Hadronic Shear Viscosity from a Microscopic Transport Model

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Outline of Talk

-Introduction: Hot Nuclear Matter: the QGP
-Motivation: Transport Coefficients at RHIC, Temperature Dependence of η/s, Necessity of Comparison Project
-Theory: Chapmann-Enskog Method, Green-Kubo formalism, UrQMD
-Results: Chiral Pion System with Current Algebra, Massive Pions with Rho Resonance
-Outlook: Investigation of Effect of Resonance Lifetime on Viscosity, Challenges with Calculating Bulk Viscosity
-Summary

"Ideal Fluid" Behavior from RHIC

- *On April 18th, 2005, BNL announced in a press release that RHIC had created a new state of hot and dense matter which behaves like a nearly perfect liquid.* K.Adcox *etal.*(PHENIX),**Nucl.Phys.A757**,184(2005)

How to Quantify Properties of "Ideal Fluid"?

Relativistic hydrodynamics equations obtained by solving conservation equations for energy/momentum and charged currents.

$$\partial_\mu T^{\mu\nu} = 0$$
$$\partial_\mu j^\mu = 0$$

Ideal fluid EM tensor: $T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu + pg^{\mu\nu}$

$$T_{ij} = T_{ij,0} - \eta\left(\partial_i u_j + \partial_j u_i - \frac{2}{3}\delta_{ij}(\nabla \cdot \vec{u})\right) - \zeta\delta_{ij}(\nabla \cdot \vec{u})$$
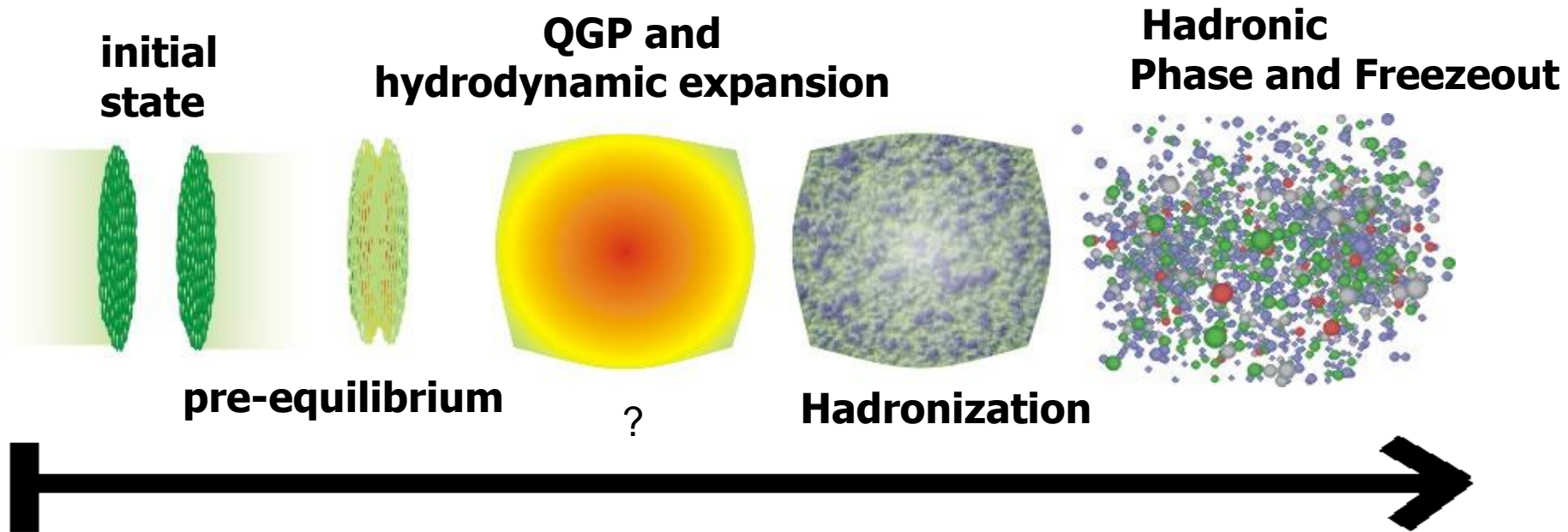
Expansion of system's stress energy tensor:
-Shear viscosity corrections.
-Bulk viscosity corrections.

# Phases of RHIC evolution.

Courtesy of S.A. Bass.

initial
state

QGP and
hydrodynamic expansion

Hadronic
Phase and Freezeout

pre-equilibrium

?

Hadronization

-viscous hydrodynamics expected QGP to exhibit low value of η/s.
-(late) hadronic phase resembles dilute gas: large value of η/s expected.
-separation of thermal/chemical freezeout in RHIC evolution.

In order to connect final state observables (i.e. v_2) with early QGP phase,
must quantify viscosity of hadronic phase! (i.e. "lift veil of hadronic phase")
- Motivated analysis for η/s calculation for hadronic gas both in and out of
chemical equilibrium!

Elliptic Flow Coefficient v_2

- two nuclei collide rarely head-on, but mostly with an offset:

Reaction plane

z

y

x
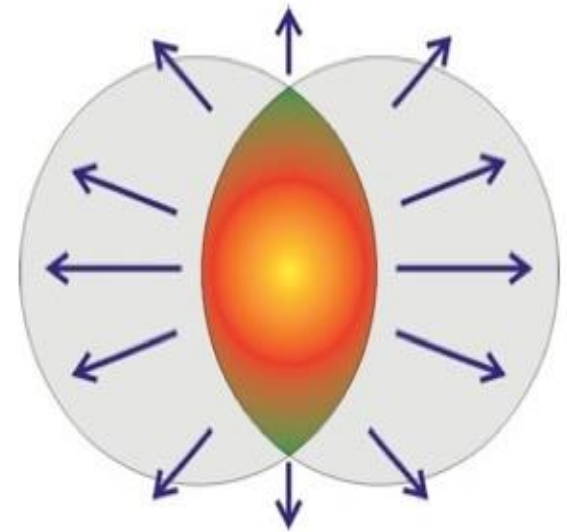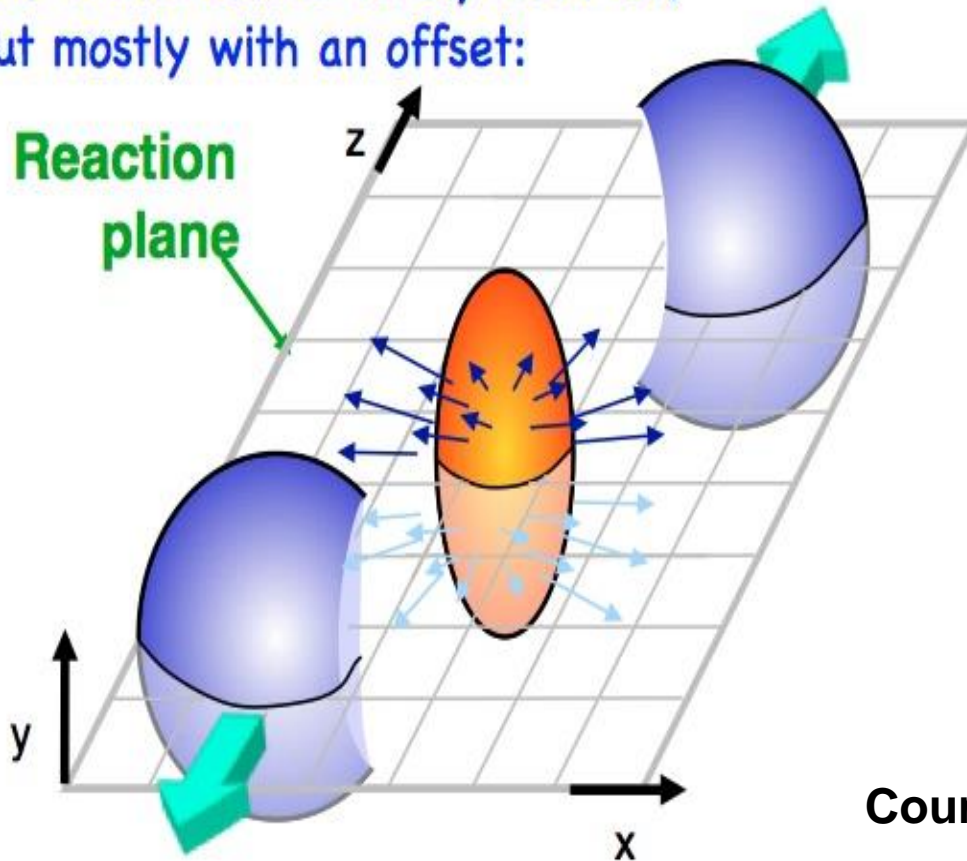
only matter in the overlap area gets compressed and heated up
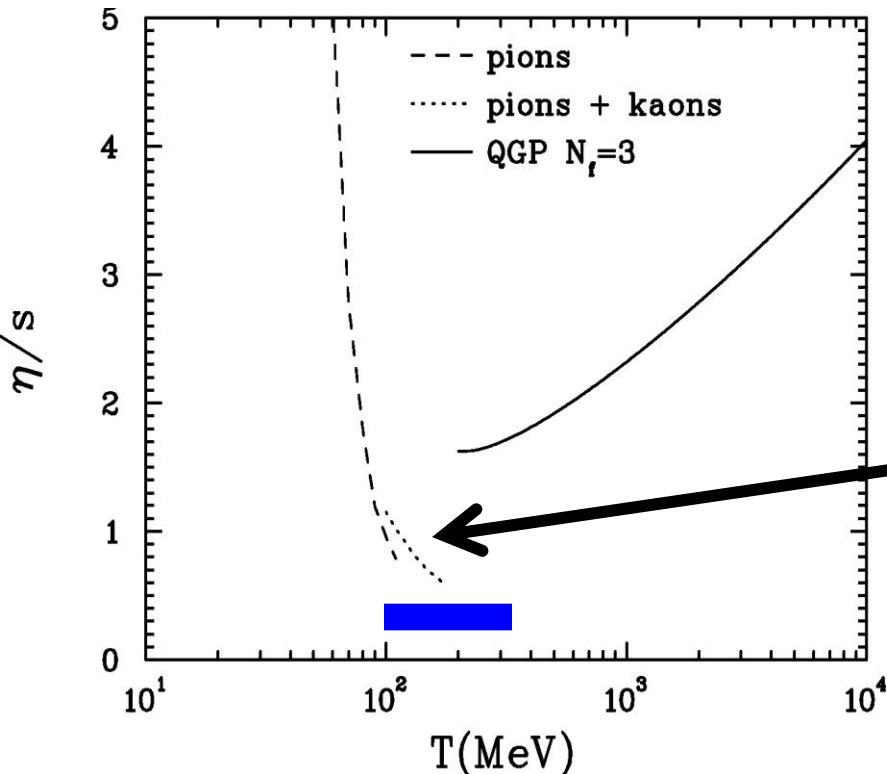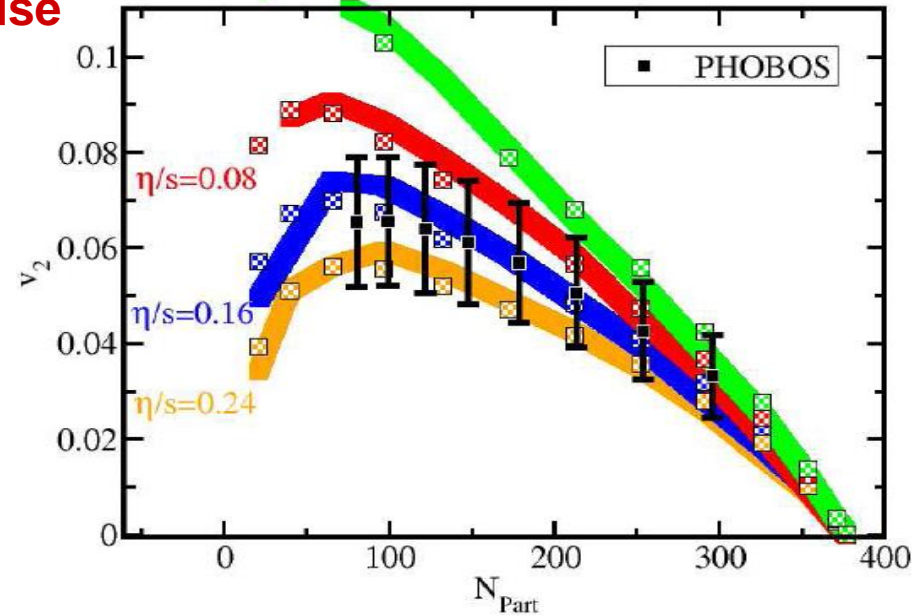
Courtesy of S.A. Bass.

Elliptic Flow Coefficient (v_2) obtained from azimuthal angle distributions of hadrons.

$$\frac{dN}{d\phi} \propto (1 + 2v_2 cos(2\phi) + ...)$$

Temperature Dependence of η/s

M.Luzum & P. Romatschke:
Phys. Rev. **C78**: 034915, 2008

- **Viscous hydro needs η/s~(0.08-0.24)**
- **Current viscous hydro calculations use fixed η/s throughout evolution of calculation.**
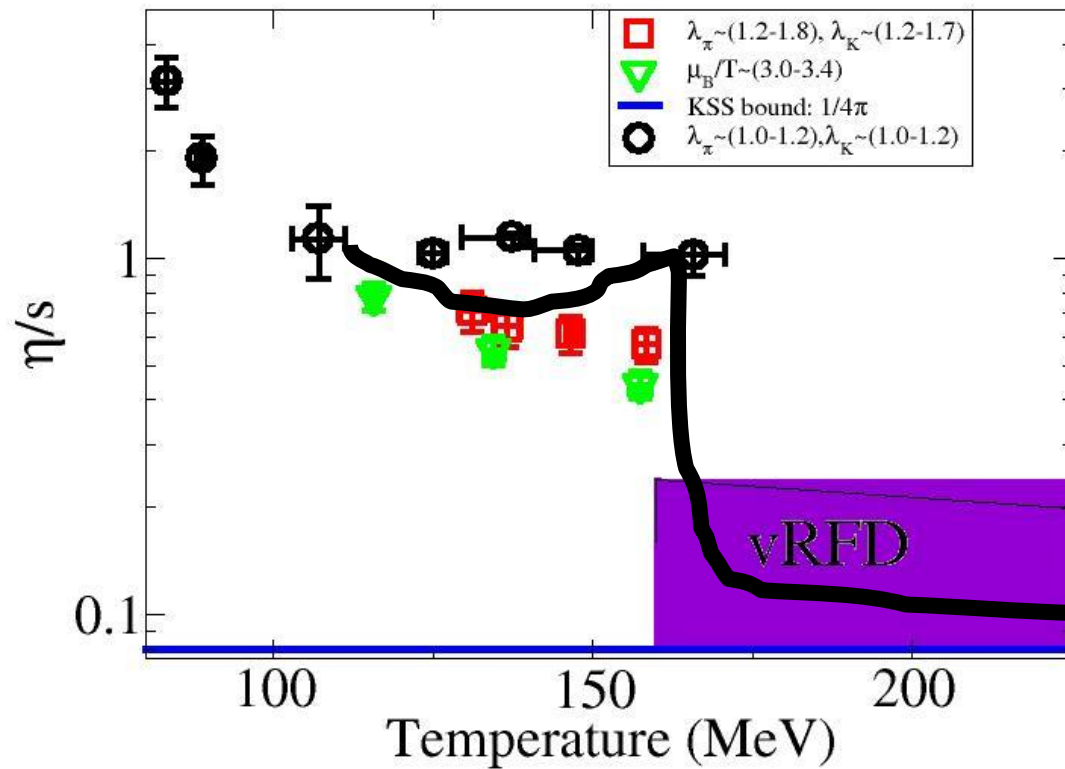- **η/s strongly temperature dependent.**

Pert. Theory N/A here.

Csernai, Kapusta, McLerran:
nucl-th/0604032 PRL **97**. 152303 (2006)
Viscous hydro needs η/s~(0.08-0.24)

Temperature Dependence of η/s, part II

Demir, Bass,
PRL 102 (2009) 172302

η/s calculations from
UrQMD via the
Green-Kubo method.

Need for systematic comparison of viscosity calculation for two methods.

- Wiranata, Prakash, **PRC 85 (2012) 054908:**

  "Our results stress the need to perform quantum molecular dynamical (UrQMD) simulations that employ Green-Kubo techniques with similar cross sections to validate the codes employed and to test the accuracy of other methods."
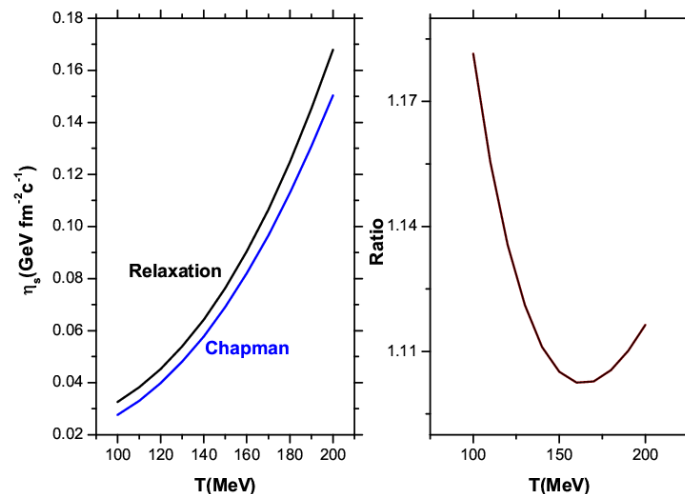
Left Panel: Shear viscosities of pion gas from the relaxation time approximation and the first order Chapman-Enskog approximation. Right Panel: Ratio of results from left panel.

What do we know thus far?

- Determining hadronic viscosity necessary to constrain viscosity of QGP.

- Several methods exist for calculating hadronic viscosity for different systems.

- Need to perform SYSTEMATIC COMPARSION BETWEEN DIFFERENT METHODS FOR SYSTEMS WITH SAME DEGREES OF FREEDOM.

- In this talk, shall show comparison of results for Chapmann-Enskog approximation with Green-Kubo method with UrQMD as the model for the hadronic medium.

Chapmann-Enskog Approximation

- For small deviations from equilibrium, the distribution function may be expressed in terms of hydrodynamic variables and their gradients.

$$T_{ij} = T_{ij,0} - \eta\left(\partial_i u_j + \partial_j u_i - \frac{2}{3}\delta_{ij}(\nabla \cdot \vec{u})\right) - \zeta\delta_{ij}(\nabla \cdot \vec{u}))$$

$$f(x,p) = f^0\left(1 + \phi(x,p)\right) \qquad T_{ij} = \int d^3 p\, f(x,p)\frac{p^i p^j}{p^0}$$

First Chapmann-Enskog Approximation:

Linearized Boltzmann equation:

$$p_\alpha \partial^\alpha f^0 = -f^0 L[\phi]$$

$$h = \frac{1}{10} T \frac{g_{00}^2}{c_{00}}$$

$$g_{00} = -10\hat{h}, \hat{h} = \frac{K_3(z)}{K_2(z)}, z = \frac{m}{T}$$

$$c_{00} = 16\left(w_2^{(2)} - \frac{w_1^{(2)}}{z} + \frac{w_2^{(2)}}{3z^2}\right)$$

$$w_i^{(2)} = \frac{2pz^3}{K_2(z)^2}\int_0^\infty dy\, \sinh^7 y \cosh^i y\, K_j(2z\cosh y)\int_0^p dq\, \sin q\, \frac{dS}{dW}(y,q)\left(1 - \cos^2 y\right)$$

Wiranata, Prakash
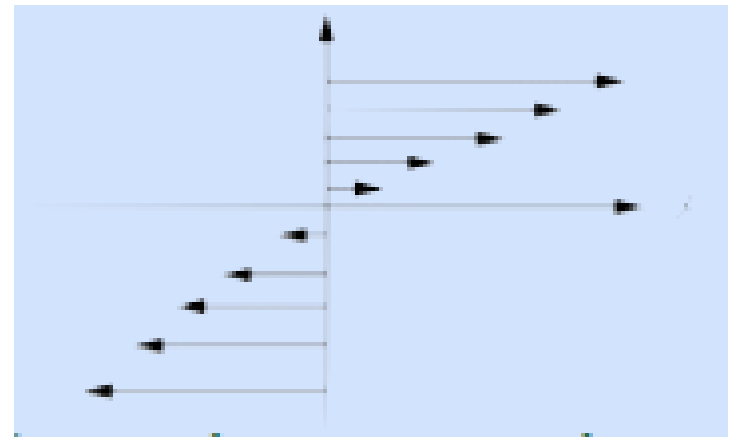***Phys.Rev. C85 (2012) 054908***

Linear Transport Coefficients & Green-Kubo Relations

Phenomenological Transport Equation: thermodynamic/mechanical flux linearly proportional to applied field in small field limit.

Examples of transport coefficients: thermal conductivity, diffusion, shear viscosity.

Shear Viscosity Coefficient:

$$P_{yx} = -\eta \frac{\partial v_x}{\partial y}$$

Green-Kubo: compute linear transport coefficients by examining near-equilibrium correlations!

$$\eta = \frac{1}{T} \int d^3r \int_0^\infty dt < \pi^{xy}(0)\pi^{xy}(t) > \qquad \pi^{\mu\nu} = \int d^3p \frac{p^\mu p^\nu}{p^0} f(x,p)$$

Microscopic Transport Models

Based upon Boltzmann Equation:

$$\left(\frac{\partial}{\partial t} + \frac{\vec{p}}{E} \cdot \nabla_r \right) f = \mathcal{C}_{coll}$$

$$\mathcal{C}_{coll} = N \int \sigma d\Omega \int d\vec{p}_2 |\vec{v}_1 - \vec{v}_2| (f(\vec{p}_1')f(\vec{p}_2') - f(\vec{p}_1)f(\vec{p}_2))$$

- Equation to evolve phase space distribution of system (note system may or may not be in equilibrium.
- Particles interact only through scattering (interparticle potential ignored).
- Straight line trajectories in phase space followed.
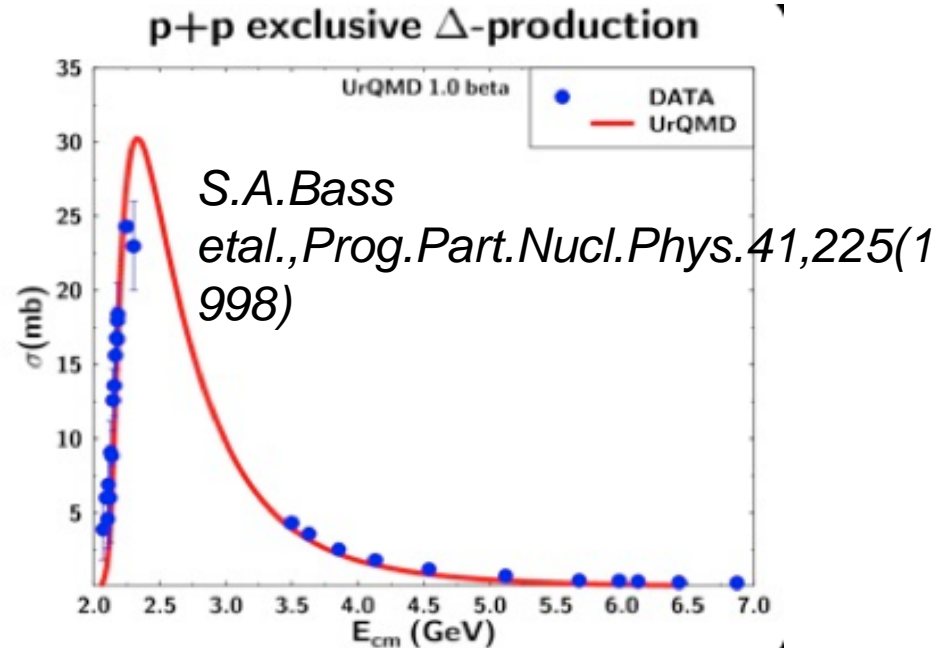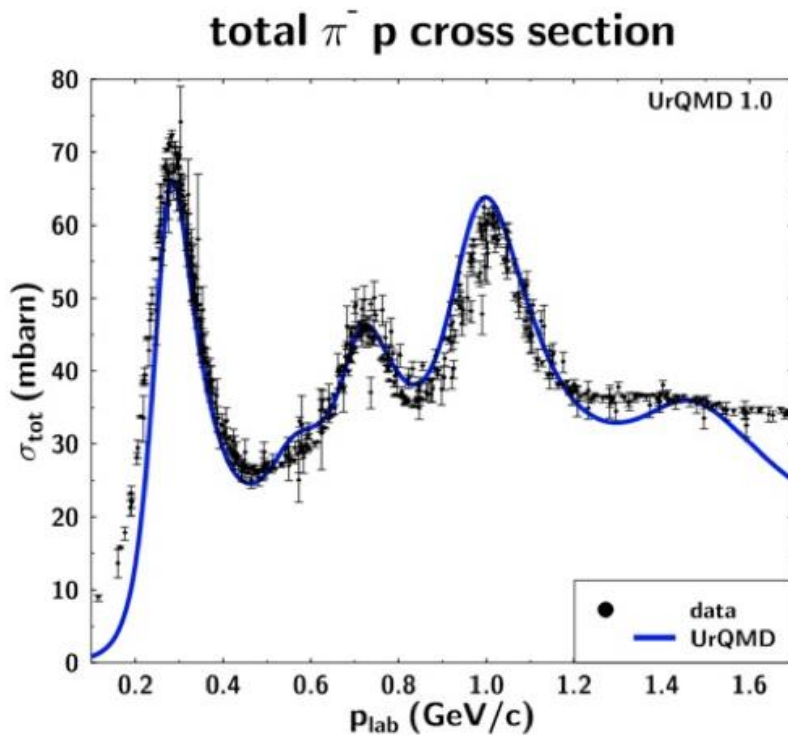- Collision criterion:

$$d_{min} \leq \sqrt{\frac{\sigma_{tot}}{\pi}}$$

d_{min} is distance of closest approach between two particles.

Modeling the Hadronic Medium:
UrQMD (Ultrarelativistic Quantum Molecular Dynamics)

Transport model with purely hadronic degrees of freedom.

- 55 baryon- and 32 meson species, among those 25 N*, Δ* resonances and 29 hyperon/hyperon resonance species

- Full baryon-antibaryon and isospin symmetry.

- Values for σ of experimentally measurable processes input from experimental data.

total π⁻ p cross section

p+p exclusive Δ-production

S.A.Bass etal.,Prog.Part.Nucl.Phys.41,225(1998)

Box Mode:Equilibriation To Study Infinite Hadronic Matter

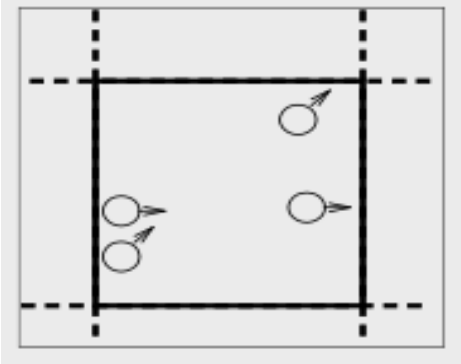Confine particles to box with periodic boundary conditions:
- Force system into equilibrium!
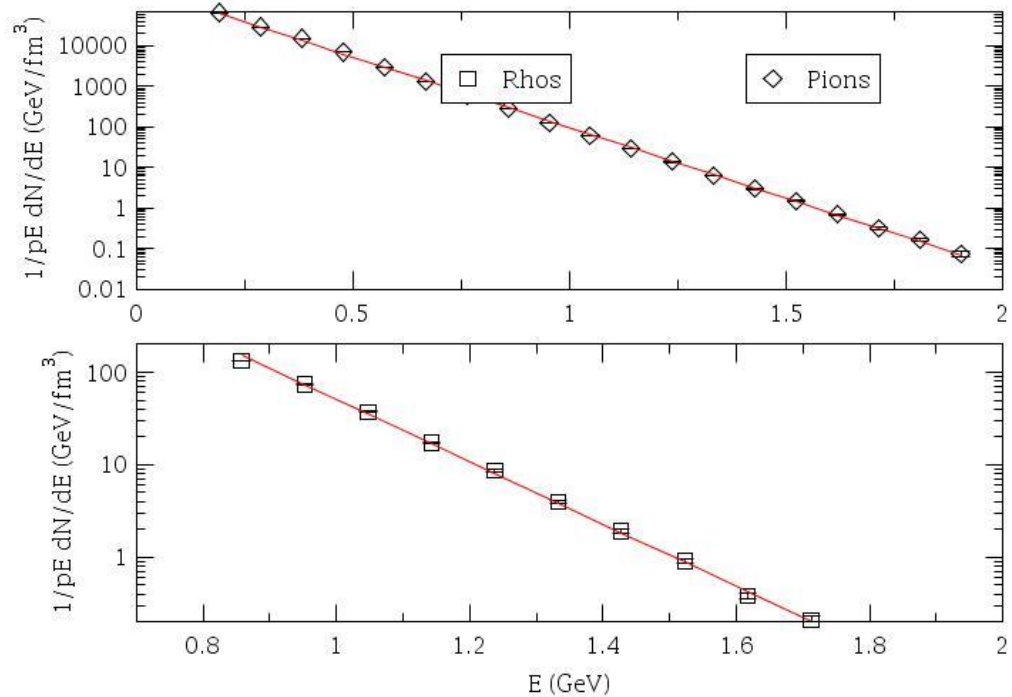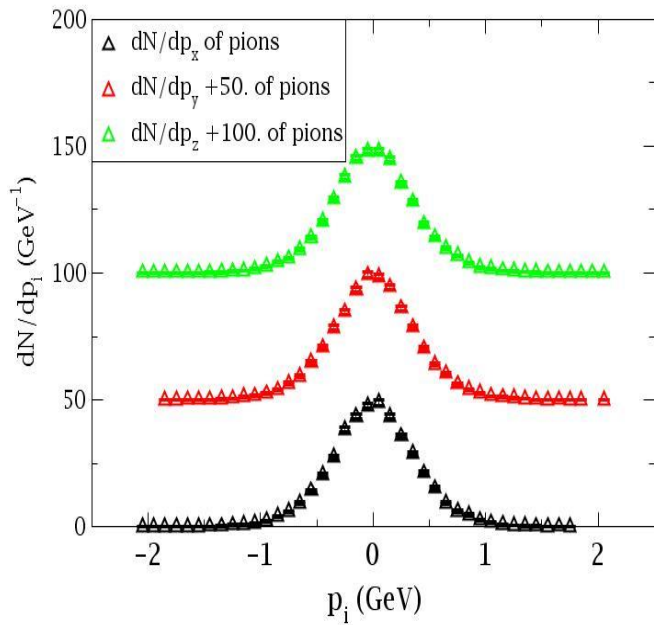
$$\frac{dN}{d^3p} = \frac{dN}{4\pi p^2 dp}$$

$$E = \sqrt{p^2 + m^2}$$

$$\frac{dE}{dp} = \frac{p}{E}$$

$$\frac{dN}{d^3p} = \frac{1}{4\pi pE}\frac{dN}{dE} \propto exp\left(-\frac{E}{T}\right)$$

T=125 MeV for pions, T=129 MeV for rhos

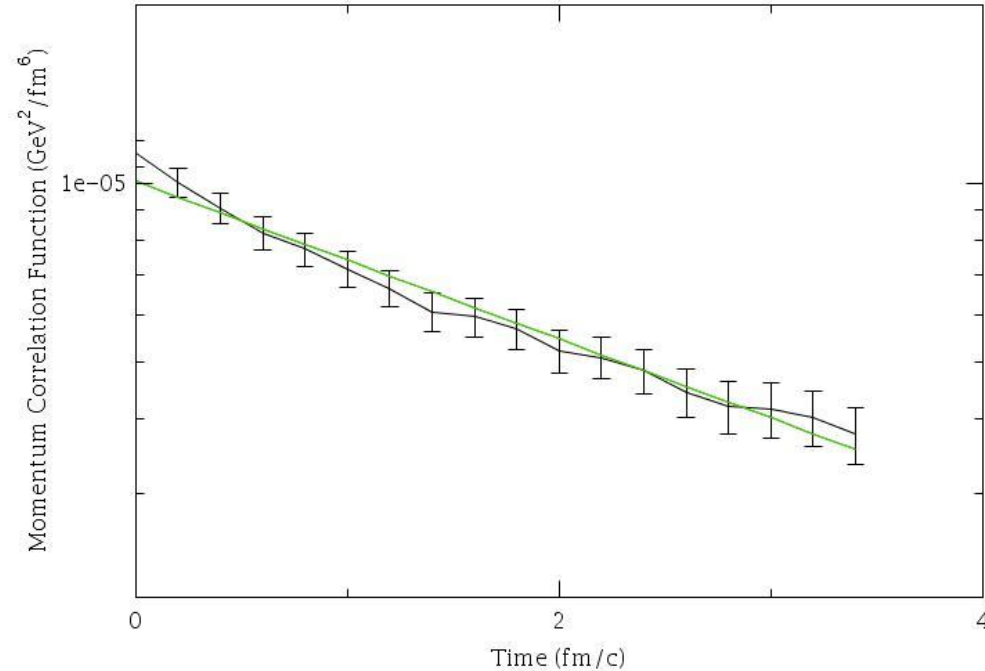Then can extract temperature from Boltzmann fit to momentum spectra!

Kubo Formalism: Calculating Correlation Functions

$$\eta = \frac{1}{T} \int d^3r \int_0^\infty dt < \pi^{xy}(0)\pi^{xy}(t) >$$

$$\pi^{\mu\nu} = \int d^3p \frac{p^\mu p^\nu}{p^0} f(x,p)$$

T= 170 MeV
$T_\pi =3.3$ fm/c

$$\pi^{xy} = \frac{1}{V} \sum_{j=1}^{N_{part}} \frac{p^x(j)p^y(j)}{p^0(j)}$$

$$< \pi^{xy}(0)\pi^{xy}(t) > \propto exp(-\frac{t}{\tau_\pi})$$

$$\eta = \frac{V}{T} \int_0^\infty dt < \pi^{xy}(0)\pi^{xy}(t) >$$

$$\eta = \frac{V\tau_\pi}{T} < \pi^{xy}(0)^2 >$$

Behavior also found in *Muronga*, **PRC 69**:044901,**2004**

- NOTE: RESULTS FOR SYSTEMS I and II ARE PRELIMINARY.

# System I: Chiral Pions with Current Algebra Cross Section
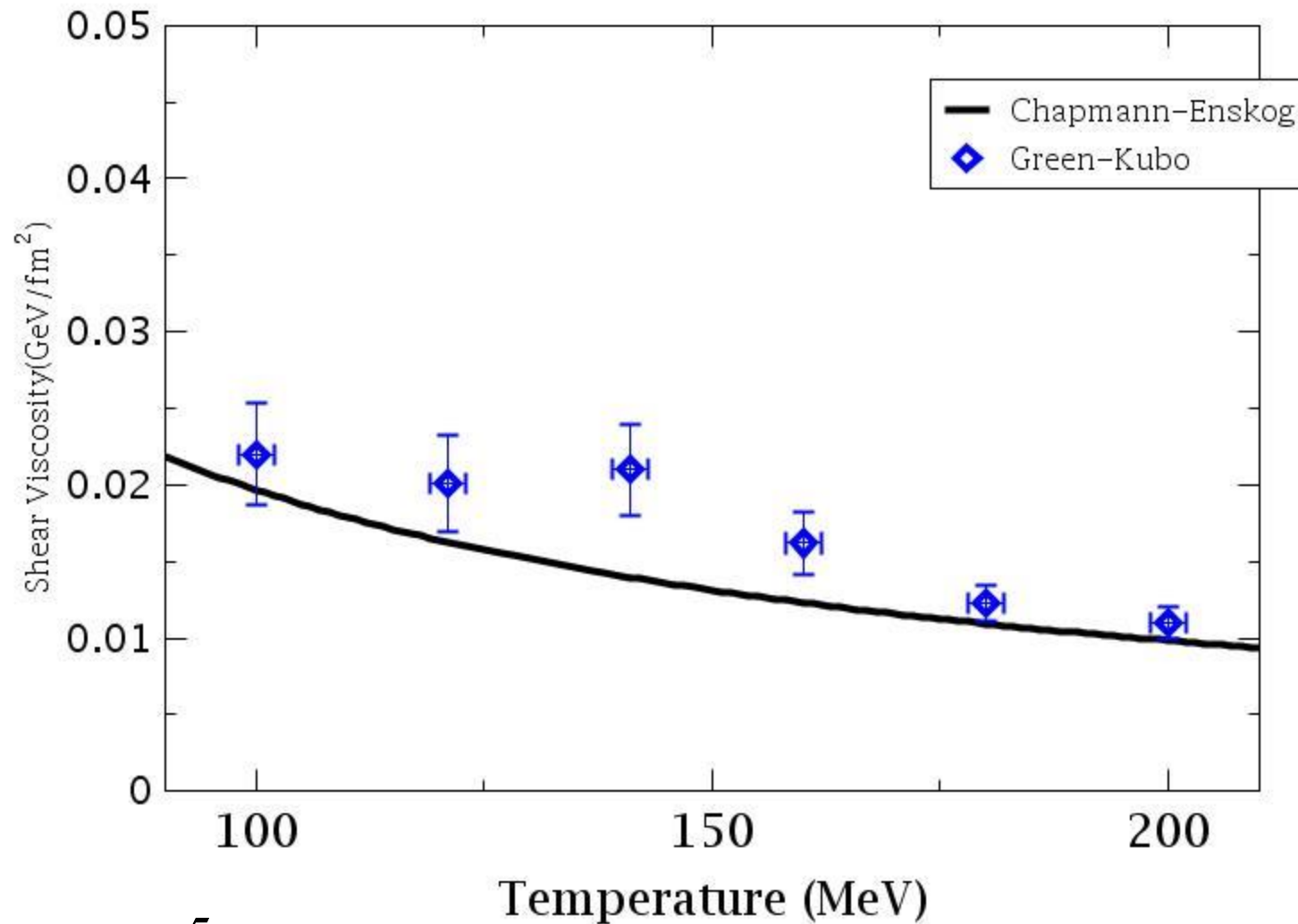
$$\sigma(\sqrt{s}) = \frac{5s}{72\pi f_\pi^{4}}$$

Chapmann-Enskog Calculation courtesy of A. Wiranata

- How to ensure appropriate "apples to apples" comparison of hadronic medium modeled by UrQMD to Chapmann-Enskog calculation of massive pions with ρ resonance enabled?

- Need to prepare UrQMD simulations for system "born in thermal equilibrium." Use statistical model with resonances. (SHARE)

- (*Torrieri, et al*.**Comput.Phys.Commun. 167 (2005) 229-251**

- **Modify SHARE tables to match those of desired system in UrQMD.**

System II: Massive Pions with ρ channel enabled
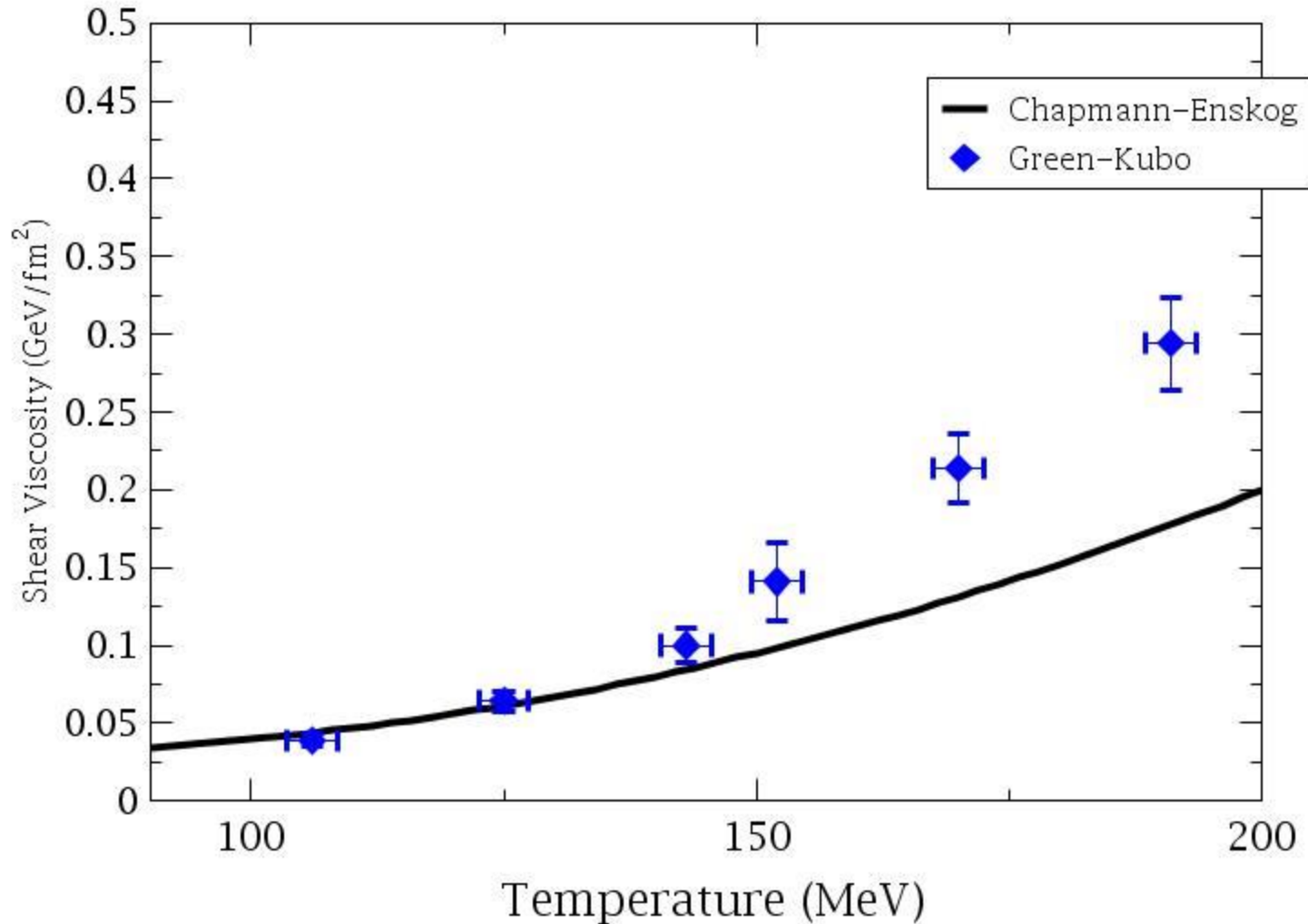
Chapmann-Enskog Calculation courtesy of A. Wiranata

Outlook: Investigating effect of modifying resonance lifetime

- We see for the pion rho mixture system, the differences between Green-Kubo and Chapmann-Enskog get larger with higher temperature (i.e. as more rho mesons populate the system).

- To better understand this, we should modify the resonance lifetime and see what possible effect this would have on the shear viscosity. Timescales associated with scattering processes are thought to have an effect on transport coefficients

  (Danielewicz, Pratt, **Phys.Rev. C53 (1996) 249-266**)

Outlook: Challenges in getting bulk viscosity from UrQMD

$$\zeta = \beta \int d^3 r \int dt \langle P(0,0) P(r,t) \rangle$$

$$P(r,t) = \frac{1}{3}(T^{xx}(r,t) + T^{yy}(r,t) + T^{zz}(r,t)) - c_s^2 \langle T^{00}(r,t) \rangle$$

-Speed of sound and equilbrium energy density need to be accurately calculated.
-Role of inelastic collisions in UrQMD, and effect of resonance lifetime on shear viscosity should be better understood. This could give insight to bulk viscosity calculation.

In Progress: Comparison with two other systems.
-System III: Constant cross section with only elastic scattering. (Massive Pions)
-System IV: Constant cross section with only elastic scattering. (Massless Pions)

Summary

- Preliminary calculation of shear viscosity from Green-Kubo method (Demir) in rough agreement with that from Chapmann-Enskog approximation (Wiranata) for two systems.

- Need to understand role of resonance lifetime and its effect on shear viscosity. (Possible insight into bulk viscosity calculation.)

- Perform comparison for two other systems with constant cross section (in progress).

BACKUP SLIDES

# Detailed Balance and Baryon/Anti-Baryon Production

Detailed Balance:

For a given cross section for process 1,2->3,4 principle of detailed balance could be used to calculate cross section for process 3,4->1,2

$$\frac{d\sigma}{d\Omega}(3,4 \to 1,2) = <j_3,m_3,j_4,m_4\|JM>^2 \lambda_{db} \frac{d\sigma}{d\Omega}(1,2 \to 3,4)$$

$$\lambda_{db} = \frac{(2S_3+1)(2S_4+1)}{(2S_1+1)(2S_2+1)} \sum_{J=J_-}^{J_+} <j_1,j_2,m_1,m_2\|JM> \frac{p_{1,2}^2}{p_{3,4}^2}$$

To ensure detailed balance, all multiparticle exit channels must be disabled.

Anti-Baryons in UrQMD:

-produced via string fragmentation, which violates detailed balance and is disabled.

-annihilation produces multi-particle final state, which violates detailed balance.

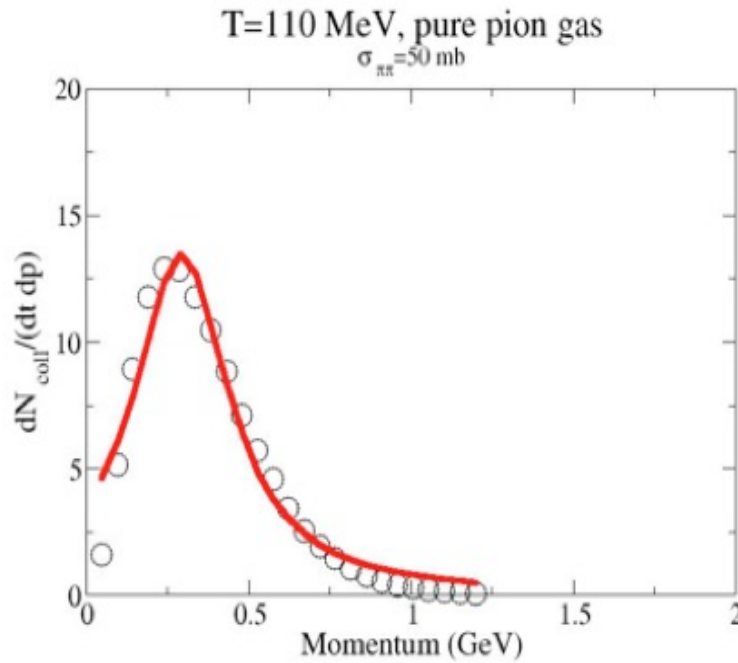-solution: produce and annihilate anti-baryons via effective 2 body process so that multiparticle f.s is achieved:

$$p + \bar{p} \leftrightarrow \rho + \omega(1420)$$
$$p + \bar{p} \leftrightarrow \rho + \phi(1020)$$
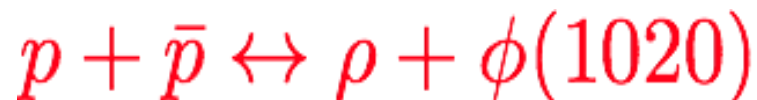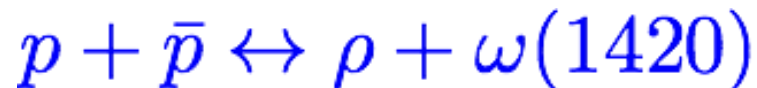
Pseudo-Knudsen # analysis

For pure pion gas with constant elastic cross sections of 10 mb.

Relaxation Time approximation

T=110 MeV, pure pion gas
$\sigma_{\pi\pi}$=50 mb
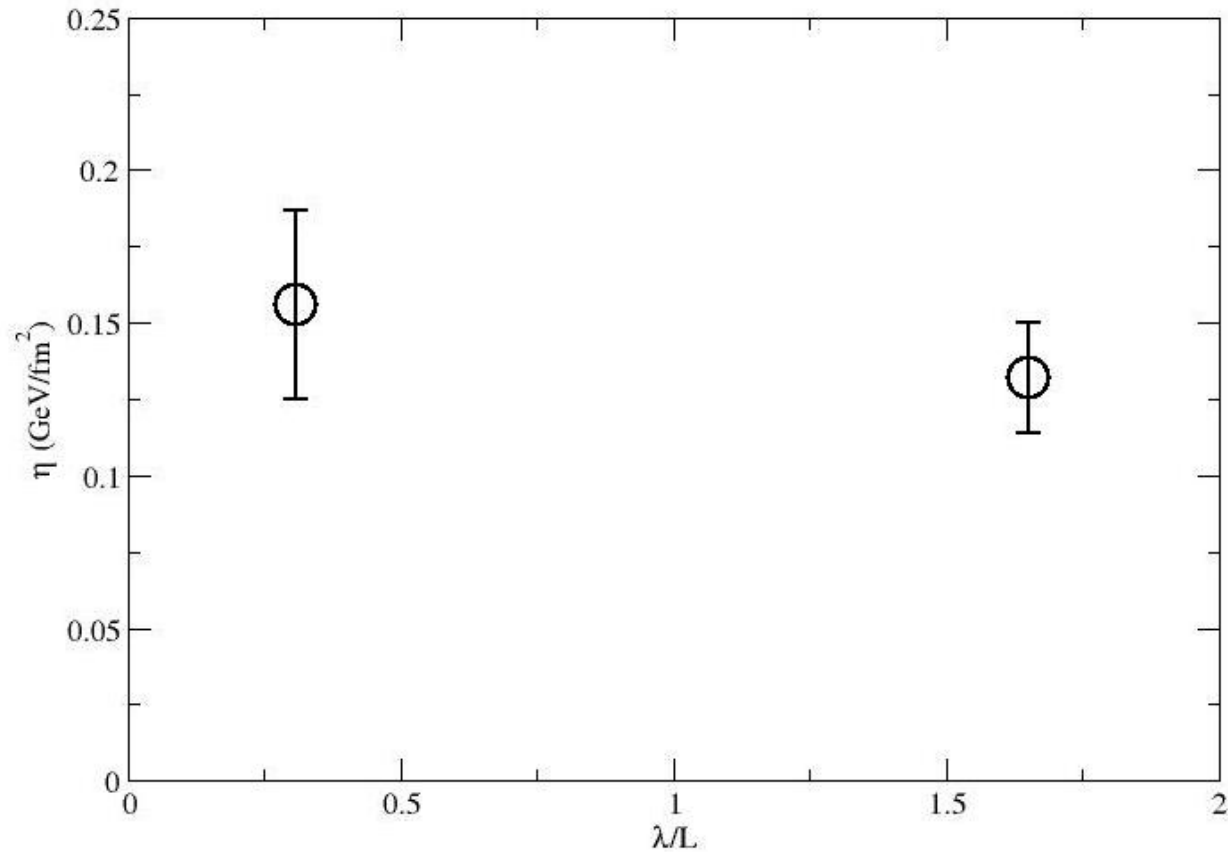
In dilute limit, relaxation time ~ collision time (Prakash et al, *Phys.Rept.227:321-366,1993.*

$$\eta = \frac{1}{15T} \sum_a \int \frac{d^3p}{(2\pi)^3} \frac{\tau_a p_a^4}{E_a^2} f_a^{eq}(E_a/T)$$

$$\zeta = \frac{1}{9T} \sum_a \int \frac{d^3p}{(2\pi)^3} \frac{\tau_a(E_a)}{E_a^2} \left[(1 - 3c_s^2) E_a^2 - m_a^2\right]^2 f_a^{eq}(E_a/T)$$

Chapman-Enskog Approximation

"For small deviations from equilibrium, the distribution function can be expresse in terms of hydrodynamic variables (f(x,p) μ(x), u(x), T(x)) and their gradients. Transport coefficients (e.g., bulk & shear viscosities) are then calculable from relativistic kinetic theory."

$$f(x,p) = f^0[1 + \varphi^1(x,p)]$$

Deviation function

$$f^{(0)}(x,p) = \frac{1}{(2\pi\hbar)^3} \exp\left[\frac{m\mu(x) + p_\alpha u^\alpha(x)}{kT}\right]$$

Equilibrium distribution function

μ(x) : Chemical potential
u(x) : Flow velocity
T(x) : Temperature

Courtesy of A. Wiranata

The Boltzmann equation

$$p_\alpha \partial^\alpha f^{(0)} = -f^{(0)} \int f_1^{(0)} \left(\varphi^{(1)} + \varphi_1^{(1)} - \varphi'^{(1)} - \varphi_1'^{(1)} \right) W' \frac{d^3 p_1}{p_1^0} \frac{d^3 p'}{p'^0} \frac{d^3 p_1'}{p_1'^0}$$

The collision integral

$$Q \partial_\alpha U^\alpha - (p_\gamma U^\gamma + mh) p_\alpha \Delta^{\alpha\beta} \left(T^{-1} \partial_\beta T + c^{-2} D U_\beta \right) + \langle p_\alpha p_\beta \rangle \langle \partial^\alpha U^\beta \rangle = -kT\mathcal{L}[\varphi^{(1)}]$$

Bulk viscosity

Heat conductivity

Shear Viscosity

the solution (deviation function) has the general structure

$$\varphi^{(1)} = A \partial_\alpha U^\alpha - B \Delta_{\alpha\beta} p^\beta \Delta^{\alpha\beta} \left(T^{-1} \partial_\beta T + c^{-2} D U_\beta \right) + C \langle p_\alpha p_\beta \rangle \langle \partial^\alpha U^\beta \rangle$$

Courtesy of A. Wiranata

Courtesy of A. Wiranata