BERGISCHE
UNIVERSITÄT
WUPPERTAL

JÜLICH
FORSCHUNGSZENTRUM

Equation of state, fluctuations and other recent results from LQCD

with: S. Borsanyi, Y. Delgado, S. Dürr, Z. Fodor, C. Hoelbling, S.D. Katz, T. Lippert, D. Nogradi, C. Ratti, K.K. Szabo, B.C. Toth, N. Trombitas (Wuppertal-Budapest collaboration)

18.02.2014| Stefan Krieg

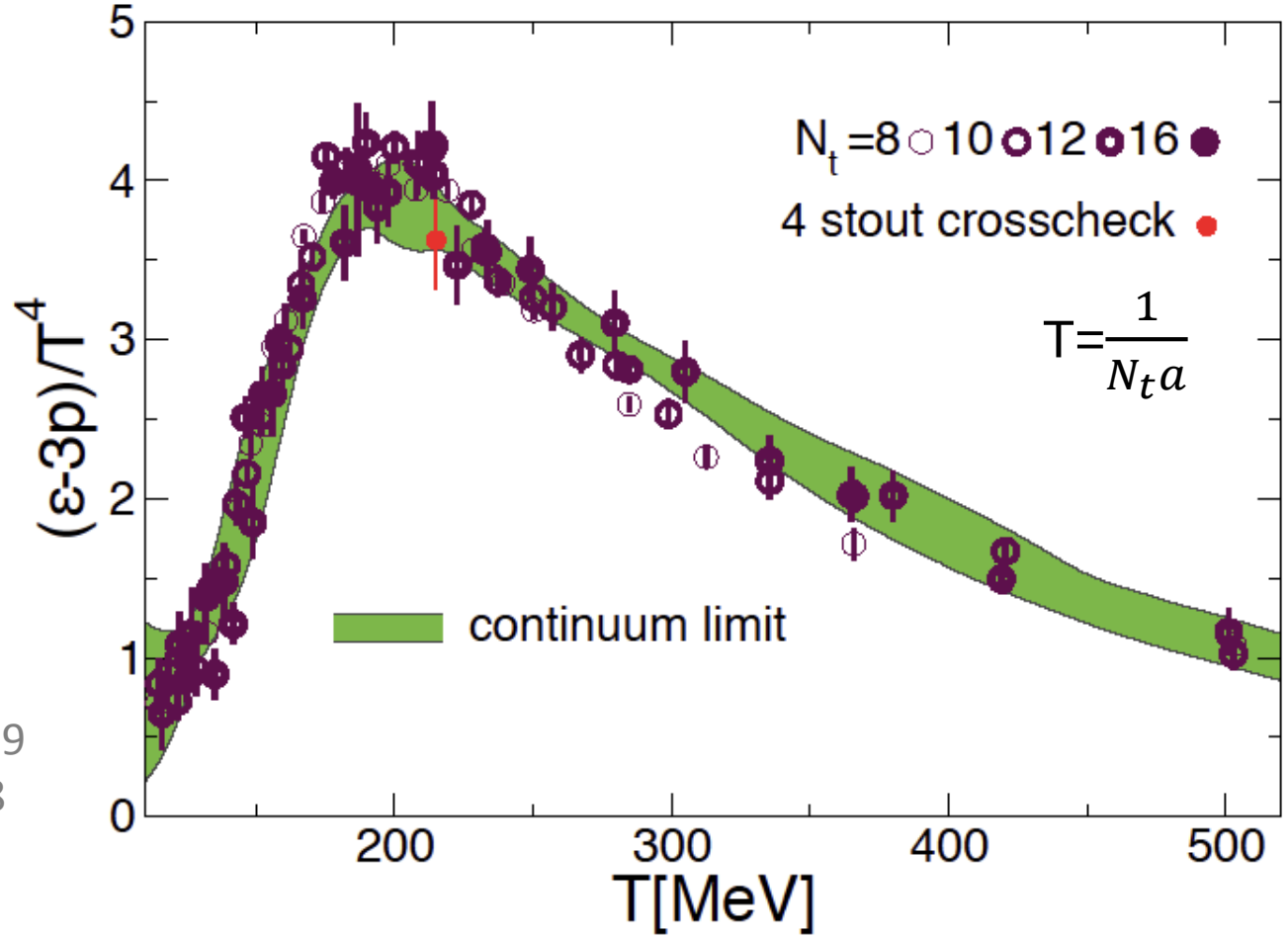
Mitglied der Helmholtz-Gemeinschaft

Overview

1. N_f=2+1 Equation of State (EoS)
2. Update on the N_f=2+1+1 EoS
3. Fluctuations
4. Crosschecks with other lattice actions
5. Conclusion and outlook

Mitglied der Helmholtz-Gemeinschaft

N_f=2+1 EoS



PLB **730**, 99
1309.5258

Mitglied der Helmholtz-Gemeinschaft

N_f=2+1 EoS

- We work at M_π = phys $\rightarrow$ no chiral extrapolation (→ 2d spline)
- No finite volume effects found in 2010/2011, 2013: larger vol

Systematics: histogram method

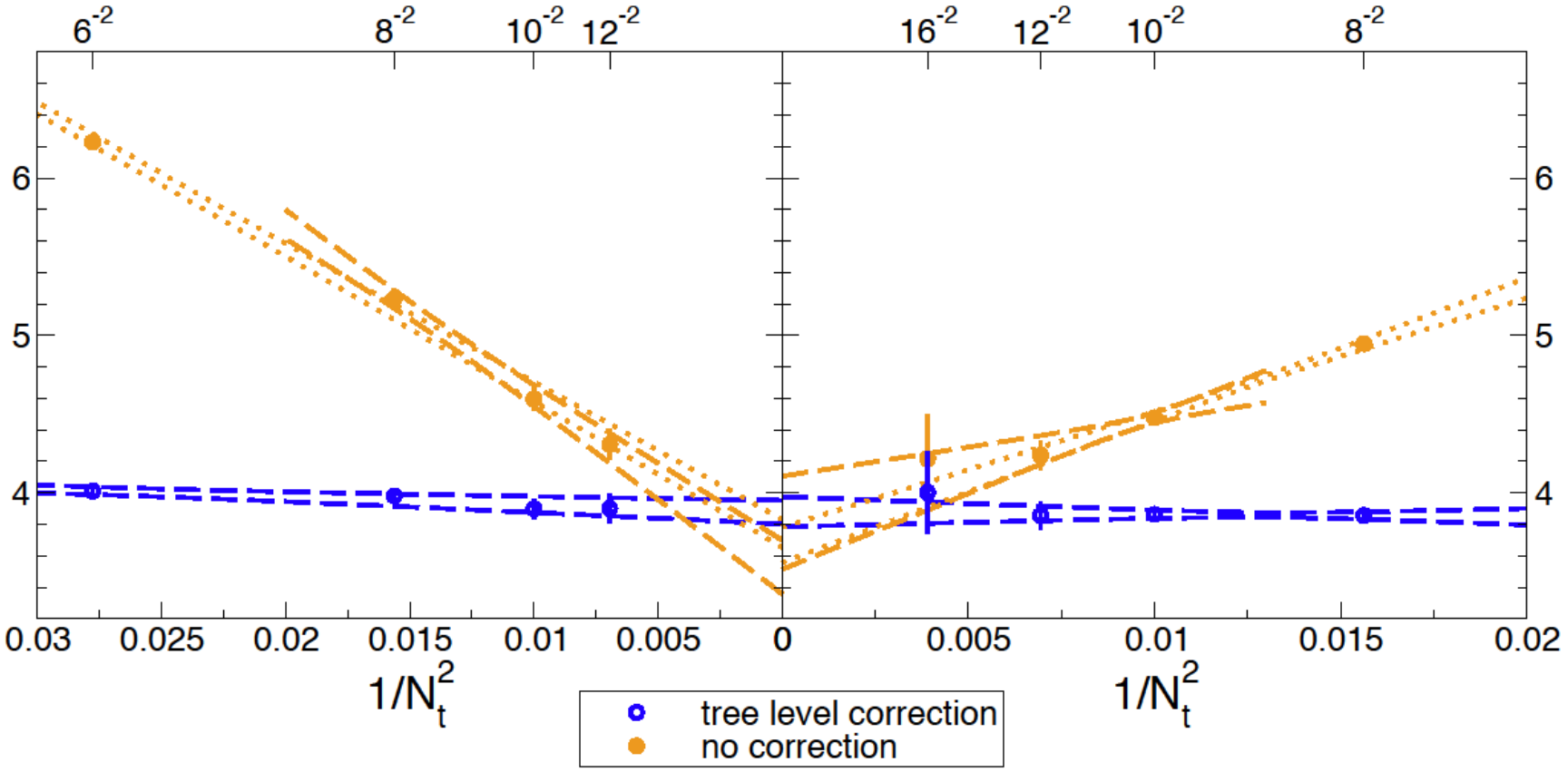
- vacuum fits, 7 different fit models (incl. direct subtr. w. interp.)
- continuum extrapolation
  - Vary node points (8 different sets)
  - Include or leave out leave N_t=6
  - With or without improvement factors
  - We use two different scale settings (f_k vs. w_0)
  - Fit includes a^2 or a^2 and a^4 terms

→ This results in $7 \times 8 \times 2 \times 2 \times 2 \times 2$ = 896 different fits

