

Fluctuations of conserved charges: lattice meets experiment

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S. Borsanyi, Z. Fodor, S. Katz, S. Krieg, C. R., K. Szabo, arXiv:1403.4576

Motivation

- ❖ We live in a **very exciting era** to understand the fundamental constituents of matter and the evolution of the Universe
- ❖ We can create the **deconfined phase of QCD** in the laboratory
- ❖ Lattice QCD simulations have reached unprecedented levels of accuracy
 - ⇒ physical quark masses
 - ⇒ several lattice spacings → continuum limit
- ❖ The joint information between **theory** and **experiment** can help us to shed light on QCD

Susceptibilities of conserved charges

- ❖ The **deconfined phase** of QCD can be reached in the laboratory
- ❖ Need for **unambiguous observables** to identify the phase transition
 - ❖ susceptibilities of conserved charges (baryon number, electric charge, strangeness)
S. Jeon and V. Koch (2000), M. Asakawa, U. Heinz, B. Müller (2000)
- ❖ A rapid change of these observables in the vicinity of T_c provides an unambiguous signal for **deconfinement**
- ❖ They can be calculated **on the lattice** as combinations of **quark number susceptibilities**
- ❖ They can be directly compared to experimental measurements

The observables under study

❖ The chemical potentials are related:

$$\begin{aligned}\mu_u &= \frac{1}{3}\mu_B + \frac{2}{3}\mu_Q; \\ \mu_d &= \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q; \\ \mu_s &= \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q - \mu_S.\end{aligned}$$

❖ susceptibilities are defined as follows:

$$\chi_{lmn}^{BSQ} = \frac{\partial^{l+m+n} p / T^4}{\partial(\mu_B/T)^l \partial(\mu_S/T)^m \partial(\mu_Q/T)^n}.$$

❖ Quadratic susceptibilities:

$$\chi_2^X = \frac{1}{VT^3} \langle N_X^2 \rangle$$

❖ Correlators between different charges:

$$\chi_{11}^{XY} = \frac{1}{VT^3} \langle N_X N_Y \rangle.$$

Physical meaning

- ❖ Diagonal susceptibilities measure the response of **quark densities** to an infinitesimal change in the **chemical potential**

$$\chi_2^X = \frac{\partial^2 p/T^4}{\partial(\mu_X/T)^2} = \frac{\partial}{\partial(\mu_X/T)} \left(n_X/T^3 \right)$$

- ➡ A **rapid increase** of these observables in a certain temperature range signals a **phase transition**

- ❖ Non-diagonal susceptibilities measure the **correlation** between different quark flavors

$$\chi_{11}^{XY} = \frac{\partial^2 p/T^4}{\partial(\mu_X/T)\partial(\mu_Y/T)} = \frac{\partial}{\partial(\mu_Y/T)} \left(n_X/T^3 \right)$$

- ➡ They can provide information about **bound-state survival** above the phase transition

Relating lattice results to experimental measurement

❖ the first four cumulants are:

$$\chi_1 = \langle(\delta x)\rangle \quad \chi_2 = \langle(\delta x)^2\rangle$$

$$\chi_3 = \langle(\delta x)^3\rangle \quad \chi_4 = \langle(\delta x)^4\rangle - 3\langle(\delta x)^2\rangle^2$$

❖ we can relate them to higher moments of multiplicity distributions:

$$\text{mean : } M = \chi_1$$

$$\text{variance : } \sigma^2 = \chi_2$$

$$\text{skewness : } S = \chi_3/\chi_2^{3/2}$$

$$\text{kurtosis : } \kappa = \chi_4/\chi_2^2$$

$$S\sigma = \chi_3/\chi_2$$

$$\kappa\sigma^2 = \chi_4/\chi_2$$

$$M/\sigma^2 = \chi_1/\chi_2$$

$$S\sigma^3/M = \chi_3/\chi_1$$

F. Karsch (2012)

Caveats

- ❖ Effects due to volume variation because of finite centrality bin width
- ❖ Finite reconstruction efficiency
- ❖ Spallation protons
- ❖ Canonical vs Grand Canonical ensemble
- ❖ Proton multiplicity distributions vs baryon number fluctuations
- ❖ Final-state interactions in the hadronic phase [J.Steinheimer *et al.*, PRL \(2013\)](#)

Caveats

- ❖ Effects due to volume variation because of finite centrality bin width
 - ➡ Experimentally corrected by centrality-bin-width correction method
- ❖ Finite reconstruction efficiency
 - ➡ Experimentally corrected based on binomial distribution [A. Bzdak, V. Koch, PRC \(2012\)](#)
- ❖ Spallation protons
 - ➡ Experimentally removed with proper cuts in p_T
- ❖ Canonical vs Grand Canonical ensemble
 - ➡ Experimental cuts in the kinematics and acceptance [V. Koch, S. Jeon, PRL \(2000\)](#)
- ❖ Proton multiplicity distributions vs baryon number fluctuations
 - ➡ Numerically very similar once protons are properly treated [M. Asakawa and M. Kitazawa, PRC \(2012\)](#), [M. Nahrgang *et al.*, 1402.1238](#)
- ❖ Final-state interactions in the hadronic phase [J. Steinheimer *et al.*, PRL \(2013\)](#)
 - ➡ Consistency between different charges = fundamental test

