Causal Baryon Diffusion and Colored Noise

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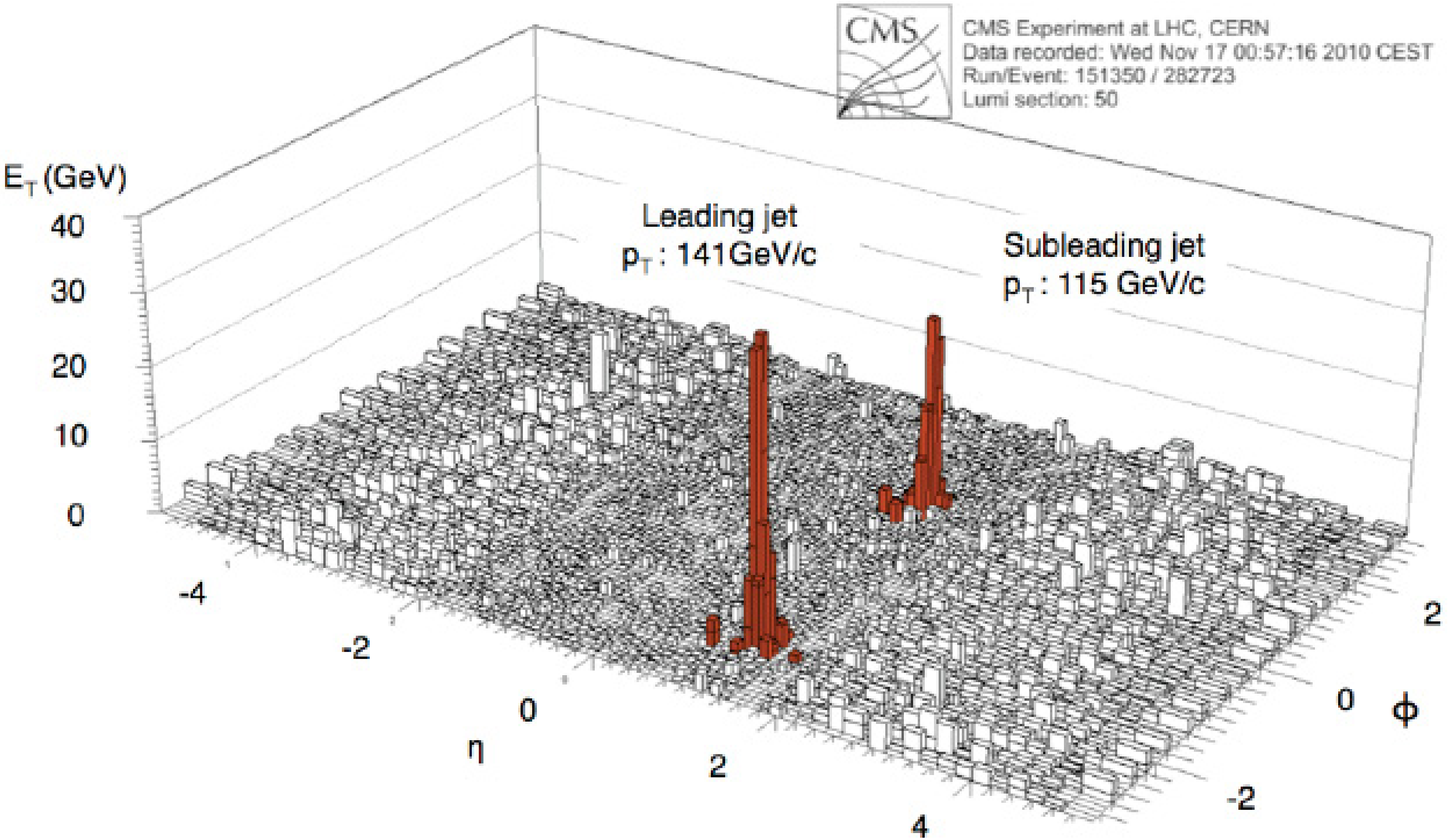
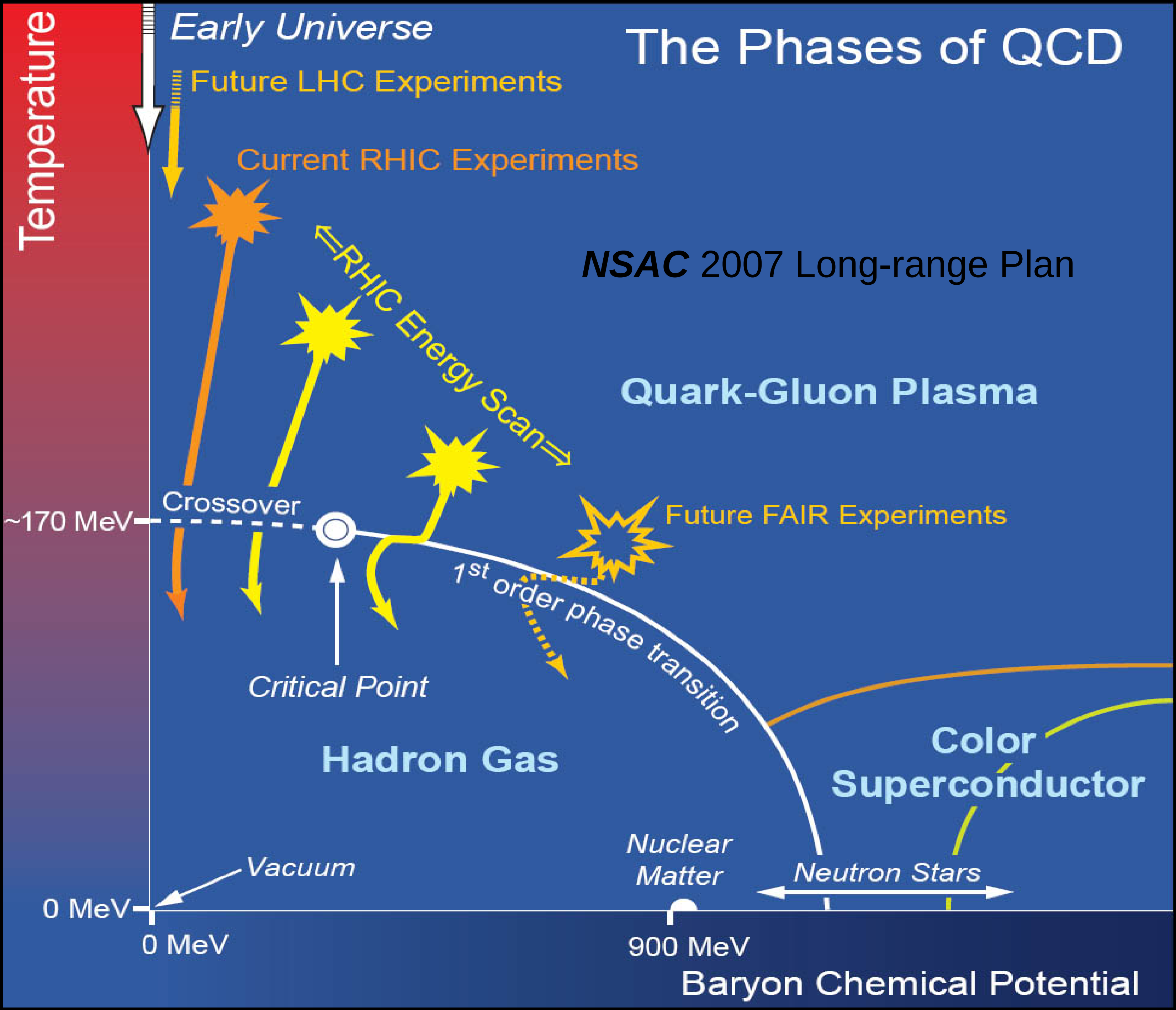