- Weighting: we consider AICc, Q, or unweighted histograms

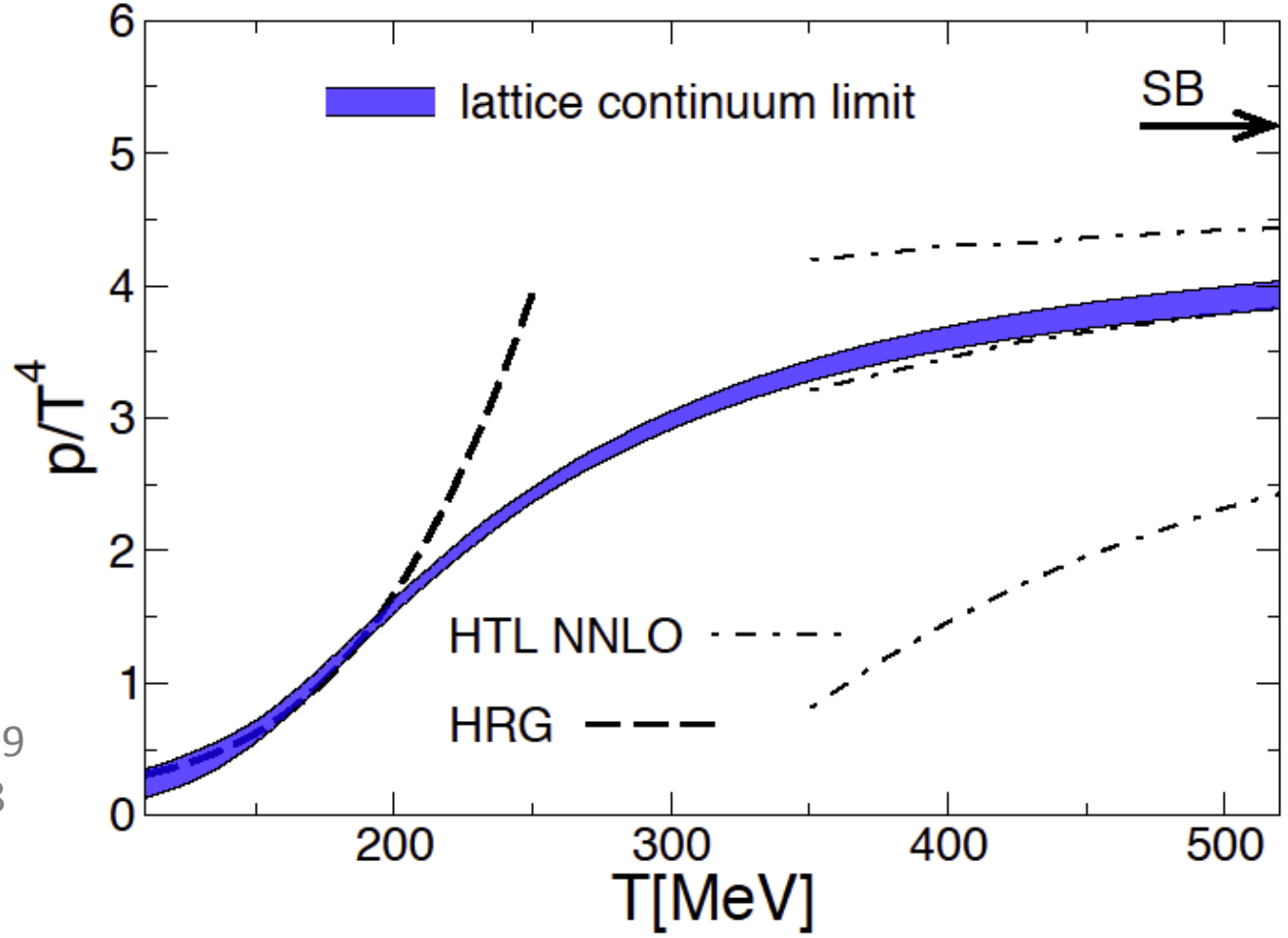
Mitglied der Helmholtz-Gemeinschaft

N_f=2+1 EoS: 214 MeV crosscheck



Mitglied der Helmholtz-Gemeinschaft

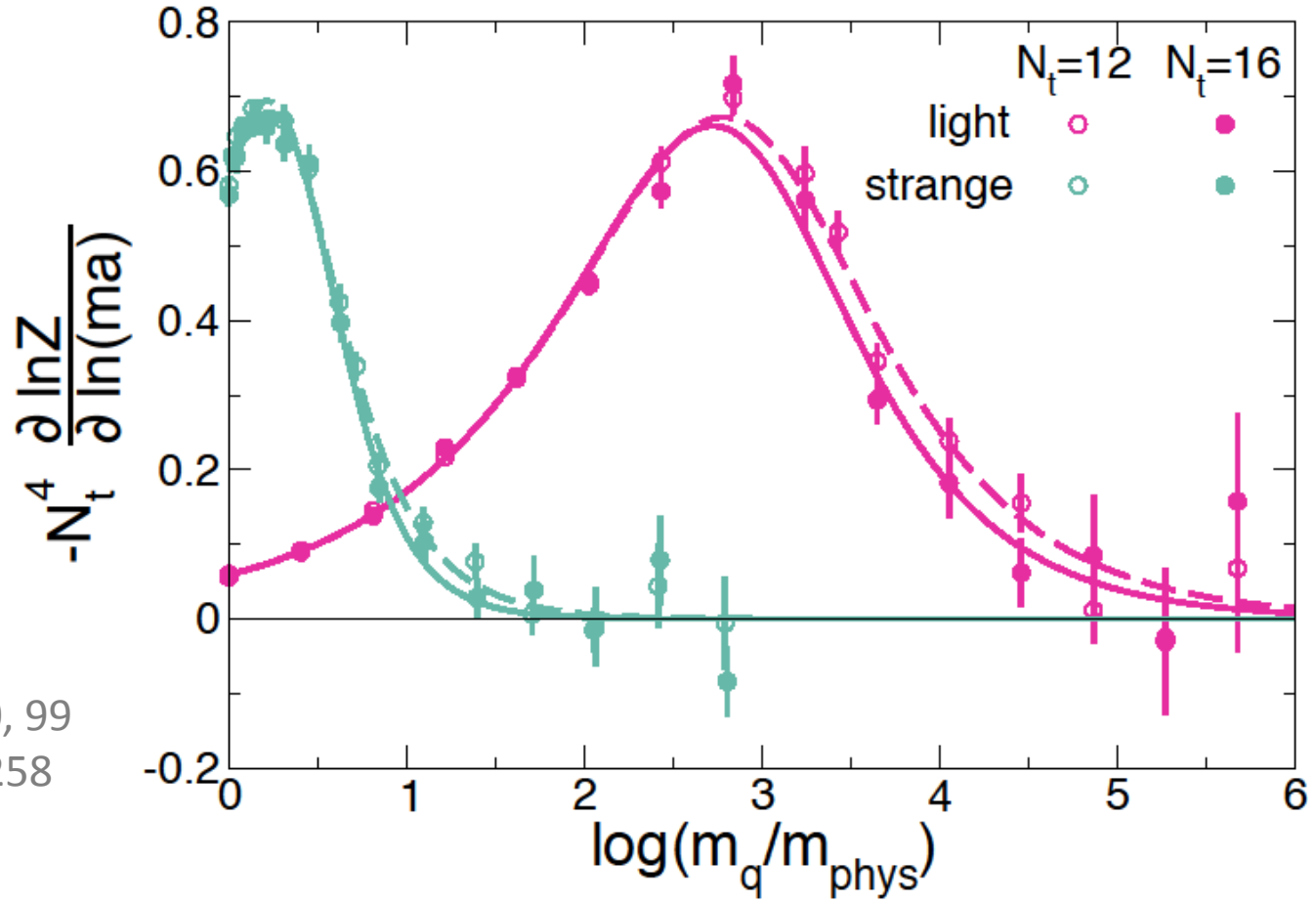
N_f=2+1 EoS: pressure et al.



