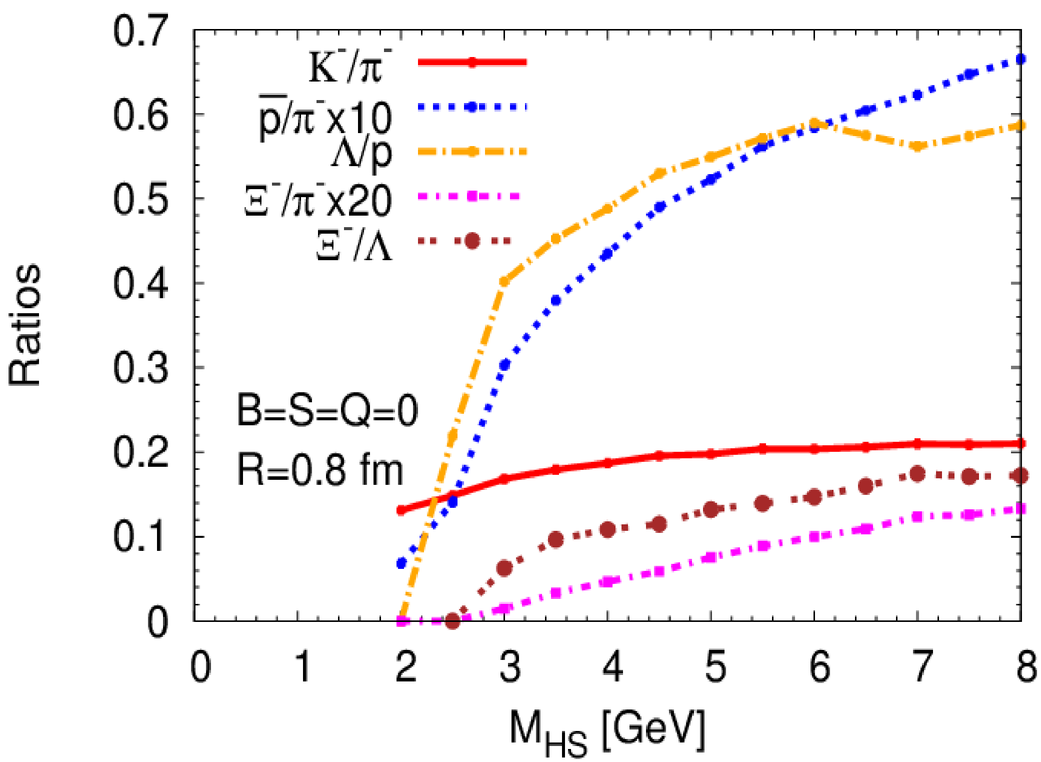


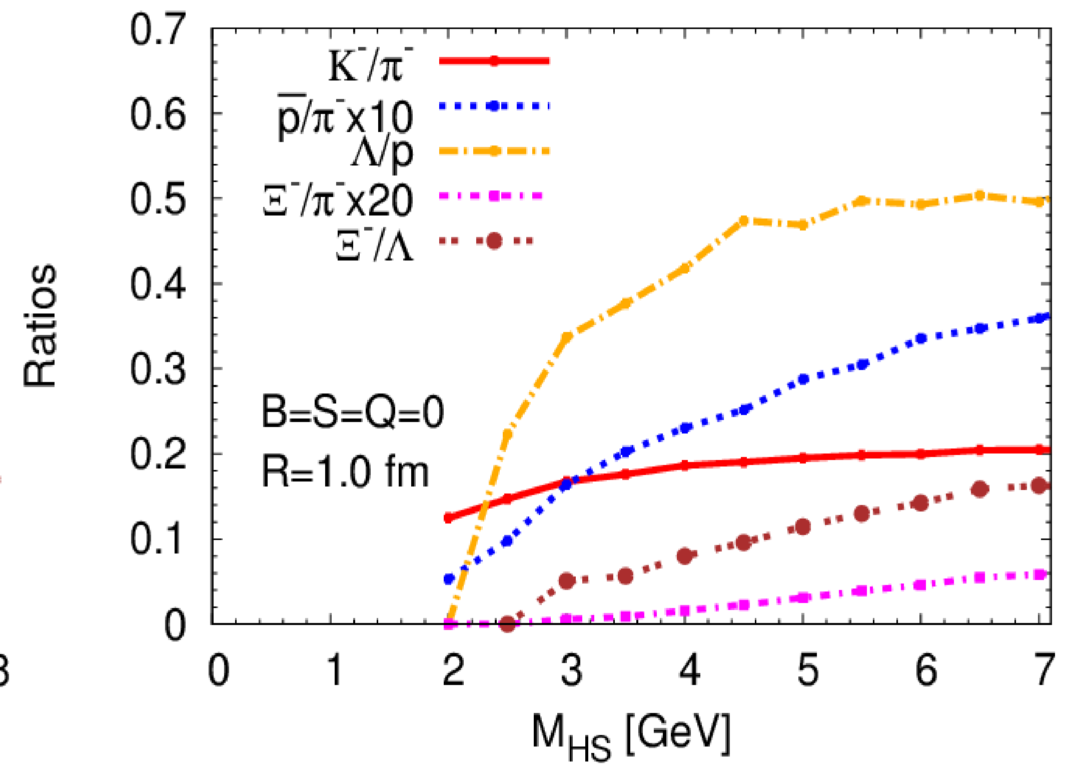
R=1.0 fm



- hadronic multiplicity magnitude order depends on phase space for (B=S=Q=0)
- for B=S=Q=0 more baryons/hyperons produced for R=0.8 fm than for R=1.0 fm due to larger Hagedorn temperature here
- if charges of Hagedorn state not all zero, then charges dictate the multiplicities in both cases