CMS
CMS Experiment at LHC, CERN
Data recorded: Wed Nov 17 00:57:16 2010 CEST
Run/Event: 151350 / 282723
Lumi section: 50
E_T (GeV)
40
30
20
10
0
Leading jet
p_T : 141GeV/c
Subleading jet
p_T : 115 GeV/c
-4
-2
0
2
4
η
2
0
-2
ϕ

Sources of Fluctuations in High Energy Nuclear Collisions

- Initial state fluctuations
- ➡ Hydrodynamic fluctuations due to finite particle number
- Energy and momentum deposition by jets traversing the medium
- Freeze-out fluctuations


The Phases of QCD
NSAC 2007 Long-range Plan
Temperature
Early Universe
Future LHC Experiments
Current RHIC Experiments
⇐RHIC Energy Scan⇒
Quark-Gluon Plasma
Crossover
~170 MeV
Future FAIR Experiments
1st order phase transition
Critical Point
Hadron Gas
Color Superconductor
Nuclear Matter
Neutron Stars
Vacuum
0 MeV
0 MeV
900 MeV
Baryon Chemical Potential

Why the need for causality?

The diffusion equation propagates information instantaneously. No good for hydrodynamic modeling of high energy nuclear collisions!

$$J^{\mu} = nu^{\mu} + \Delta J^{\mu}$$

$$\Delta J^{\mu} = \sigma T \Delta^{\mu}(\beta\mu), \quad \Delta_{\mu} = \partial_{\mu} - u^{\mu}(u \cdot \partial)$$

$$\left(\frac{\partial}{\partial t} - D\nabla^2 \right) n = 0$$

$$\sigma = D(\partial n / \partial \mu)$$

What should the current be modified to?

Why colored noise?

Add noise to the current; in the local rest frame:

$$\left\langle I^{\mu}(\mathbf{x},t)\right\rangle = 0, \quad \left\langle I^{i} I^{j}(\mathbf{x},t)\right\rangle = 2\sigma T \delta(\mathbf{x})\delta(t)\delta_{ij}$$

This is white noise (Fourier transform is a constant). It is OK for hydrodynamics if noise is treated as a perturbation, but creates havoc if it is treated nonperturbatively as there will be a dependence on the coarse-grained cell size.

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Universality Crossover of the Pinch-Off Shape Profiles of Collapsing Liquid Nanobridges in Vacuum and Gaseous Environments

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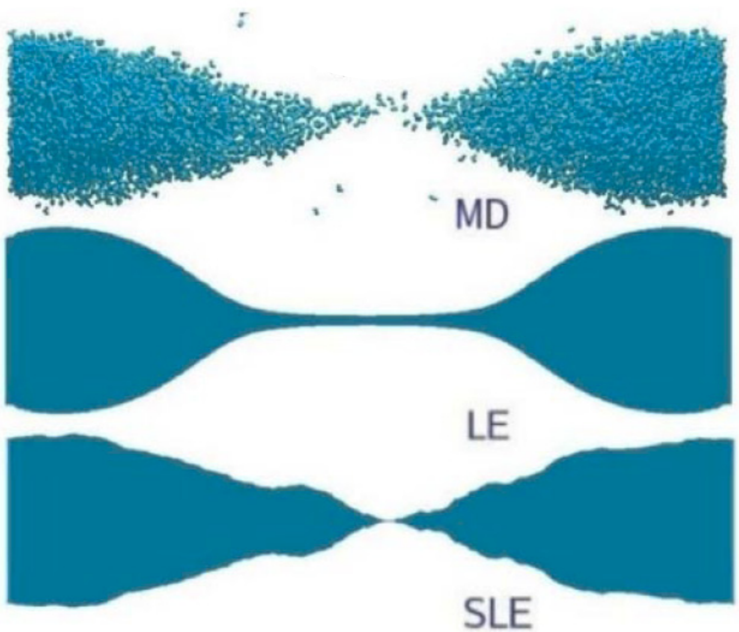
Liquid propane nanobridges were found through molecular dynamics simulations to exhibit in vacuum a symmetric break-up profile shaped as two cones joined in their apexes. With a surrounding gas of sufficiently high pressure, a long-thread profile develops with an asymmetric shape. The emergence of a long-thread profile, discussed previously for macroscopic fluid structures, originates from the curvature-dependent evaporation-condensation processes of the nanobridge in a surrounding gas. A modified stochastic hydrodynamic description captures the crossover between these universal break-up regimes.