Relations between chemical potentials

❖ μ_B , μ_S and μ_Q are NOT independent:

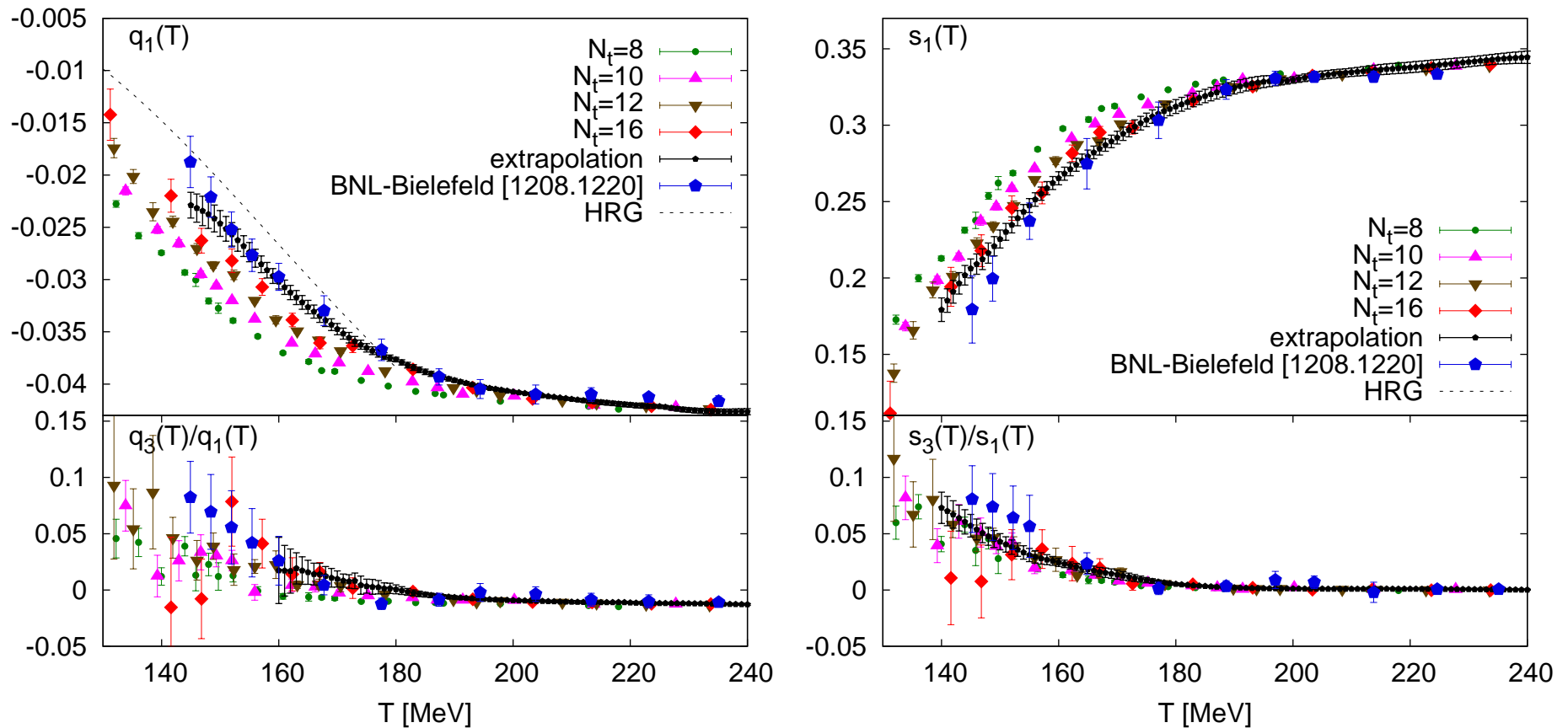
$$\langle n_S \rangle = 0 \quad \langle n_Q \rangle = \frac{Z}{A} \langle n_B \rangle \quad \Rightarrow \quad \frac{Z}{A} = 0.4$$

❖ By expanding n_B , n_S and n_Q up to μ_B^3 we get:

$$\mu_Q(T, \mu_B) = q_1(T)\mu_B + q_3(T)\mu_B^3 + \dots$$

$$\mu_S(T, \mu_B) = s_1(T)\mu_B + s_3(T)\mu_B^3 + \dots$$

Taylor coefficients: results



WB Collaboration: PRL (2013)

- ❖ μ_Q turns out to be very small
- ❖ Agreement between WB and BNL-Bielefeld collaborations

Thermometer and Baryometer

❖ R_{31}^B : thermometer

$$R_{31}^B(T, \mu_B) = \frac{\chi_3^B(T, \mu_B)}{\chi_1^B(T, \mu_B)} = \frac{\chi_4^B(T, 0) + \chi_{31}^{BQ}(T, 0)q_1(T) + \chi_{31}^{BS}(T, 0)s_1(T)}{\chi_2^B(T, 0) + \chi_{11}^{BQ}(T, 0)q_1(T) + \chi_{11}^{BS}(T, 0)s_1(T)} + \mathcal{O}(\mu_B^2)$$