Hadronic ratios from Hagedorn state cascading decay





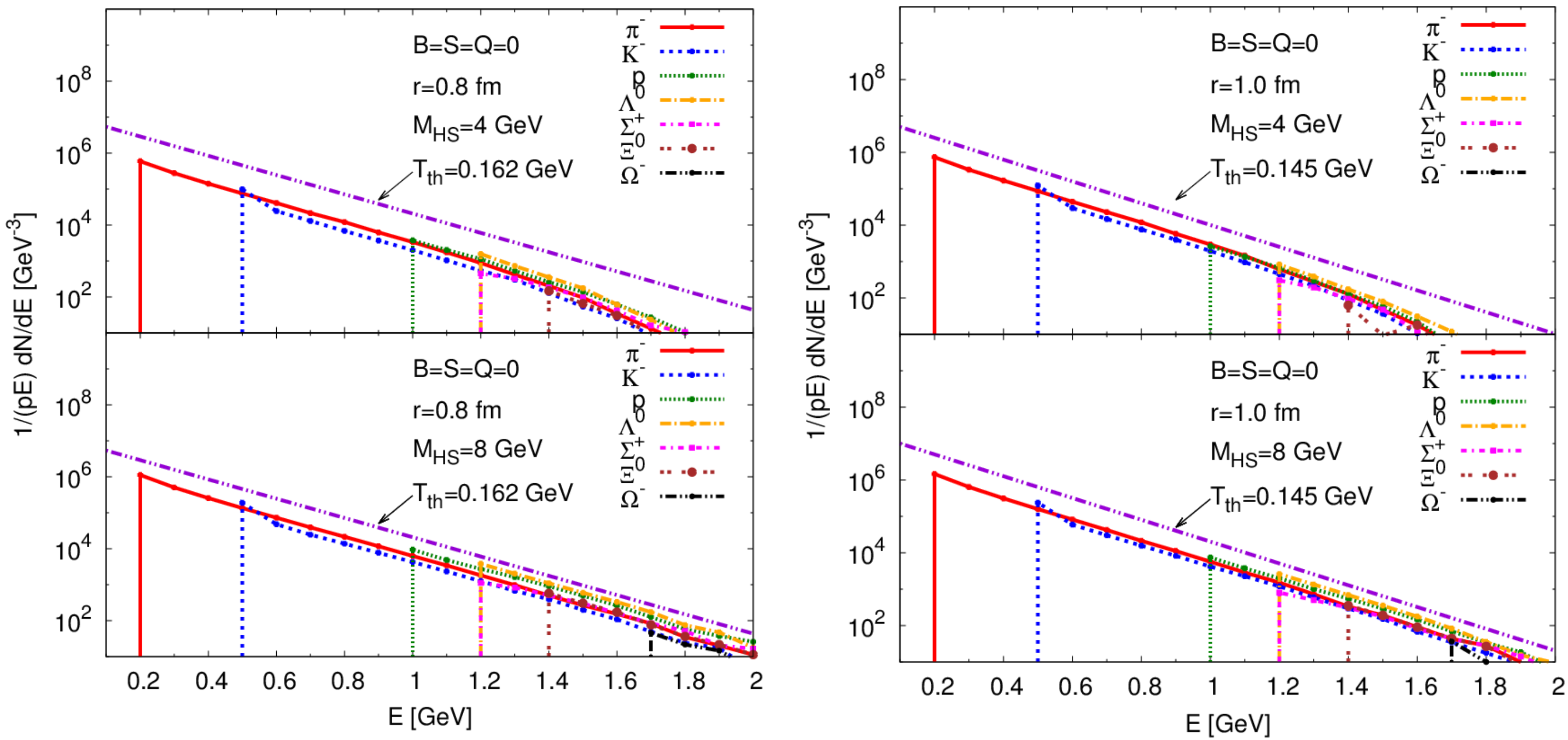
	p-p	Pb-Pb	4 GeV	8 GeV
K^-/π^-	$0.123(14)$	$0.149(16)$	0.187	0.210
$\bar{p}/\pi^-$	$0.053(6)$	$0.045(5)$	0.043	0.066
Λ/π^-	$0.032(4)$	$0.036(5)$	0.021	0.038
$\Lambda/\bar{p}$	$0.608(88)$	$0.78(12)$	0.494	0.579
Ξ^-/π^-	$0.003(1)$	$0.0050(6)$	0.0023	0.0066
$\Omega^-/\pi^- \cdot 10^{-3}$	-	$0.87(17)$	0.086	0.560