PLB **730**, 99
1309.5258

Mitglied der Helmholtz-Gemeinschaft

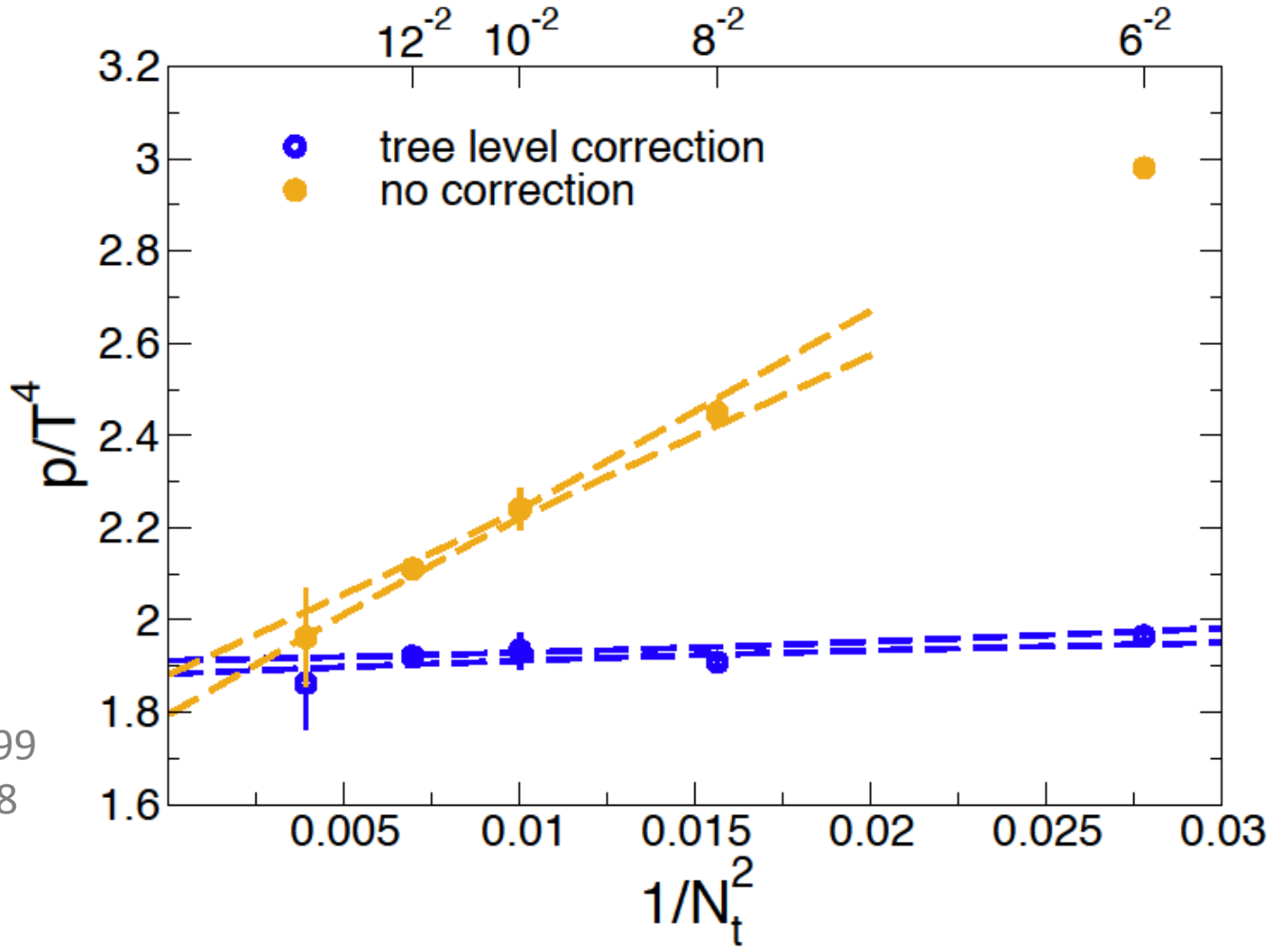
N_f=2+1 EoS: Normalization



PLB **730**, 99
1309.5258

Mitglied der Helmholtz-Gemeinschaft

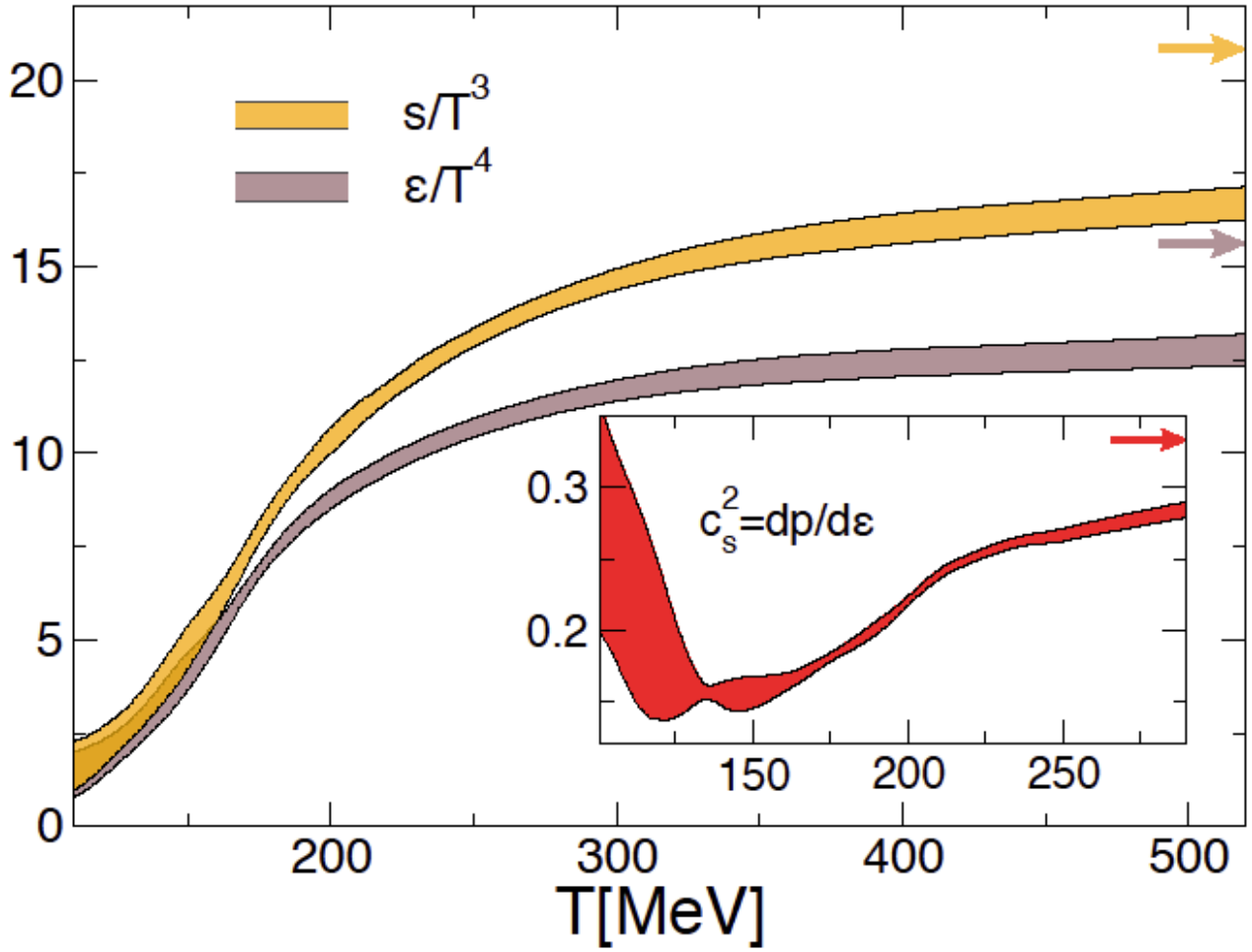
N_f=2+1 EoS: Normalization



PLB **730**, 99
1309.5258

Mitglied der Helmholtz-Gemeinschaft

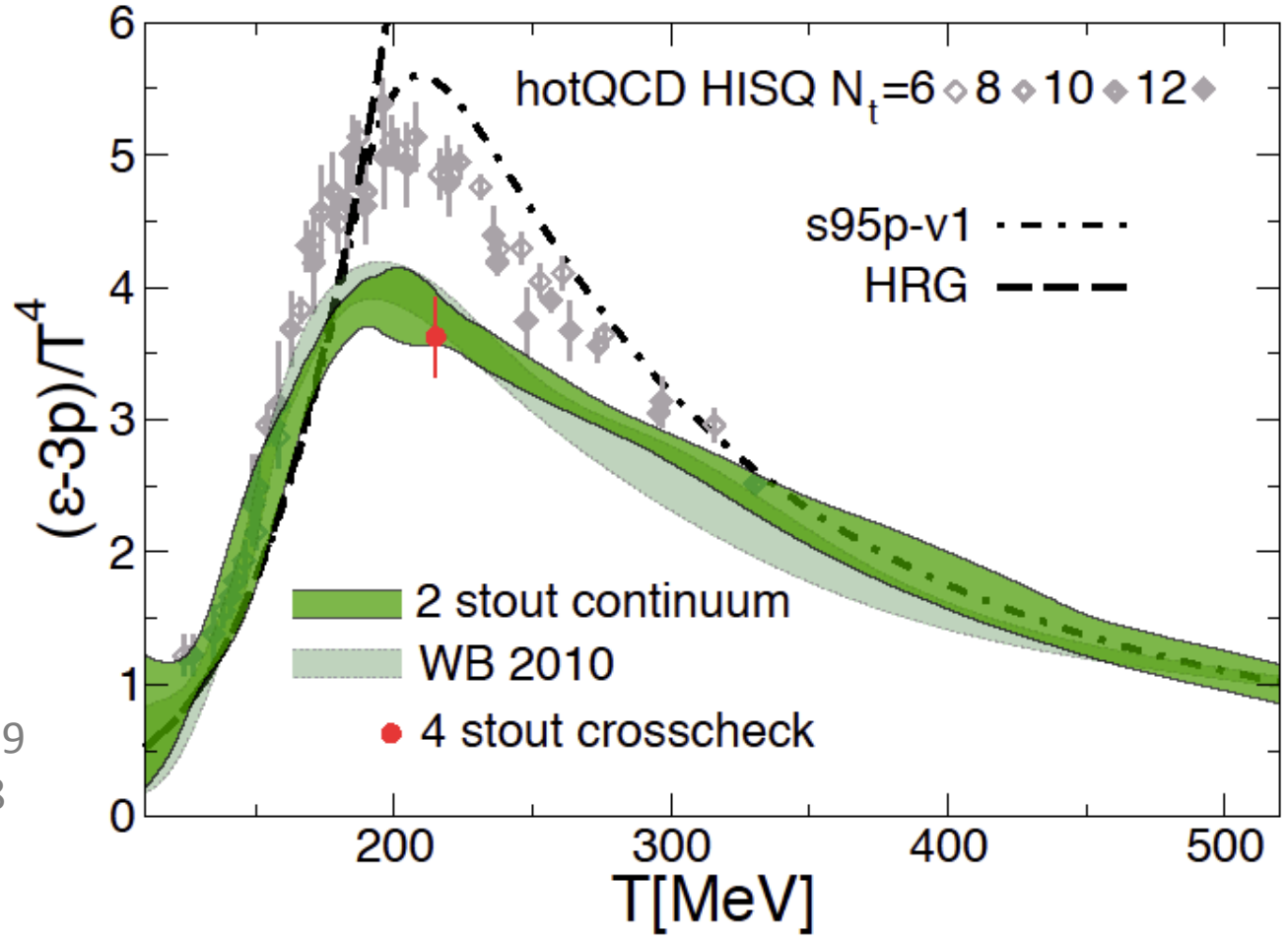
N_f=2+1 EoS: pressure et al.