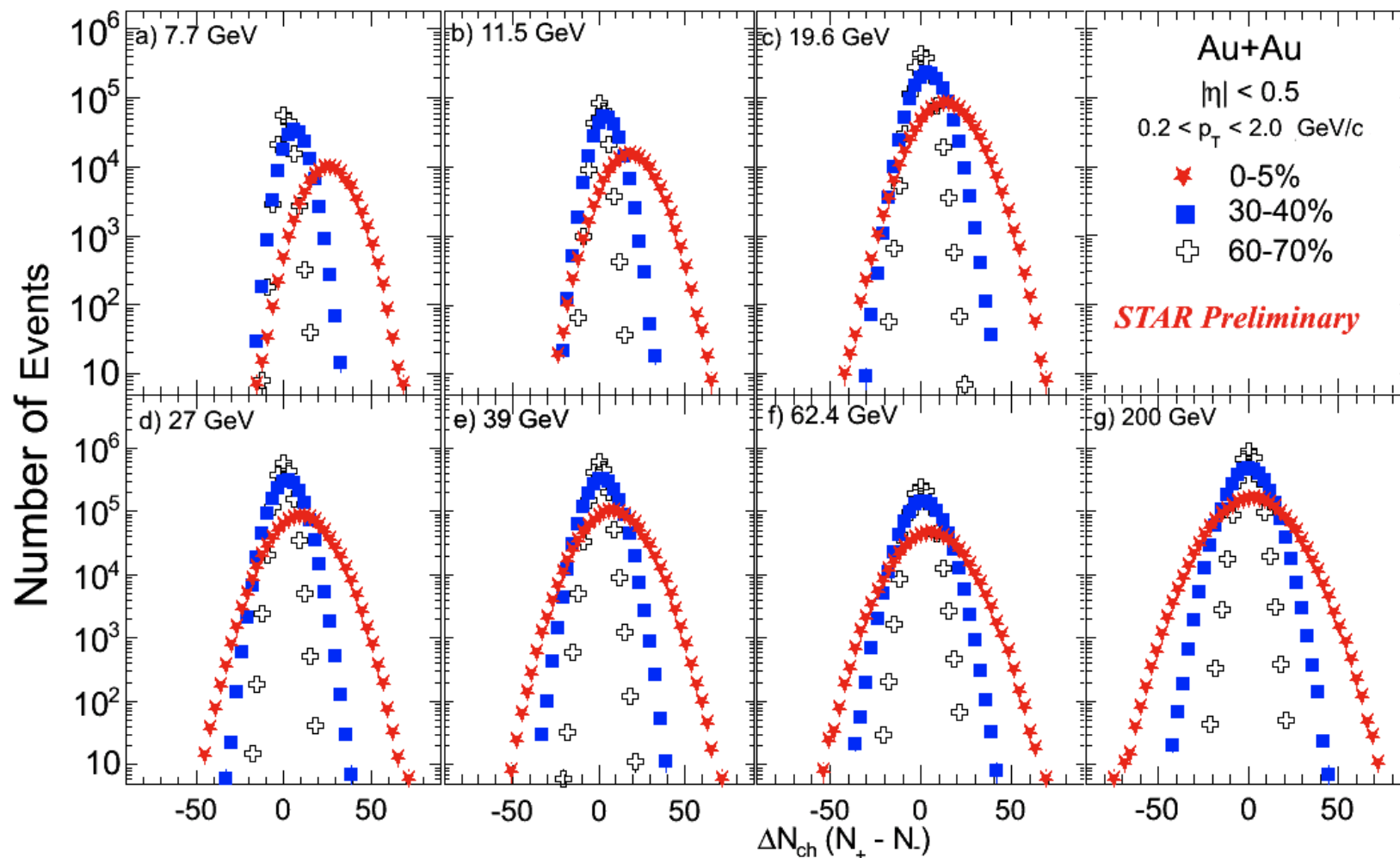
❖ Expand numerator and denominator around $\mu_B = 0$: ratio is independent of μ_B

❖ R_{12}^B : baryometer

$$R_{12}^B(T, \mu_B) = \frac{\chi_1^B(T, \mu_B)}{\chi_2^B(T, \mu_B)} = \frac{\chi_2^B(T, 0) + \chi_{11}^{BQ}(T, 0)q_1(T) + \chi_{11}^{BS}(T, 0)s_1(T)}{\chi_2^B(T, 0)} \frac{\mu_B}{T} + \mathcal{O}(\mu_B^3)$$