ALICE at LHC Ratios:

p-p @ 0.9 TeV
Pb-Pb @ 2.76 TeV

K. Aamodt et al. Eur. Phys J. C 71
B. Abelev et al. Phys Rev. C. 88
B. Abelev et al. Phys. Lett. B 728

Energy spectra of decay products in Hagedorn state cascading chain are thermal



- Thermal temperature equals Hagedorn temperature, independent of

a. Initial Hagedorn state mass
b. Hagedorn state radius
c. Hagedorn state charges

Summary

- chem. eq. times in UrQMD (box) too long
- derivation of covariant bootstrap equation
- Hagedorn spectra derived from known hadronic spectral functions
- Hagedorn state total decay width being constant in infinite mass limit
- Hadronic multiplicities from Hagedorn state cascading simulations
- Hadronic multiplicity ratios and comparison to experimental data
- Energy spectra of decay products in Hagedorn state cascading simulations are thermal

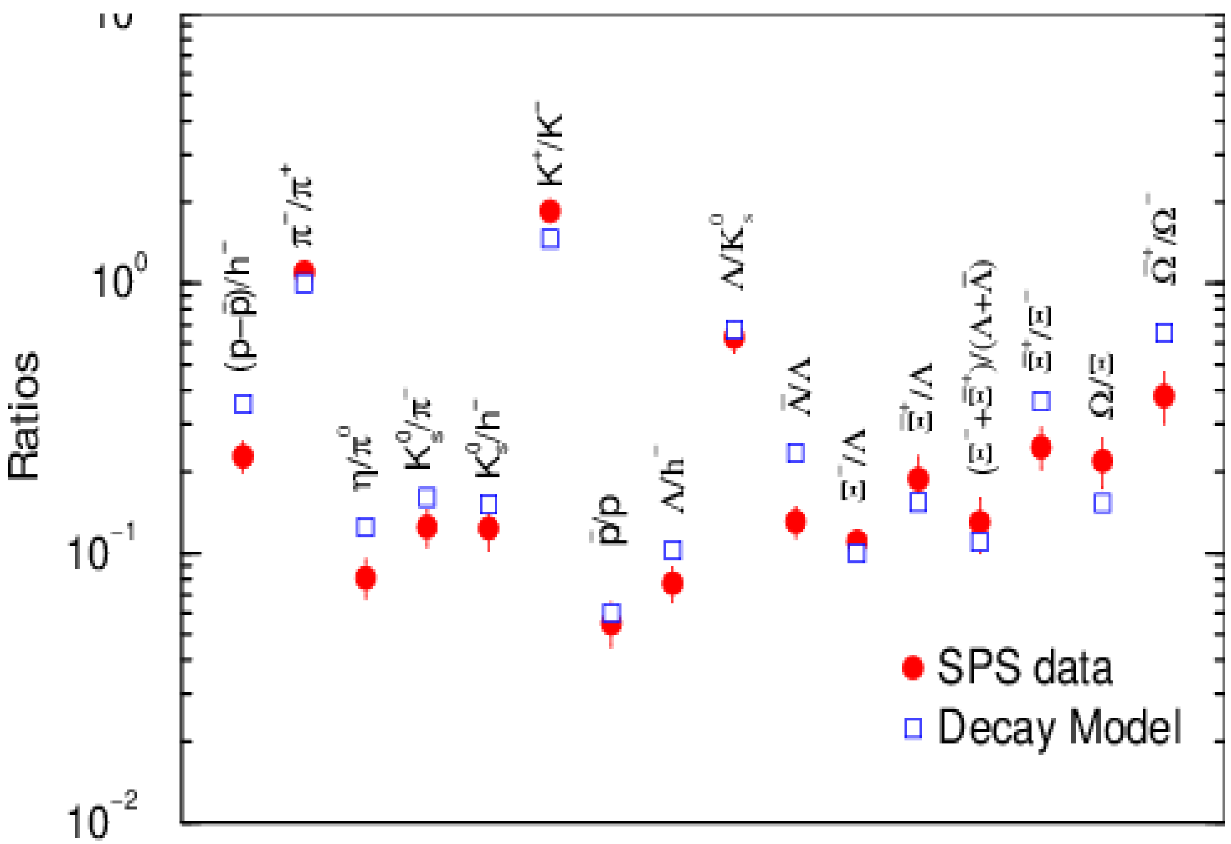
Outlook

- Hagedorn temperature is the same as thermal temperature from energy spectra fit
- our model explains the success of Statistical Hadronization Model
- Regeneration of particles will explain quick chem. eq.
- ... thus dynamical creation and decay of Hagedorn states lowers chem. eq. times
- ... and shear viscosity over entropy ratio in UrQMD

Stat. appl. of Hagedorn states

S. Pal and P.Danielewicz, PLB 627 (2005)

- One large resonance decays down in a cascade mode



SPS Pb-Pb

- central collisions
- sqrts=158 A GeV

Model assumptions

- m_0=100 GeV
- T_H=0.170 GeV
- B_0 = 26
- S_0 = 0