PLB **730**, 99
1309.5258

Mitglied der Helmholtz-Gemeinschaft

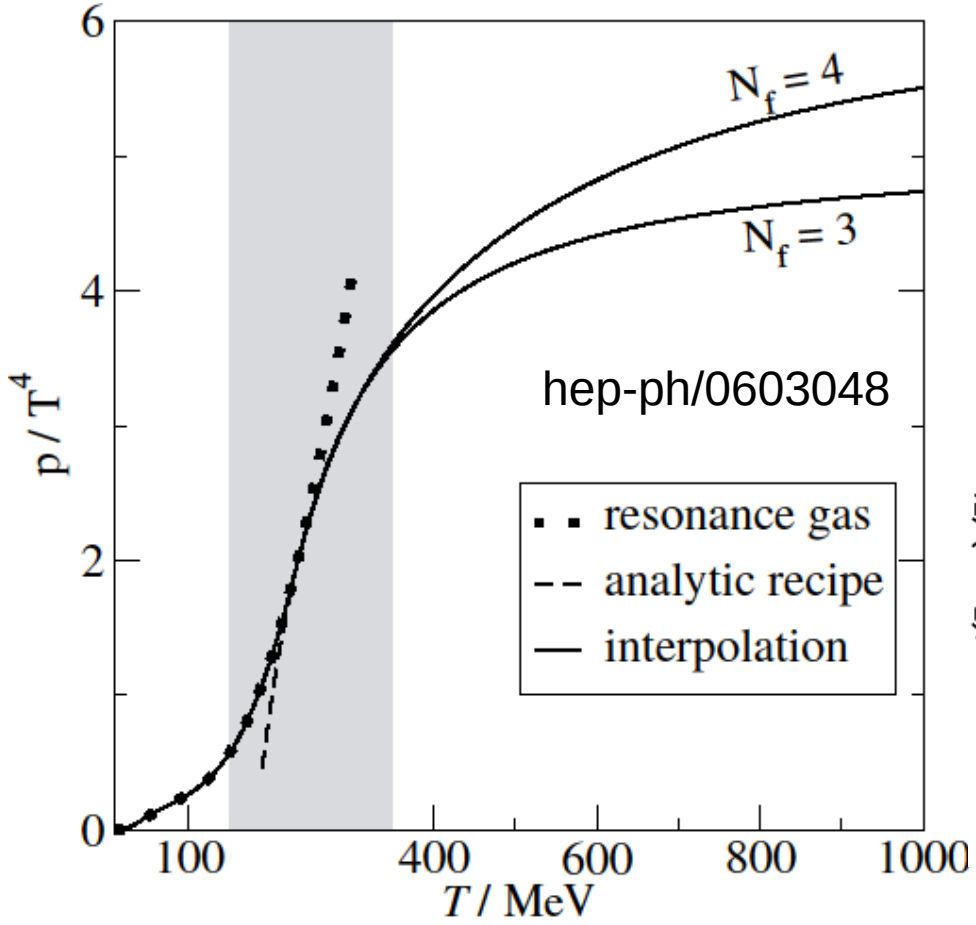
N_f=2+1 EoS: Comparison



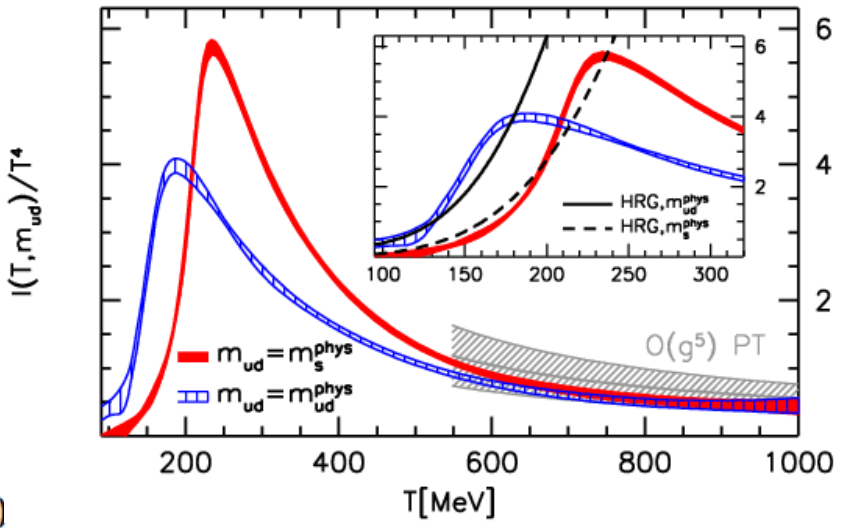
PLB **730**, 99
1309.5258

Mitglied der Helmholtz-Gemeinschaft

Update: N_f=2+1+1 EoS, perturbation therory

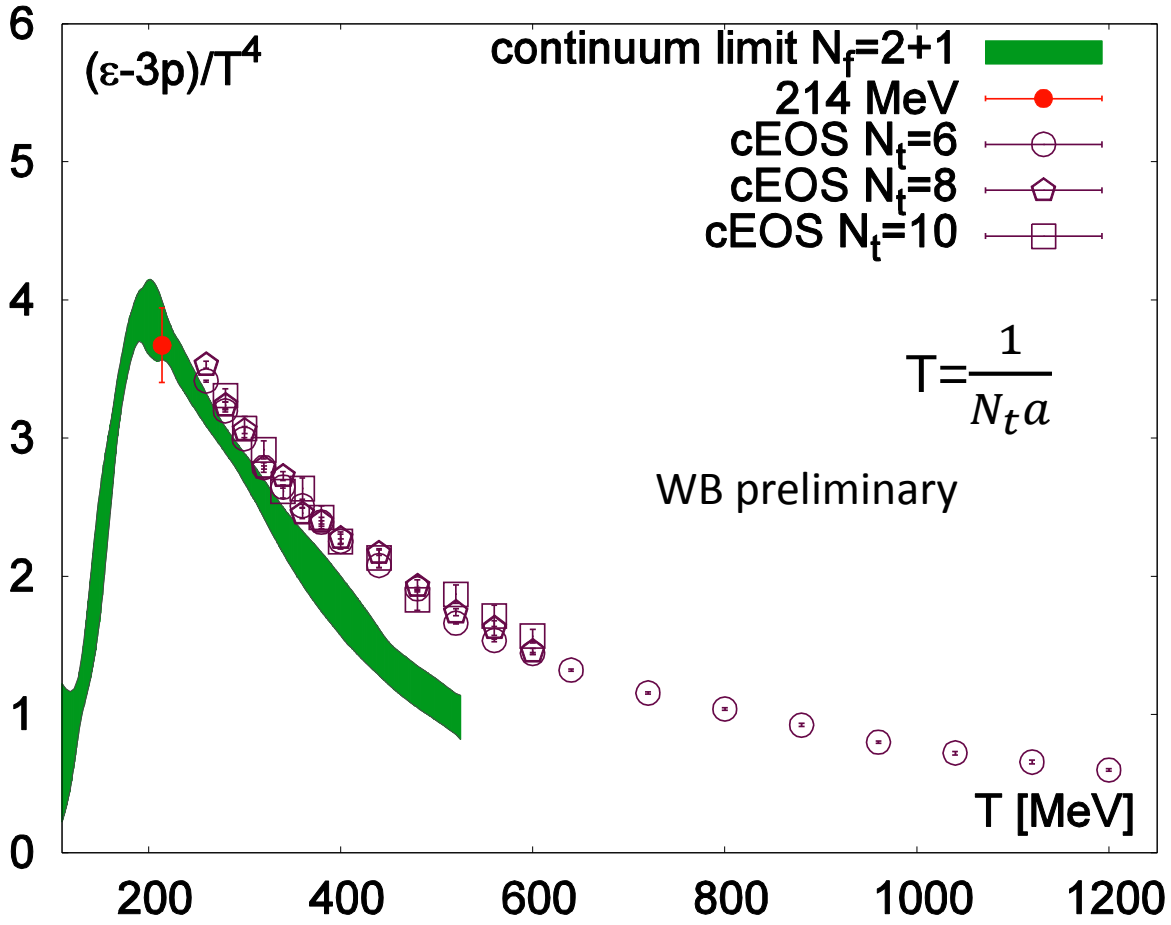


Calculate the EoS from LQCD from 100 to 1000 MeV



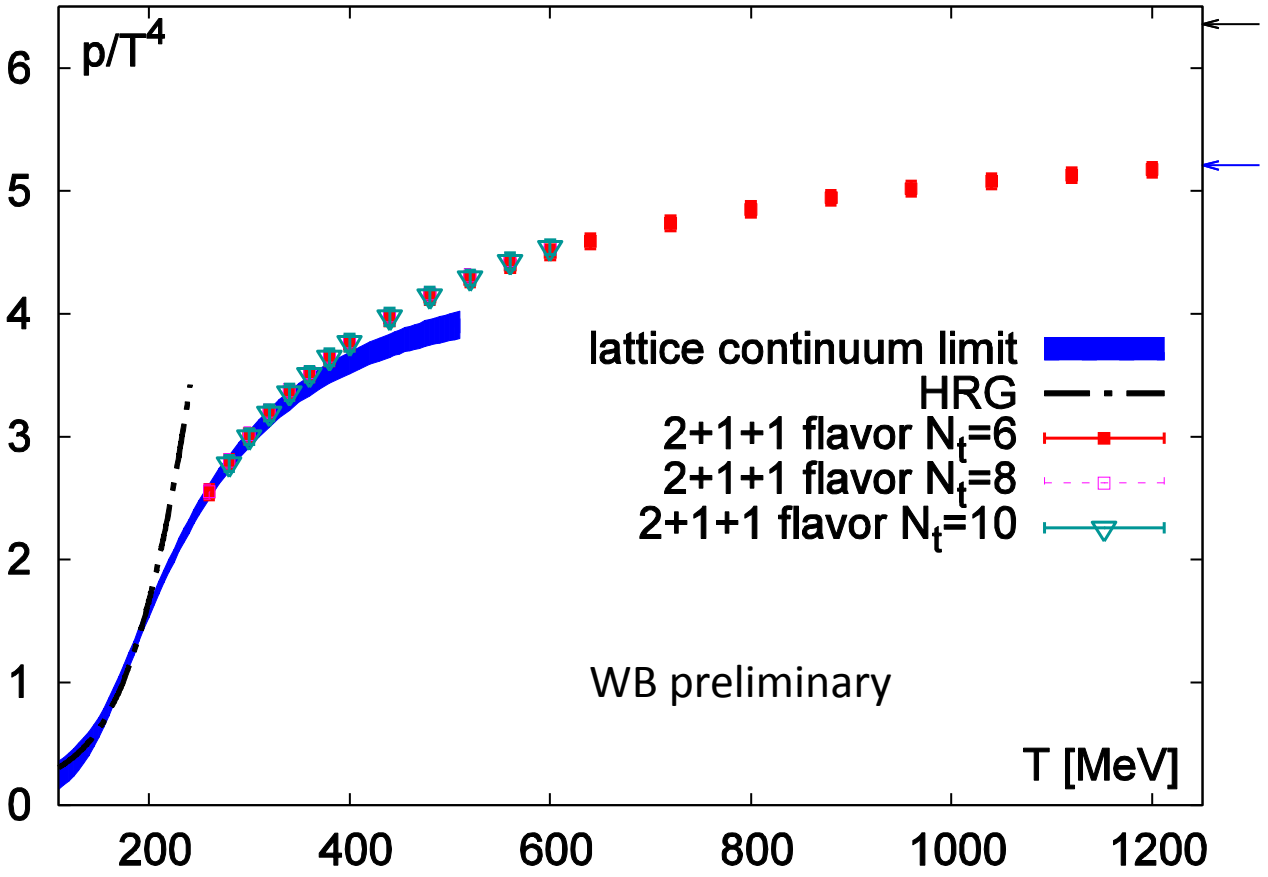
Mitglied der Helmholtz-Gemeinschaft

Update: N_f=2+1+1 EoS



Mitglied der Helmholtz-Gemeinschaft

Update: N_f=2+1+1 EoS



Mitglied der Helmholtz-Gemeinschaft

Fluctuations

- Net yields are given by $\langle N_X \rangle = -T \frac{\partial \log Z}{\partial \mu_X} \quad \langle N_X^2 \rangle - \langle N_X \rangle^2 = \frac{\partial^2 \log Z}{(\partial \mu_X / T)^2}$
- Defining: $\hat{\chi}_2^X = \frac{1}{VT^3} \frac{\partial^2 \log Z}{(\partial \mu_X / T)^2} \qquad \hat{\chi}_4^X = \frac{1}{VT^3} \frac{\partial^4 \log Z}{(\partial \mu_X / T)^4}$
- Computable at finite μ through:

$$\frac{p}{T^4} \equiv \frac{1}{VT^3} \ln Z(T, \mu_q) = \sum_{n=0}^{\infty} c_n(T, m_q) \left(\frac{\mu_q}{T}\right)^n \qquad c_n(T, m_q) = \frac{1}{n!} \frac{1}{VT^3} \frac{\partial^n \ln Z(T, \mu_q)}{\partial (\mu_q / T)^n} \bigg|_{\mu_q = 0}$$

- χ_2 requires (finite μ_B): $\hat{\chi}_{22}^{XB} = \frac{1}{VT^3} \frac{\partial^4 \log Z}{(\partial \mu_X / T)^2 (\partial \mu_B / T)^2}$