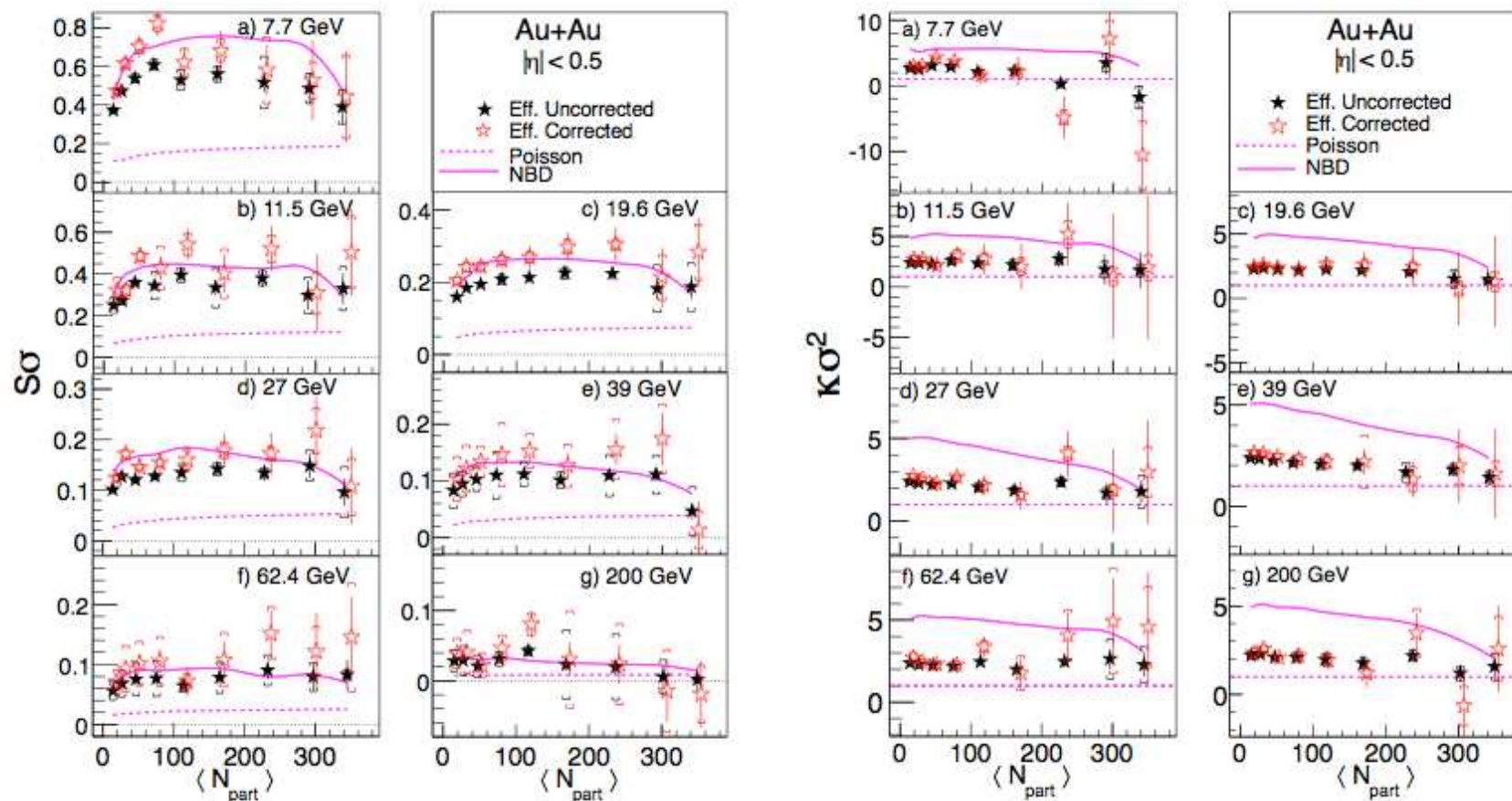
❖ Expand numerator and denominator around $\mu_B = 0$: ratio is proportional to μ_B