Molecular Dynamics

Lubrication Equation

Stochastic Lubrication Equation

Descriptions of heat conduction

Ordinary diffusion equation - 1st order

$$\left(\frac{\partial}{\partial t} - D\nabla^2\right)n = 0$$

Cattaneo equation (1948) - 2nd order

$$\left(\frac{\partial}{\partial t} - D\nabla^2 + \tau_1 \frac{\partial^2}{\partial t^2}\right)n = 0$$

Gurtin - Pipkin equation (1968) - 3rd order

$$\left(\frac{\partial}{\partial t} - D\nabla^2 + \tau_1 \frac{\partial^2}{\partial t^2} + \tau_2^2 \frac{\partial^3}{\partial t^3} - \tau_3' D \frac{\partial}{\partial t}\nabla^2\right)n = 0$$

The Associated Baryon Current

$$\Delta J^{\mu} = \sigma T \Delta^{\mu} \frac{1 + \tau_4 (u \cdot \partial)}{1 + \tau_1 (u \cdot \partial) + \tau_2^2 (u \cdot \partial)^2 + \tau_3 D \Delta^2} \beta \mu$$

Ordinary diffusion equation : $\tau_1 = \tau_2 = \tau_3 = \tau_4 = 0$

Cattaneo equation : $\tau_2 = \tau_3 = \tau_4 = 0$

Here $\tau_3' \equiv \tau_3 + \tau_4$

Fluctuation-Dissipation Theorem

$$A(\mathbf{k},\omega) \equiv \omega + \frac{iDk^2(1-i\tau_4\omega)}{1-i\tau_4\omega-\tau_2^2\omega^2+\tau_3 Dk^2}$$

Response function $$G_R(\mathbf{k},\omega) = \left(\frac{\partial n}{\partial \mu}\right)\frac{\omega}{A}$$

Density correlator $$\langle \delta n \delta n(\mathbf{k},\omega)\rangle = iT\left(\frac{\partial n}{\partial \mu}\right)\left(\frac{1}{A}-\frac{1}{A^*}\right)$$

Noise $$\frac{1}{3}k^2\langle I^l I^l(\mathbf{k},\omega)\rangle = -iT\left(\frac{\partial n}{\partial \mu}\right)\left(A-A^*\right)$$

density correlator $\Leftrightarrow$ zeros of A

noise $\Leftrightarrow$ poles of A

Ordinary Diffusion Equation

$$\left\langle \delta n \delta n(\mathbf{x},t) \right\rangle = T \left(\frac{\partial n}{\partial \mu} \right) \left(\frac{1}{4\pi D t} \right)^{3/2} \exp\left(-r^2 / 4Dt \right)$$

$$\left\langle I^i I^j(\mathbf{x},t) \right\rangle = 2\sigma T \delta(\mathbf{x}) \delta(t) \delta_{ij}$$

Cattaneo Equation

For the density correlator there is a pair of imaginary poles for $k < k_c$, and a pair of complex poles for $k > k_c$ where $k_c^2 = 1/4\tau_1 D$.

Group velocity $v_g = \dfrac{v_0 k}{\sqrt{k^2 - k_c^2}}, \quad v_0 = \sqrt{\dfrac{D}{\tau_1}}$