Challenges:
- **Derivatives come with volume penalty in the statistics**
- **Electric charge is pion-dominated → fine lattices and taste-improvement.**
- **Baryon number noisy → large statistics.**

Mitglied der Helmholtz-Gemeinschaft

Fluctuations: derivatives

$$\chi^{us}_{i,j} = \frac{T}{V}\frac{\partial^{i+j}\log Z}{(\partial\mu_u)^i(\partial\mu_s)^j}$$

$$\partial_i \log Z = \frac{1}{Z}\int \mathcal{D}U\ \partial_i e^{-S_{\text{eff}}} = \langle A_i\rangle \qquad (=0 \text{ at } \mu = 0)$$

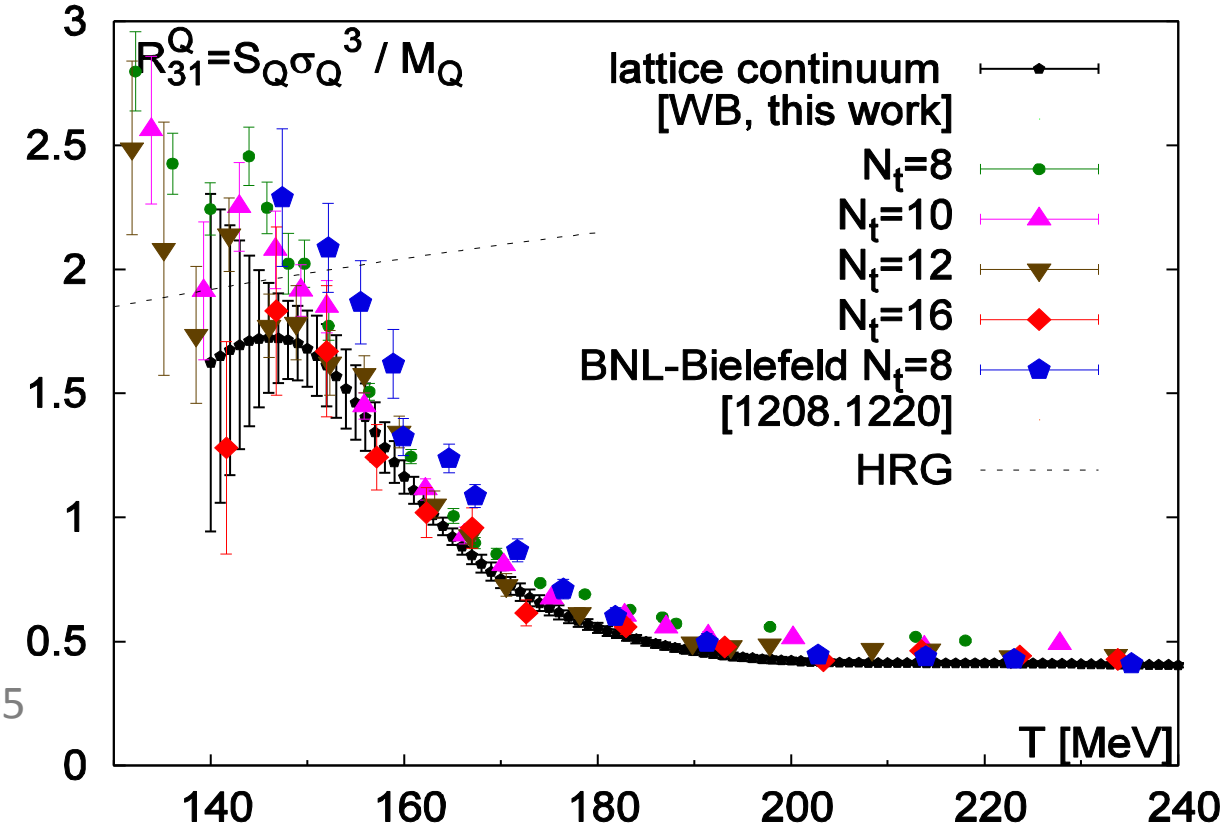
$$\begin{aligned}\partial_j \langle X\rangle &= -\langle X\rangle(\partial_j \log Z) + \left\langle X\partial_j e^{-S_{\text{eff}}}\right\rangle + \langle \partial_j X\rangle \\ &= \langle X A_j\rangle - \langle X\rangle\langle A_j\rangle + \langle \partial_j X\rangle\end{aligned}$$

$$\begin{aligned}\partial_i\partial_j \log Z &= \langle A_i A_j\rangle - \langle A_i\rangle\langle A_j\rangle + \delta_{ij}\langle B_i\rangle \\ \partial_i^4 \log Z &= \left\langle A_i^4\right\rangle - 3\left\langle A_i^2\right\rangle^2 + 3\left(\left\langle B_i^2\right\rangle - \langle B_i\rangle^2\right) \\ &\quad +6\left(\left\langle A_i^2 B_i\right\rangle - \left\langle A_i^2\right\rangle\langle B_i\rangle\right) + 4\langle A_i C_i\rangle + \langle D_i\rangle\end{aligned}$$