Experimental measurement I



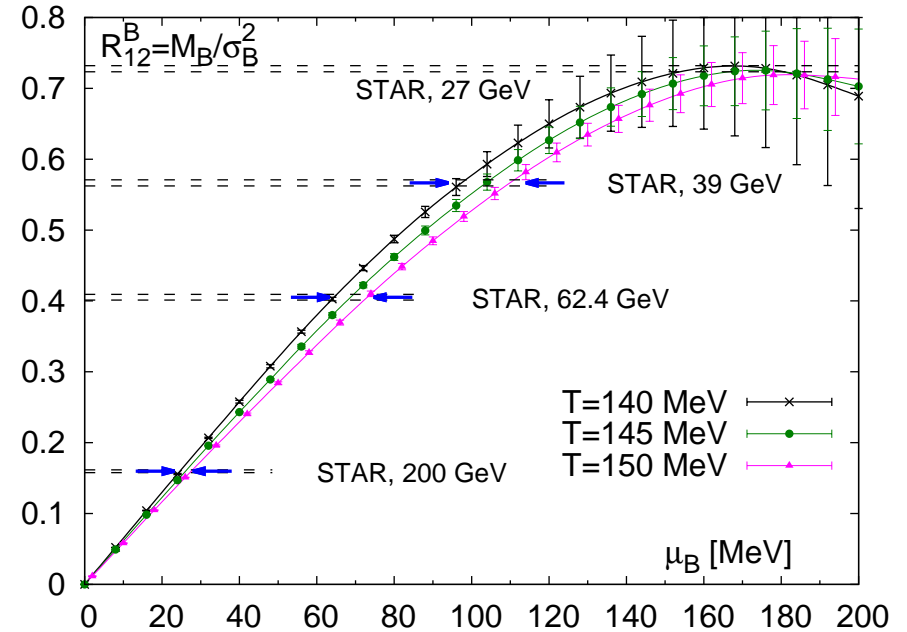
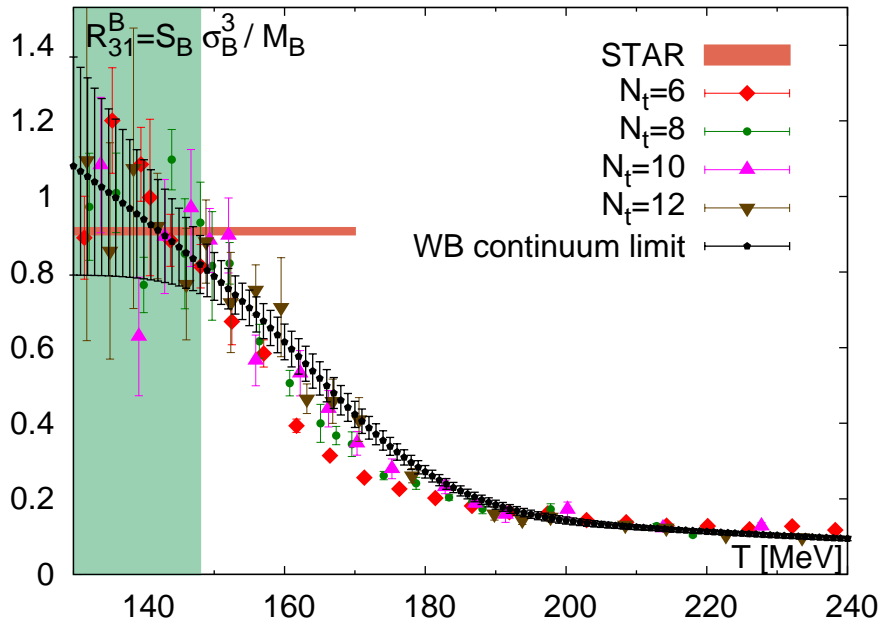
Star Collaboration: [arXiv 1212.3892](https://arxiv.org/abs/1212.3892)

Experimental measurement II



Star Collaboration: arXiv 1402.1558

Extracting freeze-out parameters from baryon number

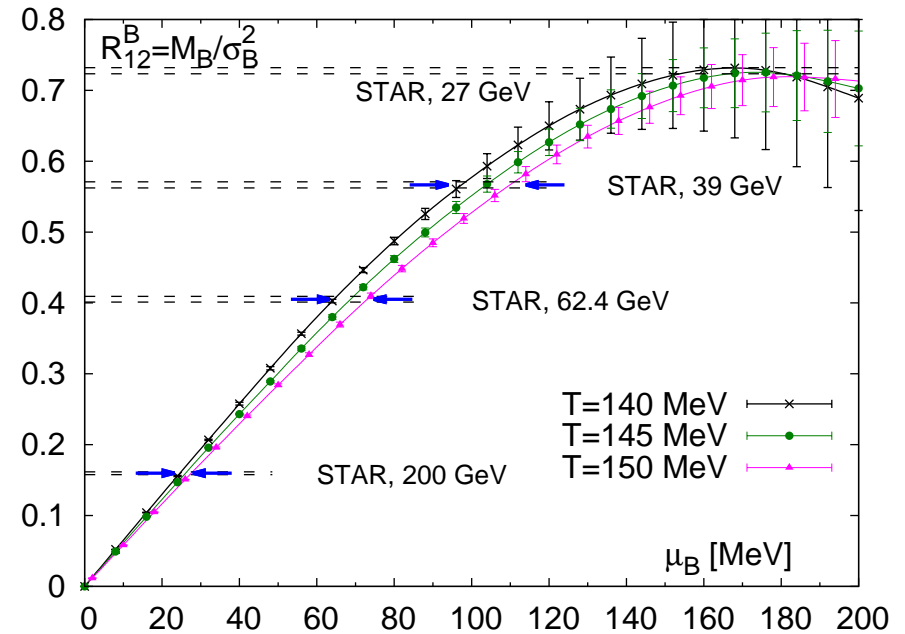
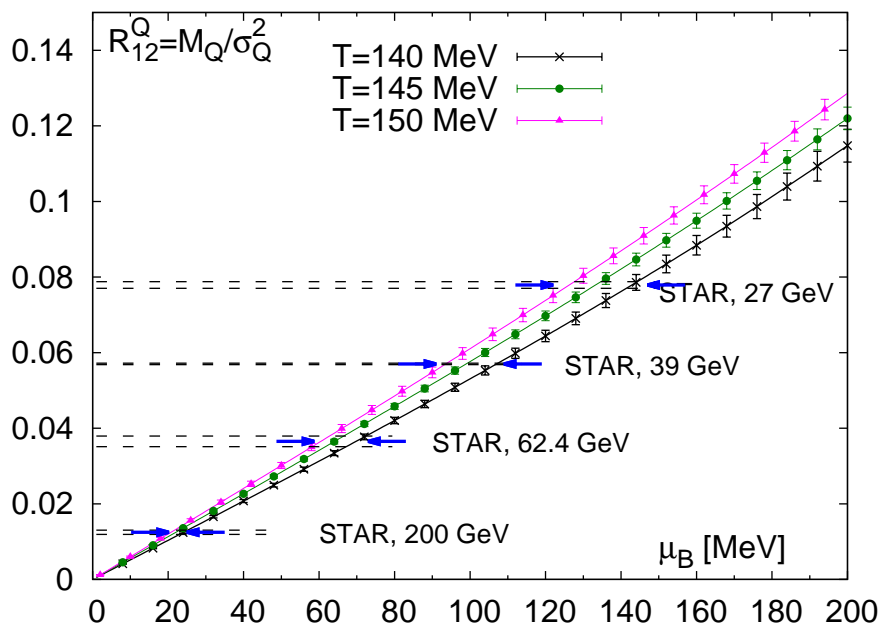