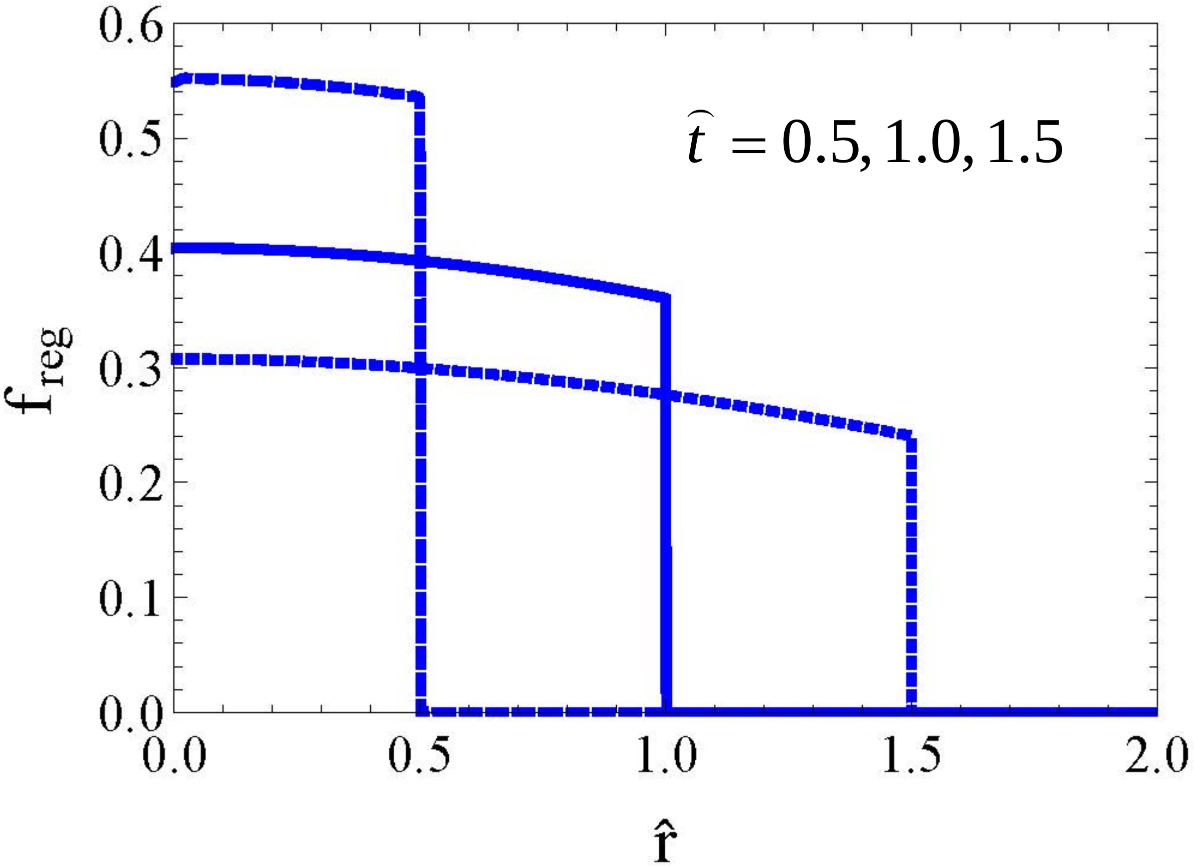
(Infinite group velocity is not an issue; Brillouin.)

$$\left\langle I^i I^j(\mathbf{x}, t) \right\rangle = \frac{\sigma T}{\tau_1} \delta(\mathbf{x}) \exp(-|t|/\tau_1) \delta_{ij}$$

Dimensionless baryon correlator

$$f(\hat{r},\hat{t}) = \frac{\pi}{2}\frac{\exp(-\hat{t})}{\hat{r}}\left[(1+\hat{t}/2)\delta(\hat{r}-\hat{t}) - \delta'(\hat{r}-\hat{t})\right]$$

$+ f_{reg}(\hat{r},\hat{t})$ where $\hat{r} = r/2v_0\tau_1$ and $\hat{t} = t/2\tau_1$.



Gurtin-Pipkin Equation

For the density correlator and for $\tau_1 > 2\tau_2$ there are 3 imaginary poles for $k < k_c$ and a pair of complex poles and 1 imaginary pole for $k > k_c$.

For $\tau_1 < 2\tau_2$ there are always a pair of complex poles and 1 imaginary pole.