Mitglied der Helmholtz-Gemeinschaft

Charge fluctuations

$$\frac{\sigma_B^2}{M_B} = \frac{\chi_{2,\mu}^B}{\chi_{1,\mu}^B}, \qquad S_B\sigma_B = \frac{\chi_{3,\mu}^B}{\chi_{2,\mu}^B}, \qquad \kappa_B\sigma_B^2 = \frac{\chi_{4,\mu}^B}{\chi_{2,\mu}^B}, \qquad \frac{S_B\sigma_B^3}{M} = \frac{\chi_{3,\mu}^B}{\chi_{1,\mu}^B}$$



WB: PRL **111** 062005
arXiv:1305.5161

Mitglied der Helmholtz-Gemeinschaft

Fluctuations: error estimation

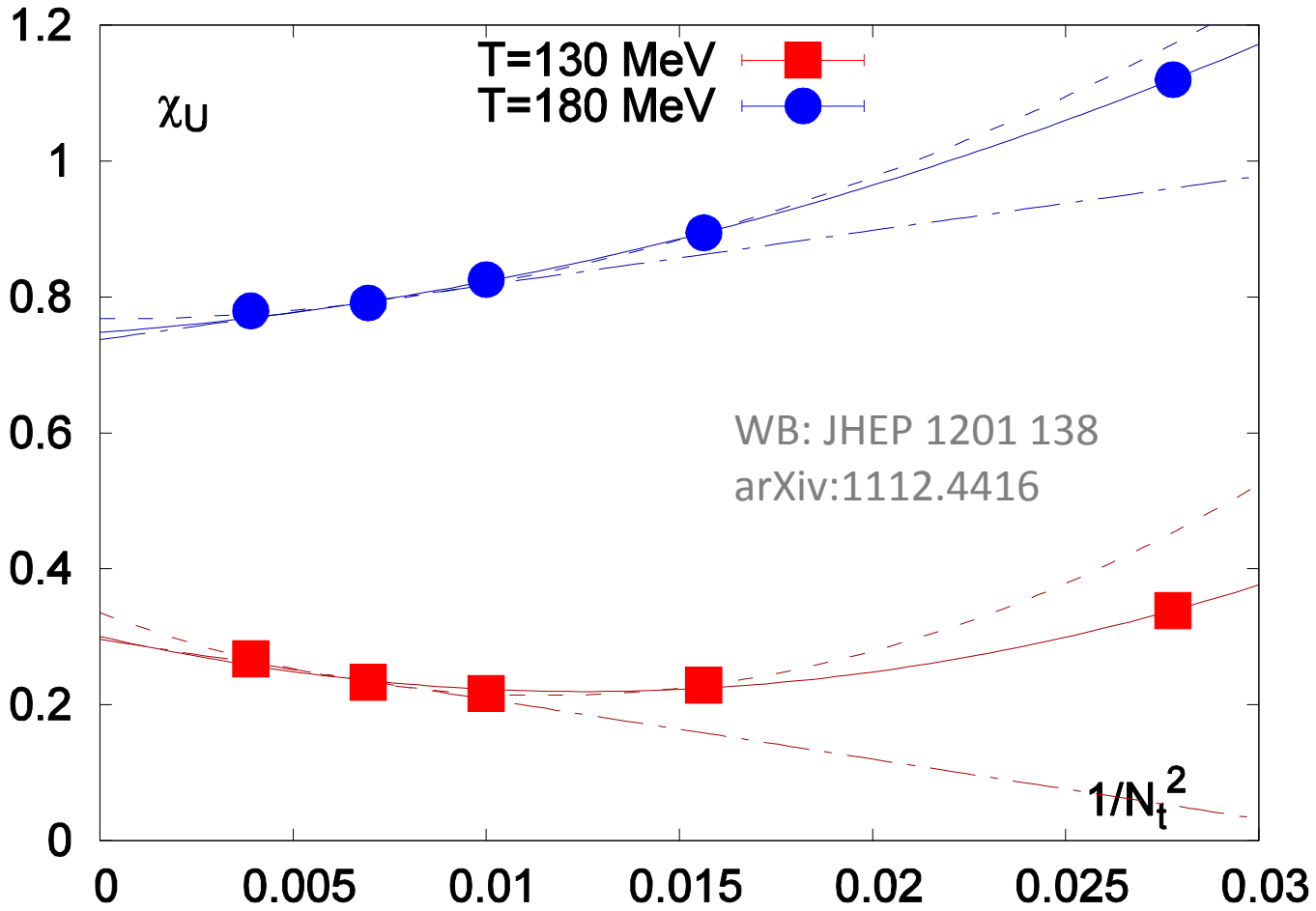
We follow our histogram ansatz as in EoS calculation

- Continuum extrapolation:
  - Multiple sets of nodepoints for spline interpolation
  - Extrapolate χ and $1/\chi$
  - Use different extrapolation formulas as in EoS
- Use jacknife estimate of deviation form mean to estimate statistical uncertainty

Systematic uncertainty is dominant

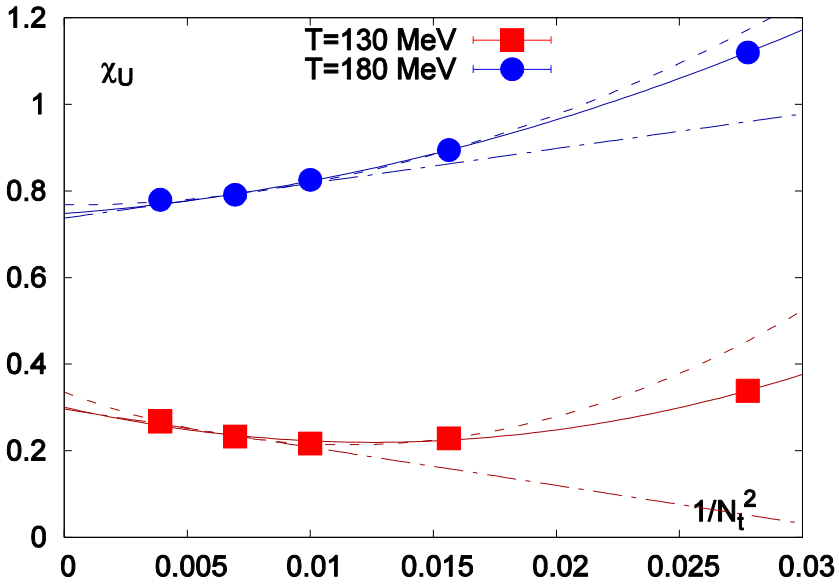
Mitglied der Helmholtz-Gemeinschaft

Fluctuations: continuum extrapolation

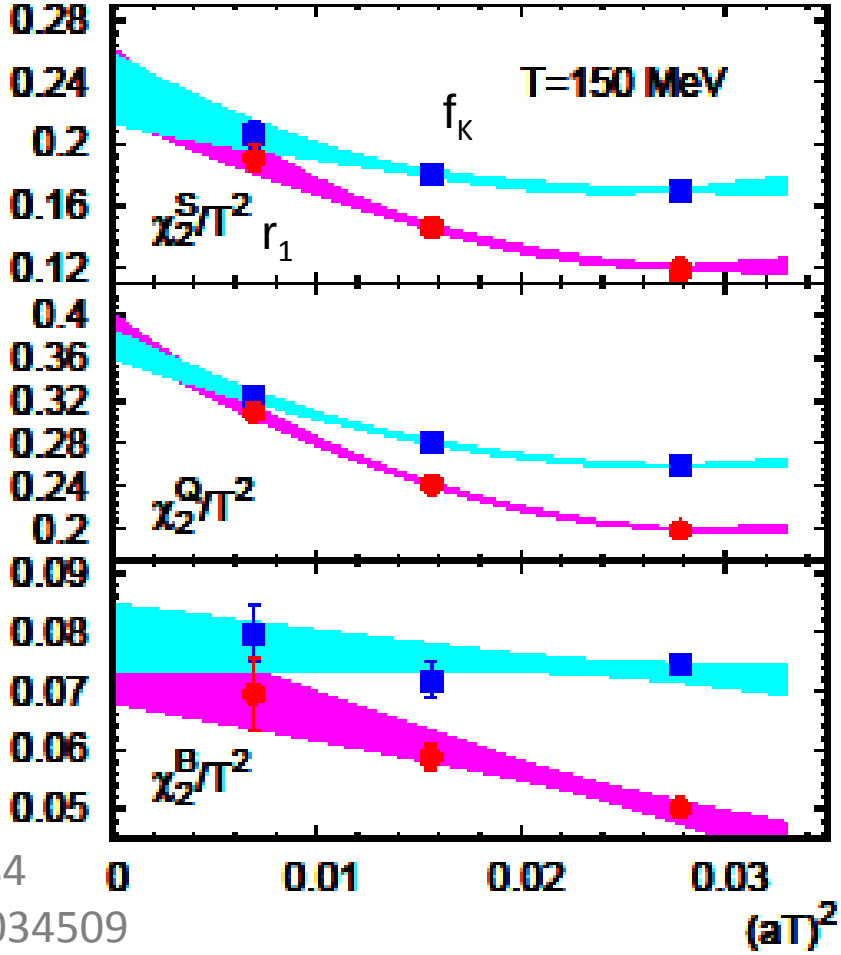


Mitglied der Helmholtz-Gemeinschaft

Fluctuations: continuum extrapolation



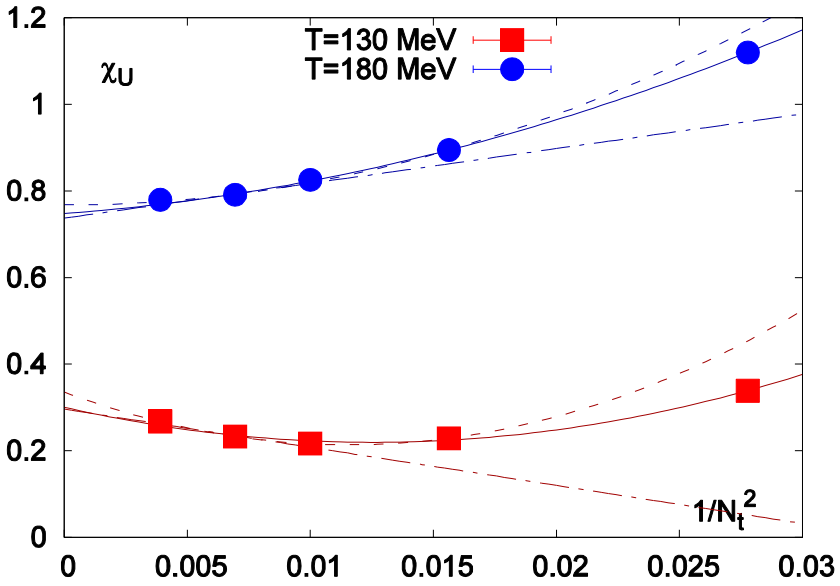
WB: JHEP 1201 138
arXiv:1112.4416



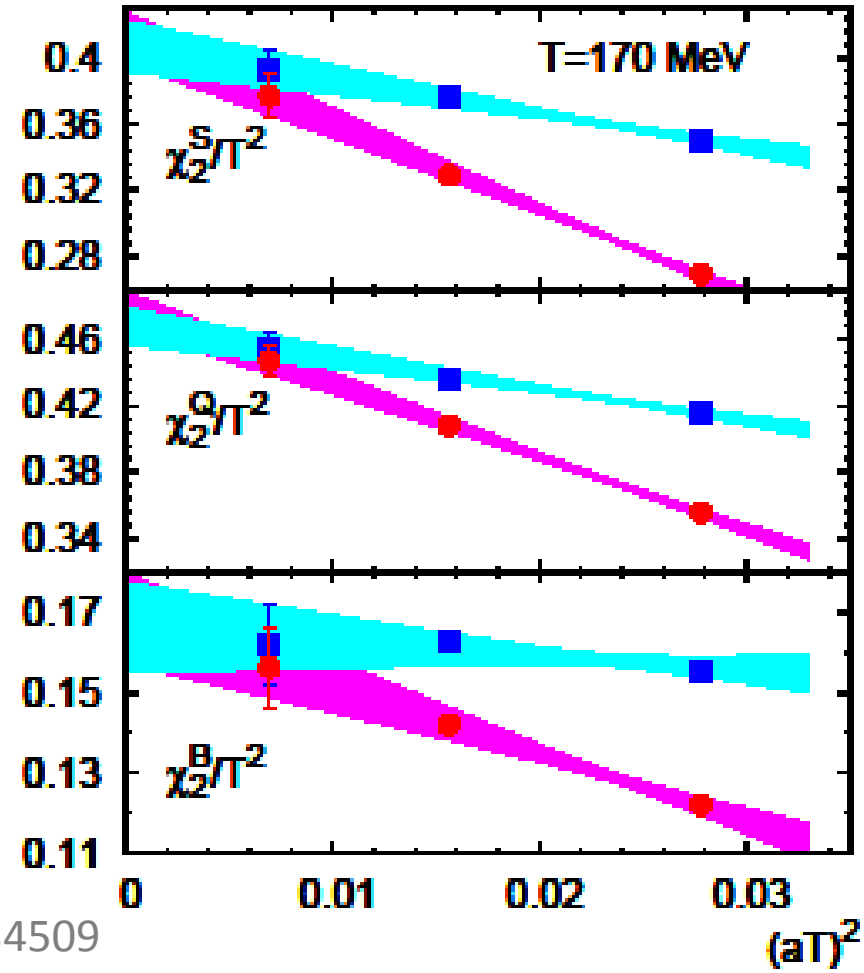
hotQCD:
1203.0784
PRD 86, 034509

Mitglied der Helmholtz-Gemeinschaft

Fluctuations: continuum extrapolation



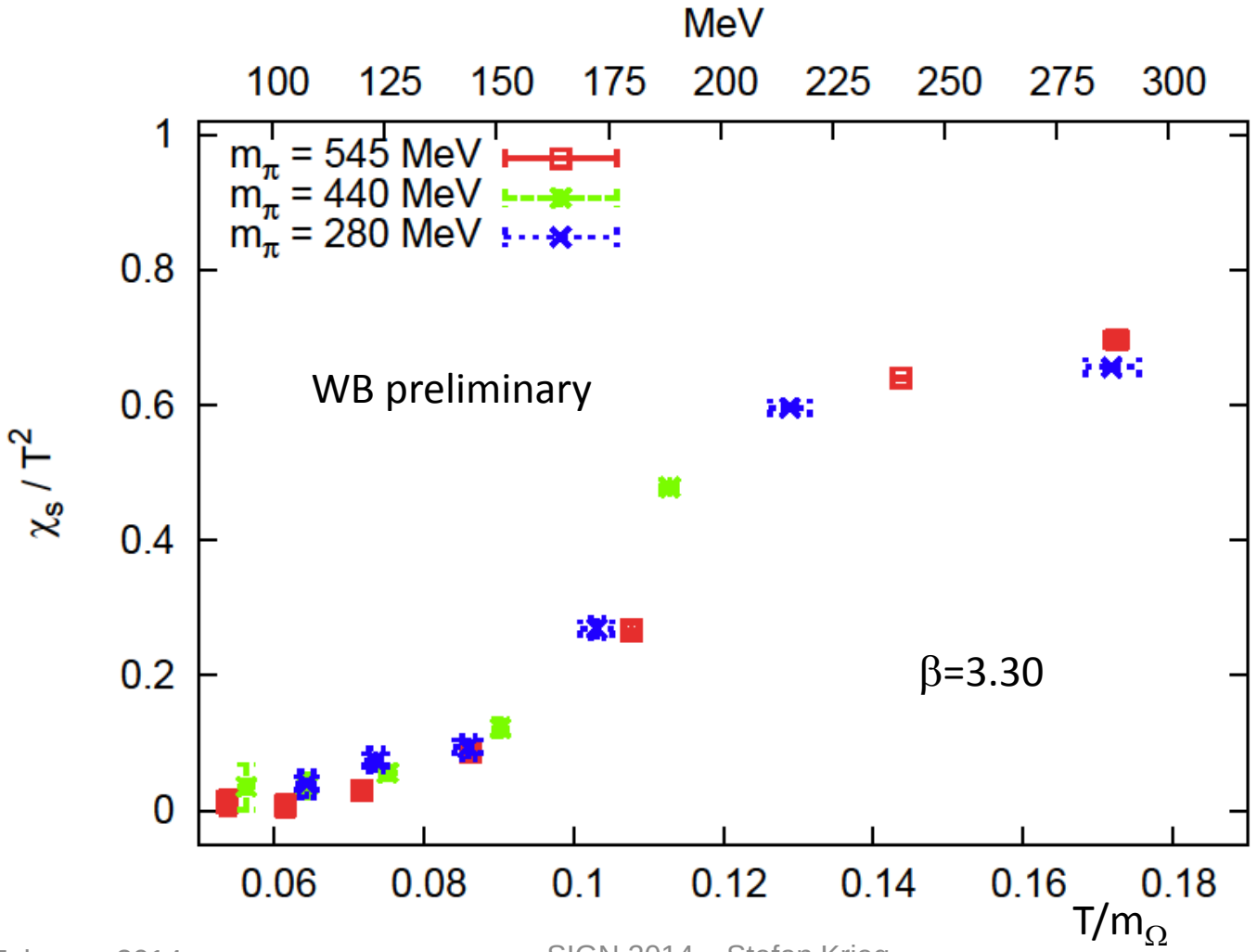
WB: JHEP 1201 138
arXiv:1112.4416



hotQCD:
1203.0784
PRD 86, 034509

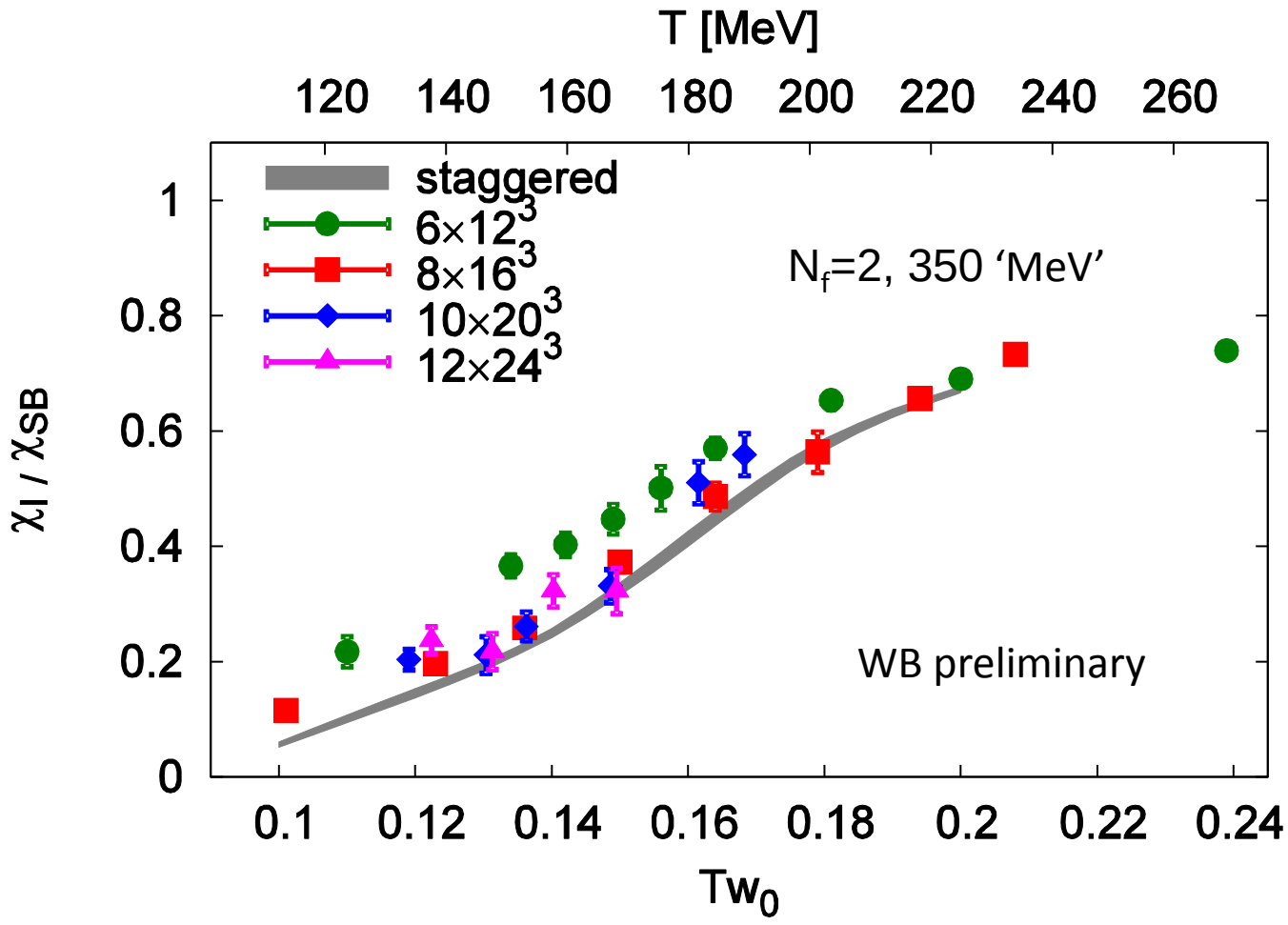
Mitglied der Helmholtz-Gemeinschaft

Updates and crosschecks: Wilson thermo



Mitglied der Helmholtz-Gemeinschaft

Updates and crosschecks: Overlap thermo



Mitglied der Helmholtz-Gemeinschaft

Conclusion and outlook

- Precision of LQCD results in thermodynamics greatly improved
- EoS for N_f=2+1 flavors available (PLB 730, 99; 1309.5258)
- Preliminary N_f=2+1+1 flavor results
- Fluctuations can be calculated with higher orders (from χ_6) being increasingly computationally costly
- Complete control of LQCD errors
- Augmented by crosschecks using other discretizations

Mitglied der Helmholtz-Gemeinschaft

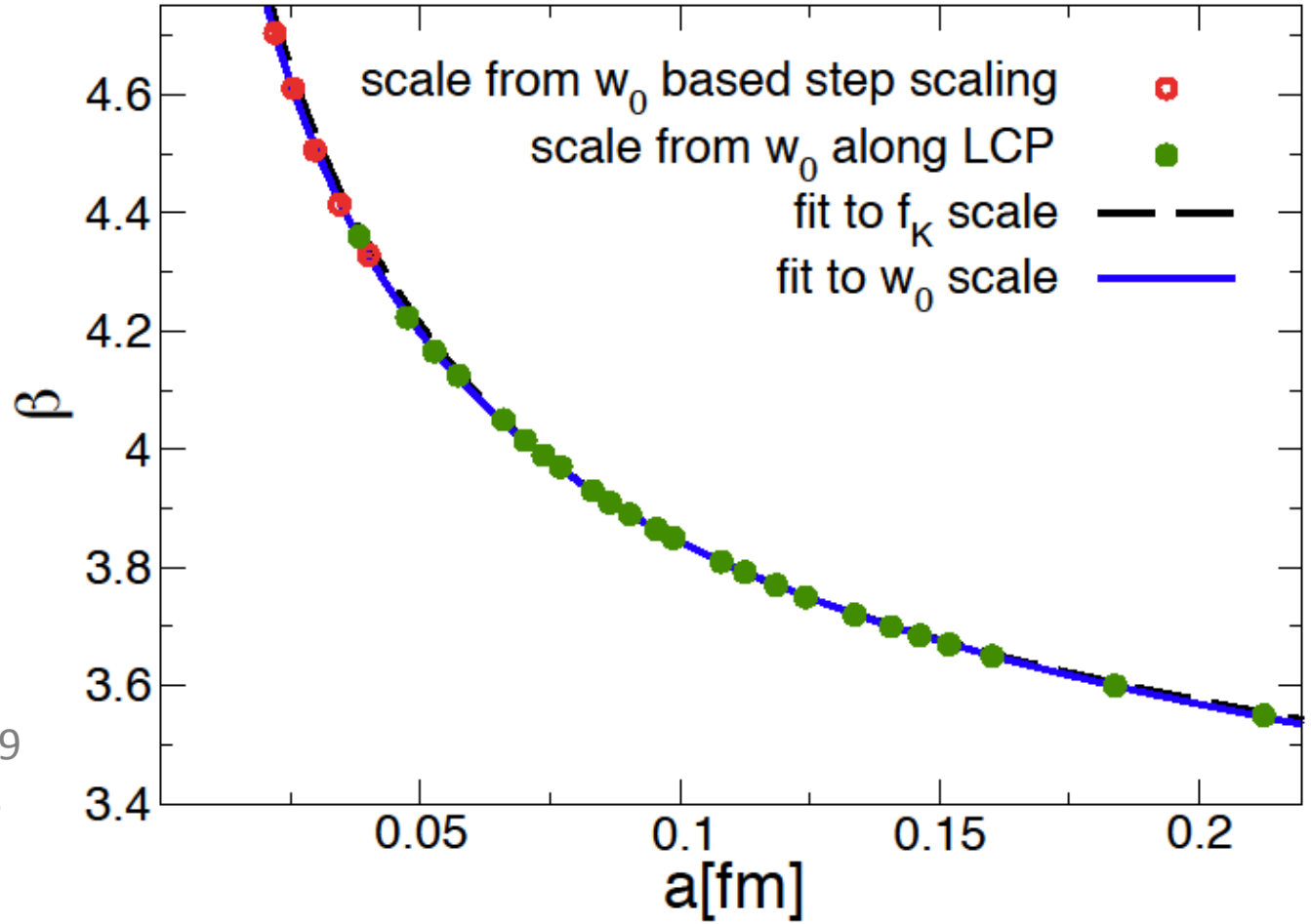
Thank You.

Mitglied der Helmholtz-Gemeinschaft

Backup

Mitglied der Helmholtz-Gemeinschaft

EoS: Scale setting



PLB **730**, 99
1309.5258

Mitglied der Helmholtz-Gemeinschaft

Fluctuations: lattice techniques

- Use SET to estimate traces (Gottlieb et al. PRL 59 (**1987**) 2247)

$$\mathtt{tr}(A) \approx \frac{1}{s} \sum_{k=1}^{s} v_k^\top A v_k$$

- Disconnected diagrams $\rightarrow$ use different sets of random vectors ($c_4 \rightarrow 4\times$ inversions)
- diagrams (e.g. $\langle A_i^2 \rangle$ and $\langle A_i^2 \cdot B \rangle$) allow reuse of inversions $\rightarrow$ increase statistics
- We use up to 1000 sources to estimate the trace or O(10k) inversions

Mitglied der Helmholtz-Gemeinschaft