WB Collaboration: arXiv 1403.4576; STAR data from 1309.5681

Upper limit: $T_f \leq 148 \pm 4$ MeV

\sqrt{s} [GeV]	μ_B^f [MeV]
200	25.6 ± 2.4
62.4	69 ± 5.7
39	104 ± 10
27	-

Extracting freeze-out parameters from electric charge

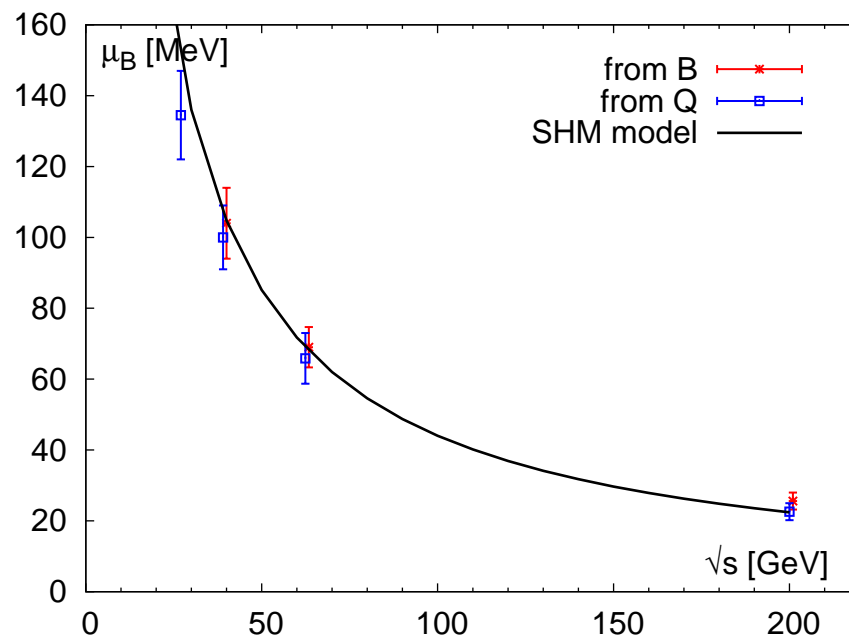


WB Collaboration: arXiv 1403.4576; STAR data from 1309.5681 and 1402.1558

- ❖ It is of fundamental importance to test the **consistency** between the freeze-out parameters obtained with **different conserved charges**
- ❖ This consistency check validates the method and shows equilibration of the medium

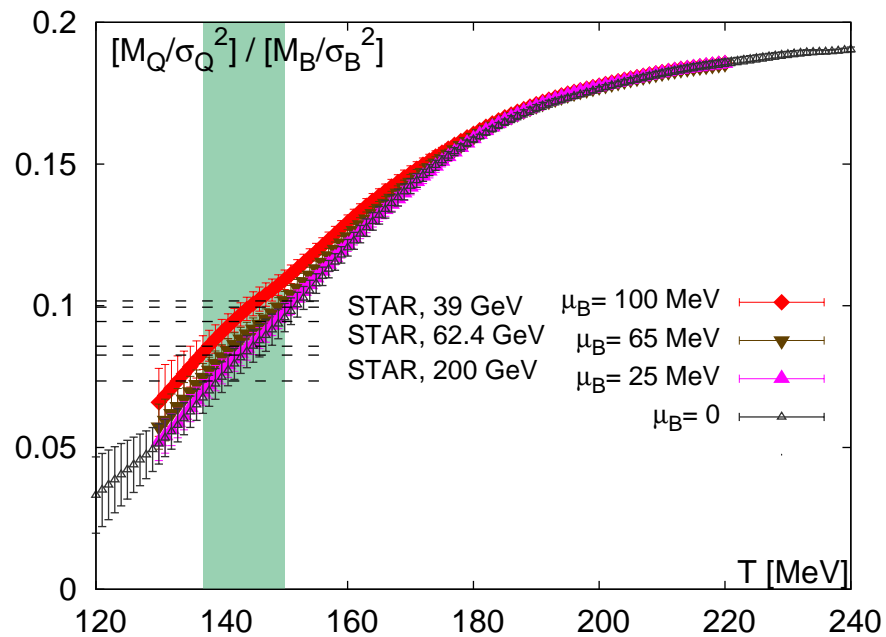
Consistency is found!

$\sqrt{s} [GeV]$	μ_B^f [MeV] (from B)	μ_B^f [MeV] (from Q)
200	25.6 ± 2.4	22.6 ± 2.4
62.4	69 ± 5.7	65.9 ± 7.2
39	104 ± 10	100 ± 9
27	-	134.5 ± 12.5



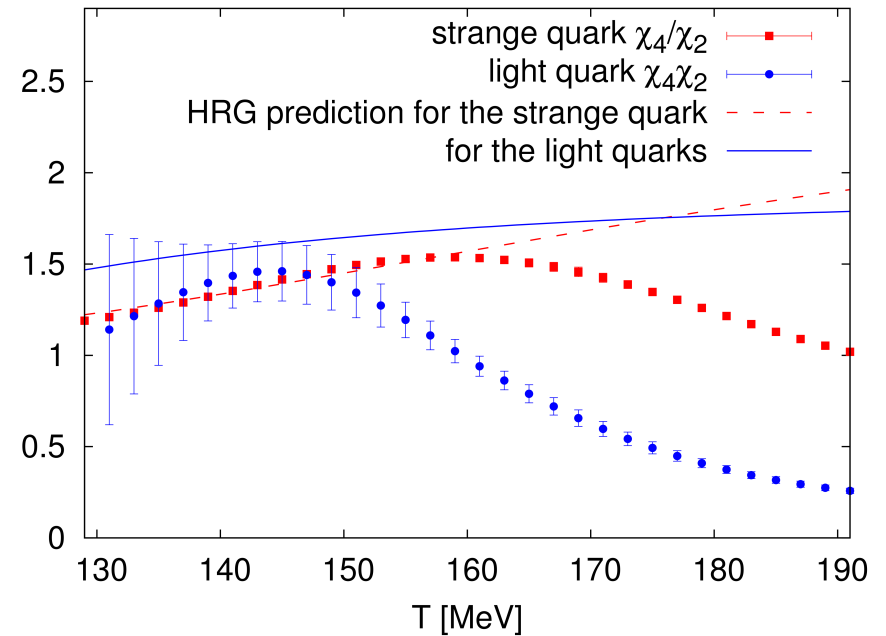
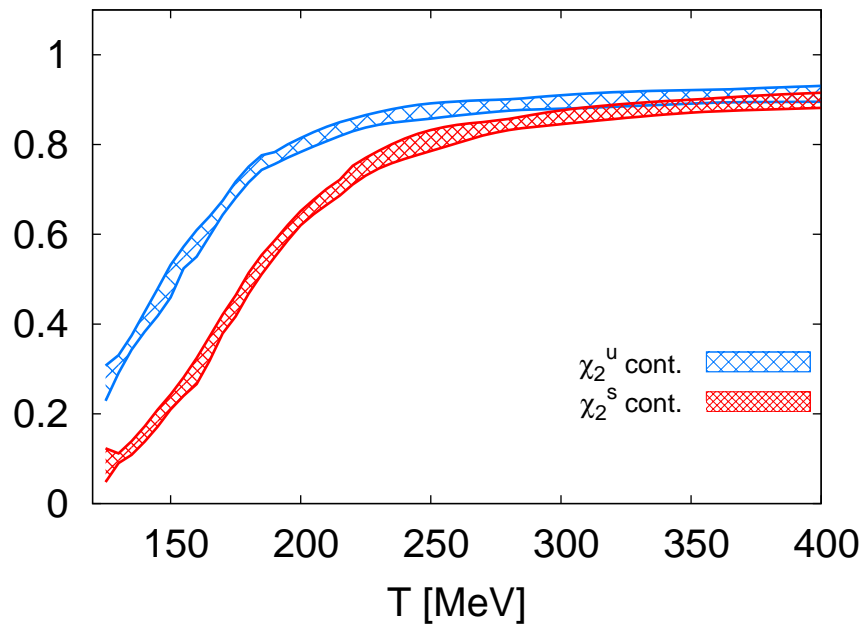
Lattice: WB Collaboration: [arXiv 1403.4576](https://arxiv.org/abs/1403.4576); SHM: Andronic *et al.*, NPA (2006)

Ratio of ratios



$$R_{12}^Q / R_{12}^B = [\chi_1^Q / \chi_2^Q] / [\chi_1^B / \chi_2^B] = [M_Q / \sigma_Q^2] / [M_B / \sigma_B^2]$$

Strange vs light thermometer

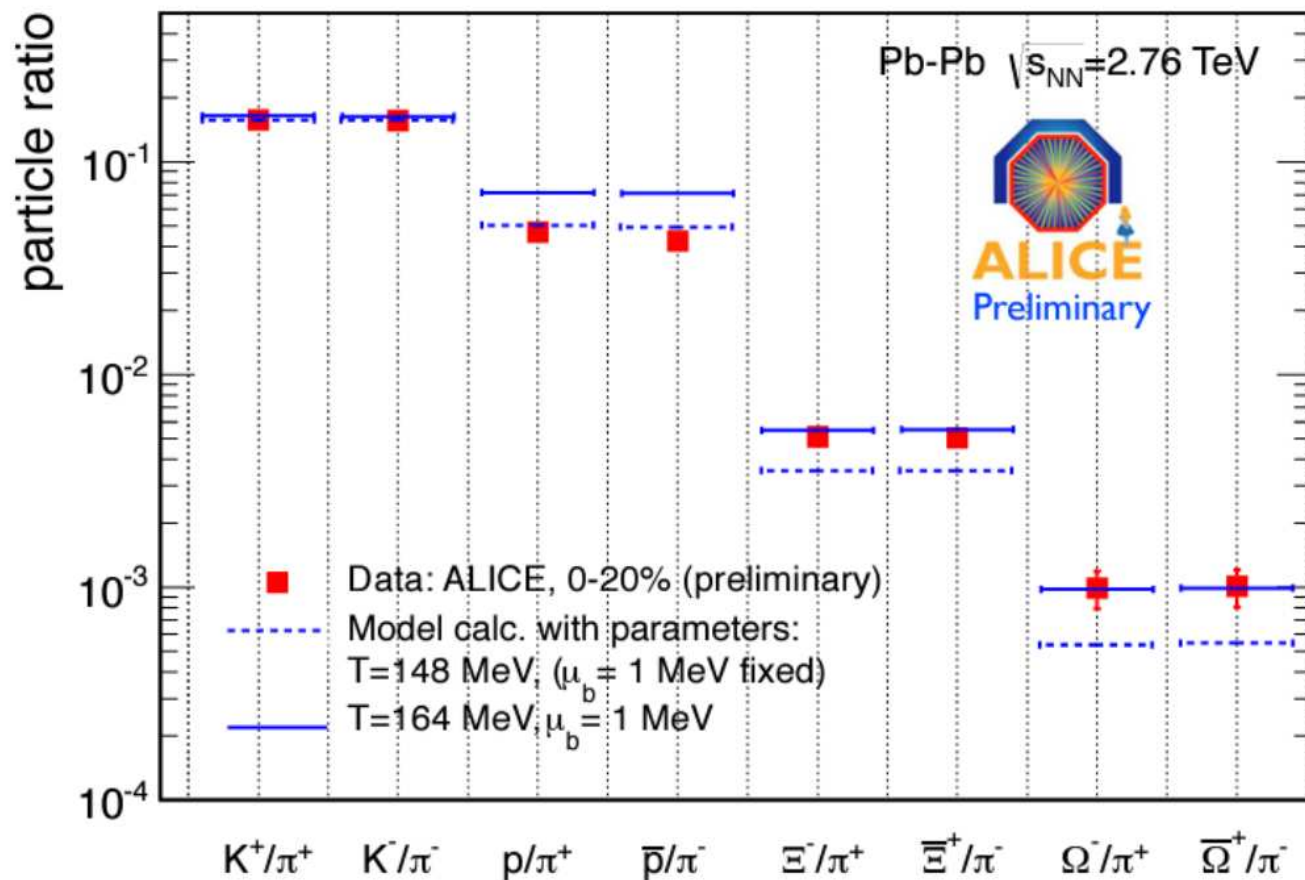


S. Borsanyi *et al.*: JHEP (2012); R. Bellwied *et al.*: PRL (2013)

- ❖ Flavor-specific fluctuations show separation between light and strange quarks
- ❖ Does it mean that light and strange quarks have different freeze-out temperatures?

Freeze-out temperature from experiment

- ❖ Fit to yields of identified particles: Statistical Hadronization Model (SHM)
- ❖ Model-dependent. Parameters: freeze-out **temperature** and **chemical potential**



R. Preghenella
for ALICE
SQM 2012
arXiv:1111.7080
Acta Phys. Pol.

Conclusions

❖ It is possible to extract freeze-out parameters from first principles

❖ Higher order fluctuations of baryon number:

$$\Rightarrow R_{31}^B(T, \mu_B) = \frac{\chi_3^B(T, \mu_B)}{\chi_1^B(T, \mu_B)}: \text{Thermometer}$$

$$\Rightarrow R_{12}^B(T, \mu_B) = \frac{\chi_1^B(T, \mu_B)}{\chi_2^B(T, \mu_B)}: \text{Baryometer}$$

❖ Higher order fluctuations of electric charge:

⇒ independent measurement

$$\Rightarrow R_{12}^Q(T, \mu_B) = \frac{\chi_1^Q(T, \mu_B)}{\chi_2^Q(T, \mu_B)}: \text{Baryometer}$$

❖ The freeze-out parameter sets obtained from B and Q are consistent with each other

❖ Looking forward to strangeness fluctuation data!