Gurtin-Pipkin Equation

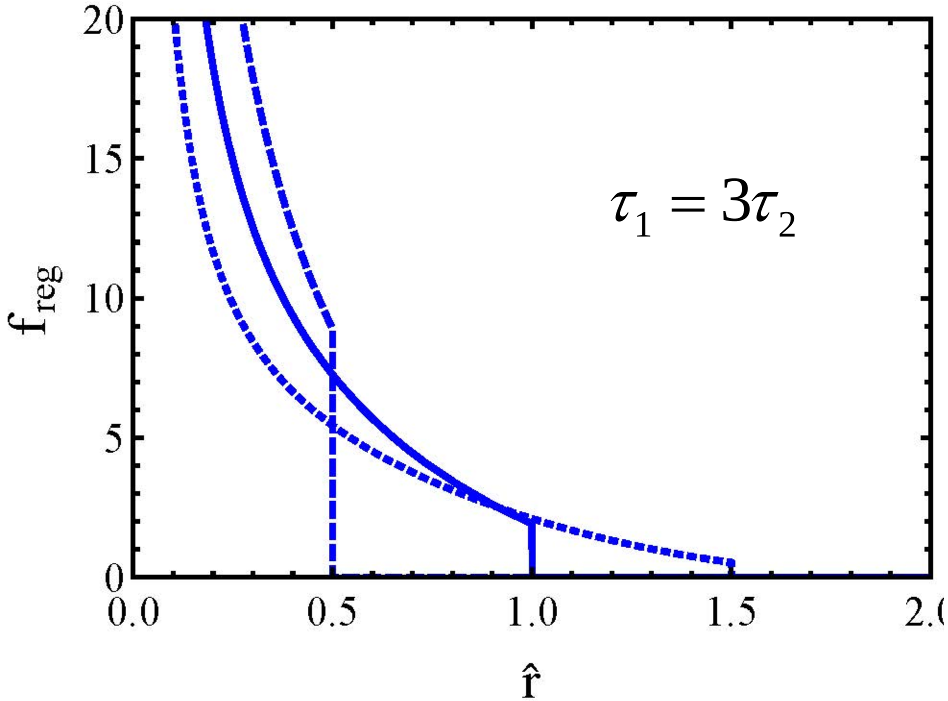
For the baryon correlation function the asymptotic group velocity is $v_0 = \sqrt{\tau_3' D / \tau_2^2}$

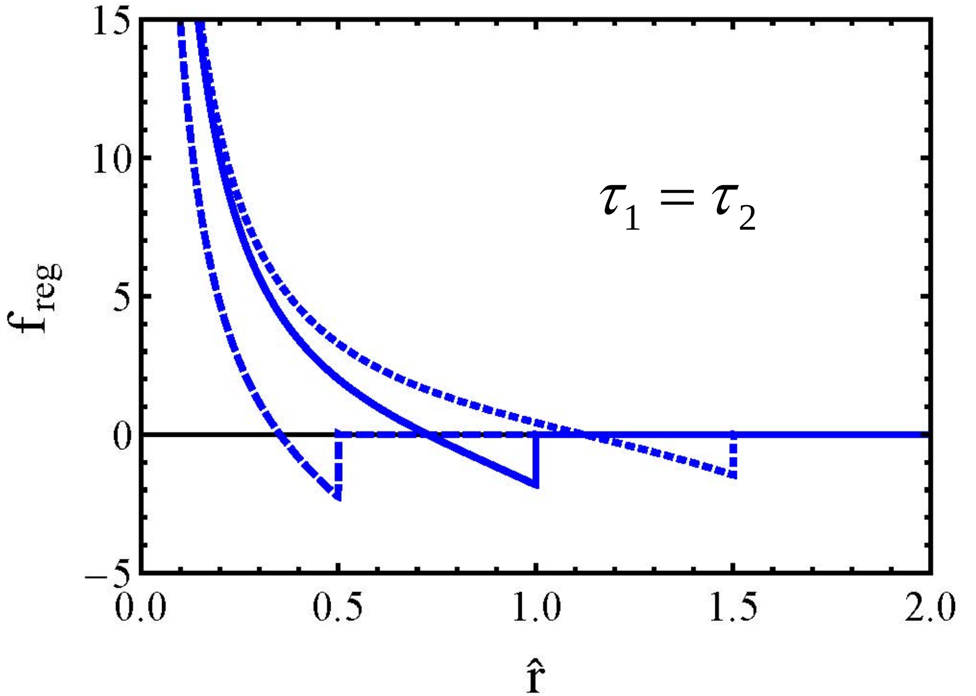
In order that the poles always lie in the lower half complex frequency plane requires $\tau_2^2 < \tau_1 \tau_3'$

Dimensionless baryon correlator

$$f(\hat{r},\hat{t}) = A\delta(\hat{\mathbf{x}})\exp(-a\hat{t}) + B\frac{\exp(-b\hat{t})}{\hat{r}}\delta(\hat{r}-\hat{t})$$

$$+ f_{reg}(\hat{r},\hat{t}) \quad \text{with } \tau_4 = 0 \text{ and } v_0^2 = 1/3.$$





Gurtin-Pipkin Equation

For the noise and for $\tau_1 > 2\tau_2$ there are a pair of imaginary poles for $k < k_c$ and a pair of complex poles for $k > k_c$ with $v_g = \dfrac{v_0 k}{\sqrt{k^2 - k_c^2}}$.

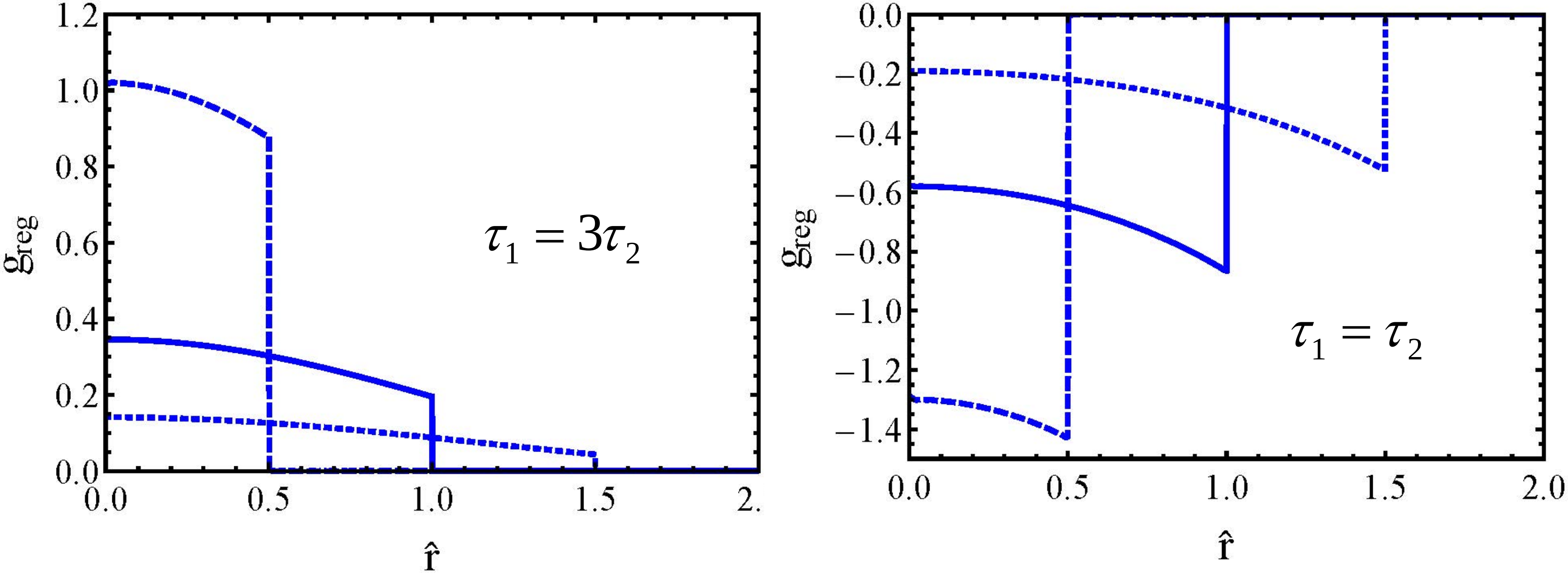
For $\tau_1 < 2\tau_2$ there are a pair of complex poles with

$$v_g = \frac{v_0 k}{\sqrt{k^2 + k_0^2}} \quad \text{where} \quad v_0 = \sqrt{\tau_3 D / \tau_2^2}.$$

Dimensionless noise

$$g(\hat{r},\hat{t}) = \frac{\pi}{2}\frac{\exp(-\tau_1\hat{t}/\tau_2)}{\hat{r}}\delta(\hat{r}-\hat{t}) + g_{reg}(\hat{r},\hat{t})$$

with $\tau_4 = 0$ and $v_0^2 = 1/3.$



When $\tau_1 = 2\tau_2$ there is no wake.

Summary

- We have studied and compared the baryon current in 1st, 2nd and 3rd order dissipative fluid dynamics.
- We computed the response function, baryon autocorrelation function, and noise.
- One needs at least 2nd order for finite propagation speed, and at least 3rd order for finite correlation lengths and times for noise.

- These results can readily be implemented in numerical hydrodynamic codes.
- Baryon transport and noise should be important even if the net baryon number is zero, and may play a crucial role near a critical point.
- Microscopic calculations are needed to compute the time constants (probably functions of temperature and density).
- Noise happens